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Parameter dependent state-feedback control of LPV time delay systems with time varying delays using a projection approach

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Abstract: This paper is concerned with the stabilization of LPV time delay systems with time varying delays by parameter dependent state-feedback. First a stability test with $\mathcal{H}_\infty$ performance is given through a parameter dependent LMI. This stability test is derived from a parameter dependent Lyapunov-Krasovskii functional combined with the Jensen's inequality. From this result we derive a state-feedback existence lemma expressed through a nonlinear matrix inequality (NMI). Using a result of the paper we are able to turn this (NMI) into a bilinear matrix inequality (BMI) involving a 'slack' variable. This BMI formulation is shown to be more flexible than the initial NMI formulation and is more adequate to be solved using algorithm such as 'D-K iteration'. The controller construction is provided by two different ways. We finally discuss on the relaxation method and we show the efficiency of our method through several examples.

Keywords: Linear parameter-varying systems; Systems with time-delays; Robust linear matrix inequalities

1. INTRODUCTION

Since several years, time-delay systems ([Moon et al., 2001, Niculescu, 2001, Zhang et al., 2001, Gu et al., 2003, Gouaisbaut and Peaucelle, 2006b, Fridman, 2006, Suplin et al., 2006]) have suggested more and more interest because of their destabilizing effects and performances deterioration. In high-speed systems, even a small time-delay may have a high effect and cannot be neglected, that is why specific stability tests and adapted controller designs must have been developed. Time-delay models appear in various problems as chemical processes, population growth... Since the advent of networks and Network Controlled Systems appears the necessity of studying systems with time-varying delays.

On the other hand, over the past recent years, LPV systems ([Apkarian and Gahinet, 1995, Apkarian and Adams, 1998, Wu, 2001, Scherer, 2001, Iwasaki and Shibata, 2001]) have been heavily studied because they offer a general way to study and control complex systems such as nonlinear systems, LTV systems, multimodel systems or switched systems. This is mainly due to the use of LMIs which provide a powerful formulation to the majority of systems and control theory problems. The LPV system and control theory is still an open problem and major improvements are needed as well for stability determination as for controller design. Several methods exist and each one has its own advantages and drawbacks (polytopic approach, gridding method and LFT method).

The control of LPV time-delay systems have also been studied in [Wu and Grigoriadis, 2001, Zhang et al., 2002, Zhang and Grigoriadis, 2005, Briat et al., 2007a] but it is still an open problem. The aim of the present paper is to find a control law based on a parameter dependent state-feedback of the form

$$u(t) = K(\rho)x(t) \tag{1}$$

which stabilizes the system

$$\begin{aligned} \dot{x}(t) &= A(\rho)x(t) + A_h(\rho)x_h(t) + B(\rho)u(t) \\ &\quad +E(\rho)w(t) \\ z(t) &= C(\rho)x(t) + C_h(\rho)x_h(t) + D(\rho)u(t)+ \\ &\quad F(\rho)w(t) \\ x(\eta) &= \phi(\eta),\ \eta \in [-h_M, 0] \end{aligned} \tag{2}$$

where $x \in \mathbb{R}^n$ is the system state, $x_h = x(t-h(t)) \in \mathbb{R}^n$ is the delayed state, $u \in \mathbb{R}^m$ is the control input, $w \in \mathbb{R}^p$ is the exogenous input, $z \in \mathbb{R}^q$ is the controlled output and $\phi(\cdot)$ is the functional initial condition.

The main contributions of the paper are

- We provide first delay-dependent stability tests for LPV time-delay systems based on the approach of [Gouaisbaut and Peaucelle, 2006a] extended to the LPV case.
- We introduce then a relaxation theorem allowing to transform nonlinear matrix inequalities (NMI) of a

particular form into equivalent bilinear matrix inequalities (BMI) involving an additional slack variable. This relaxation simplifies the problem through the additional slack variable and allows to easily find feasible initial conditions. This 'weak'-formulation will be shown to be easily solved using 'D-K iteration'-like algorithms.

- From these preliminary results we are able to construct stabilizability condition through parameter dependent matrix inequalities. Moreover, it is also possible to measure the conservatism brought by the approximation of the NMI into a BMI.
- We provide two controller reconstruction procedures, the former is based on an explicit formulation while the latter is an implicit formulation and the controller is the solution of an SDP. In the last case, the closed-loop system norm varies between the stabilization and controller construction results and we can then measure the conservatism gap.
- We show the efficiency of our method through an example with a comparison with an existing method.

1.1 Objectives

The present paper deals with the following problem

Problem 1.1. Find a parameter dependent state-feedback (1) which

1. Asymptotically stabilizes system (2): $x(t) \to 0$ as $t \to +\infty$ with $w(t) = 0$.
2. Provides a $\mathcal{H}_\infty$ performance attenuation from w to $\bar{z}(\rho)$: $||\bar{z}(\rho)||_{\mathcal{L}_2}/||w||_{\mathcal{L}_2} < \gamma(\rho)$ with $x(\eta) = 0$, $\eta \in [-h_M, 0]$.

where for a complex signal $||v||_{\mathcal{L}_2} := \int_0^{+\infty} v^*(t)v(t)dt$ and $v^*(t)$ is the complex conjugate of $v(t)$. We introduce here with a slight abuse the notation $\bar{z}(\rho)$ indicating that the signal z depends on the parameters ρ which is not integrated over the time while computing the norm as if ρ was independent of the time. A similar consideration has been made for instance in [Yu et al., 2005].

Remark 1.1. The parameter varying $\mathcal{H}_\infty$ performance index $\gamma(\rho)$ allows to give a better description of system $\mathcal{H}_\infty$ performances over the whole parameter space. Note that if γ is chosen to be constant then its value will correspond to $\max_{\rho \in U_\rho} \gamma(\rho)$.

1.2 Notations

The delay is assumed to belong to the set

$$\mathscr{H} := \left\{ h \in \mathscr{C}^1(\mathbb{R}^+, [0, h_M]), |\dot{h}| < \mu < 1 \right\} \tag{3}$$

and the parameters to

$$\mathscr{P} := \left\{ \rho \in \mathscr{C}^1(\mathbb{R}_+, U_\rho \subset \mathbb{R}^{N_p}), \underline{\nu} \leq \dot{\rho} \leq \bar{\nu} \right\} \tag{4}$$

where $\mathscr{C}^1(I, J)$ denotes the Hilbert-space of continuous functions with continuous derivatives mapping I into J, $N_p > 0$ is the number of parameters. $\underline{\nu} < 0$ and $\bar{\nu} > 0$ are respectively the lower and upper bound onto parameter derivatives. Finally let the set U_ν be

$$U_\nu := \{\underline{\nu}_1, \bar{\nu}_1\} \times \ldots \times \left\{ \underline{\nu}_{N_p}, \bar{\nu}_{N_p} \right\} \tag{5}$$

This set define a hypercube (convex set) in which all parameter derivatives values evolve.

For a real square matrix M we define $M^H := M + M^T$. For a complex matrix M, M^* stands for the conjugate transpose of M (for real matrices it is replaced by the transpose). The space of signals with finite energy is denoted by $\mathcal{L}_2$ and the energy of $v \in \mathcal{L}_2$ is $||v||_{\mathcal{L}_2} := \int_0^{+\infty} v^*(t)v(t)dt$. The set $\mathbb{S}^k_{++}$ denotes the set of real symmetric positive definite matrices of dimension k. $\mathbb{R}_+$ denotes the set of positive real numbers. $\star$ denotes symmetric terms in symmetric matrices and in quadratic forms. $\oplus$ is the direct sum of matrices.

Section 2 presents a delay-dependent stability result for LPV time-delay systems with time-varying delays and a relaxation lemma used to make more tractable conditions. Section 3 gives sufficient conditions to the existence of a parameter dependent state-feedback. Section 4 provides algorithms to construct the controller in two different ways. Section 5 propose an example showing the effectiveness of the approach.

2. STABILITY RESULT AND NMI RELAXATION LEMMA

We provide here a result of stability for LPV time-delay systems with single time-varying delay. We extend the approach of [Gouaisbaut and Peaucelle, 2006a] to time-varying delays and to the the parameter varying case. Secondly, we propose a lemma turning a family of rational matrix inequality into an equivalent BMI formulation involving a 'slack'-variable.

2.1 Delay-dependent stability result of LPV time-delay systems

The following theorem is inspired from a result of [Gouaisbaut and Peaucelle, 2006b] and we provide here the case of LPV time-delay systems with time-varying delays.

Theorem 2.1. The system (2) with no control input (ie. $u(t) = 0$) is asymptotically stable for all $h \in \mathscr{H}$ and satisfies $||\bar{z}(\rho)||_{\mathcal{L}_2}/||w||_{\mathcal{L}_2} < \gamma(\rho)$ with $\gamma : \mathbb{R}_+ \to \mathbb{R}_{++}$ if there exist a continuously differentiable matrix function $P : U_\rho \to \mathbb{S}^n_{++}$, matrices $Q, R \in \mathbb{S}^n_{++}$ such that the parameter dependent (6) holds for all $\rho \in U_\rho$ and for all $\nu \in U_\nu$ with $Q_\mu = (1 - \mu)Q$.

Proof : The proof is given in appendix A. □

2.2 NMI relaxation lemma

It is convenient to introduce here the following lemma

Lemma 2.1. Consider two symmetric positive definite matrix functions $\alpha(\cdot), \beta(\cdot)$ and a symmetric function $\delta(\cdot)$ then the following propositions are equivalent:

a) $\delta(\cdot) - \alpha^T(\cdot)\beta^{-1}(\cdot)\alpha(\cdot) < 0$

b) there exists a matrix function of appropriate dimensions $\eta(\cdot)$ such that

$$\begin{bmatrix} \delta(\cdot) + [\eta^T(\cdot)\alpha(\rho)]^H & \star \\ \beta(\cdot)\eta(\cdot) & -\beta(\cdot) \end{bmatrix} < 0 \tag{7}$$

Proof : The proof is omitted for brevity. □

This lemma states that if one has a nonlinear matrix inequality of the form a), then it can be equivalently turned

$$\begin{bmatrix} \frac{\partial P(\rho)}{\partial \rho}(\rho)\nu + [A(\rho)^T P(\rho)]^H + Q - h_M^{-1} R & \star & \star & \star & \star \\ A_h^T(\rho)P(\rho) + h_M^{-1} R & -Q_\mu - h_M^{-1} R & \star & \star & \star \\ E(\rho)^T P(\rho) & 0_{p\times n} & -\gamma I_p & \star & \star \\ C(\rho) & C_h(\rho) & 0_{q\times p} & -\gamma I_q & \star \\ h_M R A(\rho) & h_M R A_h(\rho) & h_M R E(\rho) & 0_{n\times q} & -h_M R \end{bmatrix} < 0 \quad (6)$$

into a BMI of form b) involving a slack variable. Such a BMI can be solved using the so-called 'D-K iteration' or any appropriate iterative algorithm. Another benefits of this relaxation are exposed later in section 4.2.

3. STATE-FEEDBACK DESIGN

The following assumption is required for the following.

Assumption 3.1. For the stabilization use of theorem 2.1 we assume that $C_h(\cdot) = 0$

Considering $C_h(\cdot) = 0$ is not restrictive since $z(t)$ is a virtual output chosen for design purposes and generally we are interested in controlling linear function of the current state, the control input and sometimes exogenous inputs (such as a tracking reference).

The stabilization results are based on the forward adjoint (See [Bensoussan et al., 2006]) representation of time-delay system (2) with $C_h = 0, B = 0, D = 0$:

$$\begin{aligned} \dot{\tilde{x}}(t) &= A^T(\rho)\tilde{x}(t) + A_h^T(\rho)\tilde{x}_h(t) + C^T(\rho)\tilde{z}(t) \\ \tilde{w}(t) &= E^T(\rho)\tilde{x}(t) + F^T(\rho)\tilde{z}(t) \end{aligned} \quad (8)$$

It is worth noting that stability is preserved with such a transformation (since the eigenvalues of a matrix M are the same as the ones of M^T) while stabilizability and detectability are permuted (which is not important here). Moreover, the $\mathcal{H}_\infty$ -norm of a system and its adjoint are identical (see [Suplin et al., 2006]). Finally, according to [Green and Limebeer, 1994], the $\mathcal{H}_\infty$ norm is also preserved even in the case of non-stationary systems.

The closed loop system issued from the interconnection of system (2) and controller (1) is given by

$$\begin{aligned} \dot{x}(t) &= A_{cl}(\rho)x(t) + A_h(\rho)x_h(t) \\ &\quad + E(\rho)w(t) \\ z(t) &= C_{cl}(\rho)x(t) + F(\rho)w(t) \end{aligned} \quad (9)$$

with $A_{cl}(\rho) = A(\rho) + B(\rho)K(\rho)$, $C_{cl}(\rho) = C(\rho) + D(\rho)K(\rho)$.

The adjoint system [Bensoussan et al., 2006] of system (9) is given by

$$\begin{aligned} \dot{\tilde{x}}(t) &= A_{cl}^T(\rho)\tilde{x}(t) + A_h^T(\rho)\tilde{x}_h(t) \\ &\quad + C_{cl}^T(\rho)\tilde{z}(t) \\ \tilde{w}(t) &= E^T(\rho)\tilde{x}(t) + F^T(\rho)\tilde{z}(t) \end{aligned} \quad (10)$$

Using the adjoint form, this leads to the following stabilization lemma:

Theorem 3.1. There exists a parameter dependent state-feedback control of the form $u(t) = K(\rho)x(t)$ such that the closed-loop system (9) is asymptotically stable and satisfies $||\bar{z}(\rho)||_{\mathcal{L}_2}/||w||_{\mathcal{L}_2} < \gamma(\rho)$ with $\gamma : U_\rho \to \mathbb{R}_{++}$ for $h \in \mathscr{H}$ if there exist a continuously differentiable matrix function $P : U_\rho \to \mathbb{S}_{++}^n$, matrices $Q, R \in \mathbb{S}_{++}^n$ and $\Lambda : U_\rho \to \mathbb{R}^{n\times n}$ such that the parameter dependent LMI (11) and the parameter dependent NMI (12) hold for all $\rho \in U_\rho$ and for all $\nu \in U_\nu$

$$\mathcal{N}_1^T(\rho)\Pi_1(\rho)\mathcal{N}_1(\rho) < 0 \quad (11)$$

with $Q_\mu = (1-\mu)Q$, $\mathcal{N}_1(\rho) := \mathcal{N}_{[B^T\ D^T]} \oplus I_{2n+p}$, $\mathcal{N}_{[B^T\ D^T]} := Ker\left[B^T(\rho)\ D^T(\rho)\right]$ and $\Pi_1(\rho)$ is defined in (13).

Proof : The proof is given in appendix B. □

We will discuss in the next section to use these results to obtain as accurate results as possible and compute optimal controllers.

4. CONTROLLER COMPUTATION

We detail in that part the way to correctly compute controller solutions of the problem.

4.1 Parameter dependent cost

We have defined a parameter dependent cost $\gamma(\rho)$ to minimize. Due to the varying nature it is difficult to minimize it directly. That is why it is interesting to introduce the following cost instead:

$$\mathcal{J}(\gamma_v) := \int_{U_\rho} \theta(\rho)\gamma(\rho)d\rho \quad (14)$$

with $\gamma(\rho) = \sum_{i=1}^t \gamma_i f_i(\rho)$ where $f_i(\rho)$ are basis functions, γ_i real numbers to determine and $\gamma_v = col(\gamma_i)$.

According to the choice of the weighting function $\theta(\rho)$, it is possible to shape the $\mathcal{H}_\infty$ norm of the closed-loop system or specify parameter domains where the minimization should be done.

4.2 Solving bilinear matrix inequalities of the form (12)

Note that the matrix inequality (12) is bilinear and thus cannot be solved by classical interior points algorithm. Nevertheless, it is possible to solve this problem using a D-K iteration algorithm which is very efficient in this case. A description of the generation of the initial condition is deeper explained in [Briat et al., 2007b], but we use matrix random generation.

Algorithm 1.

1. Generate a initial symmetric constant matrix Λ_0 such that $[\Lambda_0^T P(\rho)]^H < 0$.
2. Solve the optimization problem
$$\begin{aligned} &\min_{\gamma_v, P(\rho), Q, R} \mathcal{J}(\gamma_v) \\ &\text{such that } P(\rho), Q, R > 0,\ \gamma(\rho) > 0 \\ &(11) \text{ and } (12) \end{aligned}$$
If problem unfeasible then go to Step 1.
3. Solve the optimization problem
$$\begin{aligned} &\min_{\Lambda(\rho), Q, \gamma_v} \mathcal{J}(\gamma_v) \\ &\text{such that } Q > 0,\ \gamma(\rho) > 0,\ (11) \text{ and } (12) \end{aligned}$$
If stopping criterion is satisfied then STOP else go to step 2.

$$\begin{bmatrix} Q - h_M^{-1}R + h_M^{-1}[\Lambda^T(\rho)P(\rho)]^H + \dfrac{\partial P(\rho)}{\partial \rho}\nu & \star & \star & \star \\ h_M^{-1}R & -(1-\mu)Q - h_M^{-1}R & \star & \star \\ E^T(\rho) & 0 & -\gamma I_p & \star \\ R\Lambda(\rho) & 0 & 0 & -h_M R \end{bmatrix} < 0 \qquad (12)$$

$$\Pi_1(\rho) := \begin{bmatrix} \dfrac{\partial P(\rho)}{\partial \rho}\nu + [A(\rho)P(\rho)]^T + Q - h_M^{-1}R & \star & \star & \star & \star \\ C(\rho)P(\rho) & -\gamma I_q & \star & \star & \star \\ A_h(\rho)P(\rho) + h_M^{-1}R & 0_{n\times q} & -Q_\mu - h_M^{-1}R & \star & \star \\ E(\rho)^T & 0_{p\times q} & 0_{p\times n} & -\gamma I_p & \star \\ h_M RA^T(\rho) & h_M RC^T(\rho) & h_M RA_h^T(\rho) & 0_{n\times p} & -h_M R \end{bmatrix} \qquad (13)$$

4.3 Controller Computation

The controller is computed using the following SDP:

Lemma 4.1. Solve for $Q, \gamma, \bar{K}_l$ with $K = \sum_{l=1}^{s} \bar{K}_l g_l(\rho)$ for all $\rho \in U_\rho$ and $\nu \in U_\nu$.

$$\begin{aligned} \Pi_1 + U_1^T K^T V_1 + V_1^T K U_1 &< 0 \\ \mathcal{L}(K) &< 0 \end{aligned} \qquad (15)$$

where Π_1 is defined in (13), $U_1 = [P(\rho)\ 0\ 0\ 0\ h_M R]$, $V_1 = \begin{bmatrix} B^T(\rho) & D^T(\rho) & 0 & 0 & 0 \end{bmatrix}$.

The term $\mathcal{L}(K) < 0$ represent additional constraints onto the controller.

5. EXAMPLE

Consider system (2) adopted from [Wu and Grigoriadis, 2001] and modified by [Zhang and Grigoriadis, 2005] with matrices given below

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 1+\phi sin(t) \\ -2 & -3+\delta sin(t) \end{bmatrix} x(t) + \begin{bmatrix} \phi sin(t) & 0.1 \\ -0.2+\delta sin(t) & -0.3 \end{bmatrix} x_h(t) \\ &+ \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix} w(t) + \begin{bmatrix} \phi sin(t) \\ 0.1+\delta sin(t) \end{bmatrix} u(t) \\ z(t) &= \begin{bmatrix} 0 & 10 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t) \end{aligned} \qquad (16)$$

5.1 Case $\phi = 0.2$ and $\delta = 0.1$

Choosing $\rho(t) = sin(t)$ as parameter, it can be easily deduced that $\rho, \dot{\rho} \in [-1, 1]$. The parameter space is gridded over $N_p = 40$ points uniformly spaced.

Choosing, as in [Zhang and Grigoriadis, 2005], $h_M = 0.5$ and $\mu = 0.5$, we find $\gamma^* = 1.8492$ which is better than all results obtained before (See [Wu and Grigoriadis, 2001, Zhang and Grigoriadis, 2005]). In Zhang and Grigoriadis [2005], they have found $\gamma = 3.09$. In our case, the resulting a controller is given by $K(\rho) = K_0 + K_1\rho + K_2\rho^2$ where $K_0 = [-5.9172\ -16.3288]$, $K_1 = [-53.1109 - 32.4388]$ and $K_2 = [-8.4071\ 3.0878]$. It is worth noting that after computing the controller, the $\mathcal{H}_\infty$ -norm achieved is now $\gamma_r = 2.2777$ corresponding to an increase of 23.17%. Better performances should be obtained while considering more complete controllers but we are limited by the fact that we do not consider rational controllers.

The values of the gain w.r.t. parameter values are represented at the top of figure 1. The bottom of figure 1 describes the gain computed by the method of [Zhang and Grigoriadis, 2005].

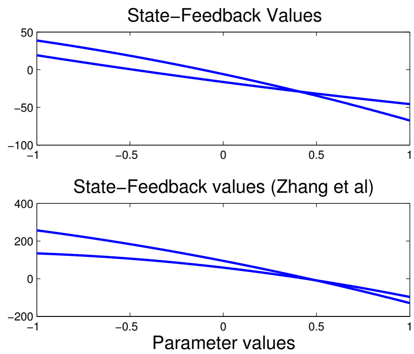

Fig. 1. Simulation 1 - Gain evolution with respect to the parameter value - theorem 3.1 (top) and method of [Zhang and Grigoriadis, 2005]

Note that we obtain better results while we have lower controller gains than in the previous approaches.

For simulation purposes let $h(t) = 0.5|\sin(t)|$ and $\rho(t) = sin(t)$. For simulation 1 we consider non zero initial conditions and $w(t) = 0$. We obtain results depicted in figures 2-4. We can see that the rate of convergence is very near but with our method the necessary input energy to make the system converges is less than for [Zhang and Grigoriadis, 2005].

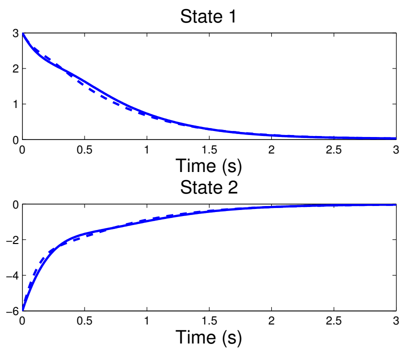

Fig. 2. Simulation 1 - State evolution - theorem 3.1 in full and [Zhang and Grigoriadis, 2005] in dashed

For simulation 2, we consider the case of zero initial conditions and an unitary step disturbance. We obtain the following results depicted in figures 5-7. We can see that our control input has smaller bounds and that the second state is less affected by the disturbance than for the method of [Zhang and Grigoriadis, 2005].

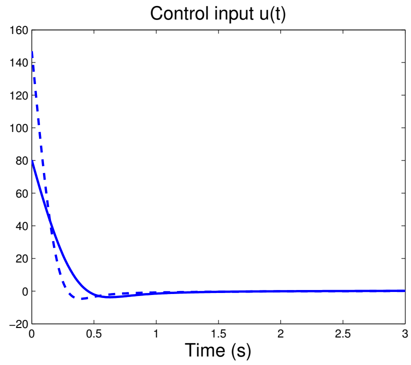

Fig. 3. Simulation 1 - Control input evolution - theorem 3.1 in full and [Zhang and Grigoriadis, 2005] in dashed

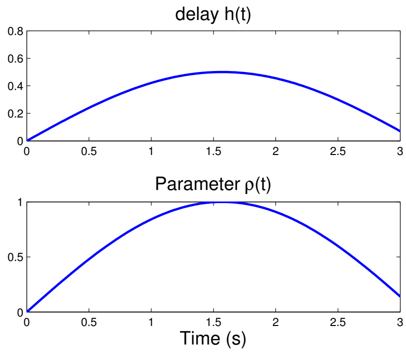

Fig. 4. Simulation 1 - Delay and parameter evolution

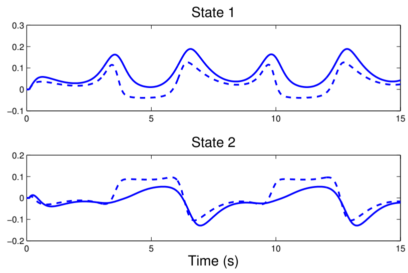

Fig. 5. Simulation 2 - State evolution - theorem 3.1 in full and [Zhang and Grigoriadis, 2005] in dashed

Then we check, the delay upper bound for which a parameter dependent stabilizing controller exists and guarantees $\gamma^* < 10$ with $\mu = 0.5$ and we find $h_M = 79.1511$, for $\gamma^* < 2$ we find $h_M = 1.750$. In [Zhang and Grigoriadis, 2005], the delay upper bound for which a stabilizing controller exist is $h_M = 1.65$. This shows that our result is less conservative.

5.2 Case $\phi = 2$ and $\delta = 1$

Now let $\phi = 2$ and $\delta = 1$. Using the results of [Zhang and Grigoriadis, 2005] no solution is found. With lemma 3.1, we find that there exists a controller such that the closed-loop system has a $\mathcal{H}_\infty$ norm lower than $\gamma = 6.4498$.

6. CONCLUSION

The current paper introduces a new approach to analyze and stabilize LPV time-delay systems using parameter dependent Lyapunov-Krasovskii functionals. We propose

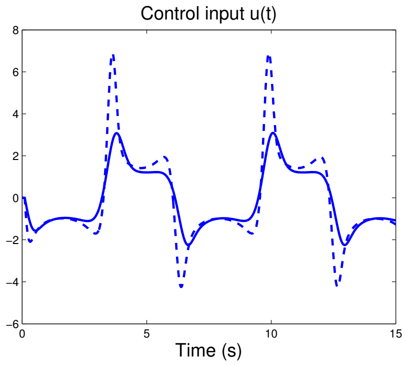

Fig. 6. Simulation 2 - Control input evolution - theorem 3.1 in full and [Zhang and Grigoriadis, 2005] in dashed

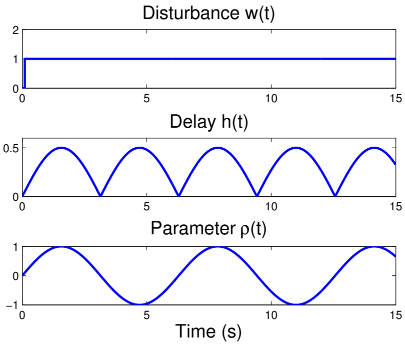

Fig. 7. Simulation 2 - Delay and parameter evolution

first a delay-dependent infinite dimensional LMI test to prove asymptotic stability of LPV time-delay systems in the $\mathcal{H}_\infty$ -norm framework. We derive from these sufficient conditions to the existence of an instantaneous state-feedback. As these conditions contain a NMI, we relax it into an equivalent BMI. This BMI enjoys some nice properties: especially it allows to be solved easily with iterative algorithms such as 'D-K iteration' algorithm. We relax then semi-infinite part by gridding the parameter space and the infinite dimensional one by choosing an explicit set of basis functions on which we project infinite dimensional decision variable. The controller computation is done either using an explicit formula or by solving an implicit SDP. The first one only works when the parameter derivatives are known or if P is chosen to be constant and has the benefits of automatically gives the controller structure. The second ones works in all the cases and allow additional constraints but the conservatism increases while choosing a basis for the controller. Finally, we show the effectiveness of our approach compared to previous ones trough an example.

Appendix A. PROOF OF THEOREM 2.1

The proof is inspired from [Gouaisbaut and Peaucelle, 2006b] and extended to the case of LPV time-delay systems with time-varying delays.

We consider the following parameter dependent Lyapunov-Krasovskii functional with parameter dependent supply function $s(w, z, \rho)$:

$$V = x^T(t)P(\rho)x(t) + \int_{t-h(t)}^{t} x^T(\theta)Qx(\theta)d\theta + \int_{-h_M}^{0}\int_{t+\theta}^{t} x^T(\eta)Rx(\eta)d\eta d\theta - \int_{0}^{t} s(w(\theta),z(\theta),\rho)d\theta \tag{A.1}$$

with

$$s(w(t),z(t),\rho) = \gamma w^T(t)w(t) - \gamma^{-1}z^T(t)z(t)$$

Taking the time derivative of V along the trajectories solution of the system (2) gives

$$\begin{aligned}\dot{V} \leq\ & x^T(t)\frac{\partial P}{\partial \rho}(\rho)\dot{\rho}(t)x(t) + \left(x^T(t)A^T(\rho)\right.\\ &\left.+x_h^T(t)A_h^T(\rho) + w^T(t)E^T(\rho)\right)P(\rho)x(t)\\ &+x^T(t)Qx(t) - (1-\dot{h})x_h^T(t)Qx_h(t)\\ &+\underbrace{h_M\dot{x}^T(t)R\dot{x}(t) - \int_{t-h(t)}^{t} x^T(\theta)Rx(\theta)d\theta}_{\mathcal{I}}\end{aligned} \tag{A.2}$$

Note that $-(1-\dot{h}) \leq -(1-\mu)$ and using Jensen's inequality ([Gu et al., 2003]) on $\mathcal{I}$ we obtain

$$\mathcal{I} \leq -h_M^{-1}\left(\int_{t-h(t)}^{t} \dot{x}^T(t)\right)R(\star)^T \tag{A.3}$$

The term $\dot{\rho}$ is relaxed considering the hypercube (polytope) defined by the set U_ν. Hence the inequality must be satisfied for each $\nu \in U_\nu$.

Then expanding the expression of $s(w,z)$ and performing two successive Schur complement onto terms $\star^T\gamma^{-1}\star$ and $\star^T(h_M R)\star$ leads to the inequality (6).

Appendix B. PROOF OF THEOREM 3.1

Injecting the closed-loop adjoint system (10) into (6) and permuting the second row/column with the third one leads to

$$\Pi_1(\rho) + \left(U_1^T(\rho)K^T(\rho)V_1(\rho)\right)^H < 0 \tag{B.1}$$

where $\Pi_1(\rho)$ is (13), $U_1(\rho) = [P(\rho)\ 0\ 0\ 0\ h_M R]$, $V_1(\rho) = \left[B^T(\rho)\ D^T(\rho)\ 0\ 0\ 0\right]$

Then applying the projection lemma leads to the two underlying LMIs

$$\mathcal{N}_1^T(\rho)\Pi_1(\rho)\mathcal{N}_1(\rho) < 0 \tag{B.2}$$

$$Ker[V_1(\rho)]^T\Pi_1(\rho)Ker[V_1(\rho)] < 0 \tag{B.3}$$

As $P(\cdot)$ and R are of full rank then a basis of the null space of $V_1(\rho)$ is given by $Ker[V_1(\rho)] = \begin{bmatrix} I \\ \hline -h_M^{-1}R^{-1}P(\rho)\ 0\ 0\ 0\end{bmatrix}$. Using this value (B.3) is equivalent to (**??**) with $Q_\mu = (1-\mu)Q$. Note that (**??**) is obviously an NMI due to the nonlinear term $-h_M^{-1}P(\rho)R^{-1}P(\rho)$. Similarly as in [Briat et al., 2007b], we use lemma 2.1 on the nonlinear term using the slack variable and we obtain (12).

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