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Suboptimal Hybrid Model Predictive Control: Application to Sewer Networks

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Abstract: This paper presents an application of the suboptimal hybrid model predictive control (HMPC) algorithm previously proposed by the authors to large scale sewer networks. HMPC relies on the on-line solution of mixed integer programs (MIP) that are known to be $\mathcal{NP}$-complete and whose worst case complexity scales exponentially with problem size. Modern MIP solvers are on the other hand highly efficient at taking advantage of problem structure and usually achieve average optimization times that are much better than the worst case predicts. But as the MIP constraints depend on the current state of the plant, complexity can vary considerably and unpredictable behavior can occur. To circumvent unpredictability and to be able to enforce hard real-time computation constraints, the number of feasible nodes in the MIP problem is limited online by adding constraints to the number of possible mode sequences over the prediction horizon. It is shown that in realistic scenarios concerning control of large scale sewer networks, depending on the value of parameters related to the mode sequence constraints (MSC), drastic reductions can be achieved in optimization time. Practical issues of the approach are also addressed.

Keywords: Model predictive control, hybrid systems, Mixed Logical Dynamics, large scale systems, sewage systems, suboptimal approach.

1. INTRODUCTION

Hybrid model predictive control (HMPC) has attracted a lot of interest recently due to the rich class of models to which it is applicable. The mixed logical dynamical (MLD) hybrid model is usually used for the MPC setup but it has been shown that MLD models are equivalent to other hybrid model classes such as piecewise affine systems (see Heemels et al. (2001)). A fundamental limitation to the use of HMPC is the complexity of the mixed integer program that is solved on-line in each sample. Explicit solutions to the HMPC control problem have been proposed as a remedy (see Borrelli et al. (2005)) but these are limited to problems of smaller scale.

But the optimization problem at the heart of HMPC is a mixed integer program (MIP), which is known to be NP-complete with a worst case optimization time that scales exponentially with problem size. Even though average calculation time can be acceptable for a particular problem, the complexity and thus calculation time can vary considerably as a function of state. This can cause unpredictable behavior of the calculation time which is highly undesirable for online implementation. The reason for this change in complexity is actually a change in the number of feasible nodes in the MIP search tree. The primary task of modern MIP solvers is to exploit structure of the problem with the aim to discard infeasible nodes of the MIP to reduce the overall computation time.

To circumvent this unpredictability and to be able to enforce hard real-time computation constraints, a method was presented in Ingimundarson et al. (2007), which limits online the number of feasible nodes in the MIP problem. This is done by adding constraints to the problem using insight into the system dynamics and in this way bound the number of feasible nodes at each sample. The prise that is paid for the reduction in computational time is suboptimality due to the added constraints. The method depends on the MIP solver to be able to take advantage of the added constraints. Notice that all implementations of HMPC that use MIP solvers depend on this exploitation of structure. If they would not, the worst case calculation time would cause only smaller problems to be practically solvable in real-time.

In Ingimundarson et al. (2007), stability of the resulting controller was proven using recent results for the stability of HMPC, see Lazar et al. (2006). In the current article the application of suboptimal HMPC to sewer networks is presented. Sewer networks are systems with complex dynamics since water flows through sewer in open channels. In sewage systems there exist several phenomena (overflows in sewers and tanks) and functional elements (redirection gates and weirs) that exhibit different behavior depending on the flow/volume in the network. These characteristics are inherently hybrid which leads naturally to the use of hybrid models in order to describe such behaviors, see Ocampo-Martinez (2007) for more details.

In Ocampo-Martínez et al. (2007) it was shown that calculation time can vary considerably as a function of the initial state when

HMPC is applied to a moderately large sewer network. The current article demonstrates how calculation time can be reduced by applying the suboptimal HMPC presented in Ingimundarson et al. (2007).

This manuscript is organized as follows. In Section 2 the suboptimal HMPC strategy is introduced. In Section 3 practical issues related to the methods are discussed. In Section 4 the case study is presented while in Section 5 the main results obtained from the case study are demonstrated and discussed. Finally the conclusions are outlined in Section 6.

2. SUBOPTIMAL HMPC

This section explains the details of the suboptimal HMPC law presented in Ingimundarson et al. (2007). Consider the Mixed Logical Dynamical (MLD) hybrid model

$$x_{k+1} = Ax_k + B_1 u_k + B_2 \delta_k + B_3 z_k \tag{1a}$$
$$E_2 \delta_k + E_3 z_k \leq E_1 u_k + E_4 x_k + E_5 \tag{1b}$$

where $x_k \in \mathbb{X} \subseteq \mathbb{R}^n$ is the state vector and $u_k \in \mathbb{U} \subseteq \mathbb{R}^m$ is the vector of manipulated variables (inputs). $\mathbb{X}$ and $\mathbb{U}$ are assumed to be polytopes that include state and input constraints, respectively, that need to be enforced by the MPC control design. The binary vector $\delta_k = [\delta_k^1, \ldots, \delta_k^{r_l}] \in \{0,1\}^{r_l}$ of dimension r_l and the continuous-valued vector $z_k \in \mathbb{R}^{r_c}$ of dimension r_c are the vectors of auxiliary variables associated with the MLD form. A specific value of the variable δ_k is referred to as a *mode* of the hybrid system. Equation (1b) collects the set of constraints on system variables as well as translations from logic propositions.

For a fixed prediction horizon N, let

$$X_k(x_k, U_k) \triangleq (x_{1|k}, x_{2|k} \ldots x_{N|k}) \in \mathbb{X}^N$$

denote the state sequence generated by the MLD system (1) from initial state $x_{0|k} \triangleq x_k$ and by applying the input sequence

$$U_k \triangleq (u_{0|k}, \ldots, u_{N-1|k}) \in \mathbb{U}^N.$$

Related to these sequences is the *mode sequence*

$$\Delta_k(x_k, U_k) = (\delta_{0|k}, \ldots, \delta_{N-1|k}) \in \{0,1\}^{r_l \times N}$$

of binary vectors δ_k uniquely defined by (1b) when U_k is applied to (1) from initial state x_k. Let

$$\bar{\Delta}_k = (\bar{\delta}_{0|k}, \ldots, \bar{\delta}_{N-1|k}) \in \{0,1\}^{r_l \times N}$$

be a *reference sequence* of binary variables $\bar{\delta}_k$ of the same dimension as Δ_k. How $\bar{\Delta}_k$ is obtained in each sample will be discussed in Section 3. The mode sequence constraints (MSC) are now defined by the following inequalities:

$$\sum_{k=0}^{N-1} |\bar{\delta}_k^i - \delta_k^i| \leq M_i \quad \text{for } i = 1 \ldots r_l \tag{2}$$

$$\sum_{i=1}^{r_l} \sum_{k=0}^{N-1} |\bar{\delta}_k^i - \delta_k^i| \leq M \tag{3}$$

where $M, M_i \in \mathbb{Z}_+$ are given bounds on the number of switches from the reference sequence. Notice that the MSC can be characterized exactly with a set of mixed integer inequalities.

The suboptimal HMPC control law is obtained in the exact same way as in Bemporad and Morari (1999). Given a cost function

$$J(x_k, U_k) = F(x_{N|k}) + \sum_{i=0}^{N-1} L(x_{i|k}, u_{i|k}) \tag{4}$$

that usually contains weighted norms (1,2 or ∞) of the performance variables, a MIP is constructed with constraints $\mathbb{X}$, $\mathbb{U}$ and (1b) at each sample over the prediction horizon. The only difference is the addition of a MSC, i.e. constraint (2) or (3) are added to the MIP optimization problem.

Notice that each node in the search tree of the MIP corresponds to a specific value of sequence Δ_k. Inequalities (2) and (3) define sets of Δ_k with a limited number of differences from the reference sequence $\bar{\Delta}_k$. Thinking of Δ_k and $\bar{\Delta}_k$ as binary strings, the inequalities in (2), (3) limit the Hamming distance between such strings. We refer to the constraints given by (2) as M_i and M constraint respectively. A simple combinatorial counting of the size of sets (cardinality) defined by the inequalities gives the following result:

$$N_{M_i} = \left(\sum_{j=1}^{M_i} \frac{N!}{j!(N-j)!} \right)^{r_l} \tag{5}$$

$$N_M = \sum_{j=1}^{M} \frac{Nr_l!}{j!(Nr_l - j)!} \tag{6}$$

The number N_M is the number of different Δ_k sequences that fulfill inequality (3). It is assumed that there exists an optimal control sequence

$$U_k^* = \left(u_{0|k}^*, u_{1|k}^*, \ldots, u_{N-1|k}^* \right) \tag{7}$$

obtained as a solution to the MIP. Using the *receding horizon* philosophy as in Maciejowski (2002), the suboptimal HMPC control law is defined as:

$$u_{\text{MPC}}(x_k) \triangleq u_{0|k}^* \tag{8}$$

where $u_{0|k}^*$ is the first element of U_k^*.

In Ingimundarson et al. (2007), it was shown that recent stability results for HMPC presented in Lazar et al. (2006) can be easily extended to the suboptimal strategy. In a remarkably compact note, Lazar et al. (2006) demonstrated that by using a terminal cost and constraint set method as in Mayne et al. (2000) but adapted to the hybrid system case, stability could be proven. Furthermore, methods to calculate the weights for the terminal cost, as well as the terminal constraint set were presented. In this paper we are concerned with presenting an application of the suboptimal approach to a realistic large scale problem which is sewer network control. The objective is to demonstrate the reduction in calculation time that can be obtained. The issue of stability will thus not be pursued further.

3. PRACTICAL ISSUES

An important practical problem in the proposed method is to find $\bar{\Delta}_k$ so that the MIP including MSC has a solution. The main tool to find this sequence is open-loop simulation. A natural candidate solution, which might be close to the optimum, is the shifted optimal sequence from the last sample

$$U_{k+1}^1 \triangleq \left(u_{1|k}^*, \ldots, u_{N-1|k}^*, h(x_{N-1|k+1}) \right) \tag{9}$$

where $u = h(x)$ is some control law that returns a control signal u that respect the constraints. This control sequence can be used to simulate the system in open loop. If all constraints are respected, U_k^1 is a feasible solution and the associated mode sequence can be used as $\bar{\Delta}_k$. If the measured state is close to the predicted state, it is reasonable to believe that this sequence provides at least a good initial guess close the the optimum.

If the open-loop simulation fails and some constraints are violated, in the worst case, the problem of finding $\bar{\Delta}_k$ is to find a feasible trajectory for the problem without MSC from the new initial state. This in turn is a MIP feasibility problem. The reduction in time that can be achieved with the presented methodology then depends on the complexity of feasibility problem compared to the optimization problem, something that is difficult to analyze a priori. This is a restriction to the presented method but if constraints related to safety or high risk are present in $\mathbb{X}$, and feasibility can not be assured within a pre-specified time-frame neither the presented method nor other hybrid MPC strategies that depend on a MIP to find a feasible solution would be applicable in practice. Indeed, in all practical applications of MPC one resorts to “softening” all the constraints that involve state variables to prevent infeasibility issues (that is, all constraints but input saturation). The difficulty of finding a $\bar{\Delta}_k$ varies according to the specific application. Some general guidelines are given in what follows.

3.1 No state constraints

If $\mathbb{X} = \mathbb{R}^n$ (no state constraints) and system (1) is stable, then using U_k^1 defined in (9) in open loop simulation from the new initial state $x_{0|k}$ results in a sequence $\bar{\Delta}_k$ that can be used in inequalities (2) and (3). If U_k^1 is not available, a feasible input sequence can be set using a local control law

$$u_k = \text{SAT}(h(x_k)); \quad x_k \in \mathbb{X}_f(N) \tag{10}$$

for $k = 0, \ldots, N-1$ and again simulate the system in open loop from the initial state $x_{0|k}$. $u_k = \text{SAT}(\cdot)$ is a function that guarantees that $u_k \in \mathbb{U}$.

3.2 State constraints

State constraints are generally related to either physical constraints of the model such as conservation equations and physical limitations of the process, or to control objectives. The way infeasibilites in the optimization problem are dealt with, or constraint management, is an important issue in constrained predictive control, see Maciejowski (2002).

As mentioned above, a common approach to deal with infeasibilities is to change constraints from “hard” to “soft”, that is, add terms containing slack variables of the constraints to the cost function. If the constraints thus changed represent physical characteristics, the resulting control signal might be of little use as the model from which the control signal is obtained might not fulfill basic physical laws. If the constraints are related to safety considerations, the resulting control signal might not be applicable either.

Constraint management is equally important in the presented scheme as a straightforward way to obtain an initial feasible solution is to change any unfulfilled constraints in $\mathbb{X}$, when U_k^1 is used in open loop simulation, into soft constraints. As mentioned previously, this approach is only appropriate if the constraints thus relaxed do not represent physical or safety characteristics of the system.

When forming the cost function containing the slack variables relates to the soft constraints, frequently, some constraints have higher priority than others. The common way to deal with distinct priorities is to assign weights to each slack variable that reflects their importance. Finding these weights is generally done with trial and error procedures involving simulations of typical disturbance and reference value scenarios. If the relative importance of the relaxed constraints is known, objective prioritization schemes implemented with propositional logic (see Tyler and Morari (1999)) represent an interesting option as these schemes are implemented with MIP solvers.

3.3 Finding a feasible solution with physical knowledge and heuristics

Physics or heuristical knowledge of the system can often be used to find a feasible solution that fulfills the physical constraints of the system. For example, in steady state, all integer variables have fixed values which could be used in the sequence $\bar{\Delta}_k$. State constraints representing physical limitations can often be incorporated into the hybrid model by using propositional logic. As an example consider a tank with an upper limit on its level and with its inflow controlled with a valve. The upper limit on the tank could be modeled by adding a constraint to the optimization problem so that any controlled signal to the valve causing the level to surpass the physical limit, would be infeasible in the optimization problem. Within the hybrid modeling framework, a logical statement could be incorporated guaranteeing that the inflow to the tank would never cause the level to surpass the physical level, irrespective of the control signal to the valve.

This hybrid modeling approach actually represents the physical behavior better and would enable the removal of a state constraint where infeasibility could occur during the open loop simulation. On the other hand it would increase the number of binary variables in the system.

4. CASE STUDY DESCRIPTION

In this section, an application will be presented that demonstrates the reduction of optimization time that can be achieved using mode sequence constraints. The application consist in the real-time control of sewer networks under a given set of performance objectives. An early reference where MPC was suggested as a control strategy for sewer networks is Gelormino and Ricker (1994). There, an implementation of linear model predictive control over the Seattle urban drainage system was presented. Their results confirmed the effectiveness of the global predictive control law relative to the conventional local automatic controls and heuristic rules that were used to control and coordinate the overall system. Other articles where predictive control ideas have been developed further are Duchesne et al. (2004); Marinaki and Papageorgiou (2005).

HMPC of sewer networks was introduced in Ocampo-Martínez et al. (2006) and Ocampo-Martinez (2007). Sewer networks are inherently hybrid and their evolution with time depends heavily on their state. An important hybrid behavior is overflow which occurs when collectors reach their limits and water starts to flow on the streets. This can cause the sewage to diverge from its original flow path. Overflow only appears under specific circumstances depending on the system state.

The sewer network under study in this paper is a portion of the Barcelona sewer network. It was considered representative as it contains the main components and characteristics of the

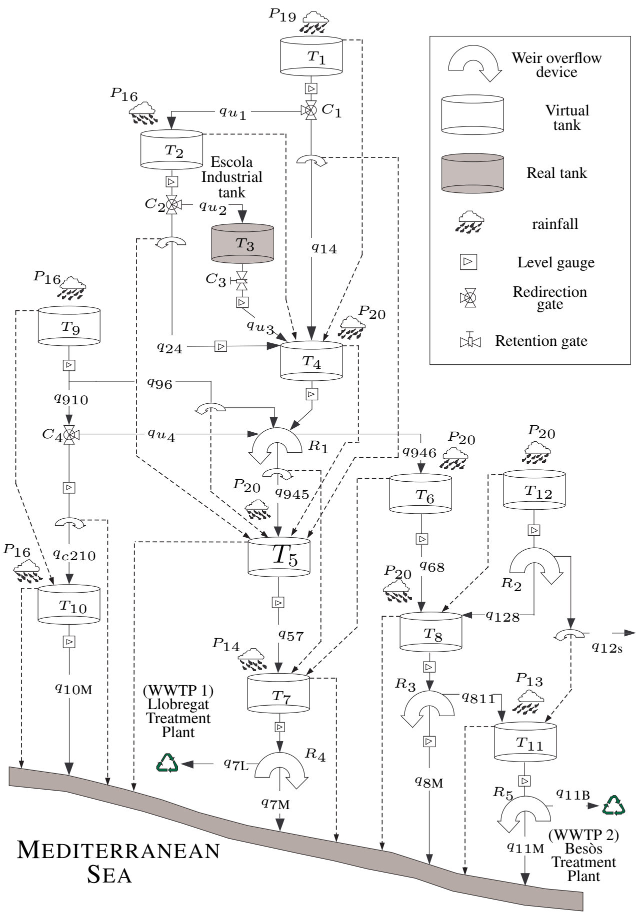


Fig. 1. BTC diagram for hybrid design.

entire network. A calibrated and validated model of the system that has been previously discussed in Ocampo-Martinez (2007) was available as well as rain gauge data for an interval of several years. The considered catchment has a surface of 22,6 Km^2. Due to its size, there is a spacial difference in the rain intensity between rain gauges. The catchment considered has 11 sub-catchments defining equal number virtual tanks, several level gauges (limnimeters), 5 flow links and two waste water treatment plants. It has 1 retention gate associated with 1 real tank and 3 redirection gates. Also, there are 5 rain-gauges in the catchment but some virtual tanks share the same rain sensor. The system is shown in Figure 1. The dashed lines represent the overflow from virtual tanks and sewers. These lines therefore represent the hybrid behavior of the network.

The system model has 12 state variables corresponding to the volumes in the 12 tanks (1 real, 11 virtual), 4 control inputs corresponding to the manipulated links and 5 measured disturbances corresponding to the measurements of rain precipitation over the virtual tanks. It is supposed that all states (virtual tank volumes) are estimated by using limnimeters. The free flows to the environment as pollution ($q_{10\text{M}}$, $q_{7\text{M}}$, $q_{8\text{M}}$ and $q_{11\text{M}}$ to the Mediterranean sea and q_{12s} to an other catchment) and the flows to the treatment plants ($Q_{7\text{L}}$ and $Q_{11\text{B}}$) are shown in the figure as well. Variable w_i are related to the rain inflow in function of one of the rain intensities P_{13}, P_{14}, P_{16}, P_{19} and P_{20} but taking into account catchment area among other things. The corresponding MLD model has 22 logical variables and 44 auxiliary variables.

4.1 Rain Episodes

The rain episodes used for the simulation of the test catchment and the design of control strategies were based on real rain gauge data obtained within the city of Barcelona between the years 1998 and 2002. These episodes were selected to represent the meteorological behavior of Barcelona, i.e., they contain representative meteorologic phenomena in the city.

4.2 Control Objectives and The Cost Function

The sewer system control problem has multiple objectives with distinct priorities, see Marinaki and Papageorgiou (2005). The control objectives considered in this case study are the following listed with decreasing priority:

(1) Minimize virtual tank and link overflow.
(2) Minimize flow to the environment.
(3) Minimize incremental actuator movements.

The second objective is closely linked to the objective of maximizing the water treated in the WWTPs. To reflect these objectives a cost function consisting of the weighted sum of the related performance variables was defined. The weights were selected to reflect the priorities between the objectives. The sewer network is a stable systems as all states are related to water volume in the sewer. Instability would be related to a divergence of these states, something unlikely to occur. The control law is therefore only aimed at improving performance.

4.3 Suboptimal Strategy Setup: Finding $\bar{\Delta}_k$

In the presented application, $\bar{\Delta}_k$ could always be found by open loop simulation in an easy manner. As the virtual tanks have no upper limits the only complication was to adjust the control signal in each sample to fulfil mass conservation for the real tank and outflows from the virtual tanks. The selected control sequence used to find $\bar{\Delta}_k$ was the shifted optimal control sequence given by (9). The prediction of rain over the prediction horizon was simplified by assuming it to be constant and equal to the last value measured as was done in Gelormino and Ricker (1994).

4.4 Simulation of scenarios

The suboptimal strategy were applied by simulating the closed-loop system for a number of rain episodes, some of which had caused flooding problems within the city. The prediction and control horizon H_p was set as 6, or equivalent to 30 minutes (with the sampling time $\Delta t = 300$ s). This selection followed the suggestions based on heuristic knowledge of the CLABSA [1] engineers and field tests made on the sewer network. Another reason for the selection of these prediction and control horizon values is that prediction provided by the sewer network model becomes less reliable for larger time horizons, see Ocampo-Martinez (2007). The duration of the simulated scenarios was determined by the duration of the rain peak and the system reaction time to that rain. The approach efficiency was measured not only regarding CPU time but also in system suboptimality for different values of M and M_i in each case. Results were obtained using the HYBRID TOOLBOX for MATLAB® (Bemporad (2006)) with MIP solver ILOG CPLEX 9.1. CPLEX parameters were set to their default values.

[1] In catalan: Clavegueram de Barcelona, SA (Barcelona sewer network company)

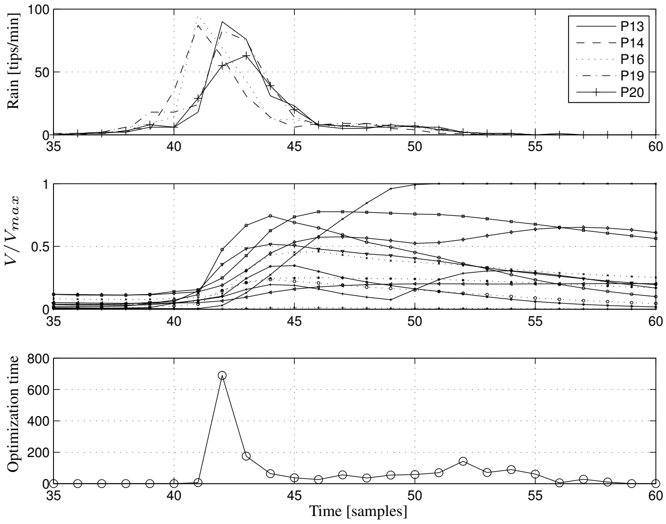


Fig. 2. Rain episode 1999-10-17. Top figure: Rain intensity. Middle figure: Virtual tank levels normalized by their maximum levels. Bottom figure: Optimization time. 2-norm used in cost function

5. RESULTS

Selected but representative simulation scenarios are now presented to demonstrate the reduction in optimization time that can be achieved by the MSC. Before that, a scenario is presented to demonstrate the high variability in optimization time that occurred in the simulation scenarios (see Figure 2). The rain episode, which was considered severe, occurred on October 17, 1999. The bottom graph shows that the optimization time in the unconstrained case rises by 2 factors of magnitude in one sample. At time 41 the optimization time was 6.6 seconds while at time 42 it was 689 seconds. As the sampling time was 300 seconds, this is an unacceptable variability in terms of implementation.

Before it was mentioned that the complexity of the optimization problem can change due to a change in the number of feasible nodes. Feasible nodes in the MIP problem correspond to feasible sequences Δ_k. A reason why there are more feasible sequences at time 42 is demonstrated in the upper two graphs. The top of the rain peak for rain gauges P_{13} and P_{19} occur at time 42. Remember that the rain is predicted to be constant over the prediction horizon. This in addition to a rising level in the tanks, depending on the control sequence, many of the tanks can overflow at some point over the prediction horizon, when the optimization problem is formed at time 42. Overflow in tanks causes a boolean variable in the optimization problem to change value which in turn represents a change in sequence Δ_k. The bottom line is that at time 42, with the combination of the rain prediction and tank levels, much more possible Δ_k sequences are feasible which in turn causes an increase in number of feasible nodes. The number of nodes explored can be returned by the CPLEX optimizer. It turned out that this number rise from 2520 at time 41 to 132365 at time 42 which is also a rise by orders of magnitude.

5.1 Reduction in optimization time

Figure 3 shows the maximum optimization time for the whole simulation scenario when M-type MSC were implemented, for

Table 1. Nodes explored as a function of M at sample 42.

M	N_M	nodes explored by CPLEX
1	133	66
2	8646	597

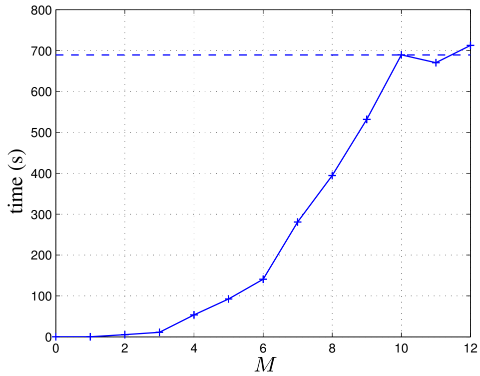
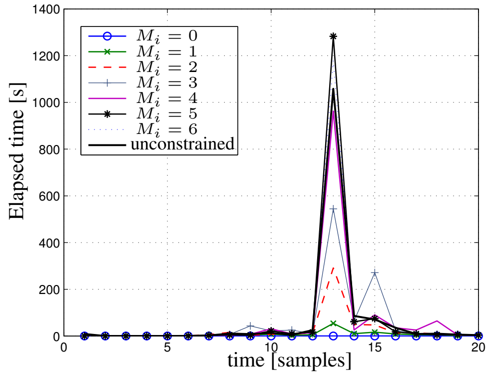


Fig. 3. Maximum CPU time in rain episode 99-10-17 for different values of M. Dashed curve (−−), optimal simulation time.

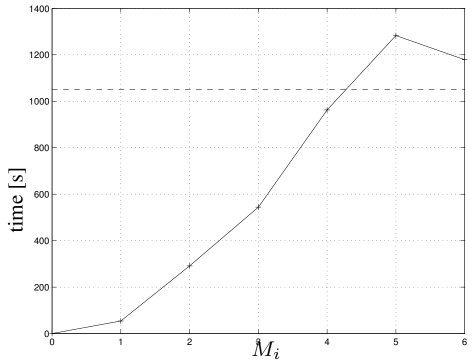
the rain episode presented in Figure 2. In most cases, the maximum occurred at time 42 as expected. The figure shows that the optimization time could be strongly reduced by implementing the MSC. For $M \leq 4$ the maximum optimization time does not surpass 50 seconds. For rising M the optimization time rises until settling around the optimization time in the unconstrained case. In terms of nodes explored, compared to the number of feasible nodes as given in (6), Table 1 demonstrates that the MSC affect strongly the number of nodes explored by the optimization software. Besides exploring considerably fewer nodes than in the unconstrained case quoted before, the nodes explored were always fewer than N_M. Figure 4(a) shows the optimization time for the unconstrained case for another rain episode as a function of sample. Also shown is the optimization time for distinct values of M_i. In the the unconstrained case, the optimization time jumps from 25 seconds at sample 12 to 1050 seconds at sample 13. With the M_i constraint it is possible to reduce optimization time considerably. This is demonstrated in Figure 4(b). Notice that the optimization time for $M_i = 5$ and $M_i = 6$ is higher than the unconstrained case. In these cases the reduction in nodes is not effective. The added constraints cause on the other hand an increase in optimization time as the optimization problem is more complex.

5.2 Level of suboptimality.

The rise in the cost function due to the MSC is on the other hand a very application specific characteristic as the optimal value of the cost function can include elements that are unaffected by changes in the mode sequence that constituents the optimal solution. For this reason, only general observations about the rise in cost will be presented for the application considered. For M constraints, the optimality of the solutions in general was not critically affected. Only when $M = 0$ and $M = 1$, the cost function could rise up to 30% for the critical samples within the scenarios. In the case of $M = 0$, this is not surprising as the optimization problem is reduced to a LP or QP and the hybrid behavior of the system is exclusively decided by the open loop simulation used to decide $\bar{\Delta}_k$. For other values of M the



(a) Optimization time for each sample.



(b) Maximum CPU time over scenario.

Fig. 4. Optimization time for different values of M_i for the rain episode 99-09-14. In (a), dashed curve (−−), optimization time in the unconstrained case time.

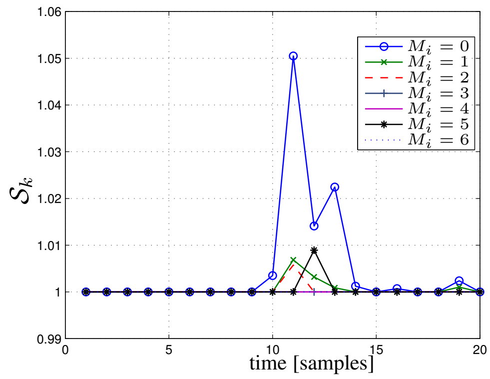


Fig. 5. Suboptimality level in rain episode 99-09-14.

increase in cost was considerably smaller. For M_i constraints, the rise in the cost function was not critically affected except for $M_i = 0$. Figure 5 shows the proportional rise at each sample during the heavy part of the rain episode presented in Figure 4. The cost function rise maximum 5% for the critical sample of the scenario.

6. CONCLUSIONS

The main conclusion drawn from the results is that the MIP solver was able to take advantage of the added mode sequence constraints (MSC) and optimization time for the large scale application presented was consistently reduced without a considerable effect on optimality. In the presented application, the optimization time was shown to be highly dependent on state. When many modes sequences were feasible, the optimization time rise considerably due to the increase in complexity of the optimization problem. It was demonstrated that the MSC were very effective at containing and limiting the effect of this increase of complexity. Systematic methods to find the sequence $\bar{\Delta}_k$ were discussed. The *apriori* selection of parameters M and M_i is still an open issue but it was shown that by counting the feasible Δ_k sequences might give an indication about the selection, at least for smaller M.

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