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Modelling and Control of Offshore Marine Pipeline during Pipelay

Gullik A. Jensen* Aksel A. Transeth* Tu Duc Nguyen*

* *Department of Engineering Cybernetics, Norwegian University of Science and Technology, Trondheim, Norway (e-mail: gullik.jensen@itk.ntnu.no).*

Abstract: A model suited for control tasks is developed for a submerged offshore pipe during the pipelay operation. The pipe is fixed in the touchdown point at the seabed in one end and attached to a pipelay vessel in the other end. The developed model is discrete and is on the form of the robot equation with minimal coordinates. Thus the methods of controller synthesis and stability analysis can be applied directly. The model constitutes a hyper redundant system and it is shown that this system is passive. A PID-controller has been suggested. The simulation results are in agreement with the theoretical results.

1. INTRODUCTION

The unprecedented demand for oil and gas coupled with high commodity prices has accelerated deep- and ultra-deepwater pipelaying through an impressive development over the last ten years. Production from oil and gas fields is currently foreseen in up to 3,500 meter water depth, and thus one of the key drivers of the offshore pipeline industry at the moment is the drive into deeper water. According to Knight and Palathingal [2007], the next five years will see 17,509 km of pipeline laid in deeper waters, compared with 9,507 km over the previous five year period. An analysis of global deepwater project numbers and lengths over the two five-year periods shows a 26% increase in the number of forecast projects, but yet a comparative increase of 56% in length.

The pipelay method S-lay is fast and economical and dominates the market for deepwater pipeline installation. The present trends in deepwater pipelay systems are well described in Heerema [2005]. Pre-fabricated pipe elements are assembled horizontally in a production facility onboard the pipelay vessel and extended over a partly submerged supporting structure known as the stinger extending from the stern of the vessel as illustrated in Figure 1. The pipe is held on to the vessel by heavy tension equipment. The pipelay vessel operates in the two modes *station keeping* and *pipe placement*, which both use dynamic positioning systems (DP). The allowable sagbend strains determine the required angle of departure β from the stinger. The departure angle, in combination with the stinger length, determines the required stinger radius r_o. This radius yields a certain overbend strain which has to be checked against design specifications to stay within limits for buckling and ovalization. To effectively utilize the tension equipment, the departure angle should be near-vertical. To limit overbend strains in the pipeline, for large pipe diameters a relatively large stinger radius is required and therefore a longer stinger. Another reason for wishing to achieve a near-vertical departure angle is to limit tension force in the pipeline on the seabed to avoid free spans in areas with uneven seabed. A steel-pipe with

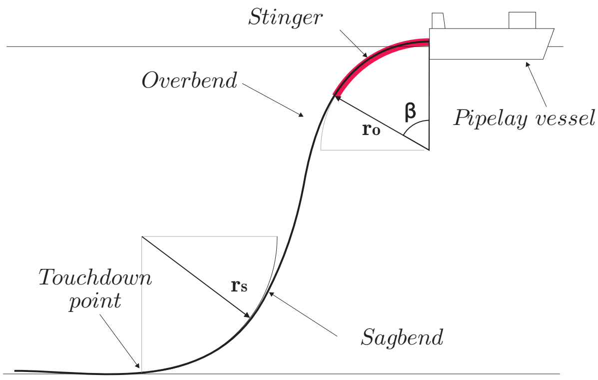

Fig. 1. The pipe configuration can be described by the radii of the overbend r_o and the sagbend r_s. These are represented by the inscribed circles. Note that $r_s > r_o$. It is desired that $\beta \to 90$.

an outer diameter of 0.76 m will require a stinger radius of approximately 240 m which would require a stinger longer than 300 m. The longest stinger in use today is 140 m, found on the pipelay vessel Solitaire owned by Allseas. Thus a near-vertical departure-angel is not always possible.

For pipe diameters and water depths where a near-vertical departure angle is not possible, control systems become of importance. Control systems may support the pipelay operation by optimizing the use of the system actuators to increase lay rates and to reach deeper waters. A model of the pipe is required to do this. There is an abundance of publications dealing with analysis of flexible pipes/risers as reviewed by Patel and Seyed [1995]. The numerical analysis of these structures are commonly classified into three main methods; *Finite-Element Method, Finite Difference Method* and *Lump-Mass Method*, where references to publications to all methods are found in Chai and Varyani [2006]. A comparison between continuous and discrete modelling is presented in Dreyer and Van Vuuren [1999]. Several commercially available tools for modelling and simulation are used to simulate the pipelay process in advance of the actual installation. These tools take as input the seabed topology, the pipe and vessel properties and external conditions such as waves and weather conditions. The usability of these models in model based controllers are limited due to the complexity.

In this paper a model for the pipe is developed as part of a system joining a pipe and a vessel, that is suited for *controller tasks* while keeping the geometric configuration and the force balance of the pipeline. Instead of discretization of catenary equations or finite element models which are disadvantageous due to their complexity, the standard robot model for a robot manipulator found in e.g. Spong and Vidyasagar [1989] or Sciavicco and Siciliano [2001] is utilized to model the pipeline. The vessel is included in the pipe model as the last link of the structure. A linked structure with many joints is termed hyper-redundant by Chirikjian and Burdick [1994]. Using a standard robot model formulation is advantageous since tools, developed for robot manipulators, for controller synthesis and stability analysis now can be applied directly. Moreover, to further extend the results, a passivity analysis for the system is performed. A controller for dynamic positioning using the angle of departure as reference is implemented and simulation results are provided to illustrate the theoretical results.

2. MATHEMATICAL MODELLING

The pipeline is modeled as a series of connected links as illustrated in Figure 2. It will move in six degrees of freedom (DOF) seen from Fossen [2002], and thus each pipe element must support longitudinal stretching, lateral bending and longitudinal rotation. Hence each element will have two rotational and a translational joint. In this paper only planar motion of the pipeline in a vertical plane is considered. Thus the rotation is ignored and the longitudinal bending is assumed to be small compared to the lateral bending so it is also ignored. The model is based on the robot equation with minimal coordinates. Kinematics of the pipe is investigated firstly, followed by the system dynamics.

2.1 Kinematics

The slender pipeline structure is divided into n elements, each denoted by i, of length l_i and mass m_i connected by 1 DOF revolute joints. Let an earth fixed frame I be used as an approximation to the inertial frame, where the origin of I is located on the seabed. Let the slender structure be fixed to the origin of I. Let an orthogonal frame B_i be attached to link i such that its x-axis is pointing toward link $i+1$. These frames are said to be in the operational space of the model. The generalized coordinates for the model are

$$\mathbf{q} = [\, q_1 \;\, q_2 \;\, \dots \;\, q_n \,]^{\mathrm{T}} \in \mathbb{R}^n, \tag{1}$$

which are in the joint space of the model. Assume that the elements are cylindrical with evenly distributed mass. The model is then simplified by using point masses located in the mass centers of the elements. Figure 2 show the first three elements of the slender structure.

The position of mass i is given by

$$\mathbf{p}_i = [\, x_i \;\, y_i \,]^{\mathrm{T}}, \tag{2}$$

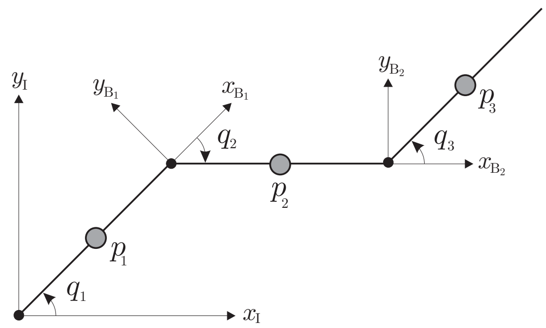

Fig. 2. The three first elements of the slender structure connected by rotational joints.

where

$$x_i = \sum_{j=1}^{i-1} l_j \cos\alpha_j + \frac{1}{2} l_i \cos\alpha_i \tag{3}$$

$$y_i = \sum_{j=1}^{i-1} l_j \sin\alpha_j + \frac{1}{2} l_i \sin\alpha_i \tag{4}$$

and

$$\alpha_i = \sum_{k=1}^{i} q_k. \tag{5}$$

The velocity of element i is given by the time derivative of the position $\mathbf{p}_i$,

$$\mathbf{v}_i = [\, \dot{x}_i \;\, \dot{y}_i \,]^{\mathrm{T}}, \tag{6}$$

where

$$\dot{x}_i = \sum_{j=1}^{i-1} \left(-l_j \sin(\alpha_j)\, \dot{\alpha}_j\right) - \frac{1}{2} l_i \sin(\alpha_i)\, \dot{\alpha}_i \tag{7}$$

$$\dot{y}_i = \sum_{j=1}^{i-1} \left(l_j \cos(\alpha_j)\, \alpha_j\right) + \frac{1}{2} l_i \cos(\alpha_i)\, \dot{\alpha}_i \tag{8}$$

and

$$\dot{\alpha}_i = \sum_{k=1}^{i} \dot{q}_k. \tag{9}$$

The Jacobian matrix $\mathbf{J}_{v_i} \in \mathbb{R}^{2\times n}$ for link i represents the mapping from the time-derivative of the generalized coordinates to the translational velocity of the mass center. Hence

$$\mathbf{v}_i = \mathbf{J}_{v_i} \dot{\mathbf{q}}, \tag{10}$$

where

$$\mathbf{J}_{v_i} = [\, (\mathbf{J}_{v_i})_1 \;\, (\mathbf{J}_{v_i})_2 \;\, \dots \;\, (\mathbf{J}_{v_i})_n \,], \tag{11}$$

and the columns of the matrix are found to be

$$(\mathbf{J}_{v_i})_k = \begin{cases} \begin{bmatrix} -\displaystyle\sum_{j=k}^{i-1} l_j \sin\alpha_j - \frac{1}{2} l_i \sin\alpha_i \\ \displaystyle\sum_{j=k}^{i-1} l_j \cos\alpha_j + \frac{1}{2} l_i \cos\alpha_i \end{bmatrix} & , k < i \\ \begin{bmatrix} -\frac{1}{2} l_i \sin\alpha_i \\ \frac{1}{2} l_i \cos\alpha_i \end{bmatrix} & , k = i \\ \begin{bmatrix} 0 \\ 0 \end{bmatrix} & , k > i. \end{cases} \tag{12}$$

The Jacobian matrix $\mathbf{J}_{\omega_i} \in \mathbb{R}^{1\times n}$ for link i is the linear mapping from time-derivative of the generalized coordinates to the rotational velocity of the center of mass of the links. That is

$$\omega_i = \mathbf{J}_{\omega_i}\dot{\mathbf{q}}, \tag{13}$$

where

$$\mathbf{J}_{\omega_i} = [\,(\mathbf{J}_{\omega_i})_1 \;(\mathbf{J}_{\omega_i})_2 \;\dots\; (\mathbf{J}_{\omega_i})_n\,], \tag{14}$$

and the columns of the matrix are found to be

$$(\mathbf{J}_{\omega_i})_k = \begin{cases} 1 & ,k \le i \\ 0 & ,k > i. \end{cases} \tag{15}$$

Both Jacobians are mappings between the joint space and operational space. This completes the kinematics of the general slender structure. Assume that the pipe is homogenous and let the links of the pipe be of equal length l and thus equal mass m and volume.

The introduction of the vessel in the system can be done in several ways. The preferred method is to connect the pipe to a real ship model as provided in Fossen [2002]. However, to simplify the model, the last link of the structure is modelled as the vessel, such that link n is the vessel and the links $1\dots n-1$ model the pipe. Thus the pipe elements are still modelled as point masses, while the vessel is not.

2.2 Dynamics

The equation of motion of the suspended pipe in the vertical plane is developed using a Lagrangian formulation and found to be a form of the robot equation of motion given in the joint space known from Sciavicco and Siciliano [2001],

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{H}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{f}(\mathbf{q}) + \mathbf{g}(\mathbf{q}) = \tau, \tag{16}$$

where

$\mathbf{M}(\mathbf{q})$	- system inertia matrix
$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$	- Coriolis-centripetal matrix
$\mathbf{H}(\mathbf{q},\dot{\mathbf{q}})$	- hydrodynamic damping matrix
$\mathbf{f}(\mathbf{q})$	- vector of spring forces in the pipe joints
$\mathbf{g}(\mathbf{q})$	- vector of gravitational/buoyancy forces
τ	- vector of control inputs

where $\mathbf{M}, \mathbf{C}, \mathbf{H} \in \mathbb{R}^{n\times n}$ and $\mathbf{g}, \mathbf{f}, \tau \in \mathbb{R}^n$. Recall that the elements modelling the pipe are point masses, thus only the last element representing the vessel has both mass and inertia. The inertia matrix $\mathbf{M}$ is

$$\mathbf{M} = m_n\mathbf{J}_{v_n}^{\mathrm{T}}\mathbf{J}_{v_n} + I_n\mathbf{J}_{\omega_n}^{\mathrm{T}}\mathbf{J}_{\omega_n} + \sum_{i=1}^{n-1}\left(m_i\mathbf{J}_{v_i}^{\mathrm{T}}\mathbf{J}_{v_i}\right), \tag{17}$$

where m_i is the mass of pipe element i and m_n and I_n is the mass and moment of inertia of the vessel. The choice of Coriolis-centripetal matrix $\mathbf{C}$ is not unique. A particular choice of $\mathbf{C}$ is where the generic element are

$$c_{ij} = \sum_{k=1}^{n} c_{ijk}\dot{q}_k, \tag{18}$$

where the coefficients

$$c_{ijk} = \frac{1}{2}\left(\frac{\partial m_{ij}}{\partial q_k} + \frac{\partial m_{ik}}{\partial q_j} - \frac{\partial m_{jk}}{\partial q_i}\right) \tag{19}$$

are termed *Christoffel symbols of the first type*. Due to symmetry of $\mathbf{M}$,

$$c_{ijk} = c_{ikj}. \tag{20}$$

By choosing the matrix $\mathbf{C}$ in this way we have that $\dot{\mathbf{M}} - 2\mathbf{C}$ is skew-symmetric, i.e.

$$\nu^{\mathrm{T}}\left[\dot{\mathbf{M}} - 2\mathbf{C}\right]\nu = 0 \quad \forall\nu \in \mathbb{R}^n. \tag{21}$$

Note that the complexity of $\mathbf{M}$ and hence $\mathbf{C}$, will grow significantly when the number of links increase. However, as the links are considered to be point masses the complexity is kept down. Also, the *Recursive Newton-Euler Algorithm* can be employed for numerical treatment of large systems since $\mathbf{C}$ is then no longer required to be found analytically.

The masses in the model are subject to the hydrostatic forces of gravity and buoyancy. Let the gravity vector be defined as

$$\mathbf{g}_0 = [\,0\; -9.81\,]^{\mathrm{T}}. \tag{22}$$

For the links with point masses representing the pipe, the gravity $\mathbf{g}_{g_i} \in \mathbb{R}^n$ and buoyancy $\mathbf{g}_{b_i} \in \mathbb{R}^n$ of an element $i \in 1\dots n-1$ both attacks in the center of gravity and are given in the inertial frame as

$$\mathbf{g}_{g_i} = m_i\mathbf{g}_0 \tag{23}$$

$$\mathbf{g}_{b_i} = -V_i\rho_w\mathbf{g}_0 \tag{24}$$

where V_i is the link volume and ρ_w is the water density. The surface vessel is at rest when buoyancy and weight balance. The total hydrostatic effects on link n representing the vessel has a translational force that keeps the vessel on the surface and a torque to keep it leveled. The force is a function of the difference between the displaced water volume of the vessel and the nominal displaced water volume when the vessel is at rest and is given as

$$\mathbf{g}_{g_n} + \mathbf{g}_{b_n} = -\rho_w A_{wp}(y_n - h)\,\mathbf{g}_0. \tag{25}$$

Small perturbations only in heave is assumed such that the waterplane area of the vessel A_{wp} is assumed to be constant. The water depth is given by h, and y_n is heave position of the center of gravity of the vessel. The model for the restoring moment in pitch $\mathbf{g}_\theta$ is given as

$$\mathbf{g}_\theta = -\phi_p(\theta - \theta_{ref}) - \phi_d\dot{\theta} \tag{26}$$

where $\theta = \alpha_n$ and $\phi_p, \phi_d \in \mathbb{R}$ are constants. θ_{ref} is the desired pitch angle, usually set to 0. The moment is highly dependent on vessel properties such as the hull shape, which justifies the use of this model. The vector $\mathbf{g}(\mathbf{q})$ in (16) is the combined restoring forces of the n links represented in the generalized coordinates, found to be

$$\mathbf{g}(\mathbf{q}) = \sum_{i=1}^{n}\mathbf{J}_{v_i}^{\mathrm{T}}(\mathbf{q})\left(\mathbf{g}_{g_i} + \mathbf{g}_{b_i}\right) + \mathbf{J}_{\omega_n}^{\mathrm{T}}\mathbf{g}_\theta. \tag{27}$$

The elasticity/bending stiffness of the pipe is modelled as static springs in the rotational joints connecting the elements. Assume that the spring is at rest when the corresponding joint angle $q_i = 0$. The spring effect in joint i is given by

$$f_i = kq_i \tag{28}$$

where $k \in \mathbb{R}$ is the bending stiffness of the pipe. As the pipe is homogeneous, it is assumed that k is the same for all joints. Let the spring forces $\mathbf{f}(\mathbf{q})$ in (16) be

$$\mathbf{f}(\mathbf{q}) = [\,f_1\; f_2\; \dots\; f_n\,]^{\mathrm{T}} \tag{29}$$

which is readily seen to be equivalent to

$$\mathbf{f}(\mathbf{q}) = k\mathbf{q} \tag{30}$$

Note that the elasticity effect is modelled directly on the states and not the mass centers of the links.

The pipe is subject to hydrodynamic forces of added mass and drag. These forces are expressed for a general body by Morison's equation, found in Faltinsen [1990]. The added mass term is only significant at high speeds and is omitted. The pipe elements have no angular momentum, due to the lumped point masses. By adopting Morison's equation, the drag force h acting on a pipe element of unit length is found to be

$$h = C_D \frac{1}{2} \rho_w d_o \left| u \right| u \tag{31}$$

where d_o is the pipe diameter, u is the undisturbed velocity normal to the element and C_D is the drag coefficient, found from experiments. Positive force is in the wave propagation direction. Let the $\mathbf{v}_{n_i}$ denote the component of $\mathbf{v}_i$ normal to the link. $\mathbf{v}_{n_i}$ is found by applying a coordinate transformation of $\mathbf{v}_i$ to the body fixed frame B_i, selecting the normal component and a new coordinate transformation back to the I frame. The formulation of $\mathbf{v}_{n_i}$ in generalized coordinates is

$$\mathbf{v}_{n_i} = \mathbf{P}_i \mathbf{J}_{v_i} \dot{\mathbf{q}} \tag{32}$$

where

$$\mathbf{P}_i = \left(\mathbf{R}^I_{B_i} \right) \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \left(\mathbf{R}^I_{B_i} \right)^{\mathrm{T}} \tag{33}$$

is a linear mapping from $\mathbf{v}_i$ to $\mathbf{v}_{n_i}$. Note that $\mathbf{P}_i^{-1}$ does not exist. The rotation matrix $\mathbf{R}^I_{B_i}$ from I to B_i is

$$\mathbf{R}^I_{B_i} = \begin{bmatrix} \cos \alpha_i & -\sin \alpha_i \\ \sin \alpha_i & \cos \alpha_i \end{bmatrix}. \tag{34}$$

Let $\gamma = \frac{1}{2} C_D \rho_w d_o l \geq 0$, and apply (31), the drag force on a link in general coordinates is

$$\boldsymbol{h}_i(\mathbf{q}, \dot{\mathbf{q}}) = \gamma \left\| \mathbf{v}_{n_i} \right\| \mathbf{P}_i \mathbf{J}_{v_i} \dot{\mathbf{q}}. \tag{35}$$

The hydrodynamic damping effects on the entire system $\mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{n \times n}$ is

$$\mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}) = \gamma \sum_{i=1}^{n} \left(\left\| \mathbf{v}_{n_i} \right\| \mathbf{J}_{v_i}^{\mathrm{T}} \mathbf{P}_i \mathbf{J}_{v_i} \right). \tag{36}$$

Applying control inputs to the actuators in the joints control the configuration of a robot manipulator. For the robot equation based slender marine structure model the joints are not actuated. Only the position of the vessel on the surface and the stinger configuration can be controlled. Let τ in (16) be given as

$$\tau = \tau_q + \tau_t \tag{37}$$

where τ_q represents control forces from the stinger and τ_t the thruster forces. The stinger configuration is adjustable, which corresponds to an actuator on the joint connecting links $n-1$ and n. Any joint that is supported by the stinger also have actuators. Assume that the stinger is shorter than the link length l_i for any link i. Hence

$$\tau_q = \left[\, 0 \, \ldots \, 0 \; \tau_{q_n} \right]^{\mathrm{T}} \in \mathbb{R}^n. \tag{38}$$

The thruster force $\mathbf{F} = \left[\, F_x \;\, 0 \right]^{\mathrm{T}} \in \mathbb{R}^2$ actuates the vessel in the surge direction in frame B_n, and is converted to general coordinates such that

$$\tau_t = \mathbf{J}_{v_n}^{\mathrm{T}} \mathbf{F}. \tag{39}$$

Now all the matrices and vectors in equation (16) are known and a complete set of equations of motions for the system of pipe and vessel has been developed.

2.3 Passivity analysis

Passivity provides a powerful tool for the stability analysis of nonlinear systems. Recall that the input is given by τ and let the output $\mathbf{y}$ be defined as

$$\mathbf{y} = \dot{\mathbf{q}}. \tag{40}$$

The model inputs are the translational and angular velocities $\mathbf{v}_n$ and ω_n of the vessel in the operational space and the stinger configuration rate of change $\dot{\mathbf{q}}_n$ in the joint space. The outputs are τ_{q_n} and thruster forces $\mathbf{F}$. For the storage function $V(\mathbf{q}, \dot{\mathbf{q}})$ defined as

$$V(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2} \dot{\mathbf{q}}^{\mathrm{T}} \mathbf{M}(\mathbf{q}) \dot{\mathbf{q}} + \frac{1}{2} k \mathbf{q}^{\mathrm{T}} \mathbf{q} + \int_0^{\mathbf{q}(t)} \mathbf{g}(\zeta) \, d\zeta \tag{41}$$

the system is input-output passive. From the previous section it is known that the inertia matrix is positive definite, $\mathbf{M}(\mathbf{q}) > 0$, $\forall \mathbf{q}$ and $k > 0$ which implies that

$$V > 0, \quad \forall \mathbf{q}, \dot{\mathbf{q}} \in \mathbb{R}^n \setminus \{0\} \tag{42}$$

Taking the time derivative of V along the system trajectories of (16)

$$\dot{V} = -\dot{\mathbf{q}}^{\mathrm{T}} \mathbf{H} \dot{\mathbf{q}} + \dot{\mathbf{q}}^{\mathrm{T}} \tau \leq \dot{\mathbf{q}}^{\mathrm{T}} \tau \tag{43}$$

as the $\mathbf{H}$ is positive semidefinite. This can be seen by reformulating (36) by inserting (33) such that

$$\mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}) = \gamma \sum_{i=1}^{n} \left(\left\| \mathbf{v}_{n_i} \right\| \mathbf{K}_i^{\mathrm{T}} \mathbf{K}_i \right) \tag{44}$$

where

$$\mathbf{K}_i = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \left(\mathbf{R}^I_{B_i} \right)^{\mathrm{T}} \mathbf{J}_{v_i} \tag{45}$$

Recall that $\gamma > 0$ and $\left\| \mathbf{v}_{n_i} \right\| > 0, \forall x \neq 0$. The matrix $\mathbf{K}_i$ is rank deficient, and $\mathbf{K}_i^{\mathrm{T}} \mathbf{K}_i$ is symmetric so it is known from Strang [1986] that $\mathbf{H}(\mathbf{q}, \dot{\mathbf{q}})$ is positive semidefinite. To get a physical understanding of this recall that the drag force defined in (31) only damps the motions normal to the pipe elements. The tangential motion is not damped and thus there will be zero damping for longitudinal velocities, thus there exists $\dot{\mathbf{q}} \neq \mathbf{0}$ such that $\mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{0}$. If surface friction of the pipe in the tangential direction is introduced, the matrix $\mathbf{K}_i$ will have full rank and $\mathbf{H}(\mathbf{q}, \dot{\mathbf{q}})$ would be positive definite, such that $\dot{\mathbf{q}}^{\mathrm{T}} \mathbf{H} \dot{\mathbf{q}} \geq \mathbf{0}, \; \mathbf{q}, \dot{\mathbf{q}} \in \mathbb{R}^n$.

Inserting (37) into (43) yields

$$\dot{V} \leq \dot{q}_n \tau_{j_n} + \dot{x}_n^{\mathrm{T}} F_x \tag{46}$$

which implies that the system is passive. By choosing a passive controller such as a PD controller the closed loop system is by Theorem 6.1 in Khalil [2002], also passive.

2.4 Comparison to the catenary

To validate the model it is geometrically compared to the solution of the classical catenary problem given in Seyed and Patel [1992] as

$$y(x) = \frac{H}{w} \left(\cosh \left(\frac{w}{H} \right) - 1 \right) \tag{47}$$

where w is the submerged pipe unit weight and H is the horizontal tension component. The catenary is a static solution while the developed model is dynamic. It is generally assumed that a pipe follows closely the curve of the catenary during pipelay except at the end points.

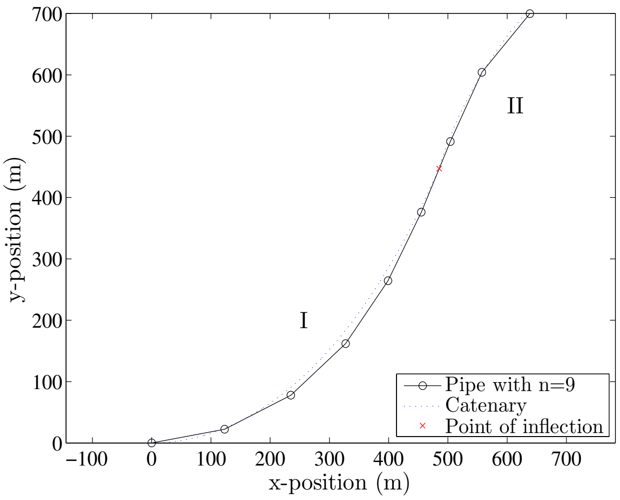

Fig. 3. Static plot of the devloped pipe model over a catenary (I) and arc (II). Note that $\tau_q = 0$, which causes the large angle q_9.

This can be compensated by applying a stiffened catenary model as presented in Plukett [1967]. Figure 3 shows the static solution of 16 plotted over a catenary from the origin to the point of inflection. For the overbend the catenary is not applicable and a circle arc is used. The curve-fit is as good as can be expected considering that the catenary assumes no bending stiffness, while $k \neq 0$ for the developed pipe model.

3. CONTROL DESIGN

It is important to prevent damage to the pipeline and to increase the operation time for the vessel, thus the lay rate. A commonly used control strategy is tension control based on the measured tension at the pipe tip. The developed model does not give this tension, so in this section a curvature based controller independent of the tension is developed. The thrusters are used to shape the curvature of the suspended pipe. This will provide optimal support for the pipe on the stinger and minimize the bending of the pipe at the stinger tip. Known measurements are the vessel and touchdown point positions, the pipe tension at the vessel, the stinger configuration and the length of the suspended pipe. An ROV is commonly positioned over the touchdown point. The actuators in the system are the thrusters of the vessel and the configuration of the stinger. Wether the latter is an actuator is arguable as it is rarely practical to modify during the pipe installation. A DP system is developed where the thrusters of the vessel are used to position the vessel such that a desired radius for the pipe at the overbend r_o is obtained. Assume that for a given depth and pipe dimension there exist an optimal pipe configuration with a overbend given as $r_{o_{ref}}$. The stinger configuration is approximated by an arc with radius r_o. It is desirable to use the thrusters to position the ship so that the pipe configuration coincides with the stinger configuration. Assume that there exist a mappings from r_o to x_n. These mappings depend on the pipe and vessel properties. To minimize the error between the stinger and pipe configurations the PID controller

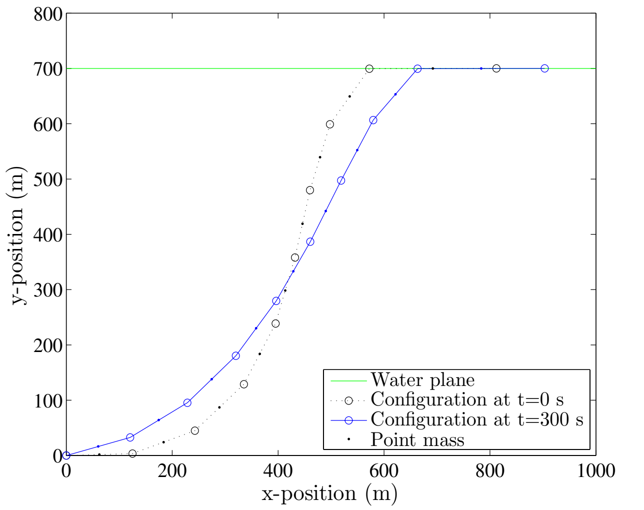

Fig. 4. Pipe configuration with and without the DP controller.

$$F_x = -k_p \tilde{x}_n - k_d v_n - k_i \int_{t_0}^{t} \tilde{x}_n(\zeta)\, d\zeta \tag{48}$$

where $\tilde{x}_n = x_n - x_{ref}$ is the error in surge direction, $k_p, k_d, k_i \geq 0$ are controller gains and x_{ref} is the desired position for x_n, is applied. This controller will control the angle q_n by applying thruster force. Let $\tau_{q_n} = 0$ as the stinger is assumed to be fixed in the optimal configuration. For $k_i = 0$ the closed loop system is known to be passive, and thus the stability properties of the closed loop are ensured. Passivity can generally not be guaranteed for a PID controller, unless the integrator action is limited. Saturation of the thrusters will probably limit the performance of the controller.

4. SIMULATIONS

To illustrate the theoretical results and the properties of the closed loop system, simulation results are presented. The numerical data utilized in the simulation are given in Appendix A. Note that $k_i = 0$ so that the controller is PD which ensures stability. The system has been simulated in Matlab with the *ode15s* solver. A nine element model is used, where the vessel starts in its equilibrium position at time $t = 0$ s, and at time $t = 5$ s the controller is enabled and moves the vessel to obtain the desired configuration. For the given numerical data $x_n(q_n)$ is found empirically. Assume also that the angles $q_i < q_c\ \forall i \in 1, \ldots, n$ where q_c is the critical angle where pipe buckling starts. Figure 4 shows the pipe configurations at times $t = 0$ s and $t = 300$ s. The DP controller reference position $x_{ref} = 790$ m.

Figure 5 shows all the states $\mathbf{q}$ of the model. The simulations confirm the passivity property found previously. The desired position was for the simulation $x_{ref} = 790$ m, while the controller moved the vessel to $x_n = 783$ m. This stationary offset can be reduced by $k_i \neq 0$. The angles has some over-shoot and oscillation, which can be reduced by better tuning of the model parameters. Let $\tau_q = 0$ for the simulation.

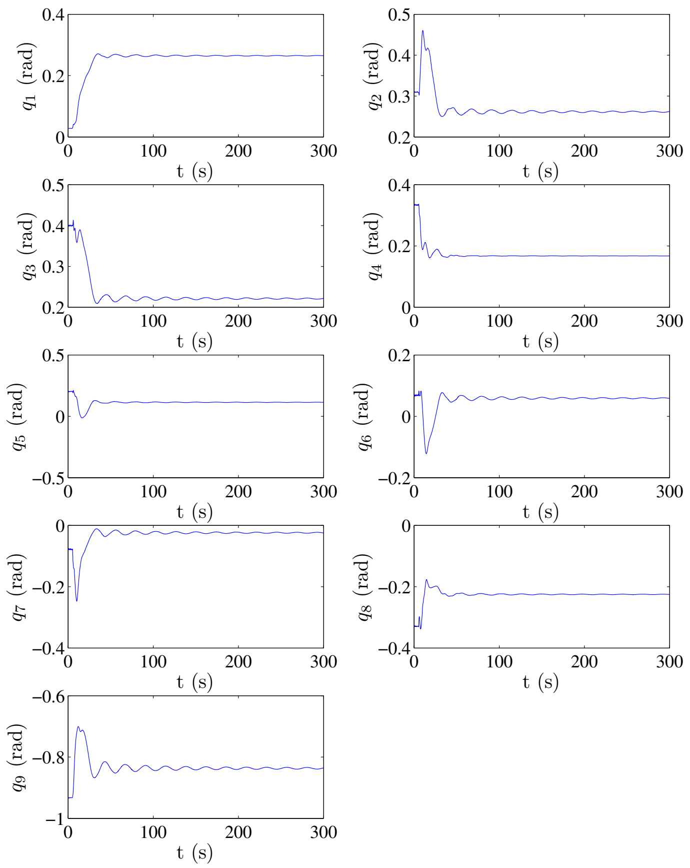

Fig. 5. The state vector $\mathbf{q}$, which are the joint angles.

5. DISCUSSION

Based on the robot equation it is possible to model a marine pipe being laid on the seabed from a pipe lay vessel by a set of ordinary differential equations. This model is suited for control applications and has advantages over the catinary pipe model, which lacks dynamics, and also over FEM models which have complexity beyond the need for control systems. The form of the model is well known in controller design groups, and numerous methods of controller design may be applied.

6. CONCLUSION

A model of a submerged offshore pipe fixed in the touch-down point at the seabed in one end and attached to a pipelay vessel in the other end is developed. The model is discrete and is on the form of the robot equation with minimal coordinates. The model structure is hyper-redundant. It is shown that the system is passive with input F_x and output $\dot{x}$. A passive PD-controller has thus been applied. The theoretical results are verified by simulations. The simulation results are in agreement with the theoretical results. Thus the model proves to be suited for control purposes in the pipelay operation.

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Appendix A. NUMERICAL DATA

Pipe length	$L = 1000$ m
Inner diameter	$d_i = 0.6900$ m
Outer diameter	$d_o = 0.7610$ m
Bending stiffness	$k = 1.4 \cdot 10^{11}$
Element length	$l_i = L/(n-1)$
Pipe density	$\rho = (\rho_a - \rho_s)(d_i/d_o)^2 + \rho_s$
Length on waterline	$l_n = 240$ m
Water plane area	$A_{wp} = 1000$ m^2
Volume displacement	$V_n = 4.96 \cdot 10^4$ m^3
Displacement	$m_n = 5.08976 \cdot 10^7$ kg
Torque gain proportional	$\phi_p = 1.0 \cdot 10^{14}$
Torque gain damping	$\phi_d = 1.0 \cdot 10^{14}$
Number of elements	$n = 9$
Water depth	$h = 700$ m
Drag coefficient	$C_d = 1.6$
Air density	$\rho_a = 1.200 \cdot 10^3$ g/m^3
Water density	$\rho_w = 1.025 \cdot 10^6$ g/m^3
Steel density	$\rho_s = 7.850 \cdot 10^6$ g/m^3
Controller gain proportional	$k_p = 0.4 \cdot 10^8$
Controller gain damping	$k_d = 0.4 \cdot 10^8$
Controller gain integrator	$k_i = 0$