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The Contribution of Control Theory to the Analysis of Economic Policy

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Abstract: In this paper, we give a survey of applications of control theory to the analysis of economic policy problems. We discuss applications of closed-loop control and of optimum control theory, including deterministic, stochastic and decentralized optimum control. A critical evaluation of these approaches shows that for an empirically useful theory of economic policy, the application of dynamic game theory (which in itself originated from control theory) seems to be the most promising.

1. INTRODUCTION

Since the beginning of the 1950s, attempts have been made to extend the theory of economic policy and its applications by incorporating methods which were originally created by control engineers and later fully developed by control theorists and applied mathematicians. From the point of view of the economist, control theory can be regarded as a collection of methods, like statistics or some other fields of applied mathematics.

A great number of economic applications of control theory relate to theoretical issues, such as growth theory, the theory of exhaustible resources and intergenerational allocation problems, etc. This work shows how control theory can be applied to a problem of economic theory, but not as a real application to a practical problem. If one is interested in practically relevant problems analyzed by means of control theory, short-term stabilization policy problems seem most appropriate, since in this field there is a long tradition of and experience in applying economic models, even numerical ones. Here stabilization policy is taken to mean economic policy with a short time horizon (up to five years) that is directed towards influencing macroeconomic variables such as output, (un-)employment and the price level (inflation), etc. Although control theory can be applied to other fields of economic policy (and to more general economic problems), we will concentrate on applications to problems of stabilization policy in this paper.

When reviewing the development of control theory applications to economic policy in this sense, we can basically distinguish between three periods, which partially overlap with certain methodological approaches and certain topics:

(1) Up to the end of the 1950s, problems concerning the stability of control systems dominated, and they were analyzed mainly by means of transfer function methods.

(2) With the discovery of the maximum principle by Pontryagin and of dynamic programming by Bellman, the era of optimum (or optimal) control theory began, which parallels the use of state-space methods. Here questions like controllability, observability and optimality of dynamic systems, which are mostly represented in the time domain as difference or differential equations, were investigated. At the beginning, this theory was only capable of solving deterministic optimization problems, but later on several methods for the analysis of stochastic economic problems became available. Due to severe criticisms raised by New Classical Macroeconomists in the 1970s, the reputation of optimum control analyses of economic problems fell during the late 1970s and after.

(3) More recently, control theorists increasingly recognized that many of these critical points can be dealt with by using and extending their methodological toolbox, in particular in the direction of dynamic game theory.

This paper attempts to survey these developments and evaluate them critically.

2. ECONOMIC POLICY AS A PROBLEM OF CLOSED-LOOP SYSTEM CONTROL

The first attempt to analyze economic policy problems from the point of view of a control theorist and electrical engineer was by Tustin (1953). He proposed starting with analogies between economic models and technical systems, tentatively applying the theory of automatic regulation. Such analogies can clearly be seen by drawing schemes of dependence for aggregate quantities, that is, by representing macroeconomic models with block diagrams as is usual in electrical circuit theory and in other applications of closed-loop control systems. In this way, Tustin investigated several macroeconomic models; notions like feedback or the stability of closed-loop systems and methods like harmonic analysis and transfer functions (especially the theory of the Laplace transform), among others, were applied to these simple models. Although all these methods had widespread applications in the physical sciences and in engineering, the possibility of applying them to problems of economic policy remained rather limited. The tendency of Tustin's work was continued only in models by A. W. Phillips (1954, 1957; cf. also Allen 1967), who became more famous for inventing the

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Phillips curve. A recent evaluation of Phillips's work, including his biography as an engineer-turned-economist, together with some of his previously unpublished (yet highly relevant) papers can be found in Leeson (2000).

The reason why only relatively little work was done on directly applying traditional methods of electrical engineering to economics may be found in the fact that the conditions of system construction differ between economists and engineers. In engineering, as in economics, the goal of achieving a stable system is attained by modifying the workings of the system, which consists either in changing the dependences within the system or adding further dependences for the purpose of stabilization. The differences arise when performing this task: while the engineer can typically modify his experimental design if he is not satisfied with its result, the economist cannot usually exert influence on the internal relations of the system.

In some cases, however, the assumption is also justified that the economist as a planner or politician has some possibilities at hand to modify even the internal relations of the system. More specifically, with respect to the problem of time lags examined by Phillips, it is possible to influence (and especially to shorten) the information lag by changing the communication structure of the planning system, to affect the decision lag by changing the process of making decisions (coordination, centralization), and to influence the execution lag by changing the political infrastructure. These and similar measures can exert direct influence on the behaviour of the system, not only on the time structure but on system relations in general; likewise, political measures with a longer time perspective may have such an effect. For the engineer, however, the question of how to change his experimental design in order to improve his results is not as important and not as complex as the corresponding question for the economist who asks how and by what means he can influence the reactions of the system to achieve an improvement. It was probably for this reason that the methods of engineers in this field did not prove fruitful for economic problems.

The other possibility of arriving at stable behaviour for a system, namely by adding further supplementary feedback loops to the system, was the only one which was developed systematically by engineers and economists alike. In the models of Phillips, this means adding government expenditures to the national income identity; these government expenditures can then take arbitrary values according to the politician's target to limit oscillations in national income. But this is also something that belongs to the class of problems relating to the "design" of a control system: by that term, engineers mean questions that are related to the planning and setting up of an experiment (a mechanism, a system) which is to have certain desired properties (in our case, stability). As an interpretation of the Phillips model, this question was generally not regarded as a problem of synthesis, but instead in the sense of an input-output scheme with a black-box structure. But this is only possible because the Phillips model, by adding the term for government expenditures to the national income identity, has become an open system, precisely because it has received an input possibility for an exogenous control variable. This addition of government expenditures may be interpreted in economic terms as the "instalment of stabilization policy" or the "abandonment of a laissez-faire attitude", as a change in the structure which makes the previously closed system (with government expenditures being one of several components of national income) an open one – this in fact can be regarded as analogue to the formal manipulation of an engineer adding a supplementary feedback loop. Thus increased stability of the system in the last resort can be said to be achieved only by manipulating the structure of the system, not by merely introducing quantitative changes of inputs.

3. ECONOMIC POLICY AS A PROBLEM OF OPTIMUM CONTROL

One of the most important results of the application of "classical" methods of control theory by Phillips was the insight that under the influence of certain kinds of "intuitive" stabilization policies, simple macroeconomic multiplier-accelerator interaction models can display undesired instabilities. This is even more probable for complicated models and in economic reality. For an analysis of larger and more realistic models, however, the approach developed by Tustin and Phillips is not well suited because it consists in "trial-and-error methods" which cannot easily be extended to more complicated models. In addition the notion of "stability" of a system, which plays a crucial role in the work of Tustin and Phillips, is not made sufficiently operational by them. Moreover, further developments in control theory showed that stability, although generally a necessary condition for a good systems design, does not in itself necessarily guarantee a design with further desirable properties.

Economists therefore realized that admissible control should also have an optimizing feature in some sense. By acknowledging this, the decisive step towards the theory of optimum control was taken. Here the optimality property of a control is defined by the minimization or maximization of a criterion function (performance measure, performance integral; in economic terms: objective function, i.e. cost or welfare function). Optimality is seen to be at least as important as the property of stability, which under rather general conditions can be shown to follow from optimality as optimized systems in general are also stable.

The optimum control problem in its deterministic version consists in choosing time paths for variables (control variables) from a given class of time paths (control set) where the time paths for the variables describing the system (state variables) are given by a set of difference or differential equations (equations of motion); this choice has to be made in such a way that a given functional which depends on the time paths of the control and state variables (objective functional) is to be maximized or minimized. The static analogue to the problem of optimal control is the problem of mathematical programming. In the discrete-time case it is possible to derive solutions for the problem of optimum control from the solution of a static programming problem by redefining the variables.

Many results are available in the control theory literature on the existence of a solution to the problem of optimal control as well as on how to find such an optimal solution. The main approaches to the solution of the optimum control problem are the calculus of variations, dynamic programming (Bellman 1957), and the maximum principle (Pontryagin et al. 1962). Using these methods, it is possible to analytically determine an optimal solution for several control problems, such as, for instance, the problem of optimizing a quadratic performance criterion with a linear system. Considerable difficulties, however, arise with a more complicated objective function and with nonlinear systems.

Among the economic applications of control theory, methods deriving from Pontryagin's maximum principle have been used most frequently for theoretical purposes, paralleled by dynamic programming. The first genuine control theoretic analysis of economic policy problems was extensions to the Phillips model, which was augmented by an objective function (Sengupta 1970, Fox et al. 1966, Turnovsky 1973). Mathematical difficulties arising at the beginning of these developments (cf. Turnovsky 1974, Preston 1972) were due to the fact that some sufficient conditions for the existence of stable optimal policies had not yet been adequately identified by economists (Aoki 1973). These were the controllability of the model, i.e. the ability of the control variables to carry the state vector of the system to any neighbouring state, and its observability, which in terms of economic policy applications can be interpreted as requiring that the objective function contain all variables "relevant" for generating optimal stabilization policies.

The notions of controllability and observability were first formulated in the control theory literature by Kalman (1960), who also derived the conditions under which these properties are satisfied for linear systems (Kalman et al. 1963). For the theory of economic policy, the possibility of interpreting controllability as a dynamic analogue to Tinbergen's (1952, 1956) notion of the existence of a policy for a given system is interesting (Preston 1974, Aoki 1975). This was later extended to develop a dynamic theory of economic policy, which makes extensive use of concepts and results from systems and control theory (Preston and Sieper 1977, Preston and Pagan 1982; see also Hughes Hallett and Rees 1983, Hughes Hallett 1989, Petit 1990). Recently, interest in this theory has re-emerged in the context of policy problems with more than one decision-maker; see Acocella and Di Bartolomeo (2008), Acocella et al. (2007).

However, optimum control theory does not only provide a generalization of the theory of economic policy to the dynamic case; it is also a tool that contributes to handling practical and empirical problems of short-term economic policy with the help of econometric models. This idea was disseminated in the practice of policy-making to such an extent that from September 1972, the scientific staff of the Federal Reserve Board (FRB) used optimum control methods, especially linear-quadratic methods (control of linear systems with quadratic criteria), in order to arrive at recommendations for economic policy-makers in the case of trade-offs between unemployment and inflation (Athans and Kendrick 1974).

The path-breaking work in this direction was carried out by Pindyck (1973b) and it quickly spread among control engineers (Pindyck 1972) and economists (Pindyck 1973a). Pindyck constructed a small quarterly linear econometric model for the USA after the Korean War with the usual macroeconomic basic variables in the Keynesian sense. Such a model can be regarded, in control-theoretic terms, as representing a linear discrete-time time-invariant system. It is used as a constraint when minimizing a quadratic cost criterion. For the cost function, it is assumed that the primary aim of stabilization policy consists not in preventing oscillations in economic variables but instead in driving the variables along "ideal" paths, for instance, with low unemployment and inflation. Preventing oscillations is a secondary goal here, which is achieved to some extent simultaneously when the economy follows the desired trajectory. As a result, Pindyck obtains not only optimal stabilization policies for different cost functions but also essential insights into the dynamic behaviour of his econometric model. These could, in principle, also be obtained by simulation experiments, but only if a sufficient number of simulations were performed; the approach of looking for optimal policies is thus more efficient and more systematic than that of simulating policies with an econometric model.

A comparison of Pindyck's model with a similar work by Livesey (1971), who investigated a nonlinear model for the UK with a quadratic objective function along these lines, also shows that a linear model, although not really supported by economic theory, is generally to be preferred to a nonlinear one because of much greater ease of computability, which was of major concern in early economic control applications (the "curse of dimensionality"). Nonlinearities were, however, introduced into optimum control analyses for econometric models by Chow (1975, 1981). His approach consists in linearizing nonlinear models along a trajectory instead of at a given time point, thus retaining as many of the nonlinearity features as possible when resorting to linearization is inevitable (as is the case with larger models in economic control applications).

Another question discussed in the economics literature relates to the assumption of a quadratic cost function as used by Pindyck and Livesey, which may be regarded as being too restrictive. Indeed, the specification of a quadratic preference or welfare function has been widely used in economics because by applying it to constraints in the form of a linear system, it yields linear decision rules (Holt 1962). But the idea of symmetry, which is often entailed by a quadratic objective function, is certainly restrictive because it implies that overshooting a target gives rise to the same costs as undershooting it by the same amount. It is, however, not clear whether "overshooting" even exists for certain economic policy targets. For instance, if we look at the problem of maintaining full employment, there is no agreement as to whether this target can be overshot, that is, whether something like "overemployment" exists in the first place. In any case, it cannot be denied that considerable numbers of unemployed and excess demand for labour to the same extent are two different social phenomena, which have also different political consequences.

Relaxing the assumption of a quadratic objective function and modifying it to more general cost functions therefore seem appropriate. B. Friedman (1974) performed such an investigation, assuming a piecewise quadratic cost function with asymmetric costs for overshooting and undershooting the targets. However, in this case as well it is questionable whether the considerably higher number of calculations arising from this modification is worthwhile. In general, it can be said that the definition of targets and preferences for a society in functional form is probably more complicated and problematic than the specification of such a function. If we believe such a definition to be possible at all, then a quadratic specification does not seem that implausible and can be justified to a certain extent, at least by computational advantages.

4. ECONOMIC POLICY APPLICATIONS OF STOCHASTIC CONTROL THEORY

Another problem arises from the fact that in the literature reported so far, a deterministic specification and solution of the optimum control problem was given, which should pertain to a deterministic economic model. In fact, however, the estimated coefficients of an econometric model, and hence the elements of the system matrices derived from them, are themselves random variables. Furthermore, each equation of the structural form of the model has an additive error term. A solution to a general stochastic control problem that takes full account of the stochastic nature of the economic model is not obtainable. There are, however, several approaches in existence, and some results for problems of stochastic control were developed by control engineers. The characteristic feature of real disturbances in engineering and economics alike is the impossibility of exactly predicting future values. Hence they cannot be represented in a model as analytical functions, but only as sequences of random variables. If disturbances are thus described as stochastic processes, the statistical concepts of time series analysis can be applied.

Stochastic control theory deals with stochastic dynamic systems, which are represented by stochastic difference or differential equations, i.e. they are subject to disturbances characterized by stochastic processes. Stochastic optimum control problems require finding a control law optimizing (maximizing or minimizing) a given criterion with a given stochastic dynamic system as a constraint. While in the theory of optimal control of deterministic systems there is no difference between a control strategy and a control program, or between the performance of a closed-loop and an open-loop system, in a stochastic framework this is different: an optimizing control has to be found as a function of the current state of the system. In addition, only in stochastic theory does it become clear that the performance of the system crucially depends on the information available at the time at which the value of the control is determined. For instance, it can be shown that a delay in observing or measuring the state makes the performance of the system deteriorate.

In stochastic control theory, it is well known that the prediction problem and the linear-quadratic stochastic control problem are mathematically dual. A link between the theories of estimating, filtering and predicting the state of the system and the theories of controlling a stochastic system is provided by results in the engineering and mathematics literature called separation theorems. They show that in LQG problems (linear system, quadratic criterion, normally or Gaussian distributed additive disturbances), the optimal control strategy can be separated into two parts: the state estimator, which produces the best estimation of the system state vector from the observations, and a linear feedback law, which gives the control vector as a linear function of the estimated state. This linear control law is the same as if there were no disturbances and the state vector were known with certainty. The first proof of such a separation theorem was given by Joseph and Tou (1961); it is interesting to note that similar results for special cases were found earlier in the econometric literature, where they were called "certainty equivalence theorems" (Simon 1956, Theil 1957).

There are several examples of applications of stochastic control theory to problems of stabilization policy. The issues discussed in the literature include, among others, the optimization of quadratic objective functions with linear econometric models whose parameters are random variables (Chow 1975, 1981; Kendrick 1981), various applications of the Kalman filter (to economic policy: Vishwakarma 1974), the optimization of nonlinear stochastic control models (Chow 1981), and comparisons between the performance of an economic policy under a deterministic and a stochastic specification respectively (Turnovsky 1973, 1977; Kendrick 1981).

A shortcoming of most applications of stochastic control theory is the following: usually only the expected value of a criterion function is maximized, without regarding higher moments, for instance. This is only useful, however, if we want to achieve our objective optimally in the long run on average. But if short-term objectives are of interest, as is especially the case for stabilization policy, then another objective function may be more reasonable. We can, for example, require economic policy to have a probability of 90% for reaching a certain target, and maximize under this constraint. It is also possible to look for a minimum variance control strategy which minimizes the variance of certain variables of the model over time. Such questions of risk aversion, which are familiar from portfolio selection analysis (Markowitz 1959), are relevant for problems of economic policy as well and have been dealt with in the literature on risk-sensitive control (cf. Whittle 2002).

As general solutions of stochastic optimum control problems are not available and approximations to the optimum are the best one can hope for, computational aspects of stochastic optimum control become very important. So far, only a few algorithms dealing with the calculation of (approximate) solutions to stochastic optimum control problems are available; in particular, one for nonlinear systems with additive disturbances (Chow 1981) and one for linear systems with a more general stochastic structure (Kendrick 1981). An attempt to combine the capabilities of both has been made by Matulka and Neck (1992). They developed the OPTCON algorithm, designed to approximate optimal solutions to stochastic control problems for nonlinear systems under multiplicative (uncertain parameters) and additive uncertainty.

The stochastic optimum control problem of the OPTCON algorithm considers an intertemporal objective function of the form

$$L=\sum_{t=S}^{T} L_t(\boldsymbol{x}_t, \boldsymbol{u}_t), \tag{1}$$

with

$$L_t(\boldsymbol{x}_t, \boldsymbol{u}_t)=\frac{1}{2}\begin{pmatrix}\boldsymbol{x}_t-\tilde{\boldsymbol{x}}_t\\ \boldsymbol{u}_t-\tilde{\boldsymbol{u}}_t\end{pmatrix}' \boldsymbol{W}_t \begin{pmatrix}\boldsymbol{x}_t-\tilde{\boldsymbol{x}}_t\\ \boldsymbol{u}_t-\tilde{\boldsymbol{u}}_t\end{pmatrix}. \tag{2}$$

$\boldsymbol{x}_t$ denotes an n-dimensional vector of state variables; $\boldsymbol{u}_t$ denotes an m-dimensional vector of control variables. The n-dimensional vector $\tilde{\boldsymbol{x}}_t$ and the m-dimensional vector $\tilde{\boldsymbol{u}}_t$ denote the given "ideal" levels of the state and control variables respectively. S denotes the initial period and T the terminal one of the finite planning horizon. The matrix $\boldsymbol{W}_t$ is defined as

$$\boldsymbol{W}_t=\begin{pmatrix}\boldsymbol{W}_t^{\boldsymbol{xx}} & \boldsymbol{W}_t^{\boldsymbol{xu}}\\ \boldsymbol{W}_t^{\boldsymbol{ux}} & \boldsymbol{W}_t^{\boldsymbol{uu}}\end{pmatrix}, t= S,...,T, \tag{3}$$

where $\boldsymbol{W}_t^{\boldsymbol{xx}}$, $\boldsymbol{W}_t^{\boldsymbol{xu}}$ $\boldsymbol{W}_t^{\boldsymbol{ux}}$ and $\boldsymbol{W}_t^{\boldsymbol{uu}}$ are ($n\times n$), ($n\times m$), ($m\times n$) and ($m\times m$) matrices respectively. Furthermore, we require $\boldsymbol{W}_t$ to be constant apart from a constant discount rate α:

$$\boldsymbol{W}_t=\alpha^{t-1}\boldsymbol{W}, t=S,...,T, \tag{4}$$

where $\boldsymbol{W}$ is a constant matrix. Without loss of generality, it is assumed that $\boldsymbol{W}$ is symmetric.

It is easy to see that with

$$\begin{pmatrix}\boldsymbol{w}_t^{\boldsymbol{x}}\\ \boldsymbol{w}_t^{\boldsymbol{u}}\end{pmatrix}=-\boldsymbol{W}_t\begin{pmatrix}\tilde{\boldsymbol{x}}_t\\ \tilde{\boldsymbol{u}}_t\end{pmatrix}, \tag{5}$$

$$w_t^c=\frac{1}{2}\begin{pmatrix}\tilde{\boldsymbol{x}}_t\\ \tilde{\boldsymbol{u}}_t\end{pmatrix}' \boldsymbol{W}_t\begin{pmatrix}\tilde{\boldsymbol{x}}_t\\ \tilde{\boldsymbol{u}}_t\end{pmatrix}, \tag{6}$$

(2) can equivalently be written as

$$L_t(\boldsymbol{x}_t, \boldsymbol{u}_t)=\frac{1}{2}\begin{pmatrix}\boldsymbol{x}_t\\ \boldsymbol{u}_t\end{pmatrix}' \boldsymbol{W}_t\begin{pmatrix}\boldsymbol{x}_t\\ \boldsymbol{u}_t\end{pmatrix}+\begin{pmatrix}\boldsymbol{x}_t\\ \boldsymbol{u}_t\end{pmatrix}'\begin{pmatrix}\boldsymbol{w}_t^{\boldsymbol{x}}\\ \boldsymbol{w}_t^{\boldsymbol{u}}\end{pmatrix}+w_t^c. \tag{7}$$

The "quadratic tracking form" (2) of the objective function is very common in economic-policy applications of stochastic control theory. It can be interpreted as requiring deviations of the state variables $\boldsymbol{x}_t$ and the control variables $\boldsymbol{u}_t$ from their "ideal" levels $\tilde{\boldsymbol{x}}_t$ and $\tilde{\boldsymbol{u}}_t$ respectively, to be punished. The "general quadratic form" (7), however, simplifies notation and computation.

The dynamic system, which corresponds to the econometric model of the economy, is assumed to be given by the system of nonlinear difference equations

$$\boldsymbol{x}_t=\boldsymbol{f}(\boldsymbol{x}_{t-1}, \boldsymbol{x}_t, \boldsymbol{u}_t, \boldsymbol{\theta}, z_t)+\boldsymbol{\varepsilon}_t,\ \ t=S,...,T, \tag{8}$$

where $\boldsymbol{\theta}$ denotes a p-dimensional vector of unknown parameters, $\boldsymbol{z}_t$ denotes an l-dimensional vector of non-controlled exogenous variables, and $\boldsymbol{\varepsilon}_t$ is an n-dimensional vector of additive disturbances. $\boldsymbol{\theta}$ and $\boldsymbol{\varepsilon}_t$, $t=S,...,T$, are assumed to be independent random vectors with known expectations ($\hat{\boldsymbol{\theta}}$ for $\boldsymbol{\theta}, \boldsymbol{0}_n$ for $\boldsymbol{\varepsilon}_t, t=S,...,T$). $\boldsymbol{f}$ is a vector-valued function, where the i-th component of $\boldsymbol{f}$(.....) is denoted by f^i(.....), i = 1,..., n. The assumption of a system of first-order difference equations in (8) is not really restrictive, as higher-order difference equations can be reduced to systems of first-order difference equations by suitably redefining lagged variables as new state variables and augmenting the state vector. The state space form of (8) apparently differs from the one used in the engineering literature but can be shown to be equivalent.

To start the algorithm, the user has to input the system function $\boldsymbol{f}(.....)$, the initial values of state variables $\boldsymbol{x}_{S-1}\equiv\overset{\circ}{\boldsymbol{x}}_{S-1}\equiv\boldsymbol{x}^*_{S-1}$, a tentative path of control variables $(\overset{\circ}{\boldsymbol{u}}_t)_{t=S}^T$, a path of exogenous variables not subject to control $(z_t)_{t=S}^T$, an estimate of the expected values of system parameters $\hat{\boldsymbol{\theta}}$, an estimate of the covariance matrix of system parameters $\boldsymbol{\Sigma}^{\boldsymbol{\theta\theta}}$ and of the covariance matrix of system noise $\boldsymbol{\Sigma}^{\boldsymbol{\varepsilon\varepsilon}}$, the weighting matrices of the objective function $\boldsymbol{W}^{\boldsymbol{xx}}, \boldsymbol{W}^{\boldsymbol{ux}}, \boldsymbol{W}^{uu}$, the discount rate of the objective function α, the target ("ideal") path for the state variables $(\tilde{\boldsymbol{x}}_t)_{t=S}^T$ and the target ("ideal") path for the control variables $(\tilde{\boldsymbol{u}}_t)_{t=S}^T$. As output, one gets the expected optimal path of the state variables $(\boldsymbol{x}_t^*)_{t=S}^T$, the expected optimal path of the control variables $(\boldsymbol{u}_t^*)_{t=S}^T$ and the resulting expected optimal welfare loss J_S^*.

The OPTCON algorithm can be summarized as follows (for details and proofs, see Matulka and Neck, 1992):

Step 1. Compute a tentative state path: Use the Gauss-Seidel or the Newton-Raphson algorithm, the tentative policy path $(\overset{\circ}{\boldsymbol{u}}_t)_{t=S}^T$, and the system equation $\boldsymbol{f}$ (.....) to calculate the tentative state path $(\overset{\circ}{\boldsymbol{x}}_t)_{t=S}^T$ according to

$$\overset{\circ}{\boldsymbol{x}}_t=\boldsymbol{f}(\overset{\circ}{\boldsymbol{x}}_{t-1}, \overset{\circ}{\boldsymbol{x}}_t, \overset{\circ}{\boldsymbol{u}}_t, \hat{\boldsymbol{\theta}}, z_t)+\overset{\circ}{\boldsymbol{\varepsilon}}_t,\ \ t=S,...,T, \tag{9}$$

with $\overset{\circ}{\boldsymbol{\varepsilon}}_t=\boldsymbol{0}$ for all $t=S,...,T$.

Step 2. Nonlinearity loop: Repeat steps (a) to (e) until convergence is reached (i.e. until the optimal control and state variables calculated do not change by more than a pre-specified amount from one iteration to the next) or the number of iterations is larger than a pre-specified number.

(a) *Initialization for backward recursion:* $\boldsymbol{H}_{T+1} = \boldsymbol{0}_{n\times n}$, $\boldsymbol{h}_{T+1}^{x} = \boldsymbol{0}_n$, $h_{T+1}^{c} = 0$, $h_{T+1}^{s} = 0$, $h_{T+1}^{p} = 0$.

(b) *Backward recursion.* Repeat the following steps (i) to (ix) for $t = T,..., S$.

(i) Compute the expected values of the parameters of the linearized system equation

$$\boldsymbol{x}_t \approx \boldsymbol{A}_t\boldsymbol{x}_{t-1} + \boldsymbol{B}_t\boldsymbol{u}_t + \boldsymbol{c}_t + \boldsymbol{\xi}_t, \quad t = S,...,T, \tag{10}$$

with

$$\boldsymbol{\xi}_t = (\boldsymbol{I}_n - \boldsymbol{F}_{\boldsymbol{x}_t})^{-1}\boldsymbol{\varepsilon}_t, \tag{11}$$

$$\boldsymbol{A}_t = (\boldsymbol{I}_n - \boldsymbol{F}_{\boldsymbol{x}_t})^{-1}\boldsymbol{F}_{\boldsymbol{x}_{t-1}}, \tag{12}$$

$$\boldsymbol{B}_t = (\boldsymbol{I}_n - \boldsymbol{F}_{\boldsymbol{x}_t})^{-1}\boldsymbol{F}_{\boldsymbol{u}_t}, \tag{13}$$

$$\boldsymbol{c}_t = \overset{\circ}{\boldsymbol{x}}_t - \boldsymbol{A}_t\overset{\circ}{\boldsymbol{x}}_{t-1} - \boldsymbol{B}_t\overset{\circ}{\boldsymbol{u}}_t, \tag{14}$$

$$\boldsymbol{\Sigma}_t^{\xi\xi} = \mathrm{cov}_{S-1}(\boldsymbol{\xi}_t,\boldsymbol{\xi}_t) = (\boldsymbol{I}_n - \boldsymbol{F}_{\boldsymbol{x}_t})^{-1}\boldsymbol{\Sigma}^{\varepsilon\varepsilon}\left[(\boldsymbol{I}_n - \boldsymbol{F}_{\boldsymbol{x}_t})^{-1}\right]', \tag{15}$$

where $\boldsymbol{I}_n$ denotes the ($n\times n$) identity matrix. Here and in the following, we require that the first and second derivatives of the system function with respect to $\boldsymbol{x}_{t-1}$, $\boldsymbol{x}_t$, $\boldsymbol{u}_t$ and $\boldsymbol{\theta}$ exist and are continuous, and we define the matrices $\boldsymbol{F}_{\boldsymbol{x}_{t-1}}$, $\boldsymbol{F}_{\boldsymbol{x}_t}$, $\boldsymbol{F}_{\boldsymbol{u}_t}$ and $\boldsymbol{F}_{\boldsymbol{\theta}}$ by their elements as

$$(\boldsymbol{F}_{\boldsymbol{x}_{t-1}})_{i,j} = \frac{\partial f^i(.....)}{\partial x_{t-1,j}}, \; i = 1,...,n, \; j = 1,...,n, \text{ etc.} \tag{16}$$

Here $\boldsymbol{x}_{t-1,j}$ denotes the j-th element of $\boldsymbol{x}_{t-1}$, etc. All derivatives are evaluated at the reference values $\overset{\circ}{\boldsymbol{x}}_{t-1}, \overset{\circ}{\boldsymbol{x}}_t, \overset{\circ}{\boldsymbol{u}}_t, \hat{\boldsymbol{\theta}}, \boldsymbol{z}_t$ and $\overset{\circ}{\boldsymbol{\varepsilon}}_t = \boldsymbol{0}_n$. The expectation and the covariance matrix of ξ_t, conditional on the information given at t–1, are denoted by $\mathsf{E}_{t-1}(\boldsymbol{\xi}_t)$ and $\mathsf{cov}_{t-1}(\boldsymbol{\xi}_t,\boldsymbol{\xi}_t)$ respectively.

(ii) The matrices $\boldsymbol{A}_t$, $\boldsymbol{B}_t$ and the vector $\boldsymbol{c}_t$ are functions of the random parameter vector $\boldsymbol{\theta}$ and are, therefore, random themselves. Both matrices can be written as collections of their column vectors:

$$\boldsymbol{A}_t = (\boldsymbol{a}_{t,1}\ldots\boldsymbol{a}_{t,n}), \quad t = S,...,T, \tag{17}$$

$$\boldsymbol{B}_t = (\boldsymbol{b}_{t,1}\ldots\boldsymbol{a}_{t,m}), \quad t = S,...,T. \tag{18}$$

All of these column vectors as well as $\boldsymbol{c}_t$ are functions of $\boldsymbol{\theta}$. While, in general, these functions will be nonlinear, we approximate them by linear functions and write

$$\boldsymbol{a}_{t,i} = \boldsymbol{D}^{\boldsymbol{a}_{t,i}}\boldsymbol{\theta}, \quad i = 1,...,n, \quad t = S,...,T, \tag{19}$$

$$\boldsymbol{b}_{t,i} = \boldsymbol{D}^{\boldsymbol{b}_{t,j}}\boldsymbol{\theta}, \quad j = 1,...,m, \quad t = S,...,T, \tag{20}$$

$$\boldsymbol{c}_t = \boldsymbol{D}^{\boldsymbol{c}_t}\boldsymbol{\theta}, \quad t = S,...,T, \tag{21}$$

where

$$\boldsymbol{D}^{\boldsymbol{a}_{t,i}} = \begin{bmatrix} \frac{\partial a_{t,1i}}{\partial\theta_1} & \cdots & \frac{\partial a_{t,1i}}{\partial\theta_p} \\ \vdots & \ddots & \vdots \\ \frac{\partial a_{t,ni}}{\partial\theta_1} & \cdots & \frac{\partial a_{t,ni}}{\partial\theta_p} \end{bmatrix}, \; i = 1,...,n, \; t = S,...,T, \tag{22}$$

etc. For the sake of future computation, we reshape these matrices and group them into two matrices and one vector:

$$\boldsymbol{D}^{\boldsymbol{A}_t} \equiv \left[\mathsf{vec}\left(\left(\boldsymbol{D}^{\boldsymbol{a}_{t,1}}\right)'\right),...,\mathsf{vec}\left(\left(\boldsymbol{D}^{\boldsymbol{a}_{t,i}}\right)'\right),...,\mathsf{vec}\left(\left(\boldsymbol{D}^{\boldsymbol{a}_{t,n}}\right)'\right)\right], \tag{23}$$

$$\boldsymbol{D}^{\boldsymbol{B}_t} \equiv \left[\mathsf{vec}\left(\left(\boldsymbol{D}^{\boldsymbol{b}_{t,1}}\right)'\right),...,\mathsf{vec}\left(\left(\boldsymbol{D}^{\boldsymbol{b}_{t,j}}\right)'\right),...,\mathsf{vec}\left(\left(\boldsymbol{D}^{\boldsymbol{b}_{t,m}}\right)'\right)\right], \tag{24}$$

$$\boldsymbol{d}^{\boldsymbol{c}_t} \equiv \left[\mathsf{vec}\left(\left(\boldsymbol{D}^{\boldsymbol{c}_t}\right)'\right)\right]. \tag{25}$$

(iii) Compute the derivatives of the parameters of the linearized system with respect to $\boldsymbol{\theta}$:

$$\boldsymbol{D}^{\boldsymbol{A}_t} \equiv \left[(\boldsymbol{I}_n - \boldsymbol{F}_{\boldsymbol{x}_t})^{-1} \otimes \boldsymbol{I}_p\right]\left[\boldsymbol{F}_{\boldsymbol{x}_t,\boldsymbol{\theta}}\boldsymbol{A}_t + \boldsymbol{F}_{\boldsymbol{x}_{t-1},\boldsymbol{\theta}}\right], \tag{26}$$

$$\boldsymbol{D}^{\boldsymbol{B}_t} \equiv \left[(\boldsymbol{I}_n - \boldsymbol{F}_{\boldsymbol{x}_t})^{-1} \otimes \boldsymbol{I}_p\right]\left[\boldsymbol{F}_{\boldsymbol{x}_t,\boldsymbol{\theta}}\boldsymbol{B}_t + \boldsymbol{F}_{\boldsymbol{u}_t,\boldsymbol{\theta}}\right], \tag{27}$$

$$\boldsymbol{d}^{\boldsymbol{c}_t} \equiv \mathsf{vec}\left[\left[\left(\boldsymbol{I}_n - \boldsymbol{F}_{\boldsymbol{x}_t}\right)^{-1}\boldsymbol{F}_{\boldsymbol{\theta}}\right]'\right] - \boldsymbol{D}^{\boldsymbol{A}_t}\overset{\circ}{\boldsymbol{x}}_{t-1} - \boldsymbol{D}^{\boldsymbol{B}_t}\overset{\circ}{\boldsymbol{u}}_t, \tag{28}$$

where all derivatives are evaluated at the same reference values as above. Here we have defined second derivatives of the vector-valued system function with respect to vectors by:

$$\boldsymbol{F}_{\boldsymbol{x}_{t-1},\boldsymbol{\theta}} = \begin{bmatrix} \begin{pmatrix} \frac{\partial f^1}{\partial x_{t-1,1}\partial\theta_1} \\ \vdots \\ \frac{\partial f^1}{\partial x_{t-1,1}\partial\theta_p} \end{pmatrix} & \cdots & \begin{pmatrix} \frac{\partial f^1}{\partial x_{t-1,n}\partial\theta_1} \\ \vdots \\ \frac{\partial f^1}{\partial x_{t-1,n}\partial\theta_p} \end{pmatrix} \\ \vdots & \ddots & \vdots \\ \begin{pmatrix} \frac{\partial f^n}{\partial x_{t-1,1}\partial\theta_1} \\ \vdots \\ \frac{\partial f^n}{\partial x_{t-1,1}\partial\theta_p} \end{pmatrix} & \cdots & \begin{pmatrix} \frac{\partial f^n}{\partial x_{t-1,n}\partial\theta_1} \\ \vdots \\ \frac{\partial f^n}{\partial x_{t-1,n}\partial\theta_p} \end{pmatrix} \end{bmatrix}, \tag{29}$$

and analogously for $\boldsymbol{F}_{\boldsymbol{x}_t,\boldsymbol{\theta}}$ and $\boldsymbol{F}_{\boldsymbol{u}_t,\boldsymbol{\theta}}$.

(iv) Compute the influence of the stochastic parameters: Compute all the matrices $\boldsymbol{\Psi}_t^{AKA}$, $\boldsymbol{\Psi}_t^{BKA}$, $\boldsymbol{\Psi}_t^{BKB}$, $\boldsymbol{\upsilon}_t^{AKc}$, $\boldsymbol{\upsilon}_t^{BKc}$ and υ_t^{cKc}, the cells of which are defined by

$$\left[\boldsymbol{\Psi}_t^{AKA}\right]_{i,j} = \mathrm{tr}\left[\boldsymbol{K}_t\boldsymbol{D}^{\boldsymbol{a}_{t,j}}\boldsymbol{\Sigma}^{\boldsymbol{\theta\theta}}\left(\boldsymbol{D}^{\boldsymbol{a}_{t,i}}\right)'\right], \; i = 1,...,n, \; j = 1,...,n, \tag{30}$$

and so on.

(v) Convert the objective function from a "quadratic-tracking" to a "general quadratic" format:

$$\boldsymbol{W}_t^{xx} = \alpha^{t-1}\boldsymbol{W}^{xx}, \tag{31}$$

$$\boldsymbol{W}_t^{ux} = \alpha^{t-1}\boldsymbol{W}^{ux}, \tag{32}$$

$$\boldsymbol{W}_t^{uu} = \alpha^{t-1}\boldsymbol{W}^{uu}, \tag{33}$$

$$\boldsymbol{w}_t^{x} = -\boldsymbol{W}_t^{xx}\tilde{\boldsymbol{x}}_t - \boldsymbol{W}_t^{xu}\tilde{\boldsymbol{u}}_t, \tag{34}$$

$$\boldsymbol{w}_t^{u} = -\boldsymbol{W}_t^{ux}\tilde{\boldsymbol{x}}_t - \boldsymbol{W}_t^{uu}\tilde{\boldsymbol{u}}_t, \tag{35}$$

$$w_t^{c} = \tfrac{1}{2}\tilde{\boldsymbol{x}}_t'\boldsymbol{W}_t^{xx}\tilde{\boldsymbol{x}}_t + \tilde{\boldsymbol{u}}_t'\boldsymbol{W}_t^{ux}\tilde{\boldsymbol{x}}_t + \tfrac{1}{2}\tilde{\boldsymbol{u}}_t'\boldsymbol{W}_t^{uu}\tilde{\boldsymbol{u}}_t. \tag{36}$$

(vi) The key idea of OPTCON is to use Bellman's principle of optimality:

$$J_t^*(\boldsymbol{x}_{t-1}) = \min_{\boldsymbol{u}_t} \mathsf{E}_{t-1}\left[L_t(\boldsymbol{x}_t, \boldsymbol{u}_t) + J_{t+1}^*(\boldsymbol{x}_t)\right], \tag{37}$$

where $J_t^*(\boldsymbol{x}_{t-1})$ denotes the loss that is expected at the end of period $t-1$ for the remaining periods $t,...,T$ if the optimal policy is implemented during these periods. $\boldsymbol{x}_{k-1}$, $k = S,...,t$, and $\boldsymbol{u}_{k-1}$, $k = S+1,...,t$, are known at the time when we have to decide about $\boldsymbol{u}_t$. It can be shown that $J_t^*(\boldsymbol{x}_{t-1})$ can be expressed as a quadratic function of $\boldsymbol{x}_{t-1}$:

$$J_t^*(\boldsymbol{x}_{t-1}) = \tfrac{1}{2}\boldsymbol{x}_{t-1}'\boldsymbol{H}_t\boldsymbol{x}_{t-1} + \boldsymbol{x}_{t-1}'\boldsymbol{h}_t^x + h_t^c + h_t^s + h_t^p, \tag{38}$$

for all periods $t = S,...,T+1$, where $\boldsymbol{H}_t, \boldsymbol{h}_t^x, h_t^c, h_t^s$ and h_t^p are defined below. Here we introduce the following simplifying assumptions:

1. Each occurrence of $\mathsf{E}_{t-1}(.)$ is substituted by $\mathsf{E}_{S-1}(.)$ and each occurrence of $\mathsf{cov}_{t-1}(..)$ is substituted by $\mathsf{cov}_{S-1}(..)$ for all $t = S+1,...,T+1$. Thus we rule out any learning about the parameters of the model.

2. Although $\boldsymbol{A}_t$, $\boldsymbol{B}_t$ and $\boldsymbol{c}_t$ are, in general, nonlinear functions of $\boldsymbol{\theta}$, we compute their expected values by evaluating the equations (17), (18) and (19) at the reference values $\overset{\circ}{\boldsymbol{x}}_{t-1}, \overset{\circ}{\boldsymbol{x}}_t, \overset{\circ}{\boldsymbol{u}}_t, \mathsf{E}_{S-1}(\boldsymbol{\theta}), \boldsymbol{z}_t$ and $\overset{\circ}{\boldsymbol{\varepsilon}}_t = \boldsymbol{0}_n$, which were true only in case of linear functions.

(vii) Compute the parameters of the function of expected accumulated loss:

$$\boldsymbol{K}_t = \boldsymbol{W}_t^{xx}\boldsymbol{H}_{t+1}, \tag{39}$$

$$\boldsymbol{k}_t^x = \boldsymbol{w}_t^x + \boldsymbol{h}_{t+1}^x, \tag{40}$$

$$\boldsymbol{\Lambda}_t^{xx} = \boldsymbol{\Psi}_t^{AKA} + \boldsymbol{A}_t'\boldsymbol{K}_t\boldsymbol{A}_t, \tag{41}$$

$$\boldsymbol{\Lambda}_t^{xu} = \left(\boldsymbol{\Lambda}_t^{ux}\right)', \tag{42}$$

$$\boldsymbol{\Lambda}_t^{ux} = \boldsymbol{\Psi}_t^{BKA} + \boldsymbol{B}_t'\boldsymbol{K}_t\boldsymbol{A}_t + \boldsymbol{W}_t^{ux}\boldsymbol{A}_t, \tag{43}$$

$$\boldsymbol{\Lambda}_t^{uu} = \boldsymbol{\Psi}_t^{BKB} + \boldsymbol{B}_t'\boldsymbol{K}_t\boldsymbol{B}_t + 2\boldsymbol{B}_t'\boldsymbol{W}_t^{xu} + \boldsymbol{W}_t^{uu}, \tag{44}$$

$$\boldsymbol{\lambda}_t^{x} = \boldsymbol{\upsilon}_t^{AKc} + \boldsymbol{A}_t'\boldsymbol{K}_t\boldsymbol{c}_t + \boldsymbol{A}_t'\boldsymbol{k}_t^x, \tag{45}$$

$$\boldsymbol{\lambda}_t^{u} = \boldsymbol{\upsilon}_t^{BKc} + \boldsymbol{B}_t'\boldsymbol{K}_t\boldsymbol{c}_t + \boldsymbol{B}_t'\boldsymbol{k}_t^x + \boldsymbol{W}_t^{ux}\boldsymbol{c}_t + \boldsymbol{w}_t^u, \tag{46}$$

$$\lambda_t^{s} = \tfrac{1}{2}\mathsf{tr}\left[\boldsymbol{K}_t\boldsymbol{\Sigma}_t^{\xi\xi}\right] + h_{t+1}^s, \tag{47}$$

$$\lambda_t^{p} = \tfrac{1}{2}\upsilon_t^{cKc} + h_{t+1}^p, \tag{48}$$

$$\lambda_t^{c} = \tfrac{1}{2}\boldsymbol{c}_t'\boldsymbol{K}_t\boldsymbol{c}_t + \boldsymbol{c}_t'\boldsymbol{k}_t^x + w_t^c + h_{t+1}^p. \tag{49}$$

(viii) Compute the parameters of the policy feedback rule:

$$\boldsymbol{G}_t = -\left(\boldsymbol{\Lambda}_t^{uu}\right)^{-1}\boldsymbol{\Lambda}_t^{ux}, \tag{50}$$

$$\boldsymbol{g}_t = -\left(\boldsymbol{\Lambda}_t^{uu}\right)^{-1}\boldsymbol{\lambda}_t^{u}. \tag{51}$$

(ix) Compute the parameters of the function of minimal expected accumulated loss:

$$\boldsymbol{H}_t = \boldsymbol{\Lambda}_t^{xx} - \boldsymbol{\Lambda}_t^{xu}\left(\boldsymbol{\Lambda}_t^{uu}\right)^{-1}\boldsymbol{\Lambda}_t^{ux}, \tag{52}$$

$$\boldsymbol{h}_t^{x} = \boldsymbol{\lambda}_t^{x} - \boldsymbol{\Lambda}_t^{xu}\left(\boldsymbol{\Lambda}_t^{uu}\right)^{-1}\boldsymbol{\lambda}_t^{u}, \tag{53}$$

$$h_t^{c} = \lambda_t^{c} - \tfrac{1}{2}\left(\boldsymbol{\lambda}_t^{u}\right)'\left(\boldsymbol{\Lambda}_t^{uu}\right)^{-1}\boldsymbol{\lambda}_t^{u}, \tag{54}$$

$$h_t^{s} = \lambda_t^{s}, \tag{55}$$

$$h_t^{p} = \lambda_t^{p}. \tag{56}$$

(c) *Forward projection*. Repeat the following steps (i) to (ii) for $t = S,...,T$.

(i) Compute the expected optimal policy:

$$\boldsymbol{u}_t^* = \boldsymbol{G}_t\boldsymbol{x}_{t-1}^* + \boldsymbol{g}_t. \tag{57}$$

(ii) Compute the expected optimal state: Use the Gauss-Seidel or the Newton-Raphson algorithm to compute $\boldsymbol{x}_t^*$ such that

$$\boldsymbol{x}_t^* = \boldsymbol{f}\left(\boldsymbol{x}_{t-1}^*, \boldsymbol{x}_t^*, \boldsymbol{u}_t^*, \hat{\boldsymbol{\theta}}, \boldsymbol{z}_t\right). \tag{58}$$

(d) *Set the new tentative paths for the next iteration:*

$$\left(\overset{\circ}{\boldsymbol{x}}_t\right)_{t=S}^{T} = \left(\boldsymbol{x}_t^*\right)_{t=S}^{T}. \tag{59}$$

$$\left(\overset{\circ}{\boldsymbol{u}}_t\right)_{t=S}^{T} = \left(\boldsymbol{u}_t^*\right)_{t=S}^{T}. \tag{60}$$

(e) *Compute the expected welfare loss:*

$$J_S^* = \tfrac{1}{2}\boldsymbol{x}_{S-1}'\boldsymbol{H}_S\boldsymbol{x}_{S-1} + \boldsymbol{x}_{S-1}'\boldsymbol{h}_{S-1}^x + h_S^c + h_S^s + h_S^p. \tag{61}$$

The OPTCON algorithm has been used to determine optimal macroeconomic policies for Austria and Slovenia, both under certainty and under the full stochastic assumptions detailed above. See Neck and Karbuz (1997), Weyerstrass et al. (2000) for some results.

5. DECENTRALIZED CONTROL SYSTEMS

In the optimum stochastic control problem, the control actions at different points in time must be set as functions of the available data. Usually it is assumed that all actions which have to be done at a certain time must be based on the same data and that all data available at time t will also be available at any later time $t' > t$, a situation which has been called a classical information pattern (structure) by Witsenhausen (1968).

In contrast to this, a problem with non-classical information pattern exists whenever the “memory” of the controller is limited; for example, it is possible to determine an optimal controller without memory such that every control action depends just on the last observation of the state. Another possibility is the interpretation of communication problems as control processes; in this case, too, the information pattern is never a classical one because there are at least two control stations (agents) who do not have access to the same data. Finally, non-classical information structures can be found whenever the system to be controlled is large or consists of several subsystems; also in these cases, the actions of the controllers at any point in time are not based on the same data, even if every control station has perfect memory, since the chains of communication between the stations are subject to lags, disturbances and cost constraints. From a methodological point of view, these chains of communication can be regarded as part of the controlled system and the communication policy as part of the control policy.

The investigation of different information patterns in the way indicated above is the subject of decentralized control theory (Singh 1981, Jamshidi 1983, Siljak 1991). There are only a few mathematical results available for special cases. However, Witsenhausen (1971) developed a rather general model showing the formulation of discrete-time decentralized control problems, i.e. of models for systems with several controllers. The essential point here is that several controllers, who have non-identical information about the structure of the system, the state vector, the parameters, etc., act together in controlling the same system.

In economics, similar problems of decisions in organizations whose members share common goals have been discussed in the literature for many years; the relevant theory is called the theory of teams. A team is a group of persons, each of whom makes decisions on different problems but who gets a common reward as a result of all these decisions (Marschak and Radner 1972). The issues discussed in team theory are very similar to those occurring in decentralized control theory; in particular, the investigation of information structures and of the influence of information on the optimal value of the criterion is of major concern in both cases. The main difference is that the theory of teams by Marschak und Radner is mostly static while decentralized control theory is essentially dynamic.

Dynamic generalizations of methods of the theory of teams and their application to problems of optimal control with a decentralized decision structure therefore appeared to be an obvious solution. However, it was necessary to set extremely restrictive assumptions in order to keep the computational efforts within reasonable limits. The main difficulties stem from the interaction between information and control, because the actual decision of an agent (controller) at any time depends on the previous actions of the other members of the team; these actions, however, are themselves part of the solution to be determined as a result of the problem. A limitation to the methods of team theory is the constraint that the same decision-maker cannot decide more than once at each point in time.

In principle, for decentralized systems with non-classical information patterns, the same questions were investigated as in classical control theory; however, solutions are only available for some special cases. Intensive investigations have been dedicated especially to the questions of stability of the controlled system, optimality of the control design, and the relation between different information patterns. Apart from this, several methods should be mentioned in this connection which were developed in order to describe complex systems and also seem to be applicable to decentralized systems, such as the theory of hierarchical systems (Singh 1977), the theory of composite systems, investigations into the decoupling and the assignment problems (Morse and Wonham 1971), and the application of the concept of aggregation to coupled systems (Aoki 1968).

The issue of decentralized planning has been recognized as a relevant problem by economists for a long time, especially since the “Socialist controversy” of the inter-war period between Mises, Hayek, Lange and others. It continued in the theory of allocation mechanisms developed after World War II (see, e.g., Hurwicz and Reiter 2006). This economic theory of decentralization, however, is fully directed towards the problems of allocation and cannot be directly applied to the problems of short-run stabilization policy over time; furthermore, these methods are mostly static ones.

On the other hand, economic policy interpretations of some models of decentralized dynamic control systems can provide new insights. The different controllers can be interpreted as policy-making persons or institutions which have to bear responsibility for different tasks of stabilization policy, like, for instance the government, the central bank, social insurance companies, etc. The questions of transmission of information, which can be analyzed by decentralized control theory, are of great practical interest as delays and disturbances in the communication and transmission of information between all these institutions are omnipresent. Some of the few economic policy applications of decentralized control theory are Aoki (1974) and Neck (1983, 1987).

Treating the problems of stabilization policy in the form of a basically team-theoretic model (with a common objective function for all controllers) is, however, only justified when only those instances are regarded as controllers that are dependent on government in some form or another. Institutions like trade unions, employers’ associations and even independent central banks with their own objectives then cannot be regarded as controllers, although they obviously have a decisive influence on economic policy decisions. In order to incorporate such institutions, it is necessary to use a game-theoretic approach, which allows for the analysis of cooperation as well as of conflicts between these agents. Dynamic game theory is the most appropriate tool for analyzing such problems – a methodology which has been developed largely by engineers and control theorists and has gradually found its way into economics (for a survey, see

Neck 2006). Within the game theory paradigm, the problem of stabilization policy is no longer regarded as a problem of optimization but of equilibrium among agents with (at least partially) conflicting interests.

Control theory methods differ from those of game theory main1y in that the former incorporate only one single decision-maker as being eventually decisive. Although this decision-maker is no longer explicitly present in problems of decentralized control theory, this approach essentially assumes the existence of somebody above all controllers (in engineering applications, for instance, the experimenter) who defines the control laws and the objective function and supervises the workings of the system. In economic terms, decentralized control can be called a topic in the organization of an economic policy; indeed, for questions of organizing an activity, the approach of team theory is well suited. Conflicts of goals or interests between several groups, on the other hand, which are very important in economic policy situations, cannot be tackled by these instruments.

6. POSSIBILITIES AND LIMITATIONS OF CONTROL THEORY MODELS IN ECONOMIC POLICY

“Economists experimenting with the decision-making approach of ‘optimal control theory’ hope that it will become fully operational in economics in the next few years. If it does, they will have at their disposal a mathematical supertool that, when used together with econometric models, could substantially advance the science of economic and financial management. Control theory has swept into the economics profession so rapidly in the past two or three years that most economists are only dimly aware that it is around. But for econometricians and mathematical economists, and for the companies and government agencies that use their skills, it promises an improved ability to manage short-run economic stabilization, long-run economic growth, investment portfolios, and corporate cash positions” (*Business Week* May 19, 1973; quoted in Athans and Kendrick 1974). This optimistic view of the possibilities of control theory from the early days of its applications to economic policy problems, under the influence of the “Lucas critique” (Lucas 1976) and the demonstration of the possible time-inconsistency of optimum control results (Kydland and Prescott 1977, Prescott 1977), gave way to a more pessimistic view a decade later, asking whether economic policy and control theory were engaging in a “failed partnership” (Currie 1985). What can be said now, another two decades or so later?

In evaluating control theory applications to problems of economic policy, one can say that in some respect the approach of control theory, especially optimum control theory, is very flexible. For instance, there are no difficulties formulating time-varying linear systems with systems matrices being dependent on time; also the weighting matrices of the objective function may be time-dependent without any problems. This also includes the case where certain variables become relevant targets only at certain points in time and others cease to be targets at certain points in time. The approach of optimum control theory is also flexible in the sense that it is not absolutely necessary to specify the ideal trajectories of the target and instrument variables for the entire planning period in advance. It is possible to feed back the “ideal” to the actual values; this would not cause principal difficulties but only computational ones.

The fact that both state variables (“targets”) and control variables (“instruments”) may be contained as arguments in the objective function also has some importance for the dichotomy between targets and instruments, which was the subject of discussions within the theory of economic policy about the so-called “teleological fallacy”. Tinbergen’s theory of economic policy is taxonomic in the sense that it distinguishes between target and instrument variables for the model under consideration; this distinction is seen by some critics of this approach as using a rather special and mostly arbitrary scheme of classification. The optimum control approach, and more generally an optimization approach, meet this criticism if it also contains the instrument variables in the objective function. Then it becomes possible for all variables of the model to become “target variables” in a wider sense (usually called “objective variables”) as their desired (optimal) values can be determined. On the other hand, it is also possible to assume as many variables as “instruments” as we like (with a minimum of one); it only has to be presumed that these variables are under the control of the planner.

On the other hand, the dichotomization between exogenous and endogenous variables is required and even essential, also for optimum control considerations. Distinguishing between endogenous and exogenous variables supposes that the system is “open”, that is to say that it has relations to an environment which is different and distinct from the system itself. In contrast to this stands the idea in systems theory (e.g., Kade et al. 1968) that observation and control compel us as observers and controllers to become part of the system ourselves: in the process of observing, some of the information necessary for observation is destroyed. Goal-seeking behaviour, which includes observation as well as control, among others, therefore has to be represented always as some sort of closed circuit. Hence the question “in which direction” the system is open is in no way trivial, and just this representation is not given in control theory. The decision-maker or controller is principally assumed to be exogenous to the system. Although being influenced by the results of the system (especially under feedback control, closed-loop control or adaptive control), the decision-maker is not himself part of the system. Apart from the exogenous variables, which are either given in a deterministic way or affect the motion of the system as additive stochastic disturbances, there are variables that are exogenous insofar as a consciously and rationally acting individual manipulates them in a certain manner freely determined by him, in order to optimize an objective function whose structure and even existence also exclusively depends on this individual.

A possible interpretation of control theory models and of optimization models in general can be given by regarding them as consistency models: if somebody sets some targets (an objective function), how do we have to specify the controls in order to guarantee optimal fulfilment of these targets? Such a consistency analysis is not, however, of much use for practical purposes unless it is also stated who shall or can bring about these conditions and who regards the goals as desirable. For problems of economic policy, the objective function is often interpreted as a “collective (social) welfare function” allegedly reflecting all costs and benefits to the society. But it is well known from social choice theory that even with plausible and not very restrictive assumptions about the preferences of the members of a society, no social welfare function exists that can be derived from these preferences. Most macroeconometric models were built for democratic societies; hence this insight is important for the optimum control approach to economic policy. The objective function cannot easily be interpreted as having been brought about by democratic methods of defining social welfare. On the other hand, this need not be required for an indirect democracy: there the citizens elect persons who, for a certain period of time, are trusted to put into effect their own goals and targets. Thus the objective function merely reflects the preferences of the politicians or planners, which for a fixed period of time can be implemented. We need not discuss whether this model of democracy adequately reflects the political realities of the countries of Western Europe or Northern America, for example; in any case, the objections raised by social choice theory against social welfare functions do not necessarily appear as impediments to applying the optimization approach to economic policy.

There is, however, another problem that is closely connected to the one just mentioned: if the objective function only reflects the preferences of the planners, who should these planners and politicians be? Is it only the elected representatives of the people, especially the government? Can these planners be modelled as one single unit, or are there conflicts between them or other reasons why we must assume a multiplicity of planners, even in a simple model? Control theory first considered one decision-maker only; in decentralized control models, several controllers are taken into consideration. But even there, one basic assumption continues: the existence of a single objective function common to all controllers. The problem then is to create an “optimal design” of the team, as is common in organization theory when organizing machines or human work in industrial firms. In doing so, the economic system itself is regarded as a variable and the goal is to organize this system in such a way that it performs optimally. The objective function is, in this sense, part of the “design” created by the “designer” of the system (the economic system mechanism).

This should rather clearly reveal the ideas underlying the control theory model; but here, as well, its inherent difficulties become clear. Apart from presuming extensive abilities to manipulate the system, it is not quite clear who in the last resort is responsible for designing the stable system, optimizing the objective function, organizing the team. An experimenter in the electrical engineering sense does not usually exist in social systems, at least if we disregard extremely powerful dictators (a situation that – alongside questions as to its desirability – does not seem realistic even for countries with very totalitarian governments and centrally planned economies). The idea of designing a system therefore seems to be more appropriate to engineering than to economics. In economic systems, there is always a variety of individuals and groups performing several control activities with different targets and goals, but these are themselves parts of the system and must be represented within it. Both in centrally planned and in market economies, agents with different aims and targets must be taken into consideration, and the institutions responsible for planning and economic policy are also integral parts of the economic system; these two aspects, however, cannot be represented in control theory models.

The high degree of uncertainty regarding policy effects has often been used as the main argument against discretionary economic policy-making; likewise the argument that high informational requirements make rational planning impossible has been advanced against economic planning. Both aspects can be captured by control theory approaches, at least conceptually; however, what cannot be captured is what might be called “objective requirements”. By this we mean that giving a theoretical foundation for an objective function in economic policy and its separation from the economic system is not easily possible. In economic policy-making, there are several planners with different targets, themselves being parts of the system. The idea of a single exogenous planner determining the design of the system is not realistic since in such a case even more “omniscience” and “omnipotence” would have to be assumed of such a planner than even in the case of completely deterministic models.

The reason why insufficient attention has been paid to this aspect in the literature so far is due to the fact that issues like transforming an economic policy into reality, exercise of power, and differences in interests have been strongly neglected in economic theory. It is a question whether the logical structure of economic models does not grasp certain relevant aspects only for ideological reasons or whether there are also reasons for this neglect that are inherent to science. In fact, the last possibility might be true; mathematical and analytical economists have taken physics and other highly developed natural sciences as a prototype in building their models and have often carried over the models of those sciences uncritically. The only mathematical model that has been developed with a specific economic (or, more generally, social science) aim might be game theory. Since control engineering also had a standard highly superior to economic theories of planning as regards formal methods, it was a natural development to occupy this theory with minor modifications for economic policy model building as well. A frequent cause for the inadequacy of some models is the requirement that the model must be solvable; often it is possible to recognize what is or could be missing, but it is impossible to formulate those aspects for the specific model or to solve an extended (and more realistic) model.

Here it is interesting that the concepts of optimization and stabilization induce a very particular focus for the science of economic policy, assuming an essential unity of policy-makers’ preferences. But how should such different policy-making institutions be modelled if not by their objective functions? In microeconomic theory, economic agents are modelled by their utility functions; a similar procedure should be applied to the theory of economic policy as regards political institutions and groups. Here concepts of game theory provide a promising alternative, especially the theory of dynamic games for dynamic problems, which has a rigorous mathematical foundation and was even initiated by control theorists and engineers. What is more, the problem of time inconsistency can be adequately treated within the framework of dynamic game theory (see, e.g., Dockner and Neck 2008). From a methodological point of view, a research program for a theory of economic policy based on dynamic game theory (of which the control-theory based approach would be a one-decision-maker special case) could even settle the old dispute between "institutionalism" and "analytical economics" because institutional problems would then be investigated by means of analytical methods. Such a theory, however, would no longer be a normative one but purely positive; it is an open question whether it could be still termed a "theory of economic policy" or rather "political economy". In any case, such a theory would hold an intermediate position between theoretical economics on the one hand and political science on the other one; it would be more useful for the problems of the stable development of an economy than those optimum control concepts derived from engineering which are interesting as consistency models but cannot provide a theoretical foundation for an empirically useful theory of economic policy.

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