

## Robust Model Predictive Control of a Diesel Engine Airpath

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**Abstract:** Model predictive control represents one of the most promising methods, also in fast industrial applications like a Diesel engine. But especially Diesel engine control meets problems of uncertainties and disturbances, thus model-plant mismatch is omnipresent, which obviously decreases the performance of model based control. This paper compares different robust predictive control strategies applied to a Diesel engine airpath.

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### 1. INTRODUCTION

Pollutant emissions of engines are the effect of an imperfect combustion, in particular during transients, and can be reduced significantly by an aftertreatment system. In the case of Diesel engines, such a system has evolved from the simple oxidation catalyst to more complex structures, *e.g.* combining an oxidation catalyst, Diesel particulate filter (DPF) and selective catalytic reduction (SCR).

Of course, it would be better to reduce the very production of pollutants in the combustion process, and improvement in this direction can be achieved both by acting on the quantity and delivery of the reactants – for instance by multiple injection (Boulouchos, *et al.*, 2000) – as well as on the combustion conditions. Also the delivery of the mixture of fresh and recirculated combusted air to the combustion process has a paramount importance and has received much attention (see *e.g.* (Nieuwstadt, *et al.*, 2000)). As the control of the single components and condition of the gas mixture is hardly possible, intermediate variables are used, typically boost pressure and fresh air-mass flow, which traditionally have proven sufficiently representative of the composition of the combustion gas, at least during steady state operation. To improve the results, other references have been tested, as in (Stefanopoulou, *et al.*, 2000) where burned gas fraction and air fuel ratio (AFR) have been used, in combination with different control strategies, or air-fuel ratio and EGR rate (Darlington, *et al.*, 2006) or boost pressure and intake charge oxygen (Nakayama, *et al.*, 2003).

While the choice of the intermediate quantities plays a critical role, also the control structure has been found out to be a key factor. Besides the control approaches included in the papers quoted above, (Ortner and del Re, 2007) has shown that model predictive control (MPC) based on a multi-linear approximation can be used to successfully control the flow and boost pressure of Diesel engine thus leading to a reduction of emissions, while (Langthaler and del Re, 2007)

have shown that MPC can be used also to control intake charge oxygen, thus allowing to some extent a combustion oriented control. Summarizing, MPC, in particular using models identified from measurements, seems a very promising technique for engine control. Up to now, MPC is possible only using linear models or very specific nonlinear ones. The approximation errors introduced by a linear or multi-linear representation add to the parameter uncertainties. All this implies that MPC must be designed taking explicitly in account the model uncertainty.

In this paper different aspects for model-plant mismatches are discussed and a closer look at different predictive control concepts is performed, comparing the nominal MPC augmented with a mixed disturbance model, against robust model predictive control (RMPC) schemes at different plant simulations.

### 2. THE PLANT AND ITS MODELLING

#### 2.1 The Plant

In Fig. 1 a representation of the Diesel engine is shown. The variable geometry turbocharger (VGT) (with control input  $u_{VGT}$  and position  $x_{VGT}$ ) uses the thermal energy of exhaust gases and loads the combustion chamber with higher pressure  $p_i$  than the ambient, one to allow injecting more fuel mass  $W_f$  without generating extensive smoke. This allows generating more torque  $M$  – or in different words – through this option it is possible to construct a smaller engine producing the same torque – so called “downsizing”. The second actuator of the airpath, the exhaust gas recirculation (EGR) valve (with input  $u_{EGR}$  and position  $x_{EGR}$ ), controls the amount of recirculated emissions. Feeding back already combusted air  $W_{xi}$  decreases the amount of oxygen  $O_{2i}$  and increases the thermal capacity of the combustion gases, lowering the combustion temperature and  $NO_x$  formation.

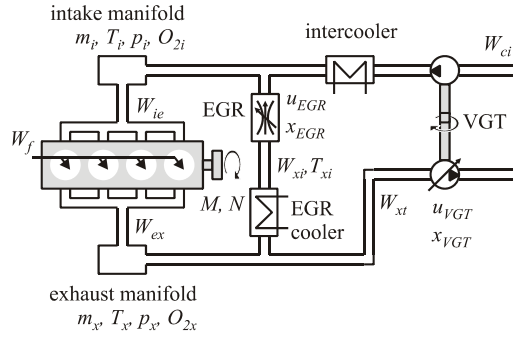


Fig. 1. Scheme of a Diesel engine

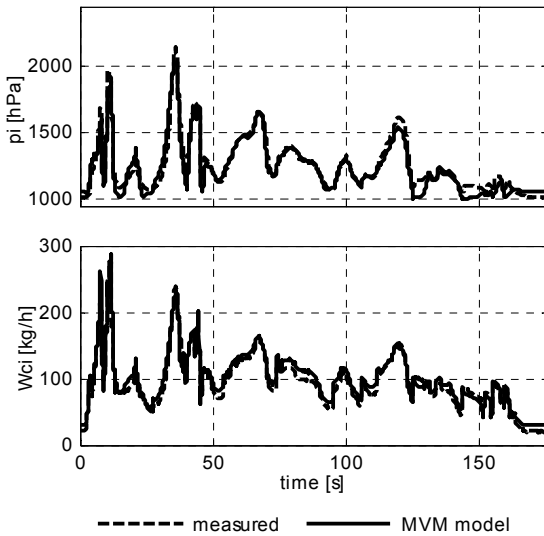


Fig. 2. Validation of the Mean Value Model

### 2.2 Mean Value Model

The behaviour of an engine can be represented with different kinds of models, in particular in function of the crankshaft angle or of time. Mean value models (MVM) are the most popular modeling technique for airpath control and simulation, and are expressed in function of time. The parameters of such nonlinear physically motivated models are typically derived by production data, analysis of components or specific measurements.

Often, steady state measurements are performed in a first step using optimization techniques to determine the parameters, for instance by nonlinear dynamic optimization via multiple shooting as in (Ferreau, *et al.*, 2006b), where also the system equations are shown. A typical approximation quality is shown by the validation measurements of Fig. 2 tested on a standard FTP driving cycle.

### 2.3 Databased Models

Standard MPC relies on linear models (Bemporad *et al.* 2002) have proposed the so called explicit approach to allow higher sampling frequencies. Because of the offline calculation of

the explicit MPC, an online linearization of the nonlinear model is not possible. So a linear identification in pre-defined different operating regions  $\mathbb{L}_p$  has been used, therefore a grid of the plants operation region is made in the input space of  $w$  and  $u$

$$u = [u_{EGR} \quad u_{VGT}]^T \quad w = [N \quad W_f]^T \quad (1)$$

where the inputs  $u_{EGR}$  and  $u_{VGT}$  are expressed in %, while  $N$  and  $W_f$  in *rpm* and *mg/st* disturbances:

$$\begin{aligned} \mathbb{L}_1 : [5 \quad 50 \quad 1500 \quad 0] \leq (u, w) \leq [25 \quad 90 \quad 2000 \quad 20]^T \\ \mathbb{L}_2 : [5 \quad 20 \quad 1500 \quad 0] \leq (u, w) \leq [30 \quad 60 \quad 2000 \quad 20]^T \\ \mathbb{L}_3 : [5 \quad 50 \quad 1500 \quad 20] \leq (u, w) \leq [25 \quad 90 \quad 2000 \quad 35]^T \\ \mathbb{L}_4 : [5 \quad 20 \quad 1500 \quad 20] \leq (u, w) \leq [30 \quad 60 \quad 2000 \quad 35]^T \end{aligned} \quad (2)$$

In previous works (Langthaler and del Re, 2007) and (Ortner and del Re, 2007), the scheduling has been performed in the full engine space. In this paper only a representative area is selected (2), but the result could be extended easily to the full space.

Using prediction error identification (Ljung, 1999) we obtain a model of the form

$$\begin{aligned} x_{t+1} &= A_p x_t + \begin{bmatrix} B_p & E_p \end{bmatrix} \begin{bmatrix} u_t \\ w_t \end{bmatrix} \\ y_t &= C_p x_t \quad \text{with } y = [p_i \quad W_{ci}]^T \end{aligned} \quad (3)$$

for every region  $\mathbb{L}_p$ . Fig. 3 shows a validation in region  $\mathbb{L}_1$  of a model identified in  $\mathbb{L}_1$ . The prediction model for MPC

$$\begin{aligned} X &= \mathbf{A}_p x_{t|t} + \mathbf{B}_p U \\ Y &= \mathbf{C}_p X \end{aligned} \quad (4)$$

can be simply derived through combination of the vectors  $u, x$  and  $y$

$$\begin{aligned} U &= [u_{t|t}^T \quad u_{t+1|t}^T \quad \dots \quad u_{t+N-1|t}^T]^T \\ X &= [x_{t|t}^T \quad x_{t+1|t}^T \quad \dots \quad x_{t+N-1|t}^T]^T \\ Y &= [y_{t|t}^T \quad y_{t+1|t}^T \quad \dots \quad y_{t+N-1|t}^T]^T \end{aligned} \quad (5)$$

and the system matrices

$$\begin{aligned} \mathbf{A}_p &= \left( I, (A_p)^T, \dots, (A_p^{N-1})^T \right)^T \\ \mathbf{C}_p &= \text{diag}(C_p, C_p, \dots, C_p) \\ \mathbf{B}_p &= \begin{pmatrix} 0 & 0 & 0 & \dots & 0 \\ B_p & 0 & 0 & \dots & 0 \\ A_p B_p & B_p & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ A_p^{N-2} B_p & \dots & A_p B_p & B_p & 0 \end{pmatrix} \end{aligned} \quad (6)$$

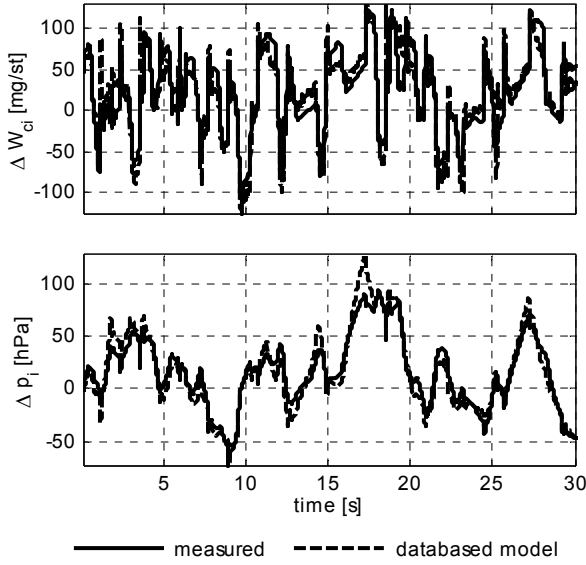


Fig. 3. Interpolation - Validation of databased model

### 3. UNCERTAINTY SOURCES

Fig. 4 shows an example of the plant model mismatches.

Plant-(control) model mismatches can have very different causes. In the case of linear MPC, three main causes are especially important.

#### 3.1 Model Uncertainties

The linear approximation of a nonlinear plant causes mainly bias in the model. Further, the limited amount of data, and the experiment design for identification itself causes some bias and variance error (Ljung, 1999). Those two errors already occur in a very limited region around the “nominal” identification point - Fig. 4.

#### 3.2 Scheduling Mismatch

Due to the very large time scale differences usually some quantities like fuel mass and engine speed are included as measurable disturbance (Langthaler and del Re, 2007). Additionally those disturbances are used for model and respectively control selection. All the models are identified around equilibrium points, such that all states (pressures and temperatures) are in a specific range. These identified models are further used for control design. During the control process, the scheduler selects the MPC (based on the according model) belonging to the actual fuel/ speed point, even though the state of the model can evolve over the boundaries of this model over the prediction horizon.

#### 3.3 Unmodelled Disturbances

The plant model does not map thermal changes of the engine nor its wearing process. Also the sub-control of the airpath

does not consider different superposed ECU (input) strategies e.g. if a particulate filter is regenerated, which causes high temperature combustion.

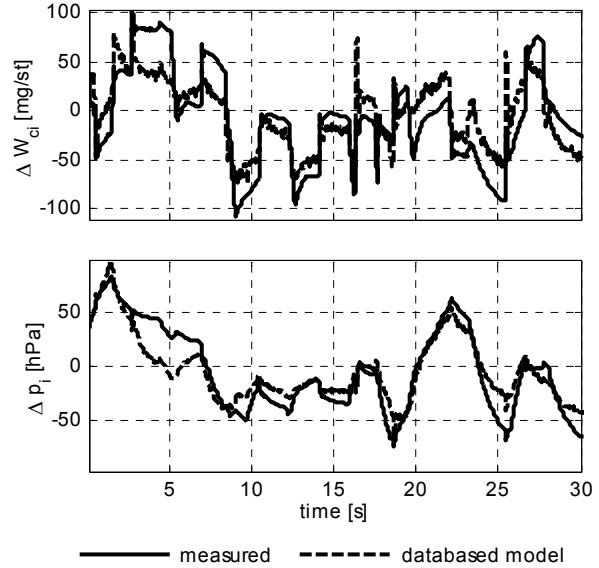


Fig. 4. Extrapolation - Validation of the databased model

### 4. MODEL PREDICTIVE CONTROL STRATEGIES

#### 4.1 Nominal MPC with (mixed) disturbance Model (MD-MPC)

State of the art in offset-free MPC is the extension of the model with an augmented constant output disturbance which can reject the disturbance. But it is also possible to use formulations where the disturbance enters the plant in input, output, state or a combination mixed disturbance models. For all of those model types, properties for detectability are given in (Muske and Badgwellb, 2002). An alternative disturbance modeling scheme is shown in (Pannocchia and Rawlings, 2001) where a mixed model is used for input and output disturbance where the disturbance state is estimated via a standard Kalman filter using such an augmentation of the system and the prediction model (4) it is possible to minimize cost (7) under constraints

$$\begin{aligned} \min_U & \|Y\|_Q^2 + \|\Delta U\|_R^2 \\ \text{s.t.} & \Delta U \in \Delta \mathbb{U}^{N \cdot Nu} \subset \mathbb{R}^{N \cdot Nu} \\ & U \in \mathbb{U}^{N \cdot Nu} \subset \mathbb{R}^{N \cdot Nu} \\ & X \in \mathbb{X}^{N \cdot Nx} \subset \mathbb{R}^{N \cdot Nx} \end{aligned} \quad (7)$$

where  $\|(\cdot)\|_S^2$  states the Euclidean-norm with weighting  $S$  and prediction horizon  $N$ . The low number of states and limitations allows the use an explicit expression of MPC (Bemporad, *et al.*, 2002) so it is possible to determine piecewise affine control law  $\Delta u(t) = F(x(t), u(t-1), r(t))$  which is calculated by the MPT toolbox (Kvasnica, *et al.*, 2004). Note that in (Ferreau, *et al.*, 2006a) an online active

set strategy for QP is shown, which allows real-time solutions also on more complex QP problems.

#### 4.2 Robust Model Formulation

Due to the identification at different regions  $\mathbb{L}_p$ , one derives a polytopic error description for different EGR, VGT and fuel settings for a certain region of speed

$$\Omega = \text{Co}\{[A_1 \ B_1], [A_2 \ B_2], \dots, [A_N \ B_N]\} \quad (8)$$

where Co defines the convex hull of the matrices  $[A_p \ B_p]$ .

For the applied control formulation applied in the next section, (8) has to be replaced by the following formulation:

$$\Sigma_p \begin{cases} x_{t+1} = A_p x_t + B_p u_t + E w_t & w_t \in \mathbb{W} \\ y_t = C_p x_t \end{cases} \quad (9)$$

where the nominal model is set to  $p=1$ . For a representation with additive disturbance, a simple estimation of the disturbance set  $\mathbb{W}$  has to be performed:

The state update difference at time  $t$  can be written as

$$w_t = [A_2 \ B_2] \begin{bmatrix} x_t \\ u_t \end{bmatrix} - [A_1 \ B_1] \begin{bmatrix} x_t \\ u_t \end{bmatrix} \quad (10)$$

Knowing the polytopes of input  $u \in \mathbb{U}$  and states  $x \in \mathbb{X}$  the maximal achievable polytope for the disturbance  $w \in \mathbb{W}$  where  $\mathbb{W} : \underline{w}_t \leq w_t \leq \bar{w}_t$  can be calculated straight forward for all combinations  $\{(\Sigma_1, \Sigma_2), (\Sigma_1, \Sigma_3), (\Sigma_1, \Sigma_4)\}$ .

In our case the maximal error input has been determined by  $\underline{w}_t = -[250 \ 30]^T$ ,  $\bar{w}_t = [250 \ 30]^T$ .

#### 4.3 Robust Open Loop MPC (OL-RMPC)

It is possible to approximate the original min-max problem

$$\min_{\Delta U} \max_W \|\mathbf{Y}\|_Q^1 + \|\Delta U\|_R^1 \quad (11)$$

which minimizes the cost on the maximal disturbed plant by semi definite relaxation (Löfberg, 2003a) which yields to optimization problem (12)

$$\begin{aligned} & \min_{\Delta U, t} t \\ & \text{s.t. } \|\mathbf{Y}\|_Q^1 + \|\Delta U\|_R^1 \leq t \\ & \Delta U \in \Delta U^{N \cdot Nu} \subset \mathbb{R}^{N \cdot Nu}, \forall W \in \mathbb{W}^{N \cdot Nx} \subset \mathbb{R}^{N \cdot Nx} \\ & U \in \mathbb{U}^{N \cdot Nu} \subset \mathbb{R}^{N \cdot Nu}, \forall W \in \mathbb{W}^{N \cdot Nx} \subset \mathbb{R}^{N \cdot Nx} \\ & X \in \mathbb{X}^{N \cdot Nx} \subset \mathbb{R}^{N \cdot Nx}, \forall W \in \mathbb{W}^{N \cdot Nx} \subset \mathbb{R}^{N \cdot Nx} \end{aligned} \quad (12)$$

with constraints, where  $\|(\cdot)\|_S^1$  states the one-norm with weighting  $S$  and the prediction model:

$$\begin{aligned} X &= \mathbf{A}x_{t|t} + \mathbf{B}\Delta U + \mathbf{E}W \\ Y &= \mathbf{C}X \end{aligned} \quad (13)$$

This model is just an augmentation of (4) with the disturbance input  $W$ . The robust optimization problem (12) (Ben-Tal and Nemerovskii, 2002) is solved by the YALMIP toolbox (Löfberg, 2004) which offers a very comfortable environment and fast solver to derive the optimal solution.

Simulation tests have shown that a shrinking of  $\mathbb{W}$  (derived in section 4.2) is possible. So  $\underline{w}_t, \bar{w}_t$  have been set to  $+/-[100 \ 100]^T$ .

#### 4.4 Robust Closed Loop Approximation RMPC (CLA-RMPC)

Löfberg suggests in (Löfberg, 2003b) an approximation approach to closed loop predictive control (called here CLA-RMPC). Instead of using feedback predictions he suggests a parameterization which feeds back the disturbance with matrix  $\alpha$

$$\Delta U = \alpha W + V \quad (14)$$

the new plant description is

$$\begin{aligned} X &= \mathbf{A}x_{t|t} + \mathbf{B}V + (\mathbf{E} + \mathbf{B}\alpha)W \\ Y &= \mathbf{C}X \\ \Delta U &= \alpha W + V \end{aligned} \quad (15)$$

For details the reader is referred to (Löfberg, 2003b).

## 5. SIMULATION RESULTS

The following simulation has been carried out on the MVM whereas all presented control schemes do not use measurable inputs of fuel and speed. The other settings can be found in table 1 where the MPCs have been tuned such that their responses are comparable despite different norms and prediction lengths. Three different cases shall point out the characteristics of the robust control schemes, where the nominal model is identified in  $\mathbb{L}_1$ :

Case 1 - step in fuel, constant fresh air-mass and boost pressure: this case does not represent a situation, an engineer would expect for a standard airpath control – because typically setpoints vary with fuel and speed. But this case shows the disturbance rejection of each controller type. The step in fuel moves the plant through polytopes  $\mathbb{L}_1, \mathbb{L}_2 \dots \mathbb{L}_4$ .

Case 2 - constant fuel and fresh air-mass, step in boost pressure: this experiment checks if the control is working near its nominal region correctly. Hereby the plant remains in polytopes  $\mathbb{L}_1$  and  $\mathbb{L}_2$ . This case should show the transient behaviour near the nominal point of the plant.

Case 3 - step in fuel, fresh air-mass, boost pressure: this step in fuel and reference values states the standard case where all variables change.

**Table 1. MPC Settings**

	MD-MPC	OL-RMPC	CLA-RMPC
<b>Horizon <math>N</math></b>	50	20	20
<b>Horizon <math>N_c</math></b>	2	2	2
<b>Weight. <math>R</math></b>	diag(0.1,0.1)	diag(0.3,0.3)	diag(0.05,0.05)
<b>Weight. <math>Q</math></b>	diag(10,20)	diag(1, 1)	diag(1, 1)
<b>Const. <math>u</math></b>	yes	yes	yes
<b>Const. <math>\Delta u</math></b>	yes	yes	yes
$\underline{w}_t, \bar{w}_t$	-	-100,100	-100,100

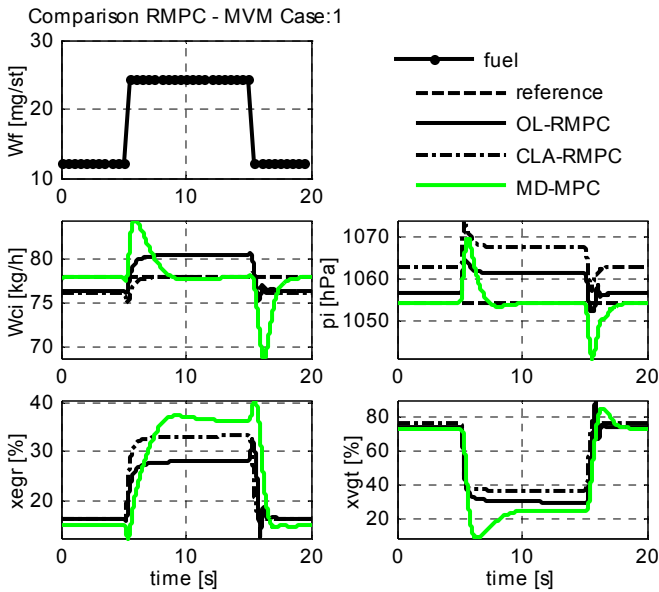


Fig. 5. Comparison of RMPC - step in fuel, constant fresh air-mass and boost pressure

Fig. 5 shows the case of input disturbance. Remarkable is the moderate reaction of RMPCs compared to the MD-MPC at  $t=5$  seconds. The MD-MPC fully closes the EGR producing a strong overshoot in  $W_{ci}$ . The RMPCs cause no overshoot on the one hand, but produce (relatively small) offsets, on the other hand. Further the behaviour of CLA-RMPC seems to be little more conservative than of OL-RMPC. But every MPC control is able to fulfil its control task correctly. Tracking without disturbance in Fig. 6 is again performed well by all the controls. But this time – if the steady state offset of RMPC is neglected – the RMPC tracks about one second faster than the MD-MPC. Comparing MD-MPC and RMPCs are compute very similar trajectories but the MD-MPC generates a totally different EGR input at  $t=5$ s.

In Fig. 7 the RMPCs show their best performance. They quickly stabilizing the plant without large overshoots, better than the MD-MPC (also with measured disturbance model). Hereby both RMPCs actuate very similar.

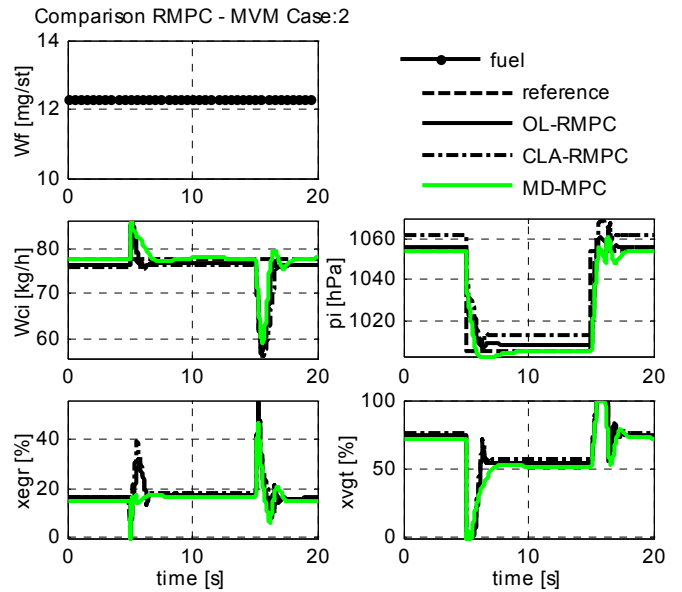


Fig. 6. Comparison of RMPC - constant fuel and fresh air-mass, step in boost pressure

## 6. CONCLUSIONS

It has been shown that different formulations of robust and robustified MPC can solve the problem of airpath control considering constraints in the inputs and rate limits. The solution under output constraints (which are relatively close to the references) causes infeasibility problems and cannot be tackled at least in the OL-RMPC. By its nature, the CLA-RMPC formulation softens the feasibility problem a little, but has much higher complexity and so computational burden than the OL-RMPC (table 2).

In this application the long prediction horizon conflicts with the feasibility of the optimization problem. On the other hand using less prediction steps and a larger sample time decreases the performance of the RMPC significantly. Additionally the long prediction horizon makes the transformation to a multiparametric problem (MP) of RMPC (Bemporad, *et al.*, 2003) with the used methods hardly computable (number of critical regions in table 2). At this time, the computational time of the MD-MPC formulation, with measured disturbances, is significantly lower.

One side effect of RMPC can be found by the fact that this kind of control is not offset free. In classical engine control strategies, often offsets to the reference values are allowed. This prevents from oscillations around reference values which would decrease the comfort of the vehicle. So, the RMPC technique represents a more sophisticated strategy to the standard engine control strategies which are tuned by application engineers.

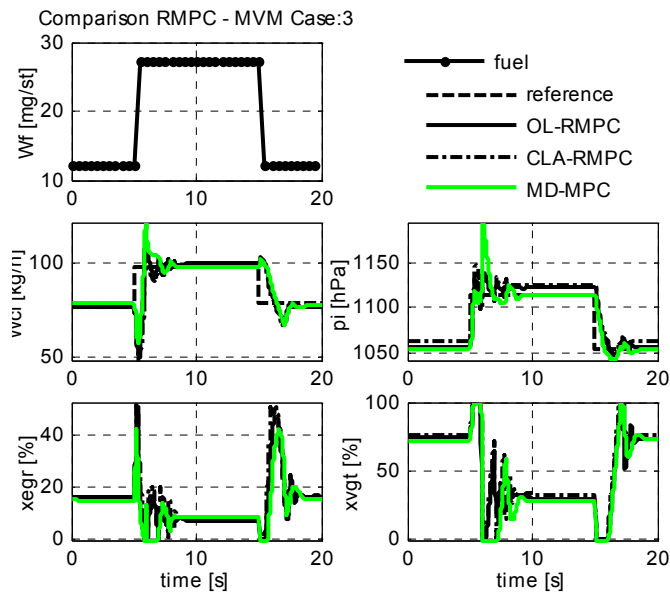


Fig. 7. Comparison of RMPC - step in fuel, fresh air-mass, boost pressure

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Table 2. MPC Control Comparison

	MD-MPC	OL-RMPC	CLA-RMPC
offset free	yes	no	no
critical regions	356	1803 (N=5)	-
tracking	fast	very fast	very fast
disturbance rejection	fast	fast	fast
exec. time [s]	2e-4	0.3	1.97

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