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Rate bounded linear parameter varying control of a wind turbine in full load operation

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Abstract: This paper considers the control of wind turbines using an LPV design technique. The controller design is done by a combination of a method that uses elimination of controller variables and a method using a congruent transformation followed by a change of variables.
An investigation is performed to understand the gap between zero rate of variation and arbitrary fast rate of variation for the selected scheduling variable. In particular it is analysed for which rate of variation, the local performance level starts to deteriorate from the performance level that can be obtained locally by LTI controllers.
A rate of variation is selected which is expected only to be exceeded outside the normal wind turbine operating conditions. For this rate of variation a controller has been designed and simulations show a performance level over the operating region which is very similar to what can be obtained by LTI designs for the specific operating condition. The LPV controller, however, works for the whole operating range with reasonably fast changes within this.

Keywords: gain scheduling; linear parameter varying (LPV) systems; Modelling, operation and control of power systems; Output feedback control; LMIs; Industrial applications of optimal control

1. INTRODUCTION

It has been shown that linear time invariant (LTI) controllers perform very well for the control of wind turbines as long as the operating condition is close to the design point. It is, however, simple to observe that the aerodynamics of wind turbines is highly nonlinear which means that designing controllers for only one operating condition does not suffice. In fact the performance will decrease significantly (if not to instability) when moving away from the design point. Because of this it has been decided to focus on the design of a gain scheduled controller for the operation of wind turbines in both partial load and full load.

Several gain scheduled controller design approaches have been investigated for the control of wind turbines. Most approaches either neglect the rate of variation of the scheduling parameter as in Cutululis et al. (2006); van Engelen et al. (2003) or alternatively controllers have been designed to allow for arbitrary fast parameter variations as in Lescher et al. (2005); Mantz et al. (2005). The nominal operating condition is essentially determined by the average wind speed together with operational settings such as rating of active power and generator speed.

With the assumption of no or insignificant parameter variations it is possible to get a high level of performance locally for all operating conditions. On the other hand, if the assumption of very slow parameter variations is violated it is unknown how the controller will perform, potentially leading to a decrease in performance level and perhaps closed loop instability.

The other extreme, allowing for arbitrary fast parameter variations has the advantage that the performance level is guaranteed for all possible rates of variation. The disadvantage is that the assumption might impose strict requirements on the controller making the local performance poor.

This paper will deal with a controller design with the scheduling parameter limited to a rate of variation between the two extreme values to give an understanding of the gap in performance level between slow and fast parameter variations. In Lescher et al. (2006) a controller design with rate bounded parameter variations is done for a piecewise affine model of a wind turbine using the multi-convexity property for this special case as described in Gahinet et al. (1996). By using multi-convexity there is a risk of conservatism and in this paper an alternative approach is taken by gridding the parameter space.

The linear paramter varying (LPV) controller will be designed to have a level of performance locally at each operating condition which is similar to what can be obtained by an LTI controller designed for the particular operating condition. At the same time the controller must maintain this level of performance even with parameter variations in a specified interval. The design method will be based on a combination of two methods in order to obtain a convex optimisation problem with low complexity and also a numerically stable algorithm for construction of the controller.

In Section 2 the considered control problem will be presented and a controller structure is selected. Then in Section 3 the LPV controller design algorithm is presented followed by a discussion of practical considerations in Section 4. In Section 5 the controller is then designed and simulation results are presented in Section 6 followed by the conclusion in Section 7.

The notation used in the paper is as follows: For real symmetric matrices, M, $M \prec 0$ is interpreted as M being negative definite, i.e. all eigenvalues are negative. In large matrix expressions the symbol $\star$ will denote terms that are induced by symmetry. Let X and Y be symmetric matrices and M and N be non-symmetric matrices then:

$$\begin{bmatrix} X + M + (\star) & \star \\ N & Y \end{bmatrix} := \begin{bmatrix} X + M + M^T & N^T \\ N & Y \end{bmatrix}$$

Also a shorthand for functional dependency will be applied when necessary for notational simplicity. A function $f(a(t), b(t), \ldots)$ will be abbreviated as $f^{a,b,\cdots}$.

2. CONSIDERED CONTROL PROBLEM

The aim is to design a full load controller that limits the drive train oscillations while tracking nominal generator speed and active power. We consider a 3 MW, three-bladed wind turbine with a rotor diameter of 90 m and a doubly-fed induction generator. The wind turbine has three pitch actuators making it possible to change the angle of attack of the blades individually, but in this paper only collective pitch is considered because the objective is regarding drive train oscillations and tracking of speed and power references. Another actuator is the generator reaction torque which can be altered by changing the current in the rotor of the generator.

In full load operation the active power should be kept close to the rated value of 3 MW with a low amount of fluctuations in order not to introduce electrical noise onto the grid. The generator speed must also be kept in the neighbourhood of the rated speed, because the generator and converter system can overheat if the generator speed exceeds the tolerated level. Regarding oscillations, the drive train is lightly damped around 10 rad/s which means that small disturbances at this eigenfrequency will lead to large oscillations and increased fatigue damage. In order to make the wind turbine profitable, means for minimising the drive train oscillations are necessary for the control of modern wind turbines. Finally the heart of the pitch system is a hydraulic actuator that can only be used to deal with the slow disturbances caused by changes in wind speed. A high frequency component in the hydraulic actuator will result in a large wear in the mechanics making it very expensive.

It has been decided to use the generator torque to dampen the drive train oscillations and the pitch system to track the generator speed reference. The reason for this split is that the drive train oscillations occur at a relatively high frequency which will lead to a high pitch activity (and thereby wear) if it is dealt with by the pitch system. The pitch system is on the other hand necessary for controlling the kinetic energy captured by the wind turbine. It has been chosen to use only pitch angle for the speed control because it is more important to limit power fluctuations than error in tracking the generator speed as long as the limits are not exceeded – and the pitch system will deal with the wind induced variations.

The main focus in this paper will be on the LPV speed controller, but for completeness the drive train damper is briefly introduced. The drive train damper is designed by a classical strategy in which a band pass filter containing the drive train eigen frequency is fed back from generator speed to generator torque. The structure of the drive train damper is then as given in (1) in which K is tuned to give a satisfactory tradeoff between drive train oscillations and power fluctuations (control effort).

$$\frac{Q_g(s)}{\omega_g(s)} = \frac{K \cdot s}{(s + \omega_0 - \Delta\omega)(s + \omega_0 + \Delta\omega)} \tag{1}$$

The speed controller is then designed as a tracking controller with integral action. A gain-scheduled LPV controller is chosen for the speed controller in order to handle the nonlinear aerodynamics and further to take into account that more control effort is accepted at lower wind speeds because tracking is harder at these frequencies. The interconnection of the wind turbine model with the controller is then as illustrated in Fig. 1 with the following signal definitions: pitch angle (β), generator speed (ω_g), rotor speed (ω_r), aerodynamic torque (Q_a), and generator torque (Q_g).

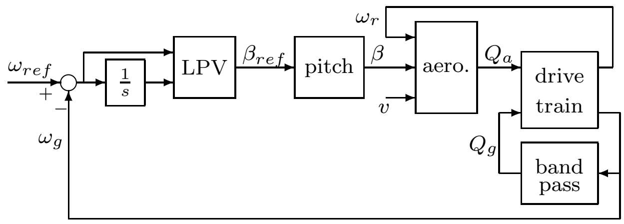

Fig. 1. Block diagram of controller structure.

When designing the LPV controller, the interconnection of the drive train with the damper can now be considered as a first order low pass filter from aerodynamic torque to generator speed and with the rotor speed proportional to the generator speed. The LPV controller can then be designed to trade off the tracking of generator speed with control effort (wear on pitch actuator).

3. LINEAR PARAMETER VARYING CONTROL

After investigations of several different approaches it has been concluded that a "classical" approach for LPV control is beneficial from a numerical point of view. The closed loop performance level γ will be measured by the energy gain (induced $\mathcal{L}_2$ gain) from a specified performance input, $w(t)$, to a chosen performance output, $z(t)$, i.e. measured by $||z(t)||_2 < \gamma \cdot ||w(t)||_2$ for all nonzero inputs with finite energy.

For notational simplicity we will describe the weighted open loop system by (2) and the objective is then to design a controller of the form (3) to satisfy an energy bound γ for the closed loop interconnection (4) with $x_{cl} = [x^T x_c^T]^T$.

$$\begin{bmatrix}\dot{x}(t)\\ z(t)\\ y(t)\end{bmatrix}=\begin{bmatrix}a(v_s(t)) & b_p(v_s(t)) & b(v_s(t))\\ c_p(v_s(t)) & d(v_s(t)) & e(v_s(t))\\ c(v_s(t)) & f(v_s(t)) & 0\end{bmatrix}\begin{bmatrix}x(t)\\ w(t)\\ u(t)\end{bmatrix} \tag{2}$$

$$\begin{bmatrix}\dot{x}_c(t)\\ u(t)\end{bmatrix}=\begin{bmatrix}a_c[v_s(t),\dot{v}_s(t)] & b_c[v_s(t),\dot{v}_s(t)]\\ c_c[v_s(t),\dot{v}_s(t)] & d_c[v_s(t),\dot{v}_s(t)]\end{bmatrix}\begin{bmatrix}x_c(t)\\ y(t)\end{bmatrix} \tag{3}$$

$$\begin{bmatrix}\dot{x}_{cl}(t)\\ z(t)\end{bmatrix}=\begin{bmatrix}A_{cl}[v_s(t),\dot{v}_s(t)] & B_{cl}[v_s(t),\dot{v}_s(t)]\\ C_{cl}[v_s(t),\dot{v}_s(t)] & D_{cl}[v_s(t),\dot{v}_s(t)]\end{bmatrix}\begin{bmatrix}x_{cl}(t)\\ w(t)\end{bmatrix} \tag{4}$$

From dissipativity arguments it is known that the closed loop system is exponentially stable and achieves an energy gain γ if there exist a symmetric, $X_{cl}(v_s(t))$, for which the following two requirements hold: $X_{cl}(v_s(t))$ is positive definite for all possible parameter values, v_s, in the interval from 15 m/s to 25 m/s, and the inequality (5) is satisfied for all possible trajectories of the plant and all possible parameter values in the interval.

$$\frac{d}{dt}x_{cl}(t)^T X_{cl}(v_s(t))x_{cl}(t)+z(t)^T z(t)<\gamma^2 w(t)^T w(t) \tag{5}$$

The inequality (5) can be formulated as an LMI which means that determining the energy gain of a closed loop system can be formulated by a convex optimisation problem. In the case of controller synthesis we plug in the open loop system and controller variables in the analysis formulation, but it turns out to be nonlinear in X_{cl} and the controller variables. In Apkarian and Gahinet (1995) it was shown that the controller variables can be eliminated from the nonlinear matrix inequality. By a partitioning of X_{cl} according to (6) we can formulate the controller synthesis as determining two symmetric matrix functions $X(v_s(t))$ and $Y(v_s(t))$ such that (7) is satisfied for all parameter values in the interval of expected wind speeds and associated rates of variation.

$$X_{cl}^v=\begin{bmatrix}X^v & M^v\\ M^{vT} & \hat{X}^v\end{bmatrix} \quad , \quad X^{v\,-1}_{cl}=\begin{bmatrix}Y^v & N^v\\ N^{vT} & \hat{Y}^v\end{bmatrix} \tag{6}$$

$$\begin{bmatrix}Y^v & I\\ I & X^v\end{bmatrix}\succ 0 \tag{7a}$$

$$\begin{bmatrix}\star\\ \star\\ \star\end{bmatrix}^T\begin{bmatrix}\dot{X}^{v,\dot{v}}+X^v a^v+(\star) & X^v b_p^v & \star\\ \star & -\gamma I & \star\\ c_p^v & d^v & -\gamma I\end{bmatrix}\begin{bmatrix}c_\perp^v & 0\\ f_\perp^v & 0\\ 0 & I\end{bmatrix}\prec 0 \tag{7b}$$

$$\begin{bmatrix}\star\\ \star\\ \star\end{bmatrix}^T\begin{bmatrix}-\dot{Y}^{v,\dot{v}}+a^v Y^v+(\star) & b_p^v & \star\\ \star & -\gamma I & \star\\ c_p^v Y^v & d^v & -\gamma I\end{bmatrix}\begin{bmatrix}b^{vT}_\perp & 0\\ e^{vT}_\perp & 0\\ 0 & I\end{bmatrix}\prec 0 \tag{7c}$$

Alternatively by a congruent transformation similar to what is done in Scherer (1995) and Chilali and Gahinet (1996) we can get an alternative formulation for the synthesis as in (8) with the variables defined as in (9), (10), and (11). This matrix inequality is still nonlinear, however with a suitable variable substitution it can be turned into an LMI.

$$\left[\begin{array}{cc|cc}Q_{11} & \star & L_{11} & L_{12}\\ Q_{21} & Q_{22} & L_{21} & L_{22}\\ \hline \multicolumn{2}{c|}{\star} & \multicolumn{2}{c}{\Delta}\end{array}\right]\prec 0 \tag{8}$$

$$\begin{aligned}Q_{11}=&-\dot{Y}^{v,\dot{v}}+a^v Y^v+b^v {d_c}^{v,\dot{v}}c^v Y^v+\\ &+b^v {c_c}^{v,\dot{v}}N^{vT}+(\star)\end{aligned} \tag{9a}$$

$$\begin{aligned}Q_{22}=&\dot{X}^{v,\dot{v}}+X^v a^v+X^v b^v {d_c}^{v,\dot{v}}c^v+\\ &+M^v {b_c}^{v,\dot{v}}c^v+(\star)\end{aligned} \tag{9b}$$

$$\begin{aligned}Q_{12}=&\dot{X}^{v,\dot{v}}Y+\dot{M}^{v,\dot{v}}N^{vT}+X^v a^v Y^v+\\ &+X^v b^v {c_c}^{v,\dot{v}}N^{vT}+M^v {b_c}^{v,\dot{v}}c^v Y^v+\\ &+M^v {a_c}^{v,\dot{v}}N^{vT}+(a^v+b^v {d_c}^{v,\dot{v}}c^v)^T\end{aligned} \tag{9c}$$

$$L_{11}=b_p^v+b^v {d_c}^{v,\dot{v}}f^v \tag{10a}$$

$$L_{22}=(c_p^v+e^v {d_c}^{v,\dot{v}}c^v)^T \tag{10b}$$

$$L_{21}=X^v b_p^v+X^v b^v {d_c}^{v,\dot{v}}f^v+M^v {b_c}^{v,\dot{v}}f^v \tag{10c}$$

$$L_{12}=(c_p^v Y^v+e^v {d_c}^{v,\dot{v}}c^v Y^v+e^v {c_c}^{v,\dot{v}}N^{vT})^T \tag{10d}$$

$$\Delta=\begin{bmatrix}-\gamma I & (d^v+e^v {d_c}^{v,\dot{v}}f^v)^T\\ d^v+e^v {d_c}^{v,\dot{v}}f^v & -\gamma I\end{bmatrix} \tag{11}$$

In this paper we choose an approach which combines the two methods. Instead of applying a variable substitution, we assume that $X(v_s(t))$ and $Y(v_s(t))$ are known from solving (7). Then we can calculate $M[v_s(t)]$ and $N[v_s(t)]$ from the relation (12) which means that the matrix inequality (8) is an LMI in the variables, $a_c[v_s(t),\dot{v}_s(t)]$, $b_c[v_s(t),\dot{v}_s(t)]$, $c_c[v_s(t),\dot{v}_s(t)]$, and $d_c[v_s(t),\dot{v}_s(t)]$.

$$M[v_s(t)]N[v_s(t)]^T=I-X[v_s(t)]Y[v_s(t)] \tag{12}$$

If we assume that (7) is satisfied then we know from the elimination lemma that there is a $d_c[v_s(t),\dot{v}_s(t)]$ such that $\Delta\prec 0$. It is therefore possible to perform a Schur complement of (8) to arrive at (13). If we then assume that we have determined a $d_c[v_s(t),\dot{v}_s(t)]$ to satisfy $\Delta<0$ it can be observed that the upper left block of (13) is only dependent on $c_c[v_s(t),\dot{v}_s(t)]$ and the lower right block only depends upon $b_c[v_s(t),\dot{v}_s(t)]$. In Gahinet (1996) it is shown for LTI systems that if (7) is satisfied it is always possible to find b_c and c_c to make the two diagonal blocks negative definite and the off-diagonal blocks zero by choosing a_c properly.

$$\begin{bmatrix}Q_{11} & Q_{12}\\ Q_{12}^T & Q_{22}\end{bmatrix}-\begin{bmatrix}L_{11} & L_{12}\\ L_{21} & L_{22}\end{bmatrix}\Delta^{-1}\begin{bmatrix}L_{11} & L_{12}\\ L_{21} & L_{22}\end{bmatrix}^T\prec 0 \tag{13}$$

The same procedure can essentially be applied for LPV systems as argued in Apkarian and Adams (1998). This means that $b_c[v_s(t),\dot{v}_s(t)]$ and $c_c[v_s(t),\dot{v}_s(t)]$ can be determined independently to satisfy (14) and (15), and $a_c[v_s(t),\dot{v}_s(t)]$ can be calculated by solving (16).

$$\begin{aligned}&\begin{bmatrix}\dot{X}^{v,\dot{v}}+X^v(a^v+b^v {d_c}^{v,\dot{v}}c^v)+\star & \star & \star\\ (b_p^v+b^v {d_c}^{v,\dot{v}}f^v)^T X^v & -\gamma I & \star\\ (c_p^v+e^v {d_c}^{v,\dot{v}}c^v) & c_p^v+e^v {d_c}^{v,\dot{v}}c^v & -\gamma I\end{bmatrix}+\\ &+\begin{bmatrix}M^v\\ 0\\ 0\end{bmatrix}{b_c}^{v,\dot{v}}\begin{bmatrix}c^v & f^v & 0\end{bmatrix}+\star\prec 0\end{aligned} \tag{14}$$

$$\begin{aligned}&\begin{bmatrix}-\dot{Y}^{v,\dot{v}}+(a^v+b^v {d_c}^{v,\dot{v}}c^v)Y^v+\star & \star & \star\\ (b_p^v+b^v {d_c}^{v,\dot{v}}f^v)^T & -\gamma I & \star\\ (c_p^v+e^v {d_c}^{v,\dot{v}}c^v)Y^v & c_p^v+e^v {d_c}^{v,\dot{v}}c^v & -\gamma I\end{bmatrix}+\\ &+\begin{bmatrix}b^v\\ e^v\\ 0\end{bmatrix}{c_c}^{v,\dot{v}}\begin{bmatrix}N^T & 0 & 0\end{bmatrix}+\star\prec 0\end{aligned} \tag{15}$$

$$
\begin{aligned}
-M^v {a_c}^{v,\dot{v}} N^T = {} & \dot{X}^{v,\dot{v}} Y^v + \dot{M}^{v,\dot{v}} N^{vT} + \\
& + X^v a^v Y^v + (a^v + b^v d_c^v c^v)^T + \\
& + X^v b^v d_c^v c^v Y^v + X^v b^v {c_c}^{v,\dot{v}} N^{vT} + M^v {b_c}^{v,\dot{v}} c^v Y^v + \\
& + \begin{bmatrix} (X^v b_p^v + \tilde{b}_c^v f^v)^T \\ c_p^v + e^v d_c^v c^v \end{bmatrix}^T \Delta^{-1} \begin{bmatrix} (b_p^v + b^v d_c^v f^v)^T \\ c_p^v Y^v + e^v \tilde{c}_c^v \end{bmatrix} \quad (16)
\end{aligned}
$$

4. PRACTICAL CONSIDERATIONS

In order to design an LPV controller using the approach presented in the previous section it requires solving (7) for infinitely many combinations of parameter values and rates of variation. To handle this we first of all assume that the maximum wind speed acceleration (parameter rate of variation) is similar for all wind speeds in the operating region. This means that the number of synthesis LMIs is reduced to five for each wind speed in the operating region. Note that if this assumption is too restrictive, it can be relaxed by scheduling the worst case acceleration on wind speed.

Still the controller design requires solving infinitely many LMIs because (7) must be solved for all parameter values in the operating region. This issue can be resolved by using a piecewise affine approximation of the model as in Lescher et al. (2006) which uses a method based on multi-convexity, Lim (1998). With this method it suffices to test the vertices of a polytopes containing the parameter region for the finite number of operating regions. The disadvantage is on the other hand that the method has potential conservativeness introduced by the multi-convexity.

Recently an alternative method has been proposed by Wu and Dong (2006) which handles rational parameter dependency. The method is based on linear fractional transformations (LFTs) and under an assumption that the Lyapunov function can be described by a quadratic form of an LFT it is non-conservative. Further it is required only to solve the associated set of LMIs at the vertices of a convex polytope covering the parameter region. This algorithm appears very promising for the application in mind, however it is still very demanding from a numerical (and computational) point of view. It has been decided to use the classical grid based method because it is expected to be approximately two orders of magnitude faster – computational time of approximately two seconds on a standard PC for a similar sized problem. Note that this approximative approach does not give guarantees in between the design points, but it is expected that the inter-grid behaviour can be analysed by testing convergence on an increased grid size.

For the control of wind turbines the effective wind speed is not measurable with adequate precision and it must be estimated as discussed in the introduction. With available methods for estimating the effective wind speed it is not possible to obtain an estimate of the acceleration of the wind field with adequate precision. The controller variables must therefore be made independent on $\dot{v}_s(t)$.

It can be observed that $b_c[v_s(t), \dot{v}_s(t)]$, $c_c[v_s(t), \dot{v}_s(t)]$, and $d_c[v_s(t), \dot{v}_s(t)]$ are independent on the derivative term which means that $a_c[v_s(t), \dot{v}_s(t)]$ is the only controller variable that depends on the time derivative of the effective wind speed.

Note that the construction of $M(v_s(t))$ and $N(v_s(t))$ according to (12) can always be made so that one of the variables is independent of $v_s(t)$. This means that if we require $X(v_s(t))$ to be independent on $v_s(t)$ (i.e. $\dot{X}(v_s(t), \dot{v}_s(t) = 0)$ we can make $a_c[v_s(t), \dot{v}_s(t)]$ independent of $\dot{v}_s(t)$ by choosing $M(v_s(t))$ constant. Furthermore from the properties of the partitioning of $X_{cl}(v_s(t)$ we have that

$$
\dot{X}^{v,\dot{v}} Y^v + \dot{M}^{v,\dot{v}} N^{vT} = -(X^v \dot{Y}^{v,\dot{v}} + M^v \dot{N}^{v,\dot{v}T})
$$

which means that by restricting the controller design to either $X(v_s(t))$ or $Y(v_s(t))$ being constant we can make $a_c[v_s(t), \dot{v}_s(t)]$ independent on $\dot{v}_s(t)$ by choosing respectively $M(v_s(t))$ or $N(v_s(t)$ to be constant.

5. LPV CONTROL OF WIND TURBINES

In this section an LPV speed controller will be designed for the high wind speed region and throughout the design it is assumed that the power and speed rating is well-known. This means that the trajectory of equilibria for the LPV controller design can be determined uniquely from the effective wind speed which can be estimated as in Østergaard et al. (2007). We will consider a controller design for the interval of wind speeds from 15 m/s to 25 m/s.

The open loop for the controller design is determined by interconnecting the components in Fig. 1 in which the pitch system is described as a second order model from pitch reference to pitch angle and the interconnection of drive train with the damper is approximated by a first order model. The aerodynamics are assumed static nonlinear functions that are linearised along the trajectory of equilibria.

For the controller design we wish to reduce the sensitivity on wind speed variations in the tracking of generator speed while keeping the pitch activity low. The performance inputs and outputs will be scaled appropriately over frequency to to give a reasonable tradeoffs over frequency. To make high frequency components in the pitch reference more "costly" than low frequency components, a high pass filter is included in the weight for the pitch reference. This means that the weighted performance inputs and outputs can be described as

$$
w(t) = v_s(t)
$$

$$
z(t) = \begin{bmatrix} W_\omega(v_s) \displaystyle\int_0^t \omega_{ref}(\tau) - \omega_g(\tau) d\tau \\ W_\beta(v_s) \left(\dfrac{T(v_s)s + 1}{\epsilon s + 1} \right)^3 \beta_{ref}(t) \end{bmatrix}
$$

with $W_\omega(v_s(t))$, $W_\omega(v_s(t))$ being scalings that are gain scheduled on wind speed and $T(v_s(t))$ is the time constant in the high pass filter which is also gain scheduled on wind speed. The parameter values for the weights have been chosen in an iterative procedure and are illustrated in Fig. 2. In this figure it can be seen that the actuator is most expensive at high wind speeds and that the focus on tracking performance is highest in the mid wind speed range, because this range of wind speeds is the region where it is most difficult to maintain the generator speed in the tolerated range. To make the synthesis procedure applicable to practical computation of controllers and to simplify the tuning of the weights it has been decided to limit the synthesis to only four grid points: 15, 18, 21, and 25 m/s.

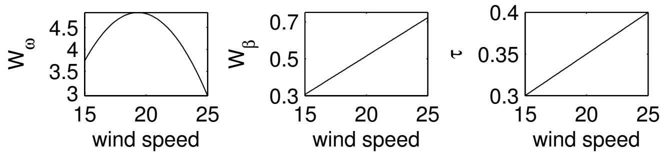

Fig. 2. Illustration of scheduled variables for the performance weights.

For the construction of suitable scalings, $X(v_s(t))$ and $Y(v_s(t))$, polynomial scalings have been investigated, i.e.

$$X(v_s(t)) = \sum_i X_i v_s(t)^i \quad \text{and} \quad Y(v_s(t)) = Y_0$$

or

$$X(v_s(t)) = X_0 \quad \text{and} \quad Y(v_s(t)) = \sum_i Y_i v_s(t)^i \quad .$$

Then to identify the size of polynomial expansion and which of the two variables that should be parameter dependent, a comparison is made with LTI synthesis (using `hinflmi`) at the chosen grid points. LPV controllers have been designed with $\dot{v}_s(t) = 0$ for three different choices of polynomial expansion:

LPV Y1: $X(v_s(t)) = X_0$, $Y(v_s(t)) = Y_0 + v_s(t)Y_1$.
LPV Y2: $X(v_s(t)) = X_0$, $Y(v_s(t)) = Y_0 + v_s(t)Y_1 + v_s(t)^2 Y_2$.
LPV X: $X(v_s(t)) = X_0 + v_s(t)X_1 + v_s(t)^2 X_2$, $Y(v_s(t)) = Y_0$.

The $\mathcal{H}_\infty$ norm of the weighted closed loop has then been calculated for the LTI controller and each of the three LPV controllers at the design points with a comparison given in Table 1. From this comparison it can be concluded that for the particular application it is advantageous to use $Y(v_s(t))$ as the parameter dependent variable and to use a second order approximation.

Table 1. Comparison of $\mathcal{H}_\infty$ synthesis and the closed loop with three different LPV controllers with zero rate of variation.

parm.	H_∞	LPV Y1	LPV Y2	LPV X
15 m/s	0.9998	1.1467	1.0031	1.5456
18 m/s	1.0012	1.2193	1.0016	1.2332
21 m/s	0.9991	1.2125	1.0035	1.1329
25 m/s	1.0012	1.0282	1.0052	1.8139

With the choice of weights and basis functions for $X(v_s(t))$ and $Y(v_s(t))$ in place it is now possible to design the controller with rate bounded parameter variations. Such a design has been done for a number of possible values of rate variation and in Fig. 3 the performance level is illustrated as a function of rate of variation. From the figure it can be seen that the performance level remains almost unchanged until a rate of variation of 0.1 m/s^2 where it starts decreasing slightly. Then in the interval from 1 m/s^2 to 100 m/s^2 it decreases rapidly until it is close to the upper limit (approximately 50% reduction in performance level) given by synthesis with arbitrary fast parameter variations.

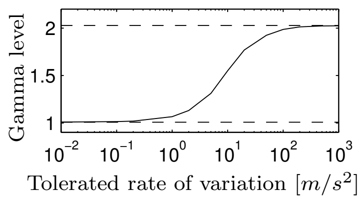

Fig. 3. Guaranteed $\mathcal{L}_2$ gain as a function of tolerated parameter rate of variation. Dashed lines indicate level for zero and arbitrary fast rate of variation.

From Fig. 3 it can be seen that it is quite inexpensive from a local performance point of view to use 1 m/s^2 as the upper limit on parameter rate of variation, which means that the local performance level is decrease by no more than 10% when comparing with LTI controllers for the specific operating point. Furthermore it is expected that the gain scheduling variable will have faster rate of variation only in extreme operating conditions which will be handled by dedicated control algorithms. It has therefore been decided to focus on the LPV controller design with a parameter rate of variation of 1 m/s^2.

6. SIMULATION RESULTS

The chosen controller has been tested in a simulation environment including tower fore-aft and sideways movement, a third order drive train, and nonlinear pitch and generator models. As wind input it has been decided to use wind specifications according to IEC (2005) with 10 minute mean wind speeds in the interval from 15 to 25 m/s. A snapshot of a simulation result is given in Fig. 4 in which the mean wind speed decreases gradually from 23 m/s to 17 m/s. For illustration of the issues with the LTI controller a comparison is made with an LTI controller designed for 23 m/s and it can be observed from Fig. 4 that the LTI and LPV controller have similar performance for the higher wind speeds whereas the performance decreases significantly (close to instability) for the lower wind speed interval.

To understand the performance of the LPV controller from a simulations point of view, local simulations have been performed at a number of operating conditions with comparison to LTI controllers for the particular operating condition. A result is given in Table 2 in which the columns represent respectively: rainflow[1] count for damage on drive train, variations in generator speed, pitch activity, and power fluctuations. The values are shown for the LPV controller relative to the LTI controllers for the particular mean wind speed. From the table it can be observed that the LPV controller is slightly more aggressive in higher wind speeds when comparing with the LTI controller. The reason for this is that with the tolerated rate of variation included in the design, the controllers will be slightly similar over the operating condition. Still it is concluded that the variation from the local design is small enough to conclude that the controller design is successful.

[1] Rainflow counting is a simulation based or experimental method for evaluating the structural damage, Matsuiski and Endo (1969).

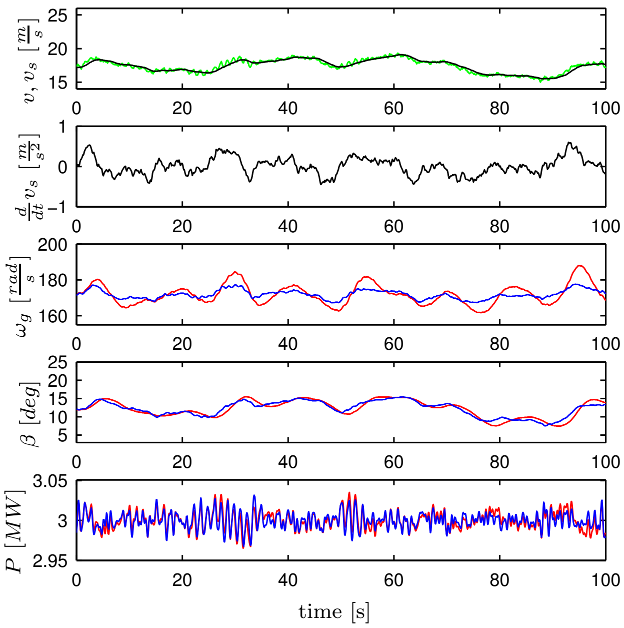

Fig. 4. Simulation results with LPV controller with rate of variation up to 0.1 m/s. Green: spatial average of wind speed. Black: scheduling variable. Blue: LPV controller. Red: LTI controller.

Table 2. Performance outputs of the LPV controller measured relative to the LTI controllers.

mean wind	RFC. drt.	gen. spd.	pitch	P std.
15 m/s	101 %	104 %	96 %	99 %
18 m/s	97 %	92 %	101 %	100 %
21 m/s	97 %	94 %	100 %	100 %
25 m/s	97 %	90 %	104 %	100 %

7. CONCLUSIONS

In this paper an LPV controller controller has been designed for the control of wind turbines in full load operation. The design method combines the benefits of two algorithms in the literature. First the two scheduled functions X and Y are determined to give an optimal performance level γ. This is done on the basis of a method eliminating the controller variables which has an advantage in terms of computational complexity in the associated convex optimisation problem. Then the controller variables are determined by solving a set of LMIs without the need for a reconstruction of the "storage" function, X_{cl}, for the closed loop. This is done by relating the result of the optimisation problem with a method that does not eliminate the controller variables and therefore has an advantage in the construction of the controller.

The controller synthesis shows that the local performance of an LTI controller can approximately be obtained with LPV controller design for the entire operating region with a rate of variation up to 0.1 m/s^2. It has been estimated that 1 m/s^2 is a suitable worst case for the tolerated rate of variation. For this case the performance level is locally decreased by no more than 10% for all operating conditions when comparing with LTI controllers designed for each operating condition.

The selected LPV controller has been simulated on a higher order simulation model and a comparison has been made to a set of local LTI controllers. From this comparison it can be seen that the performance levels of the LPV controller and LTI controllers are very similar for each investigated operating conditions.

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