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LMI-based control of vehicle platoons for robust longitudinal guidance ⋆

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Abstract: This paper presents a novel approach to control layout for longitudinal guidance of platoons with a limited number of vehicles. It accounts for both the reduction of spacing errors and a limitation in velocities and accelerations of following vehicles to avoid saturations. All criteria can be expressed using a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ problem formulation. The objectives are formulated as one set of linear matrix inequalities that are solved for the controller. The optimization is presented for different control structures and the effectiveness of reducing overshoots in velocities or accelerations is shown through simulation results. This work also considers structural constraints concerning the information available to the controller and evaluates a sequential control algorithm applying the same layout method. Finally, the effects of changing parameters of the vehicle's drivetrain are analyzed and robustness of the presented controllers is investigated.

Keywords: Automotive Control, Automated guided vehicles, H-infinity control, Optimal control, Sequential control, Vehicle dynamics

1. INTRODUCTION

Automated Highway Systems (AHS) and vehicle platooning have attracted a lot of research interest over the last two decades. A lot of work has been done especially for isolating and determining the right criteria to evaluate a platoon control law. One of the very important criteria is string stability of a platoon, describing the property that upper limits on system variables don't depend on the platoon length and especially still hold for infinitely long vehicle platoons. Most of the work concentrated on spacing errors and string stability in terms of declining errors along the platoon. To cite just some of the important work we refer to Swaroop and Hedrick [1996] and Lu et al. [2004]. Mostly, the upstream amplification of velocities and accelerations and their upper limits have not been addressed. The importance of them is profound though since saturations in both velocities or accelerations are typically not included into the controller layout. Especially for critical longitudinal maneuvers with high (positive or negative) accelerations this becomes very important.

This paper focusses on a control layout explicitly accounting for declining errors and a minimization of upper limits for velocities and accelerations of following platoon vehicles. This work was done within the ongoing project "KONVOI" which aims at the control of small-scale (up to 4 vehicles) platoons of heavy-duty trucks with constant spacing. Due to the limited platoon size a dynamic communication structure with every vehicle having access to all other vehicles' longitudinal data is possible. For the introduction into traffic only close-to-production technical equipment is used.

⋆ The work presented in this paper has been developed within the project "KONVOI" which is supported by the German Federal Ministry of Economics and Technology (BMWi).

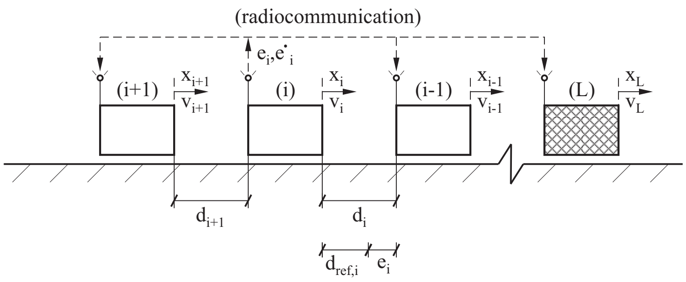

Fig. 1. Platoon setup with leading vehicle and n following vehicles.

1.1 Platoon Model

The complexity of drivetrain models governing the longitudinal dynamics is well known in literature. It was shown though that by lower level controls this dynamics can be approximated by a linear first order filter for the acceleration, see Ha et al. [1989] and Lu and Hedrick [2004]. The trucks of our project are equipped with acceleration controls by different suppliers and the above assumption could be verified by measurements.

According to figure 1 the time behavior of the platoon can then be described by the error dynamics of each vehicle i where the error is defined as the difference between the actual distance to the predecessor and a (fixed) reference distance: $e_i(t) = d_i(t) - d_{ref,i}(t)$. It holds

$$\ddot{e}_i = a_{i-1} - a_i, \quad (1)$$
$$\dot{a}_i = -1/T_i \cdot a_i + 1/T_i \cdot u_i \quad (2)$$

while the error of the first follower is described by $\ddot{e}_1 = a_L - a_1$, with a_L being the leading vehicle's acceleration.

Table 1. System variables and parameters.

Symbol	Description
d_i	distance of vehicle i to predecessor
$d_{ref,i}$	reference distance for vehicle i
e_i	distance error between vehicle $i-1$ and i
a_i	acceleration of vehicle i
v_i	longitudinal velocity of vehicle i
u_i	control output of vehicle i
T_i	time constant for drivetrain dynamics
n	number of following vehicles
x	state vector for the whole platoon

All variables refer to the nomenclature given in table 1. The acceleration dynamics is mainly governed by the drivetrain's time constant T_i which varies depending on the engine's operating point, the different dynamics of brakes and engine and finally the underlying controls given by the suppliers. This work does only focus on the upper level platoon control using u_i as reference acceleration - for a more detailed description of the whole control topology see Maschuw et al. [2007]. For convenience we summarize the vehicles' states as platoon state vector

$$x = (\cdots, e_i, \dot{e}_i, a_i, \cdots)^T, \quad x \in \mathbb{R}^{3n}. \tag{3}$$

1.2 Control Objectives

The goal of platoon control typically is the minimization of distance errors and especially the avoidance of collisions while keeping the control effort low. This so far can be considered as a problem of linear optimal control. Additionally, some structural conditions of the controlled platoon have to be met. One of the most important ones is string stability for the whole platoon - apart from the individual stability (which is a necessary but no sufficient condition). For linear systems string stability in terms of non-amplifying control errors can be expressed by their transfer functions or impulse responses. With the transfer function

$$G_i(s) = \frac{e_i(s)}{e_{i-1}(s)} \tag{4}$$

relating the distance errors of two following vehicles $i-1$ and i and its impulse response $g_i(t)$ it holds

$$||e_i(t)||_\infty \leq \int_0^\infty |g(\tau)| d\tau \cdot ||e_{i-1}(t)||_\infty. \tag{5}$$

For the $||\cdot||_1$-Norm of the impulse response given in integral form above it follows

$$||G_i(s)||_\infty \leq ||g_i(t)||_1 \leq \gamma_e \overset{!}{<} 1 \tag{6}$$

in order to achieve string stability - the additional relation for the $||\cdot||_\infty$-Norm of the transfer function follows from linear system theory, see Lu et al. [2004]. Among others Swaroop and Hedrick [1999] proved that string stability can be achieved for control laws using information on distance and relative velocity from all preceding vehicles or equivalently the position and velocity from the leading vehicle.

Opposed to distance errors a decline of maximum values along the platoon is not possible for velocities or accelerations when constant spacing policies are used. This becomes clear when one considers that distance errors are the integral of reference velocities - an initial velocity difference between two succeeding vehicles has to result in a velocity overshoot of the follower to make the integral zero again (a similar result holds for accelerations). Nevertheless, with the property of string stability it is possible to prove that there is an upper limit for velocities and accelerations of followers which is independent of the platoon length (see appendix). Another objective is then to minimize the upper bound for both variables. For this, we now extend the formulation of transfer functions to the description of velocities and accelerations. For the evaluation of critical velocities or accelerations the predominant excitation of the system described in (1)-(2) comes from the acceleration of the leading vehicle a_L. The control objective is the reduction of overshoots in velocity related to the leading vehicle $v_i(t) - v_L(t)$ and acceleration $a_i(t)$. For the transfer functions

$$F_{v,i}(s) = \frac{(v_i - v_L)(s)}{a_L(s)} \quad \text{and} \quad F_{a,i}(s) = \frac{a_i(s)}{a_L(s)} \tag{7}$$

a relation for the impulse response analogous to equation (5) holds (we are only giving the one for the accelerations here):

$$||a_i(t)||_\infty \leq \int_0^\infty |f_{a,i}(\tau)| d\tau \cdot ||a_L(t)||_\infty. \tag{8}$$

Hence, in order to keep accelerations for all followers below a certain level $\gamma_a \cdot ||a_L||_\infty$ the objective writes as

$$||F_{a,i}(s)||_\infty \leq ||f_{a,i}(t)||_1 \overset{!}{<} \gamma_a. \tag{9}$$

Besides the mentioned objectives we have to account for structural constraints concerning control information and finally robustness concerning unknown or varying parameters T_i of the drivetrain model. Both issues will be considered separately in the last two sections.

2. CONTROL DESIGN

According to the control objectives identified in the previous section the performance criteria can be expressed in terms of system norms. The optimal control weighing distance errors and control effort is a minimization of their quadratic time integrals or similarly the optimization of the corresponding $\mathcal{H}_2$-norms. If we define a mapping of the leading vehicle's acceleration a_L to errors and control effort as

$$H : a_L \mapsto (e_1, .., e_n, u_1, .., u_n)^T \tag{10}$$

the optimal control problem is similar to minimizing $||H||_2$. The additional objectives expressing string stability and upper bounds for velocities and accelerations were formulated in equations (6) and (9). Although the relation for the $\mathcal{H}_\infty$-norm of the transfer functions is only a necessary condition for the criteria to be met, we will show that it is possible to directly influence upper bounds on velocities or accelerations by minimizing this system norm. Thus, all performance criteria formulated for platoon control can be expressed as mixed $\mathcal{H}_2/\mathcal{H}_\infty$ problem:

$$\min \alpha \cdot ||F||_\infty + \beta \cdot ||H||_2 \quad \text{s.t.} \tag{11}$$

$$||G_i||_\infty < 1 \quad \forall i, \tag{12}$$

$$||F_i||_\infty < \gamma \quad \forall i. \tag{13}$$

For convenience we write F as mapping from a_L to either the velocities (F_v), accelerations (F_a) or both.

2.1 LMI formulation

The problem formulation above can be expressed in terms of linear matrix inequalities (LMI). Together with a convex objective function they form a convex optimization problem with the advantage that very efficient numerical solvers exist for this type of problem. For the LMI formulation the transfer functions given in equation (11) can be represented as state space model of a generalized plant drawn in figure 2. The system's states x are defined according to our model from equation (1)-(2).

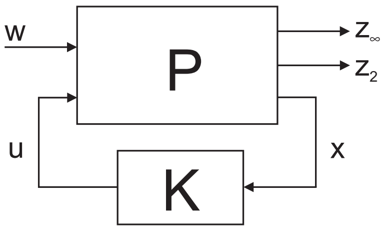

Fig. 2. Generalized Plant with input and output signals.

As mentioned earlier, the condition of string stability can be satisfied already by using information of all preceding vehicles. Thus, in this work we do not include this condition explicitly. The generalized inputs w and outputs z are defined such that the same i/o-relation as for the mapping H and F holds. To formulate the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ approach we would have $w{=}a_L$, $z_2{=}(e_1, .., e_n, u_1, .., u_n)^T$ and $z_\infty{=}(a_1, .., a_n)^T$ to bound amplifications of acceleration for example.
The state space description for the generalized plant is then

$$\dot{x} = Ax + B_1 \cdot w + B_2 \cdot u, \tag{14}$$

$$z_\infty = C_\infty \cdot x + D_{\infty,1} \cdot w + D_{\infty,2} \cdot u, \tag{15}$$

$$z_2 = C_2 \cdot x + D_{2,1} \cdot w + D_{2,2} \cdot u. \tag{16}$$

The control law we are considering for the longitudinal guidance uses feedback of all vehicles' states, i.e. the control law is given by

$$u = K \cdot x. \tag{17}$$

At first we assume full information on all vehicles (no constraints on K), later we analyze the control layout for constraints on the availability of information. The optimization with LMI constraints that corresponds to equation (11)-(13) can then be stated as

$$\min \alpha \cdot \gamma^2 + \beta \cdot \mathrm{trace}(Q) \quad \text{s.t.} \tag{18}$$

$$\begin{bmatrix} (A+B_2K)X + X(A+B_2K)^T & B_1 & X(C_\infty + D_{\infty 2}K)^T \\ B_1^T & -I & D_{\infty 1}^T \\ (C_\infty + D_{\infty 2}K)X & D_{\infty 1} & -\gamma^2 I \end{bmatrix} < 0$$

$$\begin{bmatrix} Q & (C_2 + D_{22}K)X \\ X(C_2 + D_{22}K)^T & X \end{bmatrix} > 0$$

Here, the matrices Q, X, $Y = KX$ and γ^2 are the optimization variables - the feedback matrix is then solved for by $K = YX^{-1}$, see Boyd et al. [1994] and Apkarian et al. [1996] for an exhaustive description of LMI formulations.

2.2 Results

For control optimization and simulation we analyze a truck platoon with one leading and three following vehicles. The solution to problem (18) is a 3-by-9 matrix K mapping the states to the synthetic control outputs u_i of the following vehicles according to control law (17). For all vehicles the nominal time constant is T_i=0.5s. It should be mentioned that although not explicitly stated in the optimization problem string stability was achieved for all controllers due to the information from front vehicles.
To analyze the control performance the platoon response to a velocity step of the leading vehicle is simulated. In figure 3 distance errors and velocities of all following vehicles are plotted for a pure $\mathcal{H}_2$ optimization, i.e. α was set to zero in equation (18).

Table 2. Maximum gains for system variables.

	$\|\|f_{v,3}\|\|_1$	$\|\|f_{a,3}\|\|_1$	$\|\|f_{e,1}\|\|_1$
$\mathcal{H}_2$, (K) $\alpha = 0.0$, $\beta = 1$	2.28	1.70	2.58
$\mathcal{H}_2/\mathcal{H}_\infty$, (K) $\alpha = 0.5$, $\beta = 1$	1.67	1.34	2.85
$\mathcal{H}_2/\mathcal{H}_\infty$, (K_1, K_2) $\alpha = 0.5$, $\beta = 1$	0.62	1.29	0.56
$\mathcal{H}_2/\mathcal{H}_\infty$, (LBT) $\alpha = 0.5$, $\beta = 1$	0.61	1.30	0.44

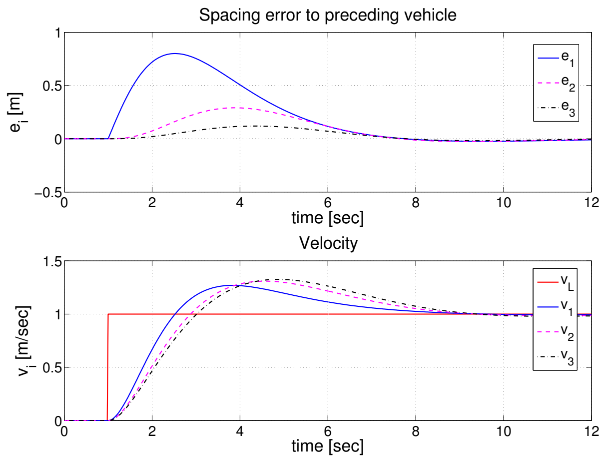

Fig. 3. Platoon response to a velocity step of 1m/s by the leading vehicle using $\mathcal{H}_2$ optimization.

While distance errors are eliminated quickly and decline along the platoon the maximum velocities increase along the platoon before they settle down to the nominal values of the leading vehicle. The worst case amplification γ can be determined by the $||\cdot||_1$ norm for velocities and accelerations respectively. For the 4-truck-platoon the worst case amplification always appears for the last (3rd) vehicle for whom the upper bound amplification is given as $||f_{v,3}||_1$ and $||f_{a,3}||_1$ respectively in table 2. Opposed to that, the maximum spacing errors occur for the front (1st) vehicle and decline along the platoon, worst case errors related to a_L can be determined similarly and are given as $||f_{e,1}||_1$ in the table above.

Now additional $\mathcal{H}_\infty$ conditions for amplifications in the velocity are used. The weighting of the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norm is set to α=0.5 and β=1. The platoon response in figure 4 changes to far lower velocities of following vehicles, on the other hand the settling time for errors and velocities increases. The tuning of both norms can be used as trade-off between both effects. Table 2 shows the reduction of upper gains when the mixed optimization is used. For example, the worst case amplification of acceleration

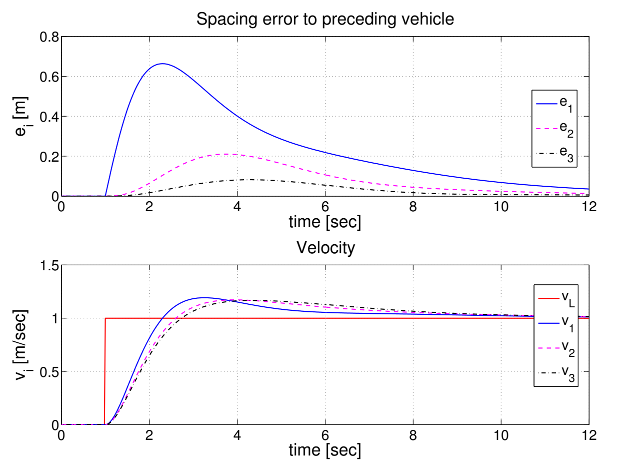

Fig. 4. Platoon response to a velocity step of 1m/s by the leading vehicle using additional $\mathcal{H}_\infty$ criteria.

settles down from 70% to 34% of the leading vehicles acceleration. Maximum gains for errors (here e_1) increase a little though.

To further reduce the upper gains on velocities and accelerations we analyze a changed control structure

$$u = K_1 \cdot x + K_2 \cdot a_L \tag{19}$$

with additional information on the leading vehicle's acceleration. The basic structural advantage is the faster control action. The LMI formulation presented in equation (18) can still be used - to parameterize this control, only the terms B_1 and $D_{\infty,1}$ are changed to $B_1{+}B_2K_2$ and $D_{\infty,1}{+}D_{\infty,2}K_2$ due to the additional feedback of $a_L{=}w$. To analyze the solution for K_1 and K_2 the 4-truck-platoon is simulated for step changes in a_L now. An emergency braking of the leading vehicle with $a_L = -7\text{m/s}^2$ at time $t = 1$s from 80km/h to halt is considered which is close to the possible limits of heavy-duty trucks. Again, the results for simple $\mathcal{H}_2$ and mixed optimization with additional $\mathcal{H}_\infty$ criteria for velocities are compared.

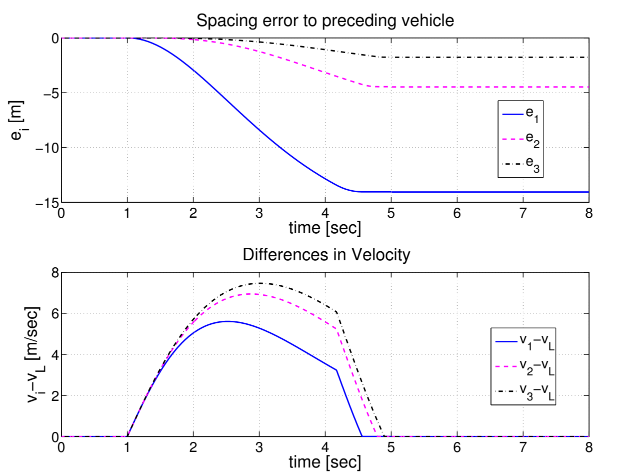

Fig. 5. Platoon response to an acceleration maneuver of the leading vehicle using $\mathcal{H}_2$ criteria.

The results for spacing errors e_i and velocity differences to the leading vehicle $v_i - v_L$ are given in figure 5 (for $\mathcal{H}_2$ optimization) and in figure 6 (for $\mathcal{H}_\infty$ optimization). For the first case, spacing errors go up to 14.1m and the maximum velocity differences go up to 7.5m/s during that maneuver. Using the additional feedback information tuned with the second approach both errors and velocity differences go down to 3.5m and 2.1m/s respectively. Due to the constant acceleration a spacing error has to remain in all cases.

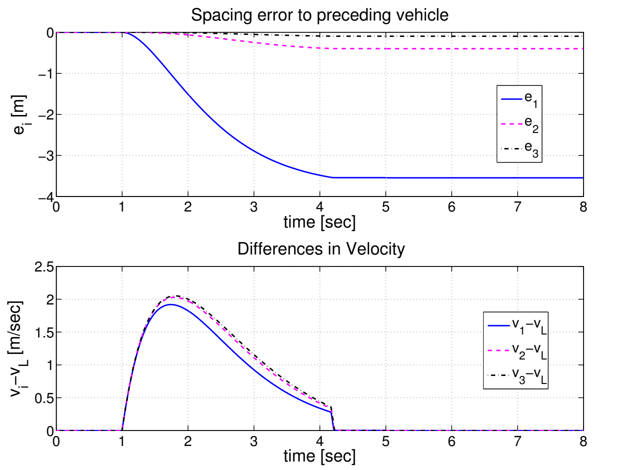

Fig. 6. Platoon response to an acceleration maneuver of the leading vehicle using additional $\mathcal{H}_\infty$ criteria.

The maximum gains in table 2 show that worst case amplifications are now 29% of the leading vehicles acceleration. Please note that due to the inequality in equation (8) the upper bound is very conservative - the simulations for acceleration steps in figure 6 show amplifications of accelerations with less than 15%. Simulations with explicit acceleration saturations (at -7m/s^2) showed that maximum errors below 6m could still be achieved with this control approach.

3. STRUCTURAL CONSTRAINTS

The motivation to analyze structural constraints of our platoon control is a security aspect. We have to consider the low reliability of radio communication networks for vehicle control. Possible strategies to overcome network failure and information loss have been presented in Maschuw et al. [2007] already. Restricting the control to only information from front vehicles guarantees decoupling from following vehicles at least for the critical braking maneuvers, i.e. errors occurring in the back of a platoon do not influence spacing errors of preceding vehicles. For the state feedback control from equation (17) this type of constraint can be described by restricting the feedback matrix K to lower block triangular (LBT) form. This constraint cannot be formulated in terms of LMIs directly though. The approaches made so far concern Youla parametrization and sequential control layout. Both have been widely used for platoon control with pure $\mathcal{H}_2$ optimization already, see e.g. Yagoubi and Chevrel [2005] and Claveau and Chevrel [2005].

We follow the approach of sequential control design but use the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ optimization for the control law $u = K_1 \cdot x + K_2 \cdot a_L$ (K_2 does not have to meet any structural conditions since a_L is only transmitted from the front to the back). The sequential application of the formulation in equation (18) leads to a K_1 with LBT form and suboptimal performance concerning integral errors and control effort and additionally meets upper bounds for maximum gains in velocities or accelerations. The simulation result for an acceleration step of the leading vehicle with -7m/s^2 is given in figure 7.

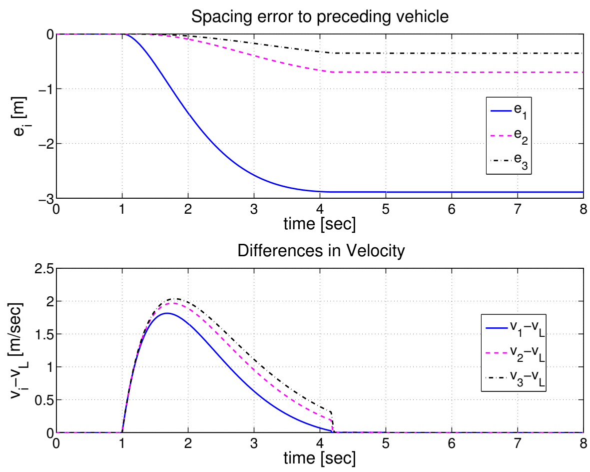

Fig. 7. Platoon response to an acceleration maneuver of the leading vehicle using control with LBT constraints on the feedback matrix.

It is obvious that the response of spacing errors and velocity differences almost resembles the results of the unconstrained optimization before. The maximum gains in table 2 even show a slight reduction of maximum velocity differences and maximum errors.
To compare all control approaches discussed so far the platoon response to acceleration maneuvers of the leading vehicle is given in figure 8. The plots correspond to simple $\mathcal{H}_2$ design, mixed $\mathcal{H}_2/\mathcal{H}_\infty$ design, additional feedback of the leading vehicles acceleration and finally the sequential design. Since maximum spacing errors occur in the front and maximum velocity overshoots in the back only e_1 and $v_3 - v_L$ are plotted for the 4-truck-platoon. It can be seen clearly that the control performance improves a lot using a mixed criteria formulation with either an unconstrained or constrained feedback matrix. Both the spacing errors and velocity differences decrease to roughly 25% of the values achieved when only $\mathcal{H}_2$ criteria are formulated.

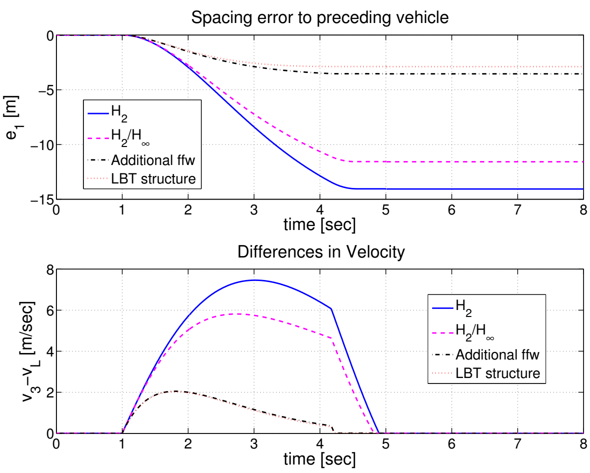

Fig. 8. Comparison of platoon responses to an acceleration maneuver of the leading vehicle using different control layouts.

4. ROBUSTNESS

The given vehicle model in equation (2) lacks from the fact that the drivetrain dynamics can only be identified to a certain extent by a first order filter. In fact the time constant T_i strongly depends on the actual operating point of the engine. Moreover, time constants for braking are much smaller than for accelerating due to the different actuators - the non-symmetry leads to faster (better) performance concerning small gaps arising in (the critical) braking maneuvers compared to positive accelerations. From measurements with our trucks we could derive a maximum range of 0.1s< T_i <1s.
This section is dedicated to the test of robust stability for the controls derived in the previous sections and an explicit robust control design, both can be done using the formulation of LMIs again. Therefore we represent the generalized plant as affine model depending linearly on the parameters $p_i = 1/T_i$. Since every following vehicle may have a different time constant we have to consider the parameters p_i for each vehicle. For the case of three following vehicles with independent time constants the parameters p_1, p_2 and p_3 span a box in $\mathbb{R}^3$. Due to the dependence on operating conditions we assume the parameters to be time varying (in worst case infinitely fast). Since both matrices A and B_2 from equation (14) depend on p_i we can write the closed-loop system matrix $\bar{A} = A + B_2K_1$ in affine form

$$\bar{A}(p) = \bar{A}_0 + p_1(t)\bar{A}_1 + p_2(t)\bar{A}_2 + p_3(t)\bar{A}_3 \tag{20}$$

with $\bar{A}_i = A_i + B_{2,i}K_1$. For the closed-loop system we are seeking a quadratic Lyapunov function $V(x) = x^T P x$ with a symmetric positive definite matrix P to guarantee asymptotic stability at all possible trajectories $p_i(t)$ that lie within the prescribed box. For the affine description it is sufficient to satisfy a set of LMIs at each corner of the parameter box denoted by Π_j, see Sommer [2001] and Gahinet et al. [1995]:

$$P\bar{A}(\Pi_j) + \bar{A}(\Pi_j)^T P < 0, \quad \text{with} \quad j = 1,..,8. \tag{21}$$

The additional LMI condition can either be used a priori for a robust control layout or a posteriori to check robust stability of the designed control law. For all controllers of the previous sections that were optimized for a nominal time constant T_i =0.5s the above condition was fulfilled for infinitely fast varying time constants in the range 0.1s< T_i <1s. The closed-loop system is even stable for time constants above T_i =1s. The critical time constants marking the stability margin of the closed loop are given in table 3 for the different control layouts.

Table 3. Time constants of drivetrain at the stability margin of the closed loop.

	T_i for stability margin
$\mathcal{H}_2$,(K) $\alpha = 0.0$, $\beta = 1$	2.4s
$\mathcal{H}_2/\mathcal{H}_\infty$,$(K)$ $\alpha = 0.5$, $\beta = 1$	2.6s
$\mathcal{H}_2/\mathcal{H}_\infty$,$(K_1, K_2)$ $\alpha = 0.5$, $\beta = 1$	2.5s
$\mathcal{H}_2/\mathcal{H}_\infty$,(robust) $\alpha = 0.5$, $\beta = 1$	2.9s

To include condition (21) into the optimization problem and solve for K_1 and K_2 with guaranteed robustness, the condition needs to be formulated in different terms. If $X = P^{-1}$ the above relation is equivalent to

$$\bar{A}(\Pi_j)X + X\bar{A}(\Pi_j)^T < 0, \quad \text{with} \quad j = 1,..,8. \tag{22}$$

If we plug in $\bar{A} = A + B_2K_1$, eight additional LMI conditions for the optimization variables X and $Y = K_1X$ can be directly added to the setup of equation (18). The a priori inclusion of condition (22) leads to closed-loop systems with slightly better stability margins. For the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control with additional robustness conditions the platoon's closed-loop eigenvalues are given in figure 9. The eigenvalues are plotted as grid for different time constant starting at 0.1s and going up to 2s - a shading from dark to light is used for increasing time constants.

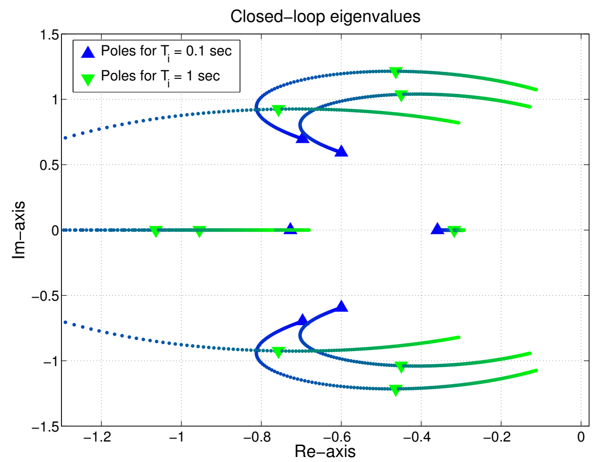

Fig. 9. Closed-loop eigenvalues for robust $\mathcal{H}_2/\mathcal{H}_\infty$ layout with α =0.5, β =1; 0.1s< T <2s.

It can be seen that stability always becomes critical for great time constants with two pairs of eigenvalues wandering towards the imaginary axis. Table 3 shows that the (explicit) robust layout for a nominal plant with T_i =0.5s leads to an upper time constant of T_i =2.9s at the stability margin which is slightly better than the one for controls with an a posteriori test of robust stability. It should be noted that robust control design is also possible for larger time constants by simply choosing a bigger nominal time constant (this will diminish performance for smaller time constants though).

5. CONCLUSION

This paper presents a direct LMI formulation for minimization of velocity and acceleration amplifications and finally robustness with respect to the drivetrain dynamics; it herewith enlarges the actual LMI concepts in longitudinal vehicle guidance concentrating on spacing errors so far. The advantages of additional $\mathcal{H}_\infty$ criteria were shown clearly. So far, only two heavy-duty trucks have been equipped and tested with the automation hard- and software. To validate the results of this paper, road trials with four vehicles are planned for the close future.

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Appendix A. PROOF FOR AN UPPER BOUND

We will only show the existence of upper limits for accelerations, a similar approach is valid though for the velocities. As stated earlier we assume that for all vehicles

$$||g_i(t)||_1 \leq \gamma_e < 1 \tag{A.1}$$

holds. The maximum acceleration of a vehicle at position $i + k$ can be expressed in terms of the acceleration at position i using the triangular inequality:

$$||a_{i+k}||_\infty \leq ||a_i||_\infty + \sum_{j=1}^{k} ||a_{i+j} - a_{i+j-1}||_\infty. \tag{A.2}$$

For the transfer function relating the acceleration differences it holds

$$\frac{(a_i - a_{i-1})(s)}{(a_{i-1} - a_{i-2})(s)} = \frac{e_i(s)}{e_{i-1}(s)} = G_i(s). \tag{A.3}$$

Hence, using the inequality from (A.1) we can write

$$||a_i - a_{i-1}||_\infty \leq \gamma_e ||a_{i-1} - a_{i-2}||_\infty \tag{A.4}$$

an plug this into equation (A.2) to get

$$||a_{i+k}||_\infty \leq ||a_i||_\infty + \sum_{j=0}^{k-1} \gamma_e^j ||a_{i+1} - a_i||_\infty. \tag{A.5}$$

Since $\gamma_e < 1$ the limit of the infinite geometric series exists and for $k \to \infty$ we have the upper limit

$$||a_\infty||_\infty \leq ||a_i||_\infty + \frac{1}{1 - \gamma_e} ||a_{i+1} - a_i||_\infty \tag{A.6}$$

that only depends on a certain vehicle i but not on the platoon length.