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# Trajectory tracking control of Skid-Steering Robot – experimental validation ⋆

**D. Pazderski** * **K. Kozłowski** **

* *Chair of Control and System Engineering, Poznan University of Technology, ul. Piotrowo 3a, 60-965 Poznan, Poland (e-mail: Dariusz.Pazderski@put.poznan.pl).*
** *Chair of Control and System Engineering, Poznan University of Technology, ul. Piotrowo 3a, 60-965 Poznan, Poland (e-mail: Krzysztof.Kozlowski@put.poznan.pl).*

**Abstract:** In this paper authors consider the problem of practical stabilization of wheeled mobile robot equipped with skid-steering drive (also know as SSMR). The kinematic model of SSMR is approximated by kinematics of unicycle including small perturbation term which describes limited skidding effect. It is justified that SSMR can be regarded as a system with non-stationary first order nonholonomic constraint. Based on this result smooth control scheme robust to limited skidding is developed. The control law ensures practical stabilization in regulation and trajectory tracking case, i.e. position and orientation errors are bounded to the assumed but nonzero values. The effectiveness of control solution is justified and illustrated by experimental results.

## 1. INTRODUCTION

In robotic applications nonholonomic systems play very important role. This is due to the fact that the most vehicles used for transportation tasks are subjected to non-integrable velocity or acceleration constraints. Hence, from a mechanical point of view, we can distinguish systems with nonintegrable kinematics or nonintegrable dynamics. In the first case it is common to say that such systems are nonholonomic while in the second case the name "under-actuted systems" is usually used. However it is justified to consider systems with nonintegrable dynamics as the second order nonholonomic systems.

Taking into account mobile robots equipped with wheels usually velocity constraints play significant role. This is a result of limited slippage between wheels and surface (see for example Campion et al. [1996]) which is observed during normal operation (i.e. assuming particular kinematic condition of motion). Then robots with unicycle-like or car-like structure can be considered as kinematic systems only.

However, there exist overconstrained robots with multi wheels for which no-slipping assumption in general is not justified. The example is Skid-Steering Mobile Robot (SSMR) with differential drive mechanism. Observing motion of such robot one can conclude that in spite of slipping and skidding its motion properties are quite similar to the first order nonholonomic robot equipped with two wheels (i.e. unicycle-like robot).

From a theoretical point of view SSMR is indeed the second order nonholonomic system and it cannot be reduced to the smooth kinematic system without losing knowledge of admissible trajectories (see for example Lewis [1999]). Some reduction is possible but it in general leads to kinematic system with changed structure which was considered by Murphey [2006].

In this paper it is formally shown that kinematics of SSMR can be approximated by kinematics of unicycle-like robot. Such problem have not been properly addressed in the robotics literature. Previously, in some papers (see Caracciolo et al. [1999] and Kozłowski and Pazderski [2004]) for control purposes authors assumed an ideal nonholonomic constraint and used dynamic model of SSMR with Lagrange multipliers which cannot be justified taking into account physical properties of the system.

Here we show that it is possible to consider SSMR at kinematic level assuming non-stationary nonholonomic constraint. In the case of slow motion one can obtain simple approximation of admissible trajectories.

Similarity between SSMR and unicycle kinematics gives possibility to use analogous control solution in both cases. In this paper in order to solve trajectory tracking problem for SSMR we propose kinematic control law based on tunable oscillator (see Dixon et al. [1999]) and transverse functions (see Morin and Samson [2003]) which is robust to bounded lateral skidding. Taking into account the lateral dynamics we give a condition of stable motion with respect to position of instantaneous center of rotation.

Next, we consider control problem assuming that linear and angular velocities can be treated as auxiliary control inputs and neglect the task of enforcing these velocities by actuators. The extension to the dynamic level is possible via backstepping technique and both adaptive and robust control schemes can be relatively easy realized in real applications.

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The proposed control solution is verified experimentally in trajectory tracking case using four-wheeled SSMR robot built in our laboratory (named MMS). According to authors' knowledge such research with respect to SSMRs is not met in the literature.

The paper is organized as follows. In Section II kinematic and dynamic model of SSMR is presented and approximation of admissible trajectories of SSMR at kinematic level is discussed. In the next section the control law using tunable oscillator is developed with respect to limited lateral skidding velocity. In Section IV experimental results are presented. Concluding remarks are given in Section V.

## 2. SSMR MODEL

### 2.1 Kinematics

In this paper we consider an example of SSMR equipped with Four Wheel Drive (4-WD) moving on the plane (Fig. 1) in the inertial frame $X_gY_g$. A local frame $x_ly_l$ is attached to its center of mass (COM). Let $\boldsymbol{q} = [q_1\ q_2\ q_3]^T \triangleq [\theta\ X\ Y]^T \in \mathbb{S}^1 \times \mathbb{R}^2$ denote generalized coordinates describing robot's position, $X$ and $Y$, in the inertial frame and orientation, $\theta$, of the local frame with respect to the inertial one.

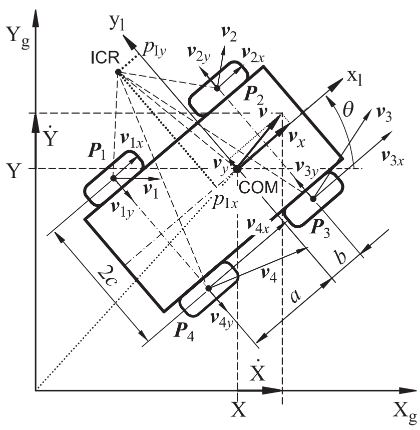


Fig. 1. Kinematics of SSMR

In the local frame one can describe robot motion using vector $\eta \triangleq [\omega\ v_x\ v_y]^T \in \mathbb{R}^3$, where $\omega$, $v_x$ and $v_y$ denote angular, longitudinal and lateral velocities of the robot, respectively. From Fig. 1 one can easily find the following map

$$\dot{\boldsymbol{q}} = \boldsymbol{\Theta}(\boldsymbol{q})\,\boldsymbol{\eta}, \tag{1}$$

where

$$\boldsymbol{\Theta}(\boldsymbol{q}) = \boldsymbol{\Theta}(q_1) \triangleq \begin{bmatrix} 1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{R}(\theta) \end{bmatrix} \tag{2}$$

and $\boldsymbol{R}(\theta) \in \mathrm{SO}(2)$. Taking into account SSMR kinematics (1) one can rewrite it in the form similar to unicycle kinematics

$$\dot{\boldsymbol{q}} = \boldsymbol{S}(\boldsymbol{q})\,\boldsymbol{\eta}^* + \boldsymbol{d}(\boldsymbol{q}), \tag{3}$$

with

$$\boldsymbol{S}(\boldsymbol{q}) \triangleq \begin{bmatrix} 1 & 0 \\ 0 & \cos\theta \\ 0 & \sin\theta \end{bmatrix}, \tag{4}$$

$\boldsymbol{\eta}^* \triangleq [\omega\ v_x]^T$ and $\boldsymbol{d}(\boldsymbol{q}) \triangleq [0\ -\sin\theta\ \cos\theta]^T v_y$. The term $\boldsymbol{d}$ can be considered as a disturbance which is dependent on lateral velocity $v_y$ resulting from skidding.

Taking into account position of instantaneous center of rotation (ICR) one can find the following first order constraint

$$\boldsymbol{A}(\boldsymbol{q}, p_{Ix})\,\dot{\boldsymbol{q}} = 0, \tag{5}$$

where $\boldsymbol{A}(\boldsymbol{q}, p_{Ix}) \triangleq [p_{Ix}\ -\sin\theta\ \cos\theta]$ is a constraint matrix dependent on current value of ICR $x$-coordinate expressed in the local frame. Equation (5) is not integrable, hence it describes first order but non-stationary nonhololonomic constraint in the case when $|p_{Ix}| \in \mathcal{L}_\infty$. Indeed evolution of $p_{Ix}$ cannot be derived from kinematics equation since non-skidding condition between wheels and surface is generally violated. As a result such system cannot be accurately reduced to a smooth kinematic system (see Lewis [1999]).

### 2.2 Dynamics

Taking into account the position of the local frame origin and assuming that mass distribution of the vehicle is homogeneous inertia matrix takes the following form: $\boldsymbol{M} = diag\{I, m, m\}$, while $m$, $I$ represent the mass and inertia, repectively. Then the dynamics equation expressed in the local frame can be written as

$$\bar{\boldsymbol{M}}\dot{\boldsymbol{\eta}} + \bar{\boldsymbol{C}}\boldsymbol{\eta} = \bar{\boldsymbol{B}}\boldsymbol{\tau} + \bar{\boldsymbol{Q}}_R, \tag{6}$$

where

$$\bar{\boldsymbol{M}} = \boldsymbol{M}, \qquad \bar{\boldsymbol{Q}}_R = [M_r\ F_{rx}\ F_{ry}]^T, \\ \bar{\boldsymbol{C}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -m \\ 0 & m & 0 \end{bmatrix}\omega, \qquad \bar{\boldsymbol{B}} = \frac{1}{r}\begin{bmatrix} -c & c \\ 1 & 1 \\ 0 & 0 \end{bmatrix}, \tag{7}$$

where $\boldsymbol{\tau} = [\tau_L\ \tau_R]^T \in \mathbb{R}^2$ is an control input determining torques produced by pairs of wheels on the left and right side of the vehicle, $\bar{\boldsymbol{Q}}_R \in \mathbb{R}^3$ is a vector of resistive forces which mainly result from wheels-ground interaction, and $r = r_i$ (it is supposed that radius of each wheel is the same). Here $F_{rx}$ is used in order to describe resultant resistive force in longitudinal direction including rolling resistant of wheels, motors and gears. The term $F_{ry}$ denotes the resultant constraint force in lateral direction which is hard to model accurately as a result of complicated wheel-ground interactions phenomena (in general tyre-ground model may be considered – see for example Wong [2001]). However, for simplicity in references by Economou [1999], Wong [2001] it is assumed that in the case of skid-steering vehicles lateral force $F_{ryi}$ for $i$-th wheel can be described using Coulomb friction model as follows

$$F_{ryi} \triangleq -\mu_i N_i\,\mathrm{sgn} v_{yi}, \tag{8}$$

where $\mu_i$ is a friction coefficient, $N_i$ is the wheel ground contact force which results from gravity and $v_{yi}$ is the lateral velocity of wheel (as indicated in Fig. 1).

Taking into account (6) one can write the following second order nonintegrable constraint

$$m\dot{v}_y + mv_x\omega = F_{ry} \tag{9}$$

which describes so-called lateral dynamics.

In the case of normal operation for SSMR significant skidding effect is undesirable. Hence, velocity $v_y$ should be limited. Considering (9) and interaction model (8) similarly to Kozłowski and Pazderski [2006] we may formulate the following proposition

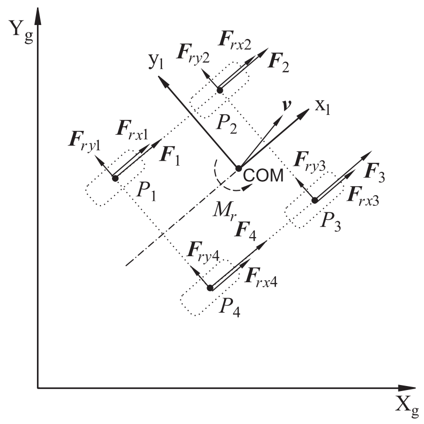


Fig. 2. Active and resistive forces

*Proposition 1.* Assuming that linear and angular velocities of the vehicle satisfy

$$|\omega v_x| \leq g \sum_{i=1}^{4} \gamma_i \mu_i \tag{10}$$

where $g$ is the value of gravity,

$$\gamma_i \triangleq \frac{1}{a+b} \begin{cases} b \text{ for } i = 1, 4 \\ a \text{ for } i = 2, 3 \end{cases}, \tag{11}$$

the motion of the vehicle is stable in the sense that $v_y \in \mathcal{L}_\infty$ and $x$-coordinate of ICR is bounded as

$$-a \leq p_{Ix} \leq b. \tag{12}$$

This proposition can be understood that as long as we consider slow motion of SSMR the skidding effect is limited and the motion stability is guaranteed.

### 2.3 Approximation of admissible trajectories

From a practical point of view it is hard to model or measure interaction forces. Therefore it is almost impossible to prepare reference trajectory (including position and orientation) off-line which is feasible for SSMR. Instead of it one can consider approximation based on kinematics of unicycle-like robot. Moreover for low velocities (i.e. limited value of product $|v_x\omega|$) value of $v_y$ is highly reduced. As a result one may introduce the following definition:

*Definition 1.* The trajectory $\boldsymbol{q}$ which is a solution to the following kinematic equation

$$\dot{\boldsymbol{q}} = \boldsymbol{S}(\boldsymbol{q})\boldsymbol{\eta}^* \tag{13}$$

is called *almost admissible trajectory* for the system (3) if only value of $|v_x\omega|$ is small enough.

## 3. CONTROL LAW

### 3.1 Tracking error definition

In this paper we use so called left-invariant operation (see Bullo and Murray [1999], Morin and Samson [2003]) which takes into account symmetry of the control system (3) described on SE(2) Lie group and it is defined as follows

$$\boldsymbol{g} \circ \boldsymbol{h} \triangleq \boldsymbol{g} + \boldsymbol{\Theta}(g_1)\boldsymbol{h}, \tag{14}$$

where $\boldsymbol{g} \triangleq [g_1\ g_2\ g_3]^T \in \mathbb{S}^1 \times \mathbb{R}^2$, $\boldsymbol{h} \triangleq [h_1\ h_2\ h_3]^T \in \mathbb{S}^1 \times \mathbb{R}^2$ are elements of Lie group and $\boldsymbol{\Theta}(\cdot)$ is defined by (2). The inverse of SE(2) group element $\boldsymbol{g}$ can be defined as

$$\boldsymbol{g}^{-1} \triangleq -\boldsymbol{\Theta}^T(g_1)\boldsymbol{g}. \tag{15}$$

Following Morin and Samson [2003] we can define so-called transformed tracking error with respect to the moving frame as

$$\tilde{\boldsymbol{q}} \triangleq \left[\tilde{\theta}\ \tilde{\boldsymbol{p}}^T\right]^T \triangleq \boldsymbol{q}_r^{-1} \circ \boldsymbol{q}, \tag{16}$$

where $\boldsymbol{q}_r \triangleq \left[\theta_r\ \boldsymbol{p}_r^T\right]^T \triangleq [\theta_r\ X_r\ Y_r]^T$ denotes reference orientation and position. Next, taking the time derivative of (16) and using (3) one can obtain

$$\dot{\tilde{\boldsymbol{q}}} = \boldsymbol{S}(\tilde{\boldsymbol{q}})\boldsymbol{\eta}^* + \boldsymbol{f}_d(\tilde{\boldsymbol{q}}, \boldsymbol{q}_r, \dot{\boldsymbol{q}}_r) + \boldsymbol{d}(\tilde{\boldsymbol{q}}), \tag{17}$$

where

$$\boldsymbol{f}_d(\tilde{\boldsymbol{q}}, \boldsymbol{q}_r, \dot{\boldsymbol{q}}_r) = -\boldsymbol{\Theta}^T(\theta_r)\left(d\boldsymbol{\Theta}(\theta_r)\tilde{\boldsymbol{p}}\omega_r + \dot{\boldsymbol{p}}_r\right) \tag{18}$$

is the drift term dependent on reference trajectory with $\omega_r \triangleq \dot{\theta}_r$ and

$$d\boldsymbol{\Theta}(\cdot) \triangleq \begin{bmatrix} 0 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}(\cdot)\boldsymbol{J} \end{bmatrix}, \tag{19}$$

where $\boldsymbol{J} \triangleq \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

### 3.2 Control law development

Here we use a concept of practical stabilization originally introduced by Dixon et al. [1999] and next developed and generalized by Morin and Samson [2003]. The control task at kinematic level can be formulated as follows:

*Definition 2.* Find bounded controls $v_x(t), \omega(t)$ for kinematics (1) such, that for initial condition $\tilde{\boldsymbol{q}}(0)$ the Euclidean norm of the error $\tilde{\boldsymbol{q}}(t)$ tends to some constant $\varepsilon > 0$ as $t \to \infty$:

$$\lim_{t\to\infty} \|\tilde{\boldsymbol{q}}(t)\| \leq \varepsilon, \tag{20}$$

where $\varepsilon$ is an assumed error envelope, which can be made arbitrary small.

In order to facilitate the control solution one can define auxiliary error taking into account left-invariant operation (14) as follows

$$\boldsymbol{z} \triangleq \tilde{\boldsymbol{q}} \circ \boldsymbol{x}_d^{-1} = \tilde{\boldsymbol{q}} - \boldsymbol{\Theta}(\tilde{q}_1 - x_{d1})\boldsymbol{x}_d \tag{21}$$

where $\boldsymbol{x}_d \triangleq [x_{d1}\ x_{d2}\ x_{d3}]^T \in \mathbb{R}^3$ is a vector containing harmonic-like signals (according to terminology used by Morin and Samson $\boldsymbol{x}_d$ is a transverse function). Such approach allows to render desired trajectory at each direction in the state space (i.e. approximates it with desired accuracy).

Taking the time derivative of (21) and using (17) one has

$$\dot{\boldsymbol{z}} = \boldsymbol{S}(\tilde{\boldsymbol{q}})\boldsymbol{\eta}^* - \boldsymbol{\Theta}(z_1)\dot{\boldsymbol{x}}_d - d\boldsymbol{\Theta}(z_1)\boldsymbol{x}_d(\omega - \dot{x}_{d1}) + \tag{22}$$
$$+\boldsymbol{f}_r(\tilde{\boldsymbol{q}}, \boldsymbol{q}_r, \dot{\boldsymbol{q}}_r, \boldsymbol{x}_d) + \boldsymbol{d}(\tilde{\boldsymbol{q}})$$

where $\boldsymbol{f}_r(\tilde{\boldsymbol{q}}, \boldsymbol{q}_r, \dot{\boldsymbol{q}}_r, \boldsymbol{x}_d) \triangleq \boldsymbol{f}_d(\tilde{\boldsymbol{q}}, \boldsymbol{q}_r, \dot{\boldsymbol{q}}_r) + d\boldsymbol{\Theta}(z_1)\boldsymbol{x}_d\omega_r$.

Now based on Dixon et al. [1999] and Morin and Samson [2003] we assume that $\boldsymbol{x}_d$ is generated using tunable linear oscillator as follows

$$\boldsymbol{x}_d \triangleq \begin{bmatrix} \boldsymbol{\Phi}_1 \\ \frac{1}{2}\boldsymbol{\xi}^T\boldsymbol{\Phi}_2 \end{bmatrix}\boldsymbol{\xi}, \tag{23}$$

where $\boldsymbol{\Phi}_1, \boldsymbol{\Phi}_2 \in \mathbb{R}^{2\times 2}$ are matrices with tuning functions and $\boldsymbol{\xi} \in \mathbb{R}^2$ is the solution of the following differential equation

$$\dot{\boldsymbol{\xi}} \triangleq \boldsymbol{J}\boldsymbol{\xi}\Omega \tag{24}$$

with $\Omega$ denoting instantaneous frequency and initial condition $\boldsymbol{\xi}(0)^T \boldsymbol{\xi}(0) = 1$. Assuming that

$$\boldsymbol{\Phi}_i(t) \triangleq \begin{bmatrix} {}^i\varphi_{11}(t) & {}^i\varphi_{12}(t) \\ {}^i\varphi_{21}(t) & {}^i\varphi_{22}(t) \end{bmatrix}, \tag{25}$$

taking the time derivative of (23), substituting the result to (22) and making some algebraic manipulations one can finally write

$$\dot{\boldsymbol{z}} = \boldsymbol{\Sigma}(\boldsymbol{z}, \boldsymbol{x}_d)\boldsymbol{H}(\boldsymbol{x}_d)\begin{bmatrix} \boldsymbol{\eta}^* \\ \Omega \end{bmatrix} + \boldsymbol{f}_\Phi(\tilde{\boldsymbol{q}}, \boldsymbol{\xi}) + $$
$$+\boldsymbol{f}_r(\tilde{\boldsymbol{q}}, \boldsymbol{q}_r, \dot{\boldsymbol{q}}_r, \boldsymbol{x}_d) + \boldsymbol{d}(\tilde{\boldsymbol{q}}), \tag{26}$$

where

$$\boldsymbol{\Sigma}(\boldsymbol{z}, \boldsymbol{x}_d) \triangleq \begin{bmatrix} 1 & 0 \\ -\boldsymbol{R}(z_1)\boldsymbol{J}\begin{bmatrix} x_{d2} \\ x_{d3} \end{bmatrix} & \boldsymbol{R}(z_1) \end{bmatrix} \in \mathbb{R}^{3\times 3}, \tag{27}$$

$$\boldsymbol{H}(\boldsymbol{x}_d) \triangleq \left[\boldsymbol{S}(\boldsymbol{x}_d) \; - \begin{bmatrix} \boldsymbol{\Phi}_1 \\ \boldsymbol{\xi}^T\boldsymbol{\Phi}_2 \end{bmatrix} \boldsymbol{J}\boldsymbol{\xi}\right] \in \mathbb{R}^{3\times 3}, \tag{28}$$

and

$$\boldsymbol{f}_\Phi(\tilde{\boldsymbol{q}}, \boldsymbol{\xi}) \triangleq \begin{bmatrix} -[1\ 0]\,\dot{\boldsymbol{\Phi}}_1 \\ \boldsymbol{R}(z_1)\left(\boldsymbol{J}\begin{bmatrix} x_{d2} \\ x_{d3} \end{bmatrix}[1\ 0]\,\dot{\boldsymbol{\Phi}}_1 - \begin{bmatrix} [1\ 0]\,\dot{\boldsymbol{\Phi}}_1 \\ \frac{1}{2}\boldsymbol{\xi}^T\dot{\boldsymbol{\Phi}}_2 \end{bmatrix}\right) \end{bmatrix}\boldsymbol{\xi}. \tag{29}$$

In order to calculate control signal $\boldsymbol{\eta}^*$ matrices $\boldsymbol{H}$ and $\boldsymbol{\Sigma}$ must be invertible. For matrix $\boldsymbol{\Sigma}$ one can show that such property is always satisfied since $\det \boldsymbol{\Sigma} \equiv 1$. However, invertibility of $\boldsymbol{H}$ is guaranteed only for properly chosen set of tuning functions ${}^i\varphi_{jk}$. Calculating determinant of $\boldsymbol{H}$ one can find the following limitations

$$\sqrt{({}^1\varphi_{11})^2 + ({}^1\varphi_{12})^2} < \frac{\pi}{2},\ {}^1\varphi_{21} \geq 0,\ {}^1\varphi_{22} > 0, \tag{30}$$

$${}^2\varphi_{11} = 0,\ {}^2\varphi_{12} = {}^2\varphi_{21} < {}^1\varphi_{11}{}^1\varphi_{22},\ {}^2\varphi_{22} \geq 0. \tag{31}$$

Next we can formulate the control solution as follows.

*Proposition 2.* The smooth control law given as

$$\begin{bmatrix} \boldsymbol{\eta}^* \\ \Omega \end{bmatrix} \triangleq (\boldsymbol{\Sigma}(\boldsymbol{z}, \boldsymbol{x}_d)\boldsymbol{H}(\boldsymbol{z}))^{-1}(-\boldsymbol{K}\boldsymbol{z} + \tag{32}$$

$$-\boldsymbol{f}_\Phi(\tilde{\boldsymbol{q}}, \boldsymbol{\xi}) - \boldsymbol{f}_r(\tilde{\boldsymbol{q}}, \boldsymbol{q}_r, \dot{\boldsymbol{q}}_r, \boldsymbol{x}_d) + \tag{33}$$
$$-\rho\bar{\boldsymbol{d}}(\tilde{\boldsymbol{q}}) f_s^a(\bar{\boldsymbol{d}}^T\tilde{\boldsymbol{q}})),$$

where $-\boldsymbol{K} \in \mathbb{R}^{3\times 3}$ is Hurwitz-stable matrix, $\rho$ is a scalar function which satisfies

$$\rho > |v_y|, \tag{34}$$

$$f_s^a(y) \triangleq \frac{\rho y}{\rho|y| + \varepsilon_s} \tag{35}$$

is an approximation of non smooth $\operatorname{sgn}(\cdot)$ function with constant $\varepsilon_s > 0$ and $\bar{\boldsymbol{d}}(\tilde{\boldsymbol{q}}) \triangleq [0\ -\sin\tilde{q}_1\ \cos\tilde{q}_1]^T$, $\|\dot{\boldsymbol{q}}_r\| \in \mathcal{L}_\infty$, $\left\|\dot{\boldsymbol{\Phi}}_i\right\| \in \mathcal{L}_\infty$ ensures practical stabilization in the sense given by (20).

*Proof 1.* Firstly, we define Lyapunov function candidate as

$$V \triangleq \frac{1}{2}\boldsymbol{z}^T\boldsymbol{z}. \tag{36}$$

Next, taking the time derivative of (36), using (22) with control (32) one has

$$\dot{V} = -\boldsymbol{z}^T\boldsymbol{K}\boldsymbol{z} + \boldsymbol{z}^T\bar{\boldsymbol{d}}(\tilde{\boldsymbol{q}})\left(v_y - \rho f_s^a\left(\bar{\boldsymbol{d}}^T(\tilde{\boldsymbol{q}})\boldsymbol{z}\right)\right). \tag{37}$$

Taking into account definition (35) and condition (34) one can find the following inequality

$$\dot{V} \leq -\boldsymbol{z}^T\boldsymbol{K}\boldsymbol{z} + \frac{\rho\left|\bar{\boldsymbol{d}}^T(\tilde{\boldsymbol{q}})\boldsymbol{z}\right|\varepsilon_s}{\rho\left|\bar{\boldsymbol{d}}^T(\tilde{\boldsymbol{q}})\boldsymbol{z}\right| + \varepsilon_s} \leq -\lambda\|\boldsymbol{z}\|^2 + \varepsilon_s \tag{38}$$

with $\lambda > 0$. Solving inequality (38) and using (36) one has

$$\|\boldsymbol{z}(t)\| \leq \sqrt{\|\boldsymbol{z}(0)\|\exp(-2\lambda t) + \frac{\varepsilon_s}{2\lambda}(1 - \exp(-2\lambda t))}. \tag{39}$$

Hence, auxiliary error in the steady-state is bounded as follows

$$\lim_{t\to\infty}\|\boldsymbol{z}(t)\| \leq \varepsilon_2, \tag{40}$$

where $\varepsilon_2 = \sqrt{\frac{\varepsilon_s}{2\lambda}}$. The steady-state error in the configuration space can be estimated as

$$\lim_{t\to\infty}\|\tilde{\boldsymbol{q}}(t)| \leq \varepsilon_1 + \varepsilon_2, \tag{41}$$

where $\varepsilon_1$ is determined by tuning functions ${}^i\varphi_{jk}$.

### 3.3 Controller tuning

In spite of stability result proved in previous subsection a good performance of the controller is related to proper selection of tuning matrices $\boldsymbol{\Phi}_i$ which influence both transient and steady-state. In this paper we use novel method based on adaptive scaling taking into account current error $\boldsymbol{z}$. In order to do that we introduce the filtered error $z_s$ which a solution of the following linear second order differential equation

$$T^2\ddot{z}_s + 2T\dot{z}_s + z_s = \sqrt{z_2^2 + z_3^2 + \epsilon^2}, \tag{42}$$

with initial condition $z_s(0) > 0$ and $\dot{z}_s(0) = 0$, $T > 0$ and $\epsilon > 0$. Taking into account (40) it is easy to show that in the steady state

$$\lim_{t\to\infty} z_s(t) \leq \varepsilon_2 + \epsilon. \tag{43}$$

Filtered error is then used to scale ${}^i\varphi_{jk}$. The details concerning tuning will not be discussed in this paper.

### 3.4 Velocity limitation

To ensure stable motion of SSMR the robot should move relatively slow, i.e. product $|v_x\omega|$ should be limited. However, the control law formulated in Proposition 2 does not take into account this limitation. Therefore we propose to use velocity and time scalling procedure similar to method given in Kozłowski and Pazderski [2006]. It should be assumed that reference trajectory is chosen in such a way that it can be approximated with desired accuracy using feasible value of velocities, i.e. that condition given in Proposition 2 has to be satisfied.

### 3.5 Control law at the dynamic level

In this paper we concentrate on control problem at kinematic level only. From a theoretical point of view dynamic description used here is only necessary to prove that within range of low velocities kinematic approximation of admissible velocity is justified. For simplicity we assume that desired velocities $\omega$ and $v_x$ generated by the controller can be realized with high accuracy. In the real application controller at dynamic level should keep velocity tracking error to be small enough. In order to do that backstepping technique can be used with adaptation to take into account dynamic uncertainty (especially related to ground-wheels interaction model).

## 4. EXPERIMENTAL VALIDATION

### *4.1 Experimental setup*

The experimental research has been conducted using Four-Wheeled MMS SSMR built in our laboratory (see Fig. 3) which is equipped with DC motors coupled mechanically and electrically. The maximum linear velocity is limited to $0.5[m/s]$. The robot's controller is built around DSP F2812 processor which is responsible for measuring and control tasks at dynamic level. The posture of the robot is given using vision system based on active LED markers. The controlling tasks are divided between PC and DSP. The PC estimates longitudinal slip based on data coming from vision system and the robot and calculates desired velocity of the robot body with respect to motors limitation (as well as the condition of stable motion). The DSP realizes the control law at dynamic level which is based on backstepping and adaptation technique in order to ensure low velocity tracking error.

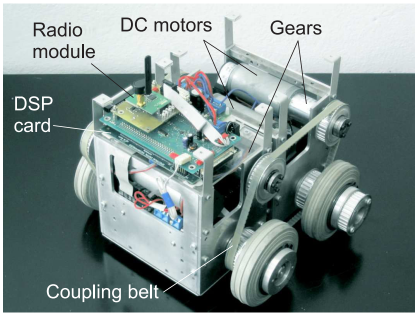


Fig. 3. MMS SSMR - experimental skid-steering robot

### *4.2 Experimental results*

In experiments described here two almost admissible reference trajectories have been used: circular

$$\begin{bmatrix} X_r(t) \\ Y_r(t) \end{bmatrix} = \begin{bmatrix} 0.7 \sin 0.4t \\ 0.7 \cos 0.4t \end{bmatrix} \tag{44}$$

and eight-like shaped

$$\begin{bmatrix} X_r(t) \\ Y_r(t) \end{bmatrix} = \begin{bmatrix} 0.7 \sin 0.4t \\ 0.8 \sin 0.2t \end{bmatrix}. \tag{45}$$

The results of circular trajectory tracking are presented if Figs. 4-6. Based on Fig. 4 one can conclude that shape of the robot path during transient states is satisfactory (i.e. no highly oscillatory behavior appears). The initial orientation and position errors are bounded to the nonzero values (see Fig. 5) as follows: $\lim_{t\to\infty} \left|\tilde{\theta}(t)\right| \leq 0.06[rad]$, $\lim_{t\to\infty} \left\{\left|\tilde{X}(t)\right|, \left|\tilde{Y}(t)\right|\right\} \leq 0.07[m]$. From Fig. 6 one can see that linear velocity is saturated which is a result of motors limitation and scaling velocity algorithm. Value of $v_y$ is rather low – only in transient states it is increased. As a result skidding effect for considered trajectory is small.

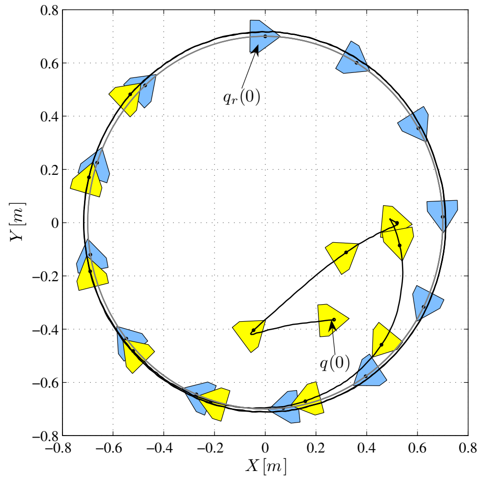


Fig. 4. Reference (■) and robot's (■) paths (stroboscopic view $\Delta t = 1[s]$)

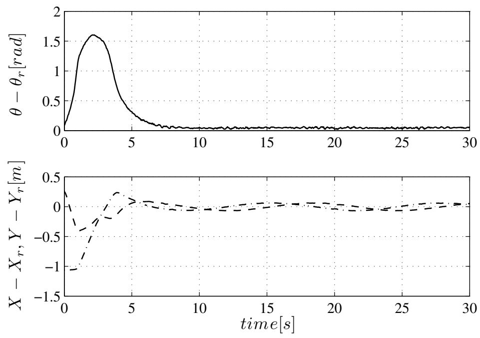


Fig. 5. Orientation and position errors: $\theta - \theta_r[rad]$ (−), $X - X_r[m]$ (−−), $Y - Y_r[m]$ (−.−).

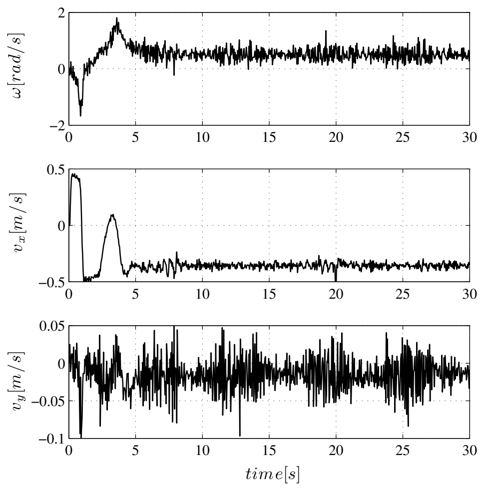


Fig. 6. Angular and linear velocities of the robot

The second reference trajectory is more demanding. The results are presented in Figs. 7-9. The posture error in the steady state are bounded as follows: $\lim_{t\to\infty} \left|\tilde{\theta}(t)\right| \leq 0.17[rad]$, $\lim_{t\to\infty} \left\{\left|\tilde{X}(t)\right|, \left|\tilde{Y}(t)\right|\right\} \leq 0.07[m]$. In this case it is quite hard to make better accuracy of tracking with respect to orientation. It can be observed that if the curvature of the path is significant orientation error

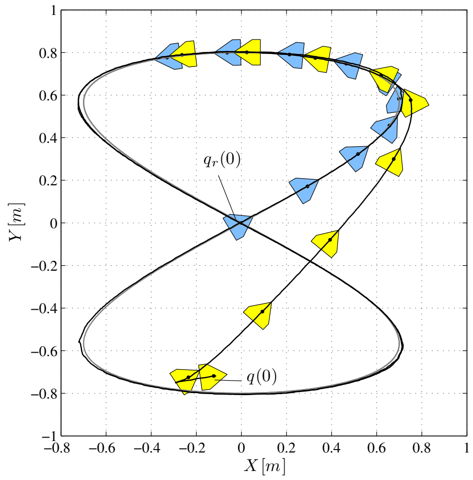


Fig. 7. Reference (■) and robot's (■) paths (stroboscopic view $\Delta t = 1[s]$)

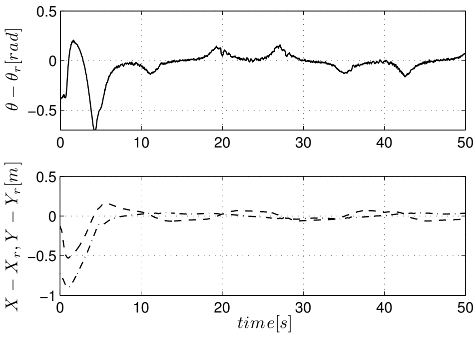


Fig. 8. Orientation and position errors: $\theta - \theta_r[rad]$ (−), $X - X_r[m]$ (−−), $Y - Y_r[m]$ (−.−).

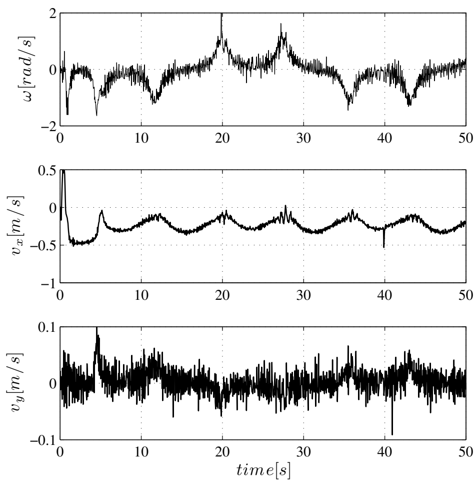


Fig. 9. Angular and linear velocities of the robot

increases. This is a result of approximation of admissible trajectory for SSMR. Indeed tracked trajectory $\boldsymbol{q}_r(t)$ is *almost* admissible. Hence, it can be tracked with limited accuracy taking into account velocity saturation and condition of stable motion which is a consequence of robot's dynamics. Moreover, forcing better accuracy of tracking by selection of small value of $\epsilon$ may lead to oscillatory behavior that in general should be avoided in practice. In Fig. 9 it can be seen that magnitude of lateral velocity $v_y$ is higher in comparison to results obtained for circular trajectory but still it remains in reasonable range.

## 5. CONCLUSIONS

In this paper approximation problem of SSMR kinematic is taken into account and the control solution which ensures practical stabilization is developed. This kind of stabilization seems to be suitable for SSMR which is the system with structural uncertainties. Moreover it was shown that using control scheme based on unicycle-like robot is justified for SSMR in the range of low velocities. The accuracy of tracking including both orientation and position cannot be very high as a result of wheels-ground interaction characteristic that is hard to predict and model. It is also worth to ask if it is necessary to force small tracking error for SSMR taking into account typical tasks for which they are used in practical applications.

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