Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Observer design for bioprocesses using a dissipative approach ⋆

Jaime A. Moreno *

* *Eléctrica y Computación, Instituto de Ingeniería, Universidad Nacional Autónoma de México (UNAM), Cd. Universitaria, 04510 Coyoacán, México DF, Mexico. JMorenoP@ii.unam.mx*

Abstract: Recently, the author has proposed a methodology for the design of nonlinear observers based on the dissipative theory. This methodology offers a systematic approach to the observer design providing great flexibility and generality. For example, several well known observer design methods, as the High-Gain and the Lipschitz Observers, can be treated and generalized in a unified manner by the Dissipative Approach. Moreover, different objectives in observation can be also unified and generalized by the Dissipative Approach, as for example the design of Unknown Input and Robust Observers. The objective of this paper is to show how this methodology can be applied in the design of observers for bioprocesses and its advantages for this kind of processes. An example illustrates the main ideas.

Keywords: Robust observers; Unknown Input Observers; Dissipative Observers; Bioprocesses;

1. INTRODUCTION

Reaction systems is a class of nonlinear dynamical systems that is widely used in areas such as chemical, biochemical and biomedical engineering, biotechnology, ecology, etc. (Robust) observation issues for this class of systems is of fundamental importance due to the limited availability of on-line sensors and the uncertainties related, in particular, to the mathematical model. It is not surprising that there is an intensive research activity to design observers (or software sensors) for these systems (Bastin and Dochain [1990], Dochain and Vanrolleghem [2001], Dochain [2003]), and different methods for uncertain reaction systems, besides the classical extended Kalman and Luenberger observers, have been proposed ([see Dochain and Vanrolleghem, 2001, for an overview]): Interval Observers (Gouzé et al. [2000]) are based on cooperative systems theory; Adaptive Observers (Dochain [2003]) assume that the uncertainties are represented by unknown parameters; Asymptotic Observers (Bastin and Dochain [1990], Dochain and Vanrolleghem [2001]) are based on the mass and energy balances without requiring the process kinetics; Practical and Parallelotopic Observers (Rapaport and Gouzé [2003]) consider uncertainties as *unknown inputs (UI)* and converge practically (not exactly) to the true state for a restricted class of systems with bounded perturbations.

For reaction systems without uncertainties several methods have been applied to design observers, as the High-Gain method (Gauthier et al. [1992]) and the Lipschitz Method (Rajamani [1998]). For uncertain reaction systems, when there are only parametric uncertainties, adaptive observers can be used. However, if stronger structural uncertainties are available the most successful method used to day are the asymptotic observers (Bastin and Dochain [1990], Dochain and Vanrolleghem [2001]). In the work (Moreno and Dochain [2008]) uncertainties are represented by *arbitrary unknown input signals* to the system, what represents a flexible way to characterize many kinds of uncertainties, and they are able to show that the asymptotic observers can be recovered and extended with their approach. A highly satisfactory result is to be able to explain, using observability/detectability arguments, why (classic) asymptotic observers converge and why their convergence rate is not assignable. Moreover, the robust observers proposed in that work can be used in more general situations and their convergence properties are completely derived from the robust observability/detectability properties of the model.

However, due to the basic linear structure of the uncertain systems considered in (Moreno and Dochain [2008]), it is not possible to consider more general situations. For example, if some reaction rates are known but others are uncertain, this leads to a nonlinear structure with unknown inputs, that cannot be treated with that approach. So a natural extension of that work is to use unknown input observer design methods for uncertain reaction systems, and this is part of the objective of this paper.

The use of systems with unknown inputs for the representation of the uncertain reaction system's family leads naturally to the study of observability and detectability concepts for this kind of systems, and the construction and existence conditions of *Unknown Input Observers (UIO)*. For linear time invariant (LTI) systems this is a very well established topic (Hautus [1983], Hou and Müller [1994]), and some advances in the design of UIOs for nonlinear systems have been obtained recently by (Rocha-Cózatl et al. [2005], Moreno [2000])

⋆ This work was supported in part by DGAPA-UNAM, project PAPIIT IN112207, and CONACyT, Project 51244.

Recently, the author has proposed (Moreno [2004, 2005]) a method to design nonlinear observers using dissipative methods. One attractive feature of this Dissipative Design is, on the one side, that it includes and generalizes many current observer design methods, and on the other side, that it is possible to design observers with unknown inputs or known inputs in a unified framework. The aim of this work is to show how the Dissipative Design Method can be used to design observers for reaction systems with or without uncertainties in a unified way. This can be seen as a first step in a more general, and far-reaching objective: to develop a methodology to design robust observers for uncertain reaction systems, in which different kinds of uncertainties are available as unknown constant parameters, unknown (bounded) disturbances, unmodeled dynamics, deterministic perturbations characterized by an internal model, etc. We believe that the Dissipative Design Method is able to reach these requirements, and this is part of active research work.

The rest of the paper is organized as follows. The basic ideas of the Dissipative Design Method of Nonlinear Observers are introduced in 2. A fairly general model of reaction systems is given in Section 3 and the proposed design strategy using the Dissipative Design is briefly introduced here. An illustrative example of the design method is given in Section 4.

2. DISSIPATIVE OBSERVER DESIGN

Motivated by the circle criterion design of nonlinear observers in Arcak and Kokotovic [2001] the author has proposed in Moreno [2004, 2005] a methodology for designing nonlinear observers for a class of nonlinear systems. This method will be briefly reviewed in this section.

2.1 Preliminaries

In this work the stability properties of dissipative systems will be used for the design of observers for systems that can be represented as the feedback interconnection of a dynamical linear time invariant (LTI) system in the forward loop and a memoryless nonlinearity in the feedback loop. From the dissipativity theory (Willems [1972], Hill and Moylan [1980]) the following results are of relevance here.

Consider the feedback interconnection

$$\begin{aligned} \dot{x} &= Ax + Bu \ , \quad x(0) = x_0 \ , \\ y &= Cx \ , \\ u &= -\psi(t, y) \ , \end{aligned} \tag{1}$$

with $x \in \mathbb{R}^n$, $u \in \mathbb{R}^q$, $y \in \mathbb{R}^m$, and quadratic *supply rates*

$$\begin{aligned} \omega(v, w) &= v^T Q v + 2 v^T S w + w^T R w \\ &= \begin{bmatrix} v \\ w \end{bmatrix}^T \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix} \ , \end{aligned} \tag{2}$$

where $v \in \mathbb{R}^r$, $w \in \mathbb{R}^s$, $Q \in \mathbb{R}^{r \times r}$, $S \in \mathbb{R}^{r \times s}$, $R \in \mathbb{R}^{s \times s}$, and Q, R symmetric.

Definition 1. The linear part (A, B, C) of system (1) is said to be *state strictly dissipative (SSD)* with respect to the supply rate $\omega(y, u)$, or for short (Q, S, R)-SSD, if there exist a matrix $P = P^T > 0$, and $\epsilon > 0$ such that

$$\begin{bmatrix} PA + A^T P + \epsilon P \ , & PB \\ B^T P & 0 \end{bmatrix} - \begin{bmatrix} C^T Q C & C^T S \\ S^T C & R \end{bmatrix} \leq 0 \ . \tag{3}$$

For quadratic systems, i.e. $m = q$, passivity corresponds to the supply rate $\omega(y, u) = y^T u$. If (A, B) controllable, (A, C) observable, then condition (3) is equivalent, by the Kalman-Yakubovich-Popov Lemma (Khalil [2002]), to the fact that the transfer matrix of Σ, i.e. $G(s) = C(sI - A)^{-1} B$, is strictly positive real (SPR). Note that this definition assures the existence of a quadratic positive definite *storage function* $V(x) = x^T P x$, and a positive definite loss function $Z(x, u) = \left(K^T x + W^T u\right)^T \left(K^T x + W^T u\right) + \epsilon x^T P x$, such that along any trajectory of the system $\dot{V}(x(t)) = \omega(y(t), u(t)) - Z(x(t), u(t))$.

Definition 2. The nonlinear part of system (1), a time-varying memoryless nonlinearity $\psi : [0, \infty) \times \mathbb{R}^m \to \mathbb{R}^q$, $u = \psi(t, y)$, piecewise continuous in t and locally Lipschitz in y, such that $\psi(t, 0) = 0$, is said to satisfy a dissipative condition in Γ with respect to the supply rate $\omega(u, y)$ (2), or for short (Q, S, R)-D in Γ, if

$$\omega(u, y) = \omega(\psi(t, y), y) \geq 0 \ , \forall t \geq 0 \ , \ \forall y \in \Gamma \subseteq \mathbb{R}^m \ ,$$

where Γ is a subset of $\mathbb{R}^m$ whose interior is connected and contains the origin. If $\Gamma = \mathbb{R}^m$, then ψ satisfies the dissipativity condition globally, in which case it is said that ψ is dissipative with respect to ω, or for short, (Q, S, R)-D.

Remark 3. Note that the classical sector conditions (Khalil [2002]) for square nonlinearities, i.e. $m = q$, can be represented in this form. If ψ is in the sector $[K_1, K_2]$, i.e. $(y - K_1 u)^T \ (K_2 u - y) \geq 0$, then it is (Q, S, R)-D, with $(Q, S, R) = \left(-I, \frac{1}{2}(K_1 + K_2), -\frac{1}{2}\left(K_1^T K_2 + K_2^T K_1\right)\right)$. If ψ is in the sector $[K_1, \infty]$, i.e. $(y - K_1 u)^T \ u \geq 0$, then it is $\left(0, \frac{1}{2} I, -\frac{1}{2}\left(K_1 + K_1^T\right)\right)$-D.

Remark 4. Classically the concept of dissipativity has been defined globally. However, it is of interest to consider the local (or non global) case, for which the local version of dissipativity of a nonlinearity has been introduced.

For the interconnected system (1) a generalization of the passivity and of the small gain theorems for non square systems can be easily obtained, and it will be used in the sequel.

Lemma 5. Consider the system (1). If the linear system (C, A, B) is $\left(-R, S^T, -Q\right)$-SSD, then the equilibrium point $x = 0$ of (1) is globally (locally) exponentially stable for every (Q, S, R)-D (in Γ for some $\Gamma \subseteq \mathbb{R}^m$) nonlinearity.

2.2 Dissipative design for certain nonlinear systems

Consider the class of systems described by a LTI subsystem with a nonlinear perturbation term, connected in feedback, i.e.

$$\Sigma : \begin{cases} \dot{x} = Ax + G\psi(\sigma, y, u) + \gamma(t, y, u) \ , \quad x(0) = x_0 \\ y = Cx \ , \\ \sigma = Hx \ , \end{cases} \tag{4}$$

or that can be brought to this form by transformations, and where $x \in \mathbb{R}^n$ is the state, $y \in \mathbb{R}^p$ is the measured output, $u \in \mathbb{R}^m$ is the input, and $\sigma \in \mathbb{R}^r$ is a (not necessarily measured) linear function of the state. $\gamma(t, y, u)$ is an arbitrary nonlinear function of the input and the output. $\psi(\sigma, y, u)$ is a q-dimensional vector that depends on σ, y, u. ψ and γ are assumed to be locally Lipschitz in σ, y, u, so that existence and uniqueness of solutions is guaranteed. It will be assumed that the trajectories of interest of Σ are defined for all future times.

An *observer* for system (4) is a dynamical system Ω that has as inputs the input u and the output y of Σ, and its output $\hat{x}$ is an estimation of the state x of Σ. A full order observer for Σ of the form

$$\Omega: \begin{cases} \dot{\hat{x}} = A\hat{x} + G\psi\left(\hat{\sigma} + N\left(\hat{y} - y\right), y, u\right) + L\left(\hat{y} - y\right) + \\ \quad +\gamma\left(t, y, u\right), \quad \hat{x}\left(0\right) = \hat{x}_0 \ , \\ \hat{y} = C\hat{x} \ , \\ \hat{\sigma} = H\hat{x} \ , \end{cases} \tag{5}$$

is proposed, where matrices $L \in \mathbb{R}^{n\times p}$, and $N \in \mathbb{R}^{r\times p}$ have to be designed. Defining the state estimation error by $\tilde{x} \triangleq \hat{x} - x$, the output estimation error by $\tilde{y} \triangleq \hat{y} - y$, and the function estimation error by $\tilde{\sigma} \triangleq \hat{\sigma} - \sigma$, $z \triangleq (H + NC)\tilde{x} = \tilde{\sigma} + N\tilde{y}$, and a new nonlinearity

$$\phi\left(z, \sigma, y, u\right) \triangleq \psi\left(\sigma, y, u\right) - \psi\left(\sigma + z, y, u\right) \ , \tag{6}$$

the dynamics of the error can be written as

$$\Xi: \begin{cases} \dot{\tilde{x}} = A_L\tilde{x} + G\nu \ , \quad \tilde{x}\left(0\right) = \tilde{x}_0 \ , \\ z = H_N\tilde{x} \ , \\ \nu = -\phi\left(z, \sigma, y, u\right) \ , \end{cases} \tag{7}$$

where $A_L \triangleq A + LC$, and $H_N \triangleq H + NC$. Note that $\phi\left(0, \sigma; y, u\right) = 0$ for all σ, y, u.

Remark 6. Note that when the plant Σ is LTI (at least up to an output injection term), then $\phi = 0$, and the error dynamics Ξ is LTI and autonomous, i.e. it does not depend on the plant state. The same is true if σ is dependent on the output y, since in this case there exists a matrix such that $\sigma = Fy$, and there exists an N such that $H_N = H + NC = 0$. In these both cases detectability of the pair (A, C) is a necessary and sufficient condition to construct an observer. However, in general, the error dynamics (7) is not autonomous, but it is driven by the system (4) through the linear function of the state $\sigma = Hx$. ϕ is therefore a time varying nonlinearity, whose time variation depends on the state trajectory of the plant.

The observer design consists in finding matrices L and N, if they exist, so that Ξ satisfies the conditions of Lemma 5. For this it is necessary to assume that the nonlinear part of (7) belongs to one or several sectors.

Assumption 1. ϕ in (7) is (Q_i, S_i, R_i)-dissipative (in Γ) for some finite set of non positive semidefinite quadratic forms $\omega_i\left(\phi, z\right) = \phi^T Q_i\phi + 2\phi^T S_i z + z^T R_i z \geq 0$, for all σ, y, u, for $i = 1, 2, \cdots, M$.

It is clear that it is necessary that the quadratic forms be independent. It is also easy to see that then ϕ is $\sum_{i=1}^{M}\theta_i\left(Q_i, S_i, R_i\right)$-dissipative (in Γ) for every $\theta_i \geq 0$, i.e. ϕ is dissipative with respect to the supply rate $\omega\left(\phi, z\right) = \sum_{i=1}^{M}\theta_i\omega_i\left(\phi, z\right)$.

Example 7. Consider a lower triangular nonlinearity

$$\psi^T\left(x, u\right) = \left[\, \psi_1\left(x_1, u\right) \cdots \psi_{n-1}\left(x_1, \cdots, x_{n-1}, u\right), \psi_n\left(x, u\right) \,\right] \tag{8}$$

with $\psi\left(0, u\right) = 0$ for all u. Assume that each component is (globally) Lipschitz, uniformly in u (or for u in a compact set). i.e. $\left\|\psi_i\left(x^i, u\right) - \psi_i\left(y^i, u\right)\right\| \leq k_i\left\|x^i - y^i\right\|$, $i = 1, \cdots, n$, where $k_i > 0$ is the Lipschitz constant of ψ_i, and $x^i = \left[\, x_1 \cdots x_i \,\right]^T$. Defining $\phi\left(z, x, u\right) = \psi\left(x, u\right) - \psi\left(x + z, u\right)$ the Lipschitz condition on ψ implies for each component of ϕ that $\left\|\phi_i\left(z^i, x^i, u\right)\right\| \leq k_i\left\|z^i\right\|$, $i = 1, \cdots, n$. Considering the Euclidean norm this implies

$$\phi_i^2\left(z_1, \cdots, z_i, x_1, \cdots, x_i, u\right) \leq k_i^2\left(z_1^2 + \cdots + z_i^2\right) \ .$$

These inequalities show that ϕ is (Q_i, S_i, R_i)-dissipative for all $i = 1, 2, \cdots, n$, with $(Q_i, S_i, R_i) = \left(-b_i b_i^T, 0, k_i\mathbb{I}_i\right)$, where b_i are the basis vectors of $\mathbb{R}^n$, $\mathbb{I}_i = \operatorname{diag}\left(I_i, 0_{n-i}\right)$, and I_p is the identity matrix of dimension p. ■

In this case the design is as follows

Theorem 8. Suppose Assumption 1 is satisfied. If there are matrices L, N and a vector $\theta = (\theta_1, \cdots, \theta_M)$, $\theta_i \geq 0$, so that the linear subsystem of Ξ is $\left(-R_\theta, S_\theta^T, -Q_\theta\right)$-SSD, with $\left(Q_\theta, S_\theta, R_\theta\right) = \sum_{i=1}^{M}\theta_i\left(Q_i, S_i, R_i\right)$, that is there exist a matrix $P = P^T > 0$, and $\epsilon > 0$ such that

$$\begin{bmatrix} PA_L + A_L^T P + \epsilon P + H_N^T R_\theta H_N \ , & PG - H_N^T S_\theta^T \\ G^T P - S_\theta H_N & Q_\theta \end{bmatrix} \leq 0 \ , \tag{9}$$

where $A_L = A + LC$, $H_N = H + NC$, then Ω is a global (local) exponential observer for Σ, i.e. there exist constants $\kappa, \mu > 0$ such that for all $\tilde{x}\left(0\right)$ (in a vicinity of $\tilde{x} = 0$) $\left\|\tilde{x}\left(t\right)\right\| \leq \kappa\left\|\tilde{x}\left(0\right)\right\|\exp\left(-\mu t\right)$.

Remark 9. The proposed method generalizes and improves several methods previously proposed in the literature. Some of them are (Moreno [2004]):

(i) The Circle criterion design: It is easy to see that this method generalizes and improves the one proposed in (Arcak and Kokotovic [2001]): our design is valid for non-square systems, the nonlinearities are of general type, and can be described by several sector conditions.
(ii) Lipschitz observer design: Proposed recently by Rajamani [1998], that is a generalization of the classical method introduced by Thau [1973].
(iii) High-Gain observer design: The well-known high-gain observer design (Gauthier et al. [1992]) is a special case of the one proposed here.

2.3 Dissipative design for uncertain nonlinear systems

One alternative to model uncertain systems consists in considering the uncertainties as completely unknown inputs to the system. The class of nonlinear systems considered for UIO design is

$$\Sigma: \begin{cases} \dot{x} = Ax + G\psi\left(\sigma, y, u\right) + \gamma\left(t, y, u\right) + Bw \ , \ x\left(0\right) = x_0 \\ y = Cx \ , \\ \sigma = Hx \ , \end{cases} \tag{10}$$

where $w \in \mathbb{R}^q$ is an arbitrary (even unbounded) unknown input. w can model an arbitrary unknown disturbance acting on the system, parametric uncertainty or unmodeled dynamics. It will be assumed that the trajectories of Σ exist and are well defined for all times, i.e. there are no finite escape times. Without loss of generality it is assumed that matrices B and C are of full rank.The objective is to design an *Unknown Input Observer (UIO)* for system Σ

(10), that is, a dynamical system that using the information of the known input $u(t)$ and the output $y(t)$ produces an state estimate $\hat{x}(t)$, that converges asymptotically to the actual state $x(t)$ of Σ, i.e. $\lim_{t\to\infty}(\hat{x}(t) - x(t)) = 0$, in spite of the lack of information on the unknown input w and derivative(s) of output y.

Remark 10. Since w is an arbitrary unknown input and an UIO of Σ is convergent independently of it, the same UIO will work for any system of the form (10) with $w = G(x, u, \tilde{w})$, where $\tilde{w}$ is a new unknown input. This represents a generalization of the considered systems. Note that the class of systems can be enlarged considering that by means of unknown input, state and/or output transformations some systems can be transformed to the particular form given by (10).

The main result of Rocha-Cózatl and Moreno [2004], Rocha-Cózatl et al. [2005] is a sufficient condition for the existence of an UIO for the plant Σ (10).

Theorem 11. Suppose that Assumption 1 is satisfied, and that there exist constant matrices $P = P^T > 0$, L, N, $\mathbb{S}$, a vector $\theta = (\theta_1, \cdots, \theta_M)$, $\theta_i \geq 0$, and a constant $\epsilon > 0$, such that (★ represent the symmetric terms)

$$\begin{bmatrix} PA_L + A_L^T P + \epsilon P + H_N^T R_\theta H_N & \bigstar & \bigstar \\ G^T P - S_\theta H_N & Q_\theta & \bigstar \\ B^T P - \mathbb{S}C & 0 & 0 \end{bmatrix} \leq 0 \ .$$

Then there exists an UIO for (10).

As it is shown in the references if this conditions are satisfied there are state $\chi = Tx$ and output transformations such that in the new coordinates the system has the form

$$\dot{\chi}_1 = \bar{A}_{11}\chi_1 + \bar{A}_{12}\chi_2 + \bar{G}_1\psi(\sigma, y, u) + \bar{\gamma}_1 - B^T PBw \tag{11}$$

$$\dot{\chi}_2 = \bar{A}_{21}\chi_1 + \bar{A}_{22}\chi_2 + \bar{G}_2\psi(\sigma, y, u) + \bar{\gamma}_2 \tag{12}$$

$$y_1 = \chi_1 \ , \ y_2 = C_2\chi_2 \ , \ \sigma = H_2\chi_2 \ .$$

Note that the states affected by the unknown input (χ_1) (11) are measured, and the estimation of the rest of the states (12), that are unaffected by the unknown input (χ_2), can be performed as in the previous subsection.

Remark 12. Note that when there is no unknown input, i.e. $w = 0$ or $B = 0$, the observer design reduces to the certain case explained in the previous paragraph 2.2. Accordingly, in this case the MI (11) reduces to the MI (9), since then one can set $B = 0$ and $\mathbb{S} = 0$. This feature of the dissipative method is very appealing, since the design of Observers with and without UIs is unified.

Remark 13. Although in the previous paragraphs it has been assumed that the system and observer matrices are constant, this is not necessary. As far as the MI 11 is satisfied with a *constant* matrix P, the design is possible. Furthermore, one can extend the method by allowing a time-varying matrix P at the cost of obtaining a Differential MI in place of the algebraic MI (11).

2.4 Matrix Inequality

In general (9, 11) are nonlinear matrix inequality feasibility problems. Under some conditions they become Linear Matrix Inequalities (LMI) feasibility problems, for which solutions can be effectively found by several algorithms in the literature (Boyd et al. [1994]). Note also that when (9, 11) are feasible, there exist in general several solutions for L and N. Replacing ϵP by ϵI, (11) is a LMI in P, PL, ϵ, $\mathbb{S}$, θ but not in N, except when $R_\theta = 0$ and $S_\theta = 0$.

One possibility to solve (9, 11) by LMI algorithms is to fix N at some value and to search for a solution. This can be made recursively until a solution is found. A particular situation arises when $N = 0$, so that the classical output injection is made.

3. MODEL OF (UNCERTAIN) REACTION SYSTEMS AND ROBUST OBSERVER DESIGN

A general state-space model of reaction systems is generally obtained from mass and energy balances (Bastin and Dochain [1990], Dochain and Vanrolleghem [2001]) and can be written in a compact and generalized form as:

$$\Sigma_R : \begin{cases} \dot{x} = K\varphi(x) - D(t)x - Q(x) + F(t) \ , \\ y = Cx \ . \end{cases} \tag{13}$$

where $y \in \mathbb{R}^m$ is the output vector, the state $x \in \mathbb{R}^n$ consists of component concentrations, volumes and temperatures, $K \in \mathbb{R}^{n\times q}$ is the constant stoichiometric coefficient matrix, $\varphi \in \mathbb{R}^q$ is the reaction rate vector, D is the (matrix) dilution rate, Q is the outflow rate vector, F is the feedrate vector. For a single reactor D is a scalar but it is a matrix when several reactors are considered.

In practice the model is usually uncertain, since the parameters and nonlinearities of the system are difficult to identify precisely and they may change over time. In particular, the reaction rates are usually poorly known. This makes the observation problem challenging. In order to deal with these uncertainties a representation of all possible behaviors of the system (13) is required. In a previous work (Moreno and Dochain [2008]) the authors have proposed to use state-affine systems with *unknown inputs*, that is

$$\Sigma_U : \begin{cases} \dot{x} = A(u, y)x + Bw + \psi(u, y) \ , \\ y = Cx \ , \end{cases} \tag{14}$$

where $w \in \mathbb{R}^p$ is a vector of (arbitrary) unknown inputs representing uncertainties, $u \in \mathbb{R}^r$ is a vector of measured inputs, and $A(u, y)$ is a continuous matrix. In this form they have been able to explain and generalize the well known asymptotic observers, that have been shown to be very useful in many practical situations (Bastin and Dochain [1990], Dochain et al. [1992], Dochain and Vanrolleghem [2001]). They are obtained when all the reaction rates are considered uncertain, $w = \varphi(x)$, but the rest of the model is assumed to be known, i.e. the uncertain system can be represented by (14) with $A(u, y) = -D(t)$, $B = K$, $\psi(u, y) = F(t) - Q(x)$.

However, due to the basic linear structure of (14) it is not possible to consider more general situations. For example, if some reaction rates are known but others are uncertain, this leads to a nonlinear structure as the one in (10), when the uncertain reaction rates are modeled as unknown inputs. It seems also natural to use (10) as a model for an uncertain reaction system. The dissipative method can then be used to design a robust observer. It is clear that the classic asymptotic observers are a special case of this approach. Moreover, the case that no uncertainties are present in the model can be treated in the same framework. This shows the great flexibility of the method.

Example 14. Consider the case that in system (13) some reaction rates $(\varphi_k(x))$ are well known but the rest is unknown $(\varphi_u(x))$. If it is assumed that Q, D and F are measured, and K is known, then the reaction system can be written as

$$\dot{x} = K_k\varphi_k(x) - D(t)x - Q(x) + K_u w + F(t) \ ,$$
$$y = Cx \ .$$

in which $w = \varphi_u(x)$. This system has the structure of (10).

4. EXAMPLE

In order to illustrate the dissipative observer design method proposed a simple biological reactor model will be considered:

$$\begin{aligned} \dot{X} &= -D(t)X + \mu(S)X \ , \\ \dot{S} &= D(t)(S_{in} - S) - \frac{1}{Y}\mu(S)X \ , \end{aligned} \tag{15}$$

where X is the biomass and S the substrate concentration, μ is the growth rate, Y the yield coefficient, S_{in} is the substrate concentration in the inflow and D is the dilution rate. The observation problem consists in estimating the substrate concentration S when the biomass concentration X is measured. Two extreme conditions on the knowledge of the reaction rate will be considered:
Case 1: The reaction rate μ is completely unknown.
Case 2: No uncertainty, i.e. the model is perfectly known.

4.1 Case 1: Unknown reaction rate

Case 1 is the standard situation for asymptotic observers, where μ is treated as an unknown input (Bastin and Dochain [1990], Dochain and Vanrolleghem [2001], Moreno and Dochain [2008]). Since there is only one reaction rate and one measurement in this example, the dissipative approach leads exactly to the classical asymptotic observer. The variable $Z = \frac{1}{Y}X + S$, whose dynamics is

$$\dot{Z} = -D(t)Z + D(t)S_{in} \ ,$$

is independent of the reaction rate μ. The asymptotic observer

$$\dot{\hat{Z}} = -D(t)\hat{Z} + D(t)S_{in} \ ,$$

converges asymptotically to the true value of Z, independently of the value of μ, when D is persistently exciting, i.e. there exist $\alpha, T > 0$ such that for all $t \geq 0$, $\int_t^{t+T} D(\tau)d\tau \geq \alpha$. The convergence of the observer cannot be assigned and depends on the behavior of D. The detectability analysis of Moreno and Dochain [2008] shows that if μ is completely arbitrary no better result can be obtained.

4.2 Case 2: Known reaction rate

For the following observer

$$\begin{aligned} \dot{\hat{X}} &= -D(t)X + \mu\left(\hat{S} + Ne_X\right)X + l_1 e_X \ , \\ \dot{\hat{S}} &= D(t)\left(S_{in} - \hat{S}\right) - \frac{1}{Y}\mu\left(\hat{S} + Ne_X\right)X + l_2 e_X \ , \end{aligned} \tag{16}$$

where $e_X = \hat{X} - X$, $e_S = \hat{S} - S$ are the observation errors, the error dynamic is given by

$$\begin{aligned} \dot{e}_X &= l_1 e_X + \phi(z,S)X \ , \\ \dot{e}_S &= -D(t)e_S - \frac{1}{Y}\phi(z,S)X + l_2 e_X \ , \\ z &= e_S + Ne_X \ , \\ \phi(z,S) &= \mu(z+S) - \mu(S) \ . \end{aligned} \tag{17}$$

It is illustrative to use (e_X, z) as state variables of the error instead of (e_X, e_S), i.e.

$$\begin{aligned} \dot{e}_X &= l_1 e_X + \phi(z,S)X \ , \\ \dot{z} &= -D(t)z + \left(N - \frac{1}{Y}\right)\phi(z,S)X + l_N e_X \ , \end{aligned} \tag{18}$$

where $l_N = Nl_1 + l_2$. In order to design the observer the sector of ϕ has to be determined. For continuous differentiable reaction rates μ this is easily done with the help of the mean value theorem. Since

$$\phi(z,S) = \frac{d\mu(S+\gamma z)}{dS}z, \gamma \in (0,1) \ ,$$

it follows that $\phi(z,S)$ is in the sector $[K_1, K_2]$, where K_1 and K_2 are the minimum and the maximum value of the derivative of μ, respectively.

Two classical classes of growth rates will be considered:

i) **The monotonic case:** The typical form is the Monod function $\mu(S) = \frac{\mu_0 S}{S+K_S}$, but other forms are possible. In this case $0 < K_1 < K_2 < \infty$. The strict positiveness of K_1 comes from the fact that in the reactor S is bounded.
ii) **The non monotonic case:** The typical form is the Haldane function $\mu(S) = \frac{\mu_0 S}{S^2/K_i + S + K_S}$, but other forms are possible. In this case $K_1 < 0 < K_2 < \infty$.

It is possible to design the observer gains by solving the Matrix Inequality (9), that, in general, has many solutions, if it is feasible. Here, for illustrative purposes, a simple storage function will be selected and the design parameters will be selected to satisfy the corresponding inequality.

Consider as Lyapunov's function candidate $V(e_X, z) = \frac{1}{2}\left(e_X^2 + \theta z^2\right)$. Its derivative along the trajectories of the observation error 18 is

$$\begin{aligned} \dot{V} = {} & l_1 e_X^2 + X\phi(z,S)e_X + \theta l_N z e_X - \theta D(t)z^2 + \\ & +\theta X\left(N - \frac{1}{Y}\right)\phi(z,S)z \ . \end{aligned}$$

The design is then as follows:

i) **The monotonic case:** Selecting $l_1 = \lambda_1 X$, $\theta > 0$, $l_N = \lambda_N X$ and $N < \frac{1}{Y}$, with λ_1 large enough, then $\dot{V} < -\epsilon V$ so that the observer converges exponentially fast, even when $D = 0$.
ii) **The non monotonic case:** Selecting $l_1 = \lambda_1 X$, $\theta > 0$, $l_N = \lambda_N X$ and $N = \frac{1}{Y}$, with λ_1 large enough, then $\dot{V}$ is negative definite, so that the observer converges asymptotically fast, if it is assumed that $D(t) \geq \epsilon > 0$.

Many more solutions can be found solving the Matrix Inequality (9). These degrees of freedom can be used to optimize certain performance criteria.

Note that setting $N = \frac{1}{Y}$ and $l_N = 0$ in the previous observer, then the asymptotic observer is recovered!

In Figure 1 some simulations illustrate the behavior of the designed observers. The growth rate is Monod and the parameters of the plant: $Y = 0.3$, $S_{in} = 10$, $\mu_0 = 0.2$, $K_S = 10$, $D = 0.4$, $X_0 = 10$, $S_0 = 5$. For both observers $\theta = 0.025$, $l_1 = -3X$, $e_{X0} = 0.5$, $e_{S0} = 10$ were used. For the asymptotic observer $N = 1/Y$ and $l_N = 0$, and for the dissipative observer $N = 1/Y - 8$, $l_N = -X/\theta$.

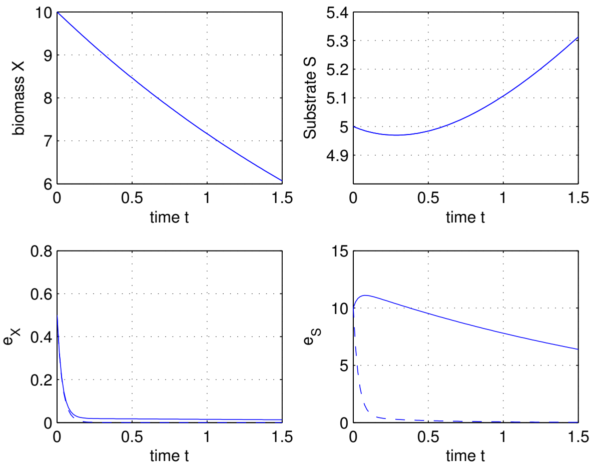

Fig. 1. Simulation of the bioreactor and the estimation errors of the asymptotic observer (continuous line) and of the dissipative observer (dotted line).

It is clear that the convergence velocity of the error of the unmeasured state (S) for the dissipative observer is much faster than that of the asymptotic observer. This is of course expected, since the model is perfectly known for the first but not for the second one. The interesting point here is that the dissipative observer methodology allows for a unified design under different uncertainty conditions.

5. CONCLUSIONS

In this work it has been shown how the Dissipative Design Method can be used to design observers for reaction systems with or without uncertainties in a unified way. Many important issues as the consideration of unknown parameters, sensor noise, consideration of trade offs between robustness and observer performance, etc. have to be addressed and this is part of active research work. We believe that the Dissipative Design Method is a methodology able to reach these requirements.

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