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Backstepping Control Design of a Single-Link Flexible Robotic Manipulator

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Abstract: In this paper, the backstepping design scheme is developed for the tip-position trajectory tracking control of single-link flexible robotic manipulator systems. An infinite dimensional dynamic model of a single-link flexible manipulator is derived through the assumed modes method associated with Lagrange approach. For simplicity, a linearized system model would be analyzed and investigated for our tracking control design with elimination of tip vibration. That is to say, the proposed backstepping controller is not only to stabilize the flexible robotic manipulator, but also to drive the trajectory tracking error and tip-deflection to converge to zero asymptotically. Furthermore, some simulation results are given to illustrate the excellent performance of the backstepping control design applied to a single-link flexible robotic manipulator.

1. INTRODUCTION

At present, flexible manipulators are commonly used in manufacturing industry. The effects of link flexibility in robot manipulator systems have received much attention among robotics and control researchers. The design of flexible link robotic manipulator is primarily motivated by the need for lightweight robot systems. In the manufacturing industry, higher productivity needs to have manipulators that can operate with higher speed, more precision, less power consumption, lower cost and improved payload handing capabilities. These requirements translate into manipulators that have structural flexibility and are lightweight. For example, a robotic manipulator that is cleaning a fragile object needs to have apparent structural flexibility so that manipulators in the position control do not produce large forces that may damage the object. Unluckily, the rigidity of lightweight link is not enough. Using a long lightweight link will lead to a position control problem since the links are subject to deflection and vibration. In many cases, their end-effectors are required to move from one place to another and follow some reference trajectories. The problem of vibration will cause inaccuracy of tracking control. Therefore, our control objects are to drive the trajectory of the system to track the reference trajectory quickly and restrain the vibration of tip position.

The problem of trajectory tracking for a flexible robotic manipulator has been discussed by many studies. Paden (1992) has integrated previous work on rigid-link manipulator control and the inverse dynamics of flexible manipulators to design a new kind of flexible manipulator control law for exponentially stable tracking. A new adaptive control scheme in Yang (1991) was designed for the tip position control of a single-link flexible manipulator handling unknown loads. This new control scheme enables the load changes to be carried out in an active state without creating any transients. The flexible manipulator is lumped to a spring mass system based on which a very simple and effective PD type controller in Ge (1996) has been developed to achieve tip tracking performance with BIBO stability. The control strategy consisted essentially of two parts in Geniele (1997). The first part is an inner control loop that incorporates a term to assign the system's transmission zeros at desired locations in the complex plane and a feedback term to move the system's poles to appropriate positions in the left-half plane. The second part is a feedback servo loop that allows tracking of the desired trajectory.

The sliding mode control, nonlinear feedback control, fuzzy control, H_∞ nonlinear control and other design schemes are commonly used in the application of tracking control of flexible manipulator systems shown in Dogan (2004); Etxebarria (2005); Ge (1997); Lee (2003); Mannani (2003); Shawky (2002); Thomas (1997); Wang (1996); Yang (2003). In addition, the aim in the modeling of a flexible manipulator system is to obtain an accurate model representing actual system behavior. It is important to recognize the flexible nature and dynamic behavior of the system and construct an appropriate mathematical structure for modeling of the system. The two most generally used modeling methods are the assumed modes method (AMM) and finite element method (FEM). The flexible manipulator was modeled using AMM in Korolov (1988) and a linear state-space model was obtained. The authors of Sakawa (1985) gave a detailed modeling process of flexible manipulator under clamped-free boundary conditions with AMM. In recent years, nonlinear and adaptive backstepping control schemes shown in Khalil (2002); Krstić (1995) are commonly used in many complex systems, such as 360-degree inverted pendulum (Tsai (2003)), active suspension system (Lin (2003)), anti-lock braking system (Lin (2007)), Furuta inverted pendulum (Fu (2005)) and permanent magnet synchronous motors (Ke (2005)). In addition, according to Chen (2004), the backstepping control design scheme has been successfully applied to a multiple-link rigid robotic manipulator to obtain excellent performance.

In this paper, a backstepping design scheme is proposed for the tip-position trajectory tracking control of a single-link flexible manipulator. The control goal is to drive the trajectory of the system and to track the reference trajectory quickly. The stability of the resulting closed-loop system is demonstrated via Lyapunov stability theory. In addition, the proposed control design scheme is applied to a linearized model of single-link flexible manipulator for simulations. As a result, the backstepping control design is not only to stabilize the single-link flexible manipulator system, but also to drive the tracking error and tip-deflection to converge to zero asymptotically. The remainder of this paper is organized as follows. In Section 2, the dynamic model of a single-link flexible manipulator is introduced and analyzed. Then a linear controller is designed to achieve our control objective with backstepping scheme in Section 3. Finally, the simulation results are illustrated in Section 4, and some concluding remarks are given in Section 5.

2. SYSTEM MODEL AND DYNAMICS

The model of a single-link flexible manipulator Martins (2003) is shown in Fig. 1. The flexible manipulator is modelled as an Euler-Bernoulli beam clamped on the rotor of a motor. A torque applied by motor rotates the beam in the horizontal plane without the gravitation influence.

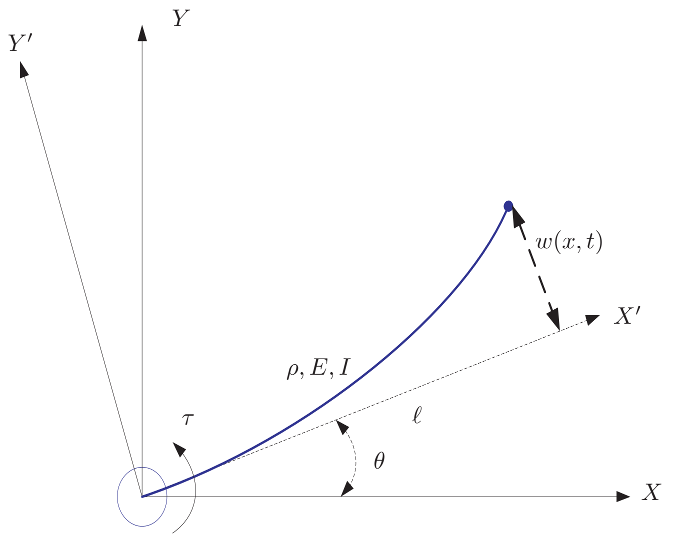

Fig. 1. A single-link flexible robotic manipulator model

Considering a single-link flexible manipulator in a horizontal plane. A fixed coordinate (X, Y) and a rotational coordinate (X', Y') are assigned as shown in Figure 1. The axis X' is rotating with the slope of the flexible link at the joint. The angle of X' with respect to X is denoted as $\theta(t)$ and the tip-deflection with respect to X' is denoted as $w(\ell, t)$. Using the assumed modes method, the solution of the tip-deflection is assumed to be a linear combination of admissible functions multiplied by time dependent generalized coordinates, and formulated as

$$w(x,t) = \sum_{i=1}^{n} \phi_i(x) q_i(t), \tag{1}$$

where $q_i(t)$ are the generalized coordinates, and $\phi_i(x)$ are admissible functions which satisfy the geometric boundary conditions and are given by

$$\phi_i(x) = \frac{\cosh \mu_i x - \cos \mu_i x - r_i(\sinh \mu_i x - \sin \mu_i x)}{\sqrt{\ell}}. \tag{2}$$

In (2), the parameters r_i and μ_i are defined as where

$$r_i = \frac{\cosh \lambda_i + \cos \lambda_i}{\sinh \lambda_i + \sin \lambda_i},\ \mu_i = \frac{\lambda_i}{\ell},\ i = 1, 2, ..., n \tag{3}$$

where λ_i is the solution of the following equation:

$$\cosh \lambda_i \cos \lambda_i + 1 = 0. \tag{4}$$

In addition, a linearized dynamic model can be obtained by AMM and Lagrange's equation. The dynamics of the single-link flexible manipulator can be written as

$$M \begin{bmatrix} \ddot{\theta} \\ \ddot{q} \end{bmatrix} + K \begin{bmatrix} \theta \\ q \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tau, \tag{5}$$

where

$$q = [\, q_1 \ q_2 \ \ldots \ q_n \,]^T .$$

In (5), the matrix M is the positive definite symmetric inertia matrix with the definition

$$M = \begin{bmatrix} J & M_{1\theta} & M_{2\theta} & \cdots & M_{n\theta} \\ M_{1\theta} & M_{11} & 0 & \cdots & 0 \\ M_{2\theta} & 0 & M_{22} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ M_{n\theta} & 0 & 0 & \cdots & M_{nn} \end{bmatrix}, \tag{6}$$

and the matrix K can be defined as

$$K = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & M_{11}\omega_1^2 & 0 & \cdots & 0 \\ 0 & 0 & M_{22}\omega_2^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & M_{nn}\omega_n^2 \end{bmatrix}, \tag{7}$$

where

$$J = \int_0^\ell \rho x^2 dx + I_H,$$

$$M_{ii} = \int_0^\ell \rho \phi_i^2 dx,$$

$$M_{i\theta} = \int_0^\ell x \rho \phi_i dx,$$

$$\omega_i^2 = \frac{1}{M_{ii}} \int_0^\ell EI(\phi_i'')^2 dx\,,$$

Note that J is the hub plus the beam inertia, ρ is the mass per unit length of the beam, $\theta(t)$ is the hub angle, τ is the input control torque applied to the hub, and ℓ is the length of the single-link flexible manipulator. In addition, ω_i are the natural frequencies of the flexible manipulator, which are given by

$$\omega_i = \mu_i^2 \sqrt{\frac{EI}{\rho}},\ i = 1, 2, ..., n \tag{8}$$

where E is the Young's modulus of the arm material, I is the inertia of the cross-section, and their multiplication EI is the flexural rigidity of the flexible beam. After all, the output of the system is chosen to be the tip-position. From Figure 1, the tip-position can be described by

$$y(x,t) = x\theta(t) + w(x,t). \tag{9}$$

For the convenience of backstepping control design, by using the following state assignments

$$\begin{aligned} x &= \left[\, \theta \;\; q \;\; \dot\theta \;\; \dot q \,\right]^T \\ &= \left[\, x_1 \;\; x_2 \;\; \cdots \;\; x_{n+1} \;\; x_{n+2} \;\; x_{n+3} \;\; \cdots \;\; x_{2n+2} \,\right]^T, \end{aligned}$$

the matrix equation (5) can be written as follows:

$$\dot x = \begin{bmatrix} 0 & 0_{1\times n} & 1 & 0_{1\times n} \\ 0_{n\times 1} & 0_{n\times n} & 0_{n\times 1} & I_{n\times n} \\ 0 & H_r & 0 & 0_{1\times n} \\ 0_{n\times 1} & H_f & 0_{n\times 1} & 0_{n\times n} \end{bmatrix} \begin{bmatrix} \theta \\ q \\ \dot\theta \\ \dot q \end{bmatrix} + \begin{bmatrix} 0 \\ 0_{n\times 1} \\ G_r \\ G_f \end{bmatrix} \tau,$$

where H_r is the $1 \times n$ vector, H_f is the $n \times n$ matrix, G_r is the 1×1 vector, G_f is the $n \times 1$ vector, and these terms are represented as follows:

$$\begin{aligned} H_r &= \frac{1}{\det M} \left[\, h_1 \;\; h_2 \;\; \cdots \;\; h_n \,\right], \\ G_f &= \frac{1}{\det M} \left[\, g_1 \;\; g_2 \;\; \cdots \;\; g_n \,\right]^T, \\ G_r &= \frac{g_r}{\det M}, \\ H_f &= \frac{1}{\det M} \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{21} & h_{22} & \cdots & h_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n1} & h_{n2} & \cdots & h_{nn} \end{bmatrix}. \end{aligned}$$

3. BACKSTEPPING CONTROL DESIGN

The control objective for the flexible robot arm system is to design a linear backstepping controller so that the hub angle and the tip-position of the single-link flexible manipulator can follow a reference trajectory quickly. In addition, the oscillation of tip-deflection should be eliminated simultaneously. Now, the backstepping scheme is applied to design the controller for achieving our control goal. The backstepping design procedure consists of the following steps:

STEP 1: The regulated variable is selected as

$$z_1 = (x_1 - x_r) + k_1[(\sum_{i=1}^{n} g_i)x_{n+2} - g_r(\sum_{i=1}^{2n-1} x_{i+3})], \tag{10}$$

where x_r is the desired reference trajectory, k_1 is a design constant. The control objective is to make the trajectory error and the vibration of the tip-deflection converge to zero. Therefore, the derivative of z_1 is computed as

$$\begin{aligned} \dot z_1 &= x_{n+2} - \dot x_r + \frac{k_1}{\det M}[(\sum_{i=n}^{n} g_i)(\sum_{i=1}^{n} h_i x_{i+1})], \\ &- \frac{k_1 g_r}{\det M}[\sum_{i=1}^{n}(h_{1i} + h_{2i} + \cdots + h_{ni})x_{i+1}]. \end{aligned} \tag{11}$$

The state x_{n+2} is used as the virtual control variable, for which the stabilizing function is chosen as

$$\begin{aligned} \alpha_1 &= \frac{k_1 g_r}{\det M}[\sum_{i=1}^{n}(h_{1i} + h_{2i} + \cdots + h_{ni})x_{i+1}] \\ &- \frac{k_1}{\det M}[(\sum_{i=1}^{n} g_i)(\sum_{i=1}^{n} h_i x_{i+1})] + \dot x_r - c_1 z_1, \end{aligned} \tag{12}$$

where c_1 is a positive design constant. The corresponding error state variable is defined as

$$z_2 = x_{n+2} - \alpha_1. \tag{13}$$

STEP 2: The derivative of z_2 is computed as follows:

$$\dot z_2 = \vartheta_1(x) + N_1 \tau, \tag{14}$$

where

$$\begin{aligned} \vartheta_1(x) &= \frac{k_1}{\det M}[(\sum_{i=1}^{n} g_i)(\sum_{i=1}^{n} h_i x_{i+n+2})] \\ &- \frac{k_1 g_r}{\det M}[\sum_{i=1}^{n}(h_{1i} + h_{2i} + \cdots + h_{ni})x_{i+n+2}] \\ &+ \frac{1}{\det M}\sum_{i=1}^{n}(h_i x_{i+1}) - \ddot x_r + c_1 \dot z_1, \\ N_1 &= \frac{g_r}{\det M}. \end{aligned}$$

The controller input τ appears in (10), so the control law is chosen as

$$\tau = \frac{1}{N_1}(-\vartheta_1(x) - z_1 - c_2 z_2), \tag{15}$$

where c_2 is a positive design constant. After finishing the linear control design procedure, now the following Lyapunov function can be considered for the analysis of system stability.

$$V = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2, \tag{16}$$

form (10)-(15), the derivative of (16) is computed as

$$\begin{aligned} \dot V &= z_1(z_2 - c_1 z_1) + z_2(\vartheta_1(x) + N_1\tau) \\ &= -c_1 z_1^2 - c_2 z_2^2 < 0. \end{aligned} \tag{17}$$

According to Lyapunov stability theorem, the resulting close-loop system (z_1, z_2) is asymptotically stable. Since there are only two steps at the backstepping design procedure, Therefore, the stability of the other states of the system cannot be guaranteed. Now, we went to analyze the internal dynamics to ensure whether the resulting closed-loop system is stable or not. In this paper, zero dynamics would be analyzed by system of two modes. The zero dynamics can be written as follows:

$$\dot{x} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -A_1 & 0 & k_1C_1 & k_1D_1 \\ 0 & -B_1 & k_1C_2 & k_1D_2 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_5 \\ x_6 \end{bmatrix}, \qquad (18)$$

where

$$A_1 = \omega_1^2,$$
$$B_1 = \omega_2^2,$$
$$C_1 = M_{11}M_{1\theta}\omega_1^2,$$
$$C_2 = M_{11}M_{2\theta}\omega_1^2,$$
$$D_1 = M_{11}M_{1\theta}\omega_2^2,$$
$$D_2 = M_{11}M_{2\theta}\omega_2^2.$$

A_1, B_1, C_1, C_2, D_1 and D_2 are positive constants, and the characteristic equation of (18) can be written as follows:

$$\Delta(s) = s^4 - k_1(C_1 + D_2)s^3 + (A_1 + B_1)s^2 \\ -k_1(B_1C_1 + A_1D_2)s + A_1B_1 = 0. \qquad (19)$$

Now, the Routh stability criterion will be used to analyze whether the resulting closed-loop system is stable or not. The Routh table can be described as follows:

$$\begin{array}{llll} s^4 & 1 & Q_2 & Q_4 \\ s^3 & -k_1Q_1 & -k_1Q_3 & 0 \\ s^2 & P & Q_4 & 0 \\ s^1 & \frac{1}{Q_1P}(-k_1C_1D_2)(A_1 - B_1)^2 & 0 & 0 \\ s^0 & Q_4 & 0 & 0 \end{array} \qquad (20)$$

where

$$Q_1 = C_1 + D_2$$
$$Q_2 = A_1 + B_1$$
$$Q_3 = B_1C_1 + A_1D_2$$
$$Q_4 = A_1B_1$$
$$P = \frac{C_1A_1 + D_2B_1}{Q_1}.$$

In (20), P is a positive constant. We can know that $k_1 < 0$ will make the closed-loop system stable. According to Lyapunov stability theorem, the resulting close-loop system (z_1, z_2) is asymptotically stable. That is to say, our backstepping controller can make z_1 and z_2 to converge to zero. Therefore, choosing suitable design constant k_1 can make the trajectory tracking error converges to zero. Moreover, the tip-deflection is also eliminated at the some time. Finally, the trajectory of single-link flexible manipulator follows the desired trajectory and the tip-deflection converges to zero.

4. SIMULATION RESULTS

A single-link flexible robotic manipulator is shown in Fig. 1. A linear model for the flexible manipulator is obtained in (5). Therefore, an infinite dimensional model for the flexible manipulator is used by considering n modes of assumed modes method. The linear model of single-link flexible manipulator is used for the simulations of the backstepping control design scheme. The reference trajectories are given in the following two cases:

- Case 1 : $x_r = 2 - 2\exp^{-t}$, and
- Case 2 : $x_r = \sin t$.

Table 1. System parameters

parameter	value	parameter	value
EI	$2Nm^2$	l	$1m$
ρ	$0.1kg/m$	J	$0.08kgm^2$
ω_1	$15.7rad/s$	ω_2	$97.6rad/s$
$\phi_1(\ell)$	2	$\phi_2(\ell)$	-2.001
M_{11}	0.1	M_{22}	0.1
$M_{1\theta}$	0.056	$M_{2\theta}$	0.009
λ_1	1.875	λ_2	4.694

Table 2. Design constants

Case 1	value	Case 2	value
c_1	7	c_1	100
c_2	7	c_2	100
k_1	-10	k_1	-2

In our simulations, the simulation parameters of a single-link flexible manipulator are listed in Table 1, design constants for the control law are listed in Table 2 and two reference trajectories are given in simulations. In our simulation results, we will describe the performance of the proposed control scheme. The simulation results are able to verify the effectiveness of the backstepping controller. The simulation results of a single-link flexible robotic manipulator are shown in Fig. 2 and Fig 3 with different reference inputs. The plots of the tip-deflection, the trajectory of tip-position, the trajectory of hub angle and the control law are shown in Fig. 2 (Case 1) and Fig. 3 (Case 2). The desired trajectory is shown by dashed lines, and the actual trajectory is shown by solid lines. In Fig. 2 (Case 1), obviously our backstepping controller will make the tip-position track the reference trajectory. We can know that the controller will make the hub angle track to desired trajectory. Moreover, the tip-deflection is eliminated by controller. Finally, the tip-deflection will converge to zero asymptotically. In Fig. 3 (Case 2), we can know that the tip-deflection will be restrained in a small range. Finally, tip-position will track to desired trajectory. That is to say, choosing suitable design constant k_1 can make the trajectory tracking error converges to zero. Moreover, the tip-deflection is also eliminated at the some time. Finally, the trajectory of single-link flexible manipulator follows the desired trajectory and the tip-deflection converges to zero.

5. CONCLUDING REMARKS

In this paper, a backstepping design scheme has been developed for the control of a single-link flexible robotic manipulator to achieve the target of position tracking control. That is to say, the movement of single-link flexible manipulator can be guaranteed to follow the reference trajectory and restrain the vibration of tip-position. According to Lyapunov stability theorem, the resulting close-loop system is asymptotically stable. From the analysis of the zero dynamics, we can know that $k_1 < 0$ will make the closed-loop system stable. In other words, choosing suitable design constant k_1 can make the trajectory tracking error converge to zero. Moreover, the tip-deflection is also eliminated at the same time. Finally, the trajectory of single-link flexible manipulator follows the desired trajectory and the tip-deflection converges to zero.

In simulation results, the backstepping design scheme is not only to drive the tip-position of single-link flexible manipulator to reach the reference trajectory, but also to make the tip-deflection converge to zero. Therefore, our controller is not only to stabilize the flexible manipulator system, but also to drive the tip-position trajectory tracking error to converge to zero asymptotically. In the further research, the tip-load of the single-link flexible manipulator should be considered to investigate the performance of the backstepping design scheme, and adaptive control design should be applied to improve system performance when the tip-load is unknown.

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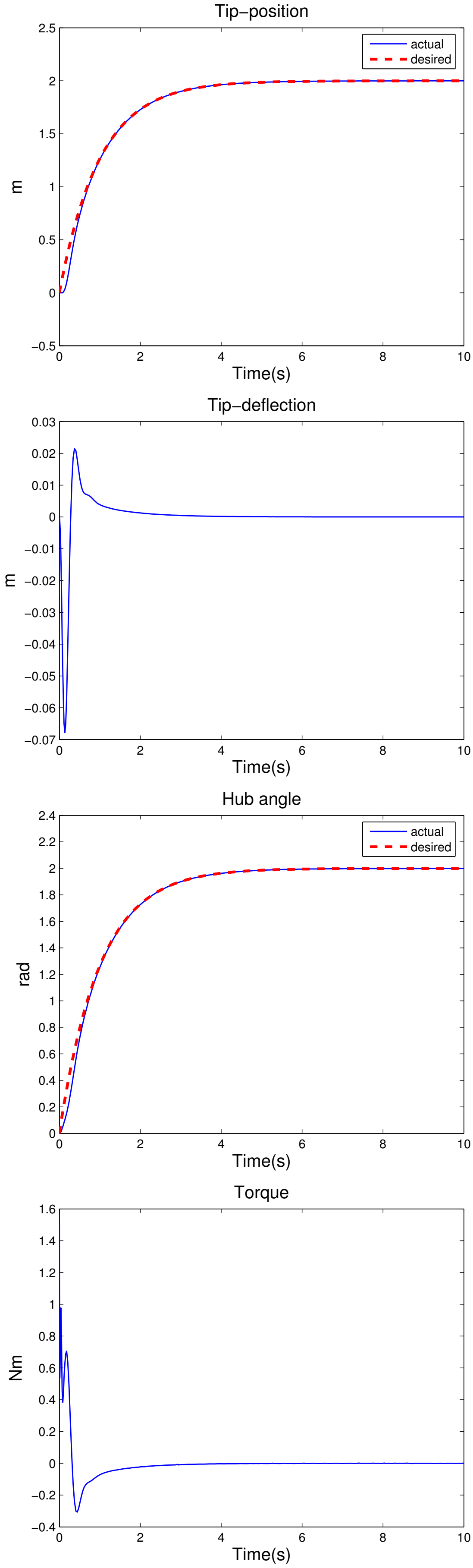

Fig. 2. The responses of a single-link flexible robotic manipulator (Case 1): actual position (— solid line) vs. desired trajectory (– – dashed line)

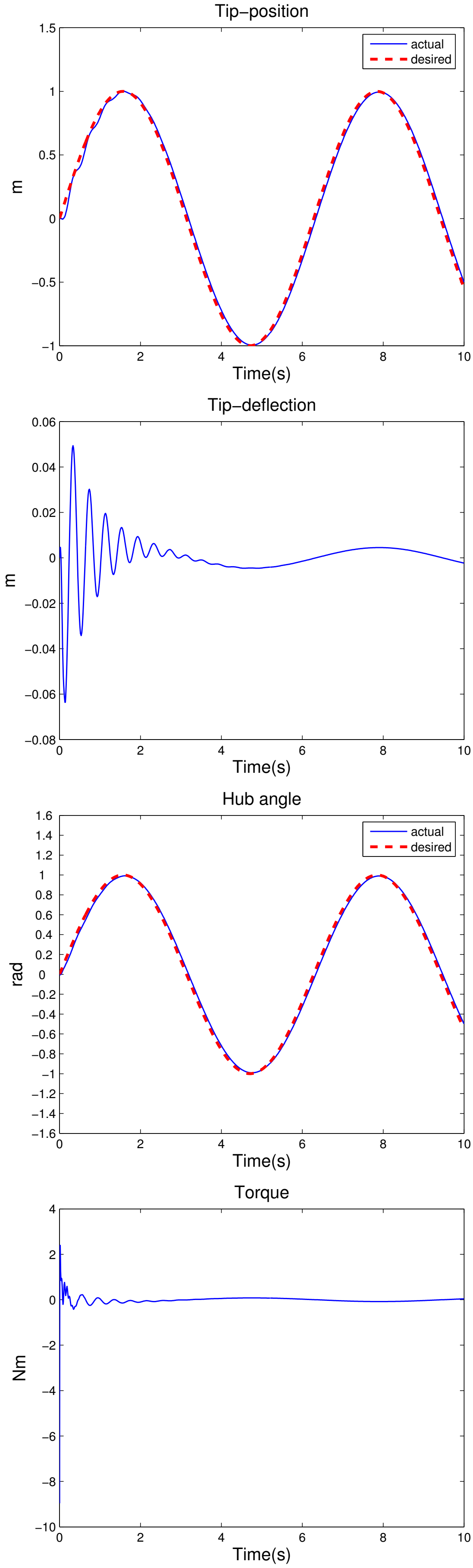

Fig. 3. The responses of a single-link flexible robotic manipulator (Case 2): actual position (— solid line) vs. desired trajectory (– – dashed line)