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On Linear Canonical Controllers Within The Unfalsified Control Framework

Osamu Kaneko *

* *Graduate School of Engineering Science, Osaka University, machikaneyama 1-3, Toyonaka, Osaka, 560-8531, JAPAN (Tel: +81-6850-6358; e-mail: kaneko@sys.es.osaka-u.ac.jp).*

Abstract: A canonical controller (cf. van der Schaft [2002]), which was proposed by van der Schaft, is a controller yielding a given specification with a plant behavior. In this paper, for a given data of a plant and a specification, we provide a synthesis of linear canonical controllers without using mathematical models of a plant. A desired canonical controller can be obtained by solving linear algebraic equations which consist of a data and a specification. We also see that a canonical controller designed by proposed method also unfalsifies the actual data and a specification, so our result is also regarded as one of synthesis of unfalsified controllers (cf. Safonov and Tsao [1997]).

Keywords: Canonical controller, Achievable behaviors, Behavioral approach, Unfalsified Control, Data-driven control systems.

1. INTRODUCTION

It is obvious that a synthesis of a controller which achieves a given specification by using available information on the dynamics of a plant is one of the central issues of control system theory. In van der Schaft [2002] and Julius et al. [2005], the achievability of a given specification was discussed within a *behavioral framework* (cf. Willems [1991], Willems [1997]) in which the central role of a synthesis of a controller is played by the set of trajectories along which the dynamics of a system evolves. Moreover, a controller yielding a given specification has been also provided as a *canonical controller*.

At the same time, since the trajectories along which the dynamics of a system evolves include fruitful information on the dynamics, it is also natural to treat such trajectories as a system itself. From this stand point, there are some researches on synthesis and analysis of control systems based on the direct use of the data without mathematical model, e.g., Markovsky et al. [2005], Markovsky and Rapisarda [2007], Fujisaki et al. [2005], Safonov and Tsao [1997], Yamamoto and Okano [2006] and so on. It might be appropriate to referred to these approaches as *data-driven control* synthesis.

From these backgrounds, this paper provide a synthesis of a linear canonical controller for a given specification based on the direct use of the data instead of using mathematical models of a linear plant. A desired canonical controller can be obtained as a Toeplitz-type linear operator by solving linear algebraic equations which consist of a data and a specification. Moreover, we also see that a canonical controller designed by proposed method also unfalsifies the actual data and a specification, so our result is also regarded as one of synthesis of unfalsified controllers.

By the way, in the case in which the structure of a controller is fixed, a control system synthesis based on the direct use of the data corresponds to the tuning of a parameterized controller. As representative methods, we can list Iterative Feedback Tuning (which is abbreviated as IFT, cf. Hjalmarsson et al. [1998]), Virtual Reference Feedback Tuning (which is abbreviated as VRFT, cf. Campi et al. [2002] and Sala [2007]) and Fictitious Reference Iterative Tuning (which is abbreviated as FRIT, cf. Souma et al. [2004] and Kaneko et al. [2005a]). IFT need many experiments for achieving the desired controller, which leads to one of the practical drawbacks. The last two methods enable one to obtain the optimal parameter of a controller by using one-shot experimental data without mathematical models of a plant. In this paper, we also show that our proposed method here includes FRIT and VRFT as special cases. Thus, our proposed method has practical advantage in the sense that only one-shot experiment yields the desired controller.

The contributions of this paper are the following. From the behavioral points of view, the result in this paper corresponds to one of the nice applications of a canonical controller and the unfalsified control. From the view points of data-driven control system theory, the proposed method is regarded as one of the new approaches to the control system synthesis without mathematical models. Particularly, our method enables one to obtain the desired controller with one-shot experiment without mathematical models, which implies that behavioral systems theory provides not only profound insight from the theoretical points of view, but also powerful concept from the practical points of view.

2. PRELIMINARIES

Let $\mathbb{R}$ and $\mathbb{Z}$ denote the set of real numbers and the integers, respectively. Let $\mathbb{R}^n$ denote the set of real vectors of size n and let $\mathbb{R}^{n\times m}$ denote the set of real matrices of size $n \times m$. Let $\mathbb{R}^{\mathtt{p}\times\mathtt{q}}[\xi]$ denote the set of polynomial matrices of size $p \times q$ with the indeterminate ξ.

Let $(\mathbb{R})^{\mathbb{Z}}$ denote the set of the maps from the time axis $\mathbb{Z}$ onto $\mathbb{R}$, i.e., $w \in (\mathbb{R})^{\mathbb{Z}}$ implies that w is a discrete time series. For $w \in (\mathbb{R})^{\mathbb{Z}}$, the value of w at the time t is denoted with w_t. For $w \in (\mathbb{R})^{\mathbb{Z}}$ and $a, b \in \mathbb{Z}$ such that $a \leq b$, $w_{[a,b]}$ denotes the finite time series of w in the time interval $[a, b]$. We regard $w_{[a,b]}$ as an element of $\mathbb{R}^{[b-a+1]}$, i.e., $w_{[a,b]} = [w_a, w_{a+1}, \cdots, w_b]^{\mathrm{T}}$. Let σ denote the shift operator defined by $\sigma w_t := w_{t+1}$ for a time series $w \in (\mathbb{R})^{\mathbb{Z}}$.

Consider a linear, time-invariant system with single-input and single-output in discrete time described by a transfer function G. Let $u_{[0,N]}$ and $y_{[0,N]}$ denote the input and output data, respectively, obtained in the interval $[0, N]$. Normally, the output y_t of an operator G with respect to the input $u_{[0,t]}$ is written by the form of $y_t = \sum_{k=0}^{t} g_k u_{t-k}$ by using the fact that G can be written by Markov parameter expression $G = \sum_{k=0}^{\infty} g_k \sigma^{-k}$. The output finite time series $y_{[0,N]} \in \mathbb{R}^{N+1}$ is the range of the following Toeplitz matrix operator with respect to $u_{[0,N]}$:

$$\begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_N \end{pmatrix} = \begin{pmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \\ g_N & \cdots & g_1 & g_0 \end{pmatrix} \begin{pmatrix} u_0 \\ u_1 \\ \vdots \\ u_N \end{pmatrix}. \tag{1}$$

We denote the Toeplitz operator of a rational function $G = \sum_{k=0}^{\infty} g_k \sigma^{-k}$ truncated at N as

$$\mathcal{T}^{G}_{[0,N]} := \begin{pmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \\ g_{N-1} & \cdots & g_1 & g_0 \end{pmatrix} \in \mathbb{R}^{(N+1)\times(N+1)}.$$

Similarly, we denote the Toeplitz matrix consisting of truncated time series $w_{[0,N]}$ as

$$\mathcal{T}^{w}_{[0,N]} := \begin{pmatrix} w_0 & 0 & \cdots & 0 \\ w_1 & w_0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \\ w_{N-1} & \cdots & w_1 & w_0 \end{pmatrix} \in \mathbb{R}^{(N+1)\times(N+1)}.$$

By using Toeplitz matrix descriptions, it is easy to see that $(Gu)_{[0,N]} = \mathcal{T}^{G}_{[0,N]} u_{[0,N]}$. In Eq.(1), the invertibility of G is equivalent to the nonsingularity of the Toeplitz matrix because of $g_0 \neq 0$.

3. PROBLEM FORMULATION

We assume that a plant is linear, time invariant in discrete time. However, we have also an assumption that *mathematical models of a plant is unknown.* We obtain the input and the output data in the finite time interval $u_{[0,N]}$ and $y_{[0,N]}$, respectively. Consider a feedback system illustrated in Fig.1 with a controller C to be designed. We are also given a specification as the set of (r, y) which evolve along the desired trajectory. Then, the purpose is to *design a controller C so as to achieve the desired trajectory by the direct use of the data without mathematical model of a plant.*

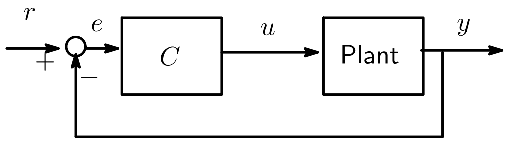

Fig. 1. A feedback loop system

4. THE BEHAVIORAL APPROACH AND UNFALSIFIED CONTROL

4.1 The basics of the behavioral theory

We give a brief review of the behavioral approach here. For more details, see the references, e.g., Willems [1991], Willems [1997], and so on. In this paper, we focus on discrete-time, linear, time-invariant systems, which is equivalent to saying that there exists a polynomial matrix $R \in \mathbb{R}^{\bullet\times\mathtt{q}}[\xi]$ such that the (manifest) *behavior*, say $\mathcal{B} \subseteq (\mathbb{R}^{\mathtt{q}})^{\mathbb{Z}}$, is described by a *kernel representation* $Rw = 0$ for all $w \in \mathcal{B}$. Here, the indeterminate of R is replaced with the shift operator σ [1]. In some cases, it would be appropriate not only to focus on the manifest behavior w but also to introduce the auxiliary behavior ℓ such that $R'w = M\ell$, where R' and M are suitable polynomial matrices, and ℓ is referred to as the *latent variables.* A pair of (w, ℓ) is referred to as the full behavior.

We assume that a plant is controllable in a behavioral sense, which is equivalent to saying that the row rank of $R(s)$ is invariant for any complex number s. If R is row rank, this condition is also equivalent to that a left prime factor of R is only a unimodular matrix. Another necessary and sufficient condition for a system to be controllable is that there exists a polynomial matrix $M \in \mathbb{R}^{\mathtt{q}\times\bullet}[\xi]$ admitting an *image representation* $w = M\ell$ for all $w \in \mathcal{B}$ with the latent variable. Without loss of generality, it is possible to take M such that $M(s)$ is full column rank for any complex number s.

4.2 Achievable behaviors and canonical controllers

Here, we give a brief review of the novel concept of achievable behaviors and canonical controllers, which have been proposed and discussed in van der Schaft [2002] and Julius et al. [2005]. See these references for more detailed issues. We here focus on the linear time invariant systems in discrete time.

Let $\mathcal{P}_f \in (\mathbb{R}^{\mathtt{q}+\mathtt{g}})^{\mathbb{Z}}$ be the full behavior of a system, that is, $(w, c) \in \mathcal{P}_f$ obey the law described by

$$R(\sigma)w = M(\sigma)c \tag{2}$$

with appropriate matrices $R \in \mathbb{R}^{\bullet\times\mathtt{q}}[\xi]$ and $M \in \mathbb{R}^{\bullet\times\mathtt{g}}[\xi]$. $\mathcal{P}_f$ is illustrated in Fig.2. Let $\mathcal{P} \in (\mathbb{R}^{\mathtt{q}})^{\mathbb{Z}}$ be the manifest behavior of $\mathcal{P}_f$. This is described by a kernel representation which is obtained by eliminating the variable c from $\mathcal{P}_f$. The variable c is used for the *partial interconnection* with another system, that is, a controller. Let $\mathcal{S} \in (\mathbb{R}^{\mathtt{q}})^{\mathbb{Z}}$ be the desired behavior, that is, the aim of the control (partial interconnection) is to achieve $\mathcal{S}$ by interconnecting $\mathcal{P}_f$

[1] In the following, the similar replacement is applied. Moreover, we often omit the notations the indeterminate ξ and the operator σ.

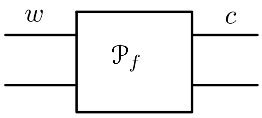

Fig. 2. A plant

and a controller. There also exists a polynomial matrix $K \in \mathbb{R}^{\bullet \times \mathtt{q}}[\xi]$ such that $w \in \mathcal{S}$ is described by

$$K(\sigma)w = 0. \tag{3}$$

A *canonical controller* is defined as the set of the trajectories described by

$$\mathcal{C}_{\mathrm{can}} = \{c \in (\mathbb{R}^{\mathtt{g}})^{\mathbb{Z}} |\ ^{\exists}\tilde{w} \text{ s.t. } (\tilde{w}, c) \in \mathcal{P}_f, \tilde{w} \in \mathcal{S}\} \tag{4}$$

and illustrated in Fig.3. The role of a canonical controller is to yield the desired behavior $\mathcal{S}$ exactly by interconnecting it to a plant. See also this concept Fig.4. Assume that $\mathcal{S}$

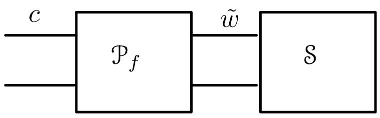

Fig. 3. A canonical controller

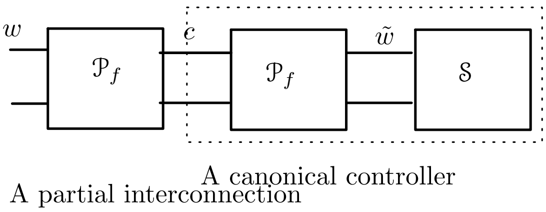

Fig. 4. A partial interconnection

is implementable via c, which is equivalent to

$$\mathcal{N} \subset \mathcal{S} \subset \mathcal{P} \tag{5}$$

(cf. Belur and Trentelman [2002]), where $\mathcal{N}$ is the kernel of R, and it is referred to as the hidden behavior. Then, a canonical controller can be parameterized by using R and M as follows. First, find a polynomial matrix L such that

$$LR = K. \tag{6}$$

Then,

$$LMc = 0 \tag{7}$$

induces a kernel representation of a canonical controller acting on the variable c.

4.3 Unfalsified control

The notion of the unfalsified control was proposed in Safonov and Tsao [1997], as one of the extensions of the behavioral interconnection to adaptive control scheme. See a conventional feedback system illustrated in Fig.1. The dynamics of a plant is unknown and the material we can obtain is only the actual data u and y evolving along the law of the dynamics of a plant. The exogenous signal r is the input of this feedback loop system while u and y are the outputs. The pair of the data u and y is unique for fixed r and C. Conversely, for the data u and y, there are infinite pairs of r' and C' so as to yield u and y. Particularly r' is referred to as a fictitious reference. Since the triple of (r', u, y) is also in the behavior of a feedback loop system with the corresponding C', it is possible to check whether a controller C' which is not implemented in the actual loop is falsified by a given specification with respect to u, y and r'. Such a falsification of a controller leads to specify the set of controller which are unfalsified by the data and the specification, that is, it is possible to obtain the set of controllers which are *not undesirable* controller. This is a core of the unfalsified control theory.

In Safonov and Tsao [1997], although this concept is used in the real-time adaptive control scheme, it is also available for off-line control system synthesis. Our result is in the latter point of view.

5. MAIN RESULTS

5.1 A linear canonical controllers for the data

Again, see a conventional feedback system illustrated in Fig.1. Moreover, a system is assumed to be with single-input and single-output for the brevity of the explanation. Though there exist coprime polynomials R_y and R_u such that $G = {R_y}^{-1} R_u$ because of the assumption that the system is linear and time-invariant, we do not know any information on these polynomials. The material we can obtain is only the finite interval data $u_{[0,N]}$ and $y_{[0,N]}$. We are also given a desired trajectory as the output of the transfer function as $y_d = T_d r$ where $T_d =: K_y^{-1} K_r$. The problem is to find a controller so as to achieve y_d with respect to r in the feedback interconnection in Fig.1.

Now we view this problem in the framework of achievable behaviors. A plant $\mathcal{P}_f$ is described by

$$\mathcal{P}_f = \left\{(w, c) \in \mathbb{R}^{\mathtt{q}+\mathtt{p}} | Rw = Mc \text{ is satisfied } \right\} \tag{8}$$

where $w := (r, y)$ and $c := (e, u)$, and

$$R = \begin{pmatrix} 1 & -1 \\ 0 & R_y \end{pmatrix}, \ M = \begin{pmatrix} 1 & 0 \\ 0 & R_u \end{pmatrix}, \tag{9}$$

which is also illustrated in Fig.5. We are also given a

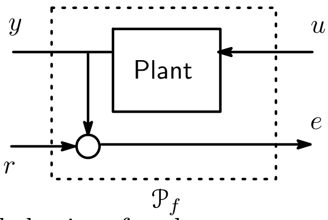

Fig. 5. The full behavior of a plant

desired behavior $\mathcal{S}$ as the kernel representation $Kw = 0$ with $K = (K_r \ \ -K_y)$. By noticing that Eq.(5) should hold for the implementability, we see that $w \in \mathcal{N}$, which is equivalent to that $r = y$ and $R_y(\sigma)y = 0$ hold, implies $K(\sigma)w = (K_r(\sigma) - K_y(\sigma))y = 0$. This conditional implication also implies that $K_r - K_y$ includes R_y as the factor, that is, K should also include R_y as the left factor. From this observation, we put the desired behavior $\mathcal{S}'$ as the kernel representation $K'w = R_y(K_r \ \ - K_y)w = 0$ instead of $\mathcal{S}$. It is also should be noted that the controllable part of $\mathcal{S}$ and $\mathcal{S}'$ are the same, which make no difference in the case in which the desired behavior is given as the set of the trajectories constrained by the transfer function.

A polynomial matrix L satisfying $LR = K'$ (cf.Eq.(6)) is described by

$$L = (\ R_y K_r\ \ K_r - K_y\ ). \tag{10}$$

Next, substituting this L into Eq.(7) with M in Eq.(9) yields that a canonical controller is described by

$$(\ R_y K_r\ \ R_u(K_r - K_y)\ ) \begin{pmatrix} e \\ u \end{pmatrix} = 0. \tag{11}$$

Of course, in the case in which we know R_y and R_y explicitly, this representation is used for the implementation of a canonical controller. However, since we assume that a mathematical model is unknown and our setting is based on the direct use the plant behavior, so we should eliminate R_y and R_u.

It is also natural to assume that a controller is controllable. Thus, we introduce an image representation of a canonical controller as

$$\begin{pmatrix} e \\ u \end{pmatrix} = \begin{pmatrix} C_e \\ C_u \end{pmatrix} \ell. \tag{12}$$

At the same time, $(e,\ u)$ should also satisfies Eq.(11), so combing Eq.(11) and Eq.(12) yields

$$(\ R_y K_r\ \ R_u(K_r - K_y)\ ) \begin{pmatrix} C_e \\ C_u \end{pmatrix} \ell = 0. \tag{13}$$

Here, it is also possible to regard Eq.(13) as one of the kernel representations with respect to ℓ. By modifying the point of view on the variable in this way, together with the assumption that a plant is single input and single output, Eq.(13) can be also written as

$$(\ C_e\ \ C_u\ ) \begin{pmatrix} K_r & 0 \\ 0 & (K_r - K_y) \end{pmatrix} \begin{pmatrix} R_y \\ R_u \end{pmatrix} \ell = 0. \tag{14}$$

Since $(R_y\ R_u)^{\mathrm{T}}\ell$ is also an image representation of the behavior (u, y) with input/output structure. Hence, we obtain

$$(\ C_e\ \ C_u\ ) \begin{pmatrix} K_r & 0 \\ 0 & (K_r - K_y) \end{pmatrix} \begin{pmatrix} u \\ y \end{pmatrix} = 0. \tag{15}$$

This equation represents one of the relationships among the trajectory of a plant, the desired behavior and a canonical controller.

Now, we consider Eq.(15) on the finite interval data. We denote the unknown rational function $C_u C_e^{-1}$ with C. Let n_a and n_b be the degree of K_r and $K_r - K_y$, respectively. Then we see $K_r u_{[0,N]} \in \mathbb{R}^{\mathtt{N-n_a+1}}$ and $(K_r - K_y) y_{[0,N]} \in \mathbb{R}^{\mathtt{N-n_b+1}}$. In the case of $n_a \neq n_b$, by denoting $\nu := \min(n_a,\ n_b)$, we truncated the vector which has a greater size than the other so as to have the same size ν. We denote these truncated signals as $K_r u_{[0,N]}|_{[0,\nu-1]} \in \mathbb{R}^\nu$ and $(K_r - K_y) y_{[0,N]}|_{[0,\nu-1]} \in \mathbb{R}^\nu$. Then Eq.(15) is described by

$$K_r u_{[0,N]}|_{[0,\nu-1]} = \mathcal{T}^C_{[0,\nu-1]} (K_r - K_y) y_{[0,N]}|_{[0,\nu-1]}. \tag{16}$$

It is also possible to expand Eq.(16) laterally such that $\mathcal{T}^{K_r u_{[0,N]}|_{[0,\nu-1]}}_{[0,N-1]}$ and $\mathcal{T}^{(K_r - K_y) y_{[0,N]}|_{[0,\nu-1]}}_{[0,N-1]}$ are upper triangle form. As a result, we also see that

$$\mathcal{T}^{K_r u_{[0,N]}|_{[0,\nu-1]}}_{[0,\nu-1]} = \mathcal{T}^C_{[0,\nu-1]} \mathcal{T}^{(K_r - K_y) y_{[0,N]}|_{[0,\nu-1]}}_{[0,\nu-1]} \tag{17}$$

holds. Summing up the above discussion, the main result of this paper can be formalized as follows

Theorem 5.1. Assume that we are given finite data $u_{[0,N]}$ and $y_{[0,N]}$. Moreover, assume that a specification $\mathcal{S}$ is described by $Kw = (K_r\ \ - K_y)w = 0$. Then C_e and C_u satisfying Eq.(15) induces a kernel representation of a canonical controller, or equivalently, $\mathcal{T}^C_{[0,N-1]}$ satisfying Eq.(17) induces Markov parameters of a transfer function of a canonical controller. □

The structures of $\mathcal{T}^{K_r u_{[0,N]}|_{[0,\nu-1]}}_{[0,N-1]}$ and $\mathcal{T}^{(K_r - K_y) y_{[0,N]}|_{[0,\nu-1]}}_{[0,N-1]}$ are upper triangle, so is $\mathcal{T}^C$. The elements of $\mathcal{T}^C$ satisfying Eq.(17) form the set of the Markov parameters of a canonical controller. Particularly, in the case of $((K_r - K_y) y_{[0,N]})(0) \neq 0$, $(\mathcal{T}^{(K_r - K_y) y_{[0,N]}|_{[0,\nu-1]}}_{[0,N-1]})^{-1}$ exists, which implies that Eq.(17) can be exactly solved with respect to $\mathcal{T}^C$ and then a canonical controller can be also realized as a proper rational function, that is, it can be implemented as a feedback controller. The method of the realization can be found in many literatures on the conventional system theory.

Another way for constructing a canonical controller is to take polynomials $C_u(\theta) = \sum_{i=0}^m \theta_i \sigma^i$ and $C_e(\rho) = \sum_{i=1}^n \rho_i \sigma^i$ with unknown parameters $\theta := (\theta_0\ \theta_1 \cdots \theta_m)^{\mathrm{T}}$ and $\rho = (\rho_1\ \rho_2\ \cdots\ \rho_n)^{\mathrm{T}}$, respectively. Eq.(15) is also described by

$$C_e(\rho) K_r u_{[0,N]} = C_u(\theta)(K_r - K_y) y_{[0,N]} \tag{18}$$

We denote $\bar{u}_{[0,N-n_a]} := K_r u_{[0,N]}$ and $\bar{y}_{[0,N-n_b]} := (K_r - K_y) u_{[0,N]}$. Then Eq.(18) is also written as

$$\Gamma \begin{pmatrix} \theta \\ \rho \end{pmatrix} = \begin{pmatrix} \bar{u}_0 \\ \vdots \\ \bar{u}_{N-n_a-m-1} \end{pmatrix} \tag{19}$$

where

$$\Gamma := \begin{pmatrix} \bar{y}_0 & \cdots & \bar{y}_n & -\bar{u}_1 & \cdots & -\bar{u}_m \\ \vdots & & \vdots & \vdots & & \vdots \\ \bar{y}_{N-n_b-n-1} & \cdots & \bar{y}_{N-n_b-1} & -\bar{u}_{N-n_a-m} & \cdots & \bar{u}_{N-n_a-1} \end{pmatrix}.$$

In most cases, $N-1$ is greater than $n+m+1$. Since it is impossible to solve the linear algebraic equation Eq.(19) in such a case, we perform the well-known least squares method alternatively. Although the controller constructed as above is an approximated canonical controller, this can be implemented as a feedback controller because of that we can fix the degree of C in advance.

5.2 An unfalsified controller

Now we observe that a linear canonical controller in our proposed method is also characterized as an unfalsified controller. Assume that y and u are the actual output and the input data. We first calculate the fictitious reference $\tilde{r}$ described by

$$\tilde{r} = C^{-1}u + y \tag{20}$$

or equivalently

$$\begin{pmatrix} C_u & -C_u & 0 & -C_e \end{pmatrix} \begin{pmatrix} \tilde{r} \\ y \\ \tilde{e} \\ u \end{pmatrix} = 0 \tag{21}$$

where $\tilde{e}$ is the fictitious error, which also yield the actual trajectory y and u together with $\tilde{r}$ and a controller C which is to be designed. Of course, $(\tilde{r}, y)$ and $(\tilde{e}, u)$ should also satisfy the plant equation Eq.(8) and (9). Since $R_y y = R_u u$ is a trivial relation because of that u and y are the data from the plant, we can eliminate this equation. Thus, the interconnected behavior between the plant and the controller is described by

$$\begin{pmatrix} 1 & -1 & -1 & 0 \\ C_u & -C_u & 0 & -C_e \end{pmatrix} \begin{pmatrix} \tilde{r} \\ y \\ \tilde{e} \\ u \end{pmatrix} = 0. \tag{22}$$

by using the actual data of a plant u and y. We want the behavior constrained in Eq.(22) to satisfy $\mathcal{S}$ exactly. In order to achieve this, we take the interconnection of the kernel of Eq.(22) and $\mathcal{S}$, that is,

$$\begin{pmatrix} 1 & -1 & -1 & 0 \\ C_u & -C_u & 0 & -C_e \\ K_r & -K_y & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{r} \\ y \\ \tilde{e} \\ u \end{pmatrix} = 0. \tag{23}$$

Here, since $\tilde{r}$ and $\tilde{e}$ are in the fictitious trajectory, we can regard this two signal as latent variables. In fact, it is easy to see that $\tilde{r}$ and $\tilde{e}$ can be eliminated from Eq.(23). Thus, the behavior on c is also described by

$$\begin{pmatrix} -K_r C_e & C_u(K_y - K_r) \end{pmatrix} \begin{pmatrix} y \\ u \end{pmatrix} = 0. \tag{24}$$

Applying the finite time data $y_{[0,N]}$ and $u_{[0,N]}$ into Eq.(24), we obtain Eq.(16). That is, the result we proposed here is also regarded as a synthesis of linear canonical controllers within the unfalsified control framework.

5.3 The relationship with VRFT and FRIT

In the case in which the structure of a controller is fixed with unknown parameters, the proposed method here is regarded as the controller parameter tuning so as to achieve the desired response with only one-shot experiment. Here we see the relationships with the other approach like VRFT, FRIT, and so on.

In VRFT (cf. Campi et al. [2002]), the virtual reference $\bar{r}$ is calculated so as to $y_{[0,N]} = T_d \bar{r}_{[0,N]}$. One takes the sum of the squared error between the actual input $u_{[0,N]}$ and the virtual input with respect to a parameterized controller $C_u(\theta)C_e(\rho)^{-1}$ as the cost function described by

$$J_V := \|u_{[0,N]} - C_u(\theta)C_e(\rho)^{-1}(\bar{r}_{[0,N]} - y_{[0,N]})\|^2 \tag{25}$$

By fixing the parameter ρ of $C_e(\rho)$, the minimization of J can be modified to that of

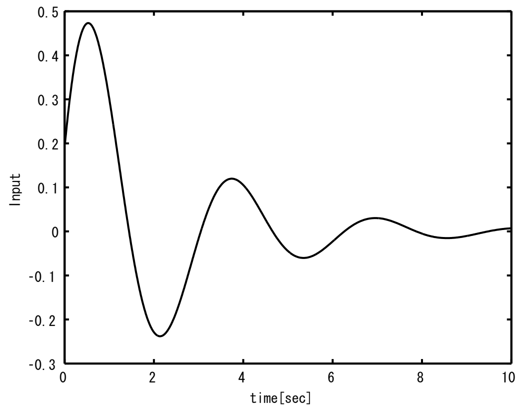

Fig. 6. The initial input data u

$$J'_V = \left\|C_e K_r u_{[0,N]} + C_u(\theta)(K_r - K_y)y_{[0,N]}\right\|^2. \tag{26}$$

which can be computed by the least squares method. Suppose that the above cost function is completely minimized , i.e., equal to zero ideally, which coincides with Eq.(19). In this sense, VRFT is also regarded as the special case of our approach. Note that only the denominator of a controller is designed in VRFT while our proposed method designs both of the numerator and the denominator.

In FRIT (cf. Souma et al. [2004]), which is also an application of the unfalsified control, we use the fictitious reference $\hat{r}$ described by Eq.(20). Then, we minimize the cost function

$$J_F := \left\|y_{[0,N]} - T_d \tilde{r}_{[0,N]}\right\|^2. \tag{27}$$

Suppose also that the above cost function is completely minimized, i.e., equal to zero ideally. Then, as shown in the unfalsified control theory, we can obtain C so as to satisfy Eq.(13). Thus, FRIT is also a special case of our proposed method. Note that FRIT should be performed in the off-line nonlinear optimization, in which there are some critical problems from the view points of numerical computations while our approach is performed by solving linear algebraic equations.

6. EXAMPLE

We show a numerical example in order to show the validity of the result. We are given input u and output y obtained in the finite interval with the sampling time 0.01[s] as Fig.6 and Fig. 7, respectively. These data have been obtained from the feedback system with a certain linear controller and unknown linear plant. In Fig.7, the desired output y_d is also plotted. The trajectory $w_d := (y_d \; r_d)^{\mathrm{T}}$ with the constant r_d is in $\mathcal{S}$, which is obtained as the kernel representation described by $(\, K_r(\sigma) \; -K_y(\sigma) \,)\, w_d = 0$, where $K_r(\sigma) = 7.19\sigma + 6.61 \times 10^{-3}$ and $K_y(\sigma) = \sigma^2 - 1.77\sigma + 0.779$. In order to obtain a linear canonical controller that achieve $(y_d, r) \in \mathcal{S}$, we apply the proposed method by using the data $(u,\, y)$ and the polynomials K_r and K_y. Here, we set $C_u = \theta_0 + \sigma$, $C_e = \rho_0 + \rho_1\sigma$ in order to guarantee that the designed controller is to be feedback one. As a result, the desired canonical controller is obtained as $\rho_0 = -0.0744$, $\rho_1 = 0.428$, and $\theta_0 = -0.806$, by solving the least squares method described as Eq.(19). The output after implementing it in the feedback loop is

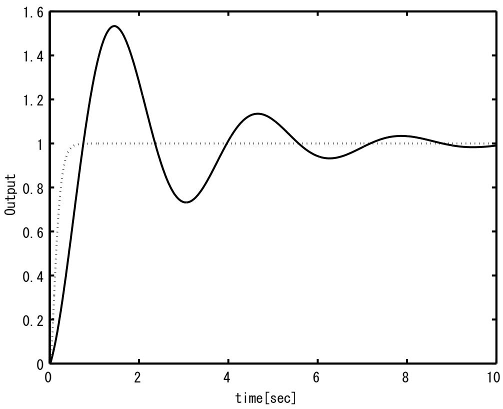

Fig. 7. Th initial output data y (The real line) and the desired output y_d (The dotted line)

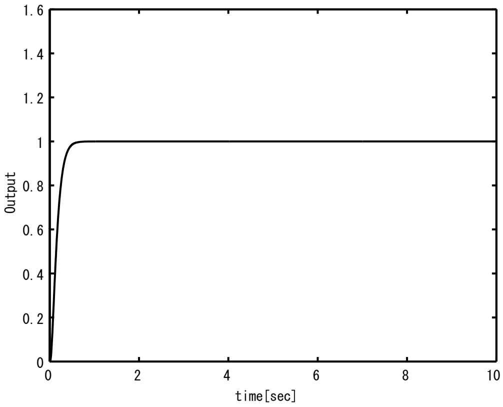

Fig. 8. The output data y (The real line) and the desired output y_d (The dotted line)

illustrated in Fig.8. In this figure, y and y_d are almost the same, thus we can see that the proposed method works effectively.

7. CONCLUSION

In this paper, we have proposed a synthesis of a canonical controller based on using the finite interval data of a plant without mathematical model. We have also seen the relationship with other data-driven control system synthesis like VRFT and FRIT. We have also given a numerical example to see the validity of the proposed method.

As future studies, we are studying the issues on the effect of noise, the extension to the multi-input and multi output case, and the stability of the interconnected systems with the proposed canonical controllers. The regularity of the implementation is also interesting problem, which would be attacked from the theoretical points of view. Moreover, our approach would be deeply related to the results on data-driven control system synthesis approach studied in Markovsky et al. [2005], Markovsky and Rapisarda [2007], Yamamoto and Okano [2006] and Fujisaki et al. [2005]. Thus, the studies on relationships between these two approaches is one of the theoretical direction.

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