Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Feedback Linearization–Based Control for a Class of Chemical Processes in Non-Standard Nonlinear Singular Perturbation Form

Atiye Sarabi-Jamab*, Mohammad Javad Yazdanpanah**

Control & Intelligent Processing Centre of Excellence;
Dept. of Electrical & Computer Eng., Faculty of Eng., University of Tehran,
Tel: +98-21-8863-3025, P.O. Box 14395/515, Tehran, IRAN
**(e-mail: asarabi@ut.ac.ir)*
***(e-mail: yazdan@ut.ac.ir)*

Abstract: This paper deals with a class of chemical process with measurable time-varying disturbances, which is modeled within the framework of singular perturbation in non-standard form. The results in singular perturbation theory consider the systems in a standard form, therefore, a transformation to change a system representation from non-standard to standard form should be found. After this transformation is made, a systematic approach to control this class of systems and disturbance rejection using feedback linearization is proposed. The application of the developed method is illustrated through a catalytic continuous stirred tank reactor.

1. INTRODUCTION

The majority of chemical processes is inherently nonlinear and is often characterized by the presence of dynamical phenomena occurring in multiple time-scales (Breusegem and Bastin 1991). Typical examples of nonlinear multiple-time scale systems include reaction networks (Breusegem and Bastin 1991), catalytic reactors (Chang and Aluko 1984), DC motor models (Kokotovic, Khalil and Oreill 1986) and electrical circuits (Khalil 1996).

Singular perturbation theory has proven to be the natural framework scale systems. This model of finite-dimensional dynamic systems, extensively studied in the mathematical literature by Tikhonov, Levinson, Vasil'eva, etc., was also the first model to be used in control and systems theory (Kokotovic, Khalil and Oreill 1986).

However, these results consider the systems in a standard form. In some cases, the singularly perturbed systems are often modeled as a non-standard form. In these cases Although there is a guideline for finding this transformation in Kokotovice et al. (Kokotovic, Khalil and Oreill 1986), but it is heuristic and especially difficult to apply in nonlinear cases (Glizer 2004).

For a class of singularly perturbed systems there have been numerous research papers for analysis and controller design (Choi, Son and Lim 2006, Glizer 2004, Krishnan and McClamroch 1994, Shao 2004, Zigang and Basar 1994). Usually singularly perturbed systems are controlled by composite control that is designed to stabilize the fast and slow subsystems (Kokotovic, Khalil and Oreill 1986). This composite-control scheme is easy to design and results in simple control structure. Feedback linearization by the main feature that reduces the nonlinear control design to a linear control is a systematic approach to the control of nonlinear singularly perturbed systems (Choi, Shin and Lim 2005).

In this paper, a class of two-time-scale nonlinear systems modeled within the non-standard singular perturbation framework, with measurable time-varying disturbances, is considered. After finding a transformation to change to standard form, by using an ε -independent diffeomorphism, nonlinear singularly perturbed system will be transformed into the linear singularly perturbed form (Choi, Shin and Lim 2005, Khorasani 1987). Fast and slow controllers are designed for each subsystem and applied to the model.

The reminder of the paper is organized as follows: Section 2 contains the problem formulation. Section 3 presents a method for changing non-standard form into standard model. In section 4 the design procedure of the feedback linearization-based controller for singularly perturbed system is introduced. The practicability of the proposed scheme is demonstrated with the control of a catalytic continuous stirred tank reactor modeled as a singularly perturbed system in non-standard form in section 5. Finally, section 6 concludes this paper.

2. PROBLEM STATEMENT

Modeling a two-time-scale process in a singularly perturbed form is in the explicit state-variable form in which the derivatives of some of the states are multiplied by a small positive scalar ε . That is,

$$\begin{aligned} \dot{x} &= f(\mathbf{x},\mathbf{z},\varepsilon,t) \quad \mathbf{x}(t_0)=x^0, \ \ \mathbf{x}\in R^n \\ \varepsilon\dot{\mathbf{z}} &= g(\mathbf{x},\mathbf{z},\varepsilon,t) \quad \mathbf{z}(t_0)=\mathbf{z}^0, \ \ \mathbf{z}\in R^m \end{aligned} \tag{1}$$

Where f and g are assumed to be sufficiently many times continuously differentiable functions of their arguments z, ε, t . The scalar ε represents all the small parameters to be neglected and is defined by taking into account the physicochemical characteristic of the process (Kokotovic, Khalil and Oreill 1986, Breusegem and Bastin 1991).

978-1-1234-7890-2/08/$20.00 © 2008 IFAC

10.3182/20080706-5-KR-1001.3117

In control and systems theory, the model (1) is a step toward reduced-order modeling, i.e. when $\varepsilon = 0$, the dimension of the state space reduces from $n+m$ to n because the second part of (1) degenerates into the

$$0 = g(\bar{\mathbf{x}}, \bar{\mathbf{z}}, 0, t) \tag{2}$$

If and if only in the domain of interest, the equation of (2) has been $k \geq 1$ distinct real root, the model (1) is in standard form because this model changed to a well-defined n-dimensional reduced model corresponds to each root. Else for using the singular perturbation theory must to find a way to convert non-standard model into the standard form.

In many chemical processes the main nonlinearities are associated with the slow variables and these two-time-scale nonlinear systems can be considered in a specific singularly perturbed system that the singular perturbation parameter ε appears only in the left-hand side of equations, while the fast variable enters in a linear fashion as the following state-space representation:

$$\begin{aligned} \dot{\mathbf{x}} &= f_1(\mathbf{x}) + Q(\mathbf{x})\mathbf{z} + g_1(\mathbf{x})u + W_1(\mathbf{x})d(t) \\ \varepsilon\dot{\mathbf{z}} &= f_2(\mathbf{x}) + Q_2(\mathbf{x})\mathbf{z} + g_2(\mathbf{x})u + W_2(\mathbf{x})d(t) \\ y &= h(\mathbf{x}) \end{aligned} \tag{3}$$

Where $\mathbf{x} \in R^n$ and $\mathbf{z} \in R^m$ denote vectors of state variables, $u \in R$ denotes the manipulated input, $d = [d_1(t), d_2(t), \cdots, d_q(t)]$ denotes the vector of disturbance inputs, which are assumed to be measurable and sufficiently smooth function of time, and $y \in R$ denotes the controlled output.

Furthermore, $f_1(x)$, $f_2(x)$, $g_1(x)$ and $g_2(x)$ are analytic vector fields, $Q_1(x)$, $Q_2(x)$ and $w_1(x)$, $w_2(x)$ are analytic matrices of dimensions $n\times m$, $m \times m$, $n \times q$, $m\times q$ respectively, and $h(x)$ is an analytic scalar function.

By setting $\varepsilon = 0$, the system (3) takes the form

$$\dot{\mathbf{x}} = f_1(\mathbf{x}) + Q_1(\mathbf{x})\mathbf{z}_s + g_1(\mathbf{x})u + W_1(\mathbf{x})d \tag{4}$$

$$f_2(\mathbf{x}) + Q_2(\mathbf{x})\mathbf{z}_s + g_2(\mathbf{x})u + W_2(x)d = 0 \tag{5}$$

Where $\mathbf{z}_s$ denotes a quasi-steady state for $\mathbf{z}$. By assuming that the system (3) is in standard form, the invariability of the matrix $Q_2(x)$ guarantees that the system of algebraic equation (5) admits a unique solution for $\mathbf{z}_s$, and the system decomposes into separate reduced-order systems evolving on different time scales. Else it must be converted into the standard form.

Performing a two-time-scale decomposition, the corresponding slow subsystem is given by

$$\begin{aligned} \dot{\mathbf{x}} &= F(\mathbf{x}) + G(\mathbf{x})u + W(\mathbf{x})d, \\ y^s &= h(\mathbf{x}) \end{aligned} \tag{6}$$

Where y^s denotes the output associated with the slow subsystem and

$$\begin{aligned} F(\mathbf{x}) &= f_1(\mathbf{x}) - Q_1(\mathbf{x})[Q_2(\mathbf{x})]^{-1} f_2(\mathbf{x}) \\ G(\mathbf{x}) &= g_1(\mathbf{x}) - Q_1(\mathbf{x})[Q_2(\mathbf{x})]^{-1} g_2(\mathbf{x}) \\ W(\mathbf{x}) &= W_1(\mathbf{x}) - Q_1(\mathbf{x})[Q_2(\mathbf{x})]^{-1} W_2(\mathbf{x}) \end{aligned} \tag{7}$$

Note the input u and the disturbance input vector d appear in an affine because of the linearity in $\mathbf{z}$ in the original system.

3. FROM NON-STANDARD TO STANDARD FORM

Suppose that the two-time-scale nonlinear system (3) is in non-standard form, i.e. systems for which the matrix $Q_2(x)$ is singular for some $x \in \mathbf{X}$. The direct consequence of it is the absence of a well-defined quasi-steady-state for the fast variable $\mathbf{z}$ (Breusegem and Bastin 1991) and thus the lack of a well-defined open-loop reduced system.

To achieve regularization of the fast dynamic, because of the fast variable is in linear fashion, appropriate feedback of the state vector $\mathbf{z}$ will be employed. Thus a control law considered of the form

$$u = \hat{u} + k^T(\mathbf{x})\mathbf{z} \tag{8}$$

Where $k^T(\mathbf{x})$ is a vector field in R^m, and $\hat{u}$ is an auxiliary input. Under the control law, the system (3) takes the form

$$\begin{aligned} \dot{\mathbf{x}} &= f_1(\mathbf{x}) + [Q_1(\mathbf{x}) + g_1(\mathbf{x})k^T(\mathbf{x})]\mathbf{z} + g_1(\mathbf{x})\hat{u} + W_1(\mathbf{x})d \\ \varepsilon\dot{\mathbf{z}} &= f_2(\mathbf{x}) + [Q_2(\mathbf{x}) + g_2(\mathbf{x})k^T(\mathbf{x})]\mathbf{z} + g_2(\mathbf{x})\hat{u} + W_2(\mathbf{x})d \end{aligned} \tag{9}$$

In addition $k^T(\mathbf{x})$ is chosen in such a manner that the matrix $Q_2(x) + g_2(x)k^T(x)$ is Hurwitz uniformly in $x \in \mathbf{X}$. Now, the new model is in standard form that can be composed into separate reduced-order systems (Breusegem and Bastin 1991, Glizer 2004).

4. DESIGN PROCEDURE

Usually singularly perturbed systems are controlled by composite control that is designed to stabilize the fast and slow subsystems (Kokotovic, Khalil and Oreill 1986).

In this part feedback linearization is used to the control of a class of nonlinear singularly perturbed systems and the ε-independent diffeomorphism is engaged to transform the nonlinear systems into the linear singularly perturbed form.

The system (1) is rewritten as follows

$$\begin{aligned} \dot{\mathbf{x}} &= f_{11}(\mathbf{x},\mathbf{z}) + g_{11}(\mathbf{x},\mathbf{z})u + f_{12}(\mathbf{x},\mathbf{z}) + g_{12}(\mathbf{x},\mathbf{z})u \\ \varepsilon\dot{\mathbf{z}} &= f_{21}(\mathbf{x},\mathbf{z}) + g_{21}(\mathbf{x},\mathbf{z})u + f_{22}(\mathbf{x},\mathbf{z}) + g_{22}(\mathbf{x},\mathbf{z})u \end{aligned} \tag{12}$$

For notational convenience the following function is defined.

$$\begin{aligned} \bar{f} &= \begin{bmatrix} f_{11}(\mathbf{x},\mathbf{z}) \\ f_{21}(\mathbf{x},\mathbf{z}) \end{bmatrix}, \quad \bar{g} = \begin{bmatrix} g_{11}(\mathbf{x},\mathbf{z}) \\ g_{21}(\mathbf{x},\mathbf{z}) \end{bmatrix}, \\ \tilde{f} &= \begin{bmatrix} f_{12}(\mathbf{x},\mathbf{z}) \\ f_{22}(\mathbf{x},\mathbf{z}) \end{bmatrix}, \quad \tilde{g} = \begin{bmatrix} g_{12}(\mathbf{x},\mathbf{z}) \\ g_{22}(\mathbf{x},\mathbf{z}) \end{bmatrix} \end{aligned} \tag{13}$$

Assume that the pair $\{\bar{f}, \bar{g}\}$ is input-state lineazable part of nonlinear system representation. By the theory that is proposed in (Choi, Shin and Lim 2005) there exists a diffeomorphism $\xi = T(\zeta)$ which transforms (1) into (14) if and only if the following conditions hold:

(a) $[\tilde{f}, adj_{\tilde{f}}^{n+m-j}\bar{g}] = \sum_{i=j}^{n} a_{ij} adj_{\tilde{f}}^{n+m-i}\bar{g}, \quad a_{ij} \in R, j = 1, \cdots, n$

(b) $[\tilde{f}, adj_{\tilde{f}}^{m-j}\bar{g}] = \frac{1}{\varepsilon}\sum_{i=j}^{n-1} b_{ij} adj_{\tilde{f}}^{m-i}\bar{g} + \frac{1}{\varepsilon}\Lambda$

Where $\Lambda \in span\{g\}, b_{ij} \in R,\ j = 1, \cdots, m-1$.

(c) $\tilde{g} \in span\{\bar{g}\}$

$$\begin{aligned}
\dot{\xi}_1 &= a_{1,1}\xi_1 + \xi_2 \\
&\vdots \\
\dot{\xi}_n &= a_{n,1}\xi_1 + \cdots + a_{n,n}\xi_n + \xi_{n+1} \\
\varepsilon\dot{\xi}_{n+1} &= b_{1,1}\xi_{n+1} + \xi_{n+2} \\
&\vdots \\
\varepsilon\dot{\xi}_{n+m-1} &= b_{m-1,1}\xi_{n+1} + \cdots + b_{m-1,m-1}\xi_{n-1} + \xi_{n+m} \\
\varepsilon\dot{\xi}_{n+m} &= v = \bar{\alpha}_0(\xi) + \tilde{\alpha}(\xi) + (\bar{\beta}_0(\xi) + \tilde{\beta}(\xi))u
\end{aligned} \tag{14}$$

Where $a_{i,j}$ and $b_{i,j}$ are real numbers.

Assuming that $\bar{\beta}_o(\xi) + \tilde{\beta}(\xi) \neq 0$ around the equilibrium point, the ε-independent feedback linearization control law can be applied.

$$u = (-\bar{\alpha}_0(\xi) - \tilde{\alpha}(\xi) + v)/(\bar{\beta}_0(\xi) + \tilde{\beta}(\xi)) \tag{15}$$

Since the slow and fast dynamics are separated, two linear controllers for each reduced subsystem can be designed ($v = v_s + v_f = \mathbf{k}_s\xi_s + \mathbf{k}_f\xi_f$).

The fast controller is designed ($\mathbf{k}_f = [k_{f1}, \cdots, k_{fn-r}]^T$) so that it stabilizes the fast dynamics. Then the fast dynamics is given by

$$\varepsilon\dot{\xi}_f = \begin{bmatrix} b_{1,1} & 1 & 0 & \cdots & 0 \\ b_{2,1} & b_{2,2} & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ k_{f1} & k_{f2} & k_{f3} & & k_{fm} \end{bmatrix}\xi_f + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} v_s \tag{16}$$

At $\varepsilon = 0$ the slow manifold is given as the following relation

$$\xi_{n+1} = -\frac{(-1)^{1+m}}{|A_f|} v_f \tag{17}$$

Where the Hurwitz matrix A_f is

$$A_f = \begin{bmatrix} b_{1,1} & 1 & 0 & \cdots & 0 \\ b_{2,1} & b_{2,2} & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ k_{f1} & k_{f2} & k_{f3} & & k_{fm} \end{bmatrix} \tag{18}$$

Then the resulting slow dynamics is given by

$$\begin{aligned}
\dot{\xi}_1 &= a_{1,1}\xi_1 + \xi_2 \\
&\vdots \\
\dot{\xi}_n &= a_{n,1}\xi_1 + \cdots + a_{n,n}\xi_n + \frac{(-1)^m}{|A_f|} v_s
\end{aligned} \tag{19}$$

This linear system is controllable and thus the stabilizing controller v_s can be designed (Choi, Shin and Lim 2005).

5. SIMULATION RESULTS

In this section, the proposed control methodology will be applied to a chemical process with time-scale multiplicity. Consider the catalytic continues stirred tank reactor shown in Fig.1, where a homogeneous reaction $A \to B$ and a catalytic $A \to C$ take place. The first reaction leads to the generation of the side-product B, while the second reaction leads to the production of the desired product C (Breusegem and Bastin 1991).

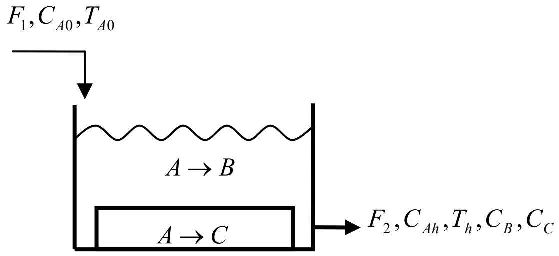

Fig. 1- A catalytic continues stirred tank reactor

The inlet stream F_1 consists of pure species A of concentration C_{A0}, and temperature T_{A0}. The process dynamic model consists of the following set of material and energy balances:

$$\frac{dV_r}{dt} = F_1 - F_2 \tag{20}$$

$$\frac{dC_{Ah}}{dt} = \frac{1}{V_r}[F_1(C_{A0} - C_{Ah}) - K_h \exp(-\frac{E_h}{RT_h})V_r - K_C A_C (C_{Ah} - C_{AC})] \tag{21}$$

$$\rho_h C_{\rho h}\frac{dT_h}{dt} = \frac{1}{V_r}[\rho_h C_{\rho h} F_1 (T_{A0} - T_h) + (-\Delta H_h) K_h \exp(-\frac{E_h}{RT_h})V_r - U_W A_W (T_h - T_W) - U_C A_C (T_h - T_C)) \tag{22}$$

$$\frac{dC_{AC}}{dt} = \frac{K_C A_C}{V_C}(C_{Ah} - C_{AC}) - K_C \exp(-\frac{E_C}{RT_C})C_{AC} \tag{23}$$

$$\rho_C C_{\rho C}\frac{dT_c}{dt} = \frac{U_C A_C}{V_C}(T_k - T_C) + (-\Delta H_C) K_C \exp(\frac{-E_C}{RT_C})C_{AC} \tag{24}$$

Where V_r denote the volume of the homogenous phase, C_{Ah}, T_h and C_{AC}, T_C denote the concentration and temperature of species A in homogeneous and catalytic phases, $k_h, k_C, E_h, E_C, \Delta H_h$ and ΔH_C denote the pre-exponential factors, the activation energies and the enthalpies of the two reactions, K_C and U_C, U_W denote mass and heat transfer coefficients of the wall and the catalyst.

The control objective is the regulation of the temperature of the catalyst by manipulating the inlet flow rate F_1, in order to remain the generation of the product species C at the desired level. The inlet concentration and temperature of the species A, C_{A0} and T_{A0}, as well as wall temperature T_W are assumed to be the measurable disturbances. The value of the system parameters and the corresponding steady-state values of the system variable are given in Table 1.

Table1. Process parameters

Parameter	Nominal value in Steady-State
F_2	$500.0\ L\min^{-1}$
$C_{\rho h}$	$0.231\ Kcal\ kg^{-1}\ K^{-1}$
$C_{\rho c}$	$2.31 Kcal\ kg^{-1}\ K^{-1}$
ρ_h	$0.9\ Kg\ L^{-1}$
ρ_C	$90.0\ Kg\ L^{-1}$
$K_C A_C$	1618.0
$U_C A_C$	$6667.0\ Kcal\ \min^{-1}\ K^{-1}$
$U_W A_r$	$3340.0 Kcal\min^{-1}\ K^{-1}$
R	$1.987 Kcal\ Kmol^{-1}\ K^{-1}$
K_h	$164.68 L\ mol^{-1}\ \min^{-1}$
E_h	$8.0\times10^3\ Kcal\ Kg^{-1}$
K_c	$2000.0\ \min^{-1}$
E_c	$9.0\times10^3\ Kcal\ Kg^{-1}$
ΔH_h	$69.2006\ Kcal\ Kmol^{-1}$
ΔH_c	$-99.0781\ Kcal\ Kmol^{-1}$
V_c	$145.1\ L$
Vs	$1000.0\ L$
C_{Ah}	$5.0\ mol\ L^{-1}$
T_h	$690\ k$
C_{Ac}	$3.75\ mol\ L^{-1}$
T_c	$720\ K$
F_{1S}	$500.0\ L\min^{-1}$
C_{A0S}	$10.0\ mol\ L^{-1}$
T_{A0S}	$305.0\ K$
T_{WS}	$310.0\ K$

The process exhibits two-time-scale behavior owing to the large heat capacity of the catalytic phase. This implies that V_r, C_{Ah}, T_h and C_{AC} are the fast process variable, while T_C is the slow process variable. In order to obtain a singularly perturbed representation of the process, the parameter ε is defined as

$$\varepsilon = \frac{1}{\rho_C C_{\rho C}} = \frac{1}{90\times 2.31} = 4.810\times10^{-3}\ \frac{K\,L}{Kcal} \tag{25}$$

Setting

$Slow: T_C = x_1,\quad Fast: V_r = z_1,\ C_{Ah} = z_2,\ T_h = z_3,\ C_{AC} = z_4$
$u = F_1 - F_{1s},\ \ d_1 = C_{A0} - C_{As},\ d_2 = T_{A0} - T_{A0s},\ d_3 = T_W - T_{Ws}$
$y = x_1,\ t_{New} = \dfrac{t}{\rho_C C_{\rho C}}$

The original set of equation can be put in the following singularly perturbed form:

$$\dot{x}_1 = \frac{UcAc}{Vc}(z_3 - x_1) + (-\Delta H_c)Kc\exp(\frac{-E_c}{Rx_1})z_4$$

$$\varepsilon\dot{z}_1 = u$$

$$\begin{aligned}\varepsilon\dot{z}_2 &= \frac{1}{z_1}[F_{1S}C_{A0S} - K_h\exp(\frac{-E_h}{Rz_3})z_1 \\ &\quad + (-F_{1S} - KcAc)z_2 + KcAc\,z_4 + (C_{A0} - z_2)u + F_{1S}d_1]\end{aligned} \tag{26}$$

$$\begin{aligned}\varepsilon\dot{z}_3 &= \frac{1}{z_1}[F_{1S}(T_{A0S} - z_3) - \frac{UwAw}{\rho_h C_{\rho h}}(z_3 - T_{WS}) \\ &\quad - \frac{UcAc}{\rho_h C_{\rho h}}(z_3 - x_1) + \frac{(-\Delta H_h)}{\rho_h C_{\rho h}}K_h\exp(\frac{-E_h}{Rz_3})z_1 \\ &\quad + (T_{A0} - z_3)u + F_{1S}d_2 + \frac{UwAw}{\rho_h C_{\rho h}}d_3]\end{aligned}$$

$$\varepsilon\dot{z}_4 = \frac{KcAc}{Vc}z_2 + [-\frac{KcAc}{Vc} - Kc\exp(\frac{-E_c}{Rx_1})z_4$$

From the structure of the differential equation for z_1 in the above system, it is clear that the fast dynamic of the process are singular. Since the process is in non-standard form, in the first step, the regularization law of the form

$$u = \hat{u} - z_1 \tag{27}$$

was used to transform the original two-time-scale system into a new one in standard form with exponentially stable dynamics.

The relative orders of output, x_1 with respect to the input, $\hat{u}$ and the disturbance input vector d is one. After that by using feedback linearization the control law for the closed loop reduced system can be designed.

$$\hat{u} = v - \beta_0 x_1 - \beta_1 \tag{28}$$

Where the parameters β_0 and β_1 were chosen to be $\beta_0 = 1.0$, $\beta_1 = 1.1$.

In simulation, the capability of the controller to keep the output of the system at the operating steady state in the presence of time-varying disturbances is evaluated. The following disturbances were imposed at $t = 0$

$$d_1(t) = 0.5 * \sin\left(\frac{2\pi}{T}t\right)\ molL^{-1}$$

$$d_2(t) = d_3(t) = 0.5 * \sin\left(\frac{2\pi}{T}t\right)\ molL^{-1}$$

Where $T = 0.2\min$

The corresponding output and input profiles for control of temperature are shown in Fig. 2 and 3.

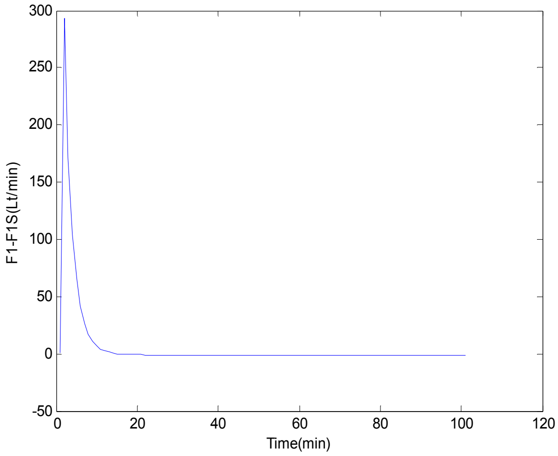

Fig. 2- control signal as inlet flow rate

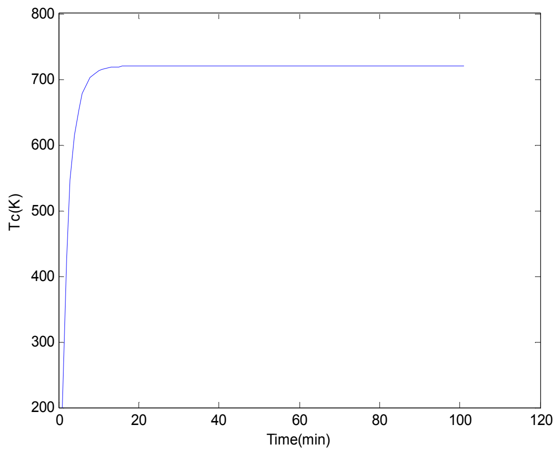

Fig. 3- Control of the temperature of the catalyst in order to remain the generation of the product at the desired level

6. CONCLUSIONS

In this paper, a class of non-standard nonlinear two-time-scale control systems with time-varing disturbances was considered.

Because of the results in singular perturbation theory, consider the systems in a standard form; a transformation to change a system representation from non-standard to standard form is proposed. After this transformation is made, a systematic approach to control this class of systems and disturbance rejection using feedback linearization is formulated.With this fotmulation, the standard nonlinear systems ared transformed into the linearized form so that a linear controller can be systematically designed. The application of the method is illustrated in the control of a catalytic continuous stirred tank reactor.

REFERENCES

Breusegem, V.V. and G. Bastin (1991). Reduced order dynamical modeling of reaction systems: a singular perturbation approach, In *Proc. 30th IEEE Conf. on Decision and Control, Brighton, U. K.*, pp. 1049-1054.

Chang, H. C. and M. Aluko (1984). Multi-scale analysis of exotic dynamics in surface catalyses reaction, In *Chemistry Engineering Science*, 39, pp. 37-50.

Choi, H.L., Y. S. Shin and J. T. Lim (2005). Control of nonlinear singularly perturbed systems using feedback linearization, In *proc. IEE Control Theory Application*, 152(1), pp. 91-94.

Choi, H.L., J.W. Son and J.T. Lim (2006). Stability analysis and control of nonstandard nonlinear singularly perturbed system, In *proc. IEE Control Theory Application*, 153(6), pp. 703-708.

Christofides, P. D. and P. Daoutidis (1996). Compensation of measurable disturbance for two-time-scale nonlinear system, *Automatica*, 12(11), pp. 1553-1573

Glizer, V.Y. (2004). on stabilization of nonstandard singularly perturbed system with small delays in state and control, In *IEEE Trans. on Automatic Control*, 46(6).

Khalil, H.K. (1996). *Nonlinear systems*, Prentice- Hall, Upper Saddle Rivre, NJ 07458.

Khorasani, K. (1987). On linearization of nonlinear singularly perturbed system, In *IEEE Trans. on Automatic Control*, AC-32(3), pp. 256-260.

Kokotovic, P. V., H. K. Khalil and J. Oreill (1986). *Singular Perturbation Methods in Control: Analysis and Design*, Academic Press, New York.

Krishnan, H. and N. H. McClamroch (1994). on the connection between nonlinear differential algebraic equation and singularly perturbed system in nonstandard form, In *IEEE Trans. on Automatic Control,* 39(5), pp. 1079-1084.

Shao, Z.H. (2004) Robust Stability of Two-Time-Scale Systems with Nonlinear Uncertainties", In *IEEE Trans. on Automatic Control*, 49(2), pp. 258-261.

Zigang P. and T. Basar (1994). H^{∞}-optimal control for singularly perturbed systems-part II: Imperfect state measurements, In *IEEE Trans. on Automatic Control*, 39(2), pp. 280-298.