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Sensor Fault Tolerant Controller for a Double Inverted Pendulum System

S. S. Yang*, J. Chen, Haider A. F. Mohamed*, M. Moghavvemi***

* Centre for Research in Applied Electronics (CRAE), University of Malaya, 50603 Kuala Lumpur, Malaysia (Tel: 603-7967-6832; e-mail:ssyang@um.edu.my).

**School of Engineering and Design, Brunel University, Uxbridge UB8 3PH, UK

Abstract: This paper presents an Internal Model Control based structure to realise fault tolerance towards sensor faults. The proposed design implicitly embeds fault detection and controller elements requirements with considerations to stability and robustness towards uncertainties besides multiple faults environment. The performance of the proposed controller is demonstrated via multi input multi output and unstable system i.e. the double inverted pendulum system. Performance of the controller is compared with the well known $H\infty$ controller. Results show the potential of this controller for implementation to handle systems with multiple sensor faults and uncertainties.

1. INTRODUCTION

In the past two decades, there has been growing demand in a fault tolerant control system (FTCS) which has ability to improve safety and reliability of technical system. There have been significant amount of research in this area (Isermann, 1997; Patton, 1997; Zhou and Frank, 1998; Blanke *et al.*, 2001). Fault-tolerant control system (FTCS) is the techniques that coordinate redundancy to increase reliability. Reliability is defined as the probability that a system will operate correctly and continuously for a defined period of time while operating under a defined set of conditions.

One of approach towards this end is to include redundancy so that if a component within the system should develop a fault, the system would tolerate that fault. Fault detection can provide indication that something has gone wrong and so that a defined reaction to the fault can take place. Tolerating a fault goes a step further and allows the system to continue to operate correctly even though a component is faulty. The objective of the use of fault tolerance is to design a system with fault tolerant that faults do not result in system failure and ensured the achievement of best performance at a lower degree of system performance (Blanke *et al*., 2001). For a redundant system to continue correct operation in the presence of a fault, the redundancy must be managed properly.

Various studies dealing with sensor FTCS are based on hardware or analytical redundancy. The hardware redundancy technique consists of switching from the failed part of the process to another achieving the same task. Nevertheless, techniques which involve hardware redundancy in a system are in a sense less reliable than the same system without the redundancy- there are more potential faults when the system has more components.

Analytical redundancy is an alternative to solve the fault-tolerant control problem avoiding the disadvantaged of the hardware redundancy such as higher probability of faults due to increased number of components in addition to additional costs and space (Isermann, 1997; Patton, 1997).

Model based schemes for sensor fault tolerant controller design framework developed by Yang and Chen, 2007; Yang and Chen, 2006; Zhou and Ren, 2001; Niemann and Stoustrup, 2005; Niemann and Stoustrup, 1997; Patton 1997, exploits the concept of analytical redundancy through the use of the process model. In addition it allows fault detection and control to be implicitly designed; the fault indicating residual is generated and utilized as a function of control. The residual signals act as weighting factors which put corresponding emphasis on nominal controller and fault accommodating controller. The structure allows the plant to be controlled by a nominal controller that ensures the achievement of best performance objectives when sensor faults are not present while preserving the stability at a lower degree of system performance in the presence of sensor faults.

This paper proposes the extension of the Generalized Internal Model Controller (GIMC) architecture to handle sensor faults and uncertainties. The GIMC for sensor fault tolerant controller design framework developed by Zhou and Ren (2001) exploits the concept of analytical redundancy through the use of the process model. The nominal controller and the fault accommodating controller are designed based on the synthesis of an $H\infty$ robust controller; by assuming that faults are uncertainties introduced as additive disturbance to the sensor inputs.. In addition it allows fault detection and control to be implicitly designed with the fault indicating residual generated utilised as a function of control. The residual signals act as weighting factors which put corresponding emphasis on nominal controller and fault accommodating controller. The structure allows the plant to be controlled by a nominal controller that ensures the achievement of best performance objectives when sensor faults are not present while preserving the stability at a lower degree of system performance in the presence of single or multiple sensor faults.

The following section describes how the fault accommodating controller can be formulated as a robust control problem and thus synthezised accordingly. Later sections present the results of evaluating the FTCS performance on a double inverted pendulum system with sensor faults. A comparison is made with the performance of the double inverted pendulum system without the augmentation of a fault accommodating controller.

2. CONTROLLER SYNTHESIS

Sensor fault symptoms can be observed as measurements that are unavailable, incorrect or unusually noisy. These faults may occur individually or concurrently or simultaneously, resulting in total system failure or degradation in performance. Significant information about the influence of faults on a process cannot be known without the inclusion of its model in the design. Additive faults provide a suitable framework for sensor faults and are modelled as additional input signals to a system (Chen and Patton, 1999),

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y'(t) = Cx(t) + Du(t) + f_s(t) \end{cases} \tag{1}$$

where $f_s(t) \in R^m$ denote sensor faults. Hence

$$y(s) = G_p(s)u(s) \tag{2}$$

2.1 Sensor fault modelling

The variable $y(s)$ denotes all available sensor outputs. When output sensor faults occur in the plant as shown in (1), the measured outputs becomes

$$y'(s) = y(s) + f_s(s) \tag{3}$$

Due to the existance of fault represented by $f_s(s)$, a conventional controller cannot usually satisfy required performance and the closed-loop control system may even become unstable. A sensor fault-compensating controller can be introduced to augment a nominal controller designed for best performance. However, since the structure of the system as seen in Fig. 1 is virtually an Internal Model Controller (Morari and Zafiriou, 1989), conditions for physical realizability needs to be observed; to ensure that the fault-compensating controller, Q is well defined and proper, the transfer matrix representation from $f_s(s)$ to controller output, *u(s)* must exists and is also proper. By appropriate use of input weight, $W_s(s)$, the input $f_s'(s)$ can be normalised and transformed into the physical input, $f_s(s)$. Therefore,

$$f_s(s) = W_s(s) f_s'(s) \tag{4}$$

Consideration of such sensor fault models has been shown to be suitable for use in formulating the FTCS objectives for rejection of sensor faults as an optimisation problem. Uncertainties affecting the sensors can also be classified as a subset of $f_s(s)$. Fig. 1 shows the block diagram illustrating the interconnections assumed for the formulation H_∞ problem associated with the proposed FTCS design.

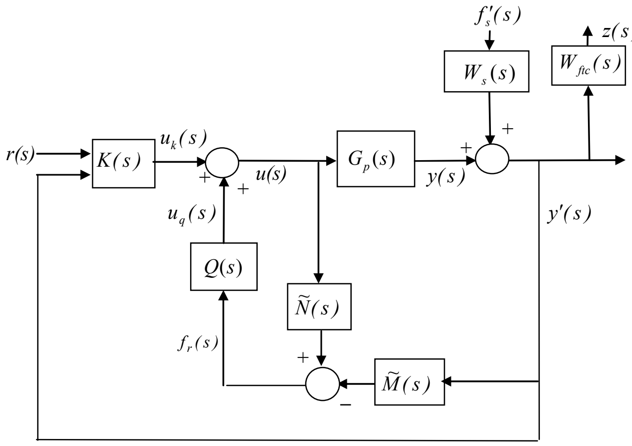

Fig. 1. Block diagram representation of H_∞ problem formulation for the proposed FTCS design

2.2 Generation of fault indicating residuals

The presence of sensor faults and uncertainty vectors defined in the previous section can be reflected by a fault indicating residual since a filtered estimation can be obtained via coprime factorisation of the plant model, $G_p(s)$ (Campos-Delgrado and Zhou, 2001; Zhou and Ren, 2001). Let

$$G_p(s) = \tilde{M}^{-1}(s)\tilde{N}(s) \tag{5}$$

Hence, from (2), (3), (4) and (5), the fault indicating residual denoted by $f_r(s)$ can be defined as

$$\begin{aligned} f_r(s) &= \tilde{N}(s)u(s) - \tilde{M}(s)\left[y'(s) + f_s(s)_s\right] \\ &= -\tilde{M}W_s(s)f'(s) \end{aligned} \tag{6}$$

It can be observed that $f_r(s)$ reflects the presence of $f_s'(s)$ (*i.e.* faults and uncertainty), thus can be utilised as an input to the fault accommodating controller. The perturbations caused are minimised by control actions augmented by the fault accommodating controller.

2.3 Synthesizing the fault accommodating controller

The control signal vector can be expressed as follows.

$$u(s) = u_k(s) + u_q(s) \tag{7}$$

where $u_k(s) = K(s)e(s)$ and, $u_k(s)$ denotes output of nominal controller *K(s)* and $u_q(s)$ denotes output of sensor fault accommodating controller *Q(s)*. Error from feedback is denoted by *e(s)* whereby *r(s)* denotes input demand. From Fig. 1, the fault indicating residual $f_r(s)$ is fed into *Q(s)* to produce the control signal $u_k(s)$ to augment to nominal controller output, $u_q(s)$.

$$u_q(s) = Q(s)f_r(s) = -Q\widetilde{M}(s)W_s(s)f_s'(s) \tag{8}$$

Thus, the relationship with *u(s)* is established as,

$$\begin{aligned} u(s) &= \left(I + K(s)G_p(s)\right)^{-1} \times \ldots \\ &\left\{K(s)r(s) - \left(K(s) + \left[Q(s)\widetilde{M}(s)\right]\right)W_s(s)f_s'(s)\right\} \end{aligned} \tag{9}$$

Finally the output of the plant, *y'(s)* can be written as,

$$\begin{aligned} y'(s) &= G_p(s)\left(I + K(s)G_p(s)\right)^{-1} \times \ldots \\ &\left\{K(s)r(s) - \left(K(s) + \left[Q(s)\widetilde{M}(s)\right]\right)W_s(s)f_s'(s)\right\} \end{aligned} \tag{10}$$

The plant output expression in (10) shows that in the absence of sensor faults and uncertainties, the output closed loop system is only reliant on the nominal controller *K(s)*, allowing for high performance during healthy operation. Note that the fault detection scheme generating the above mentioned fault indicating residual does not need to be made robust since the fault indicating residual, is mainly used as an activating signal for *Q(s)*. It is thus not essential to identify nor estimate the source of the faults. Even if the presence of $f_r(s)$ is due to uncertainties and not faults in the sensors, *Q(s)* will still provide the appropriate control signals to compensate for such perturbations thereby introducing robustness to the system. A post fault performance weight, $W_{ftc}(s)$ need to be defined to complete the design formulation. The corresponding solution for achieving *Q(s)* is by minimising the following optimisation criterion,

$$\gamma = \min_{Q(s)} \left\| F_l\left[P_f(s), Q(s)\right] \right\|_\infty \tag{11}$$

$W_{ftc}(s)$ contains post fault performance requirements which can be relaxed accordingly if solution of *Q(s)* from (11) cannot be obtained. Note that the original design proposal for GIMC did not allow for such a feature and hence may result in non realizability of *Q(s)*. The equivalent linear fractional transformation (LFT) block diagram for the H_∞ problem stated above is shown in Fig. 2. If the controller, *Q(s)* in (11) is found, then the closed loop system is said to have robust performance towards $f_s'(s)$ (Zhou *et al.,* 1996). Thus if the linear fractional transformation of *P* over *F* is defined as

$$F_l(P, F) = P_{11} + P_{12}F\left(I - P_{22}F\right)^{-1} P_{21} \tag{12}$$

where *F* is assumed to have appropriate dimensions and $\left(I - P_{22}F\right)^{-1}$ is well defined (Zhou *et al.,* 1996), then

$$\begin{bmatrix} z(s) \\ f_r(s) \end{bmatrix} = \underbrace{\begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}}_{P_s(s)} \begin{bmatrix} f_s'(s) \\ u_q(s) \end{bmatrix} \tag{13}$$

From (6), $P_{21}(s)$ and $P_{22}(s)$ can be derived as

$$P_{21}(s) = -\widetilde{M}(s)W_s(s) \tag{14}$$

$$P_{22}(s) = 0 \tag{15}$$

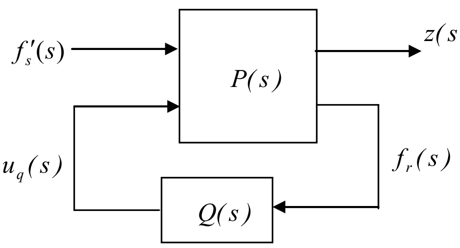

Fig. 2. The LFT representation of proposed FTCS

Now note that,

$$\begin{aligned} u_k(s) &= K(s)\left(r(s) - y'(s)\right) \\ &= K(s)\left(r(s) - G_p(s)u(s) - W_s(s)f_s'(s)\right) \\ &= K(s)r(s) - K(s)G_p(s)\left[u_k(s) + u_q(s)\right] \\ &\ldots - K(s)W_s(s)f_s'(s) \end{aligned} \tag{16}$$

and thus,

$$\begin{aligned} u_k(s) &= \left(I + K(s)G_p(s)\right)^{-1} K(s)r(s) - \\ &\ldots\left(I + K(s)G_p(s)\right)^{-1} K(s)W_s(s)f_s'(s) \\ &\ldots - \left(I + K(s)G_p(s)\right)^{-1} K(s)G_p(s)u_k(s) \end{aligned} \tag{17}$$

Also,

$$\begin{aligned} z(s) &= W_{ftc}(s)y'(s) \\ &= W_{ftc}(s)\left[G_p(s)u(s) + W_s(s)f_s'(s)\right] \\ &= W_{ftc}(s)G_p(s)u_k(s) + W_{ftc}(s)G_p(s)u_q(s) \\ &\ldots + W_{ftc}(s)W_s(s)f_s'(s) \end{aligned} \tag{18}$$

Substituting (17) into (18),

$$\begin{aligned} z(s) &= W_{ftc}(s)G_p(s)\left(I + K(s)G_p(s)\right)^{-1} K(s)r(s) - \\ &\ldots W_{ftc}(s)G_p(s)\left(I + K(s)G_p(s)\right)^{-1} K(s)G_p(s)u_q(s) \\ &\ldots + W_{ftc}(s)G_p(s)u_q(s)\ldots \\ &- W_{ftc}(s)G_p(s)\left(I + K(s)G_p(s)\right)^{-1} K(s)W_s(s)f_s'(s) \\ &\ldots + W_{ftc}(s)W_s(s)f_s'(s) \end{aligned} \tag{19}$$

Ignoring the reference input *r(s)*, we have

$$P_{11}(s) = -W_{ftc}(s)G_p(s)\left(I + K(s)G_p(s)\right)^{-1}K(s)W_s(s)$$
$$\ldots + W_{ftc}(s)W_s(s)$$
$$= W_{ftc}(s)\left\{I - G_p(s)\left(I + K(s)G_p(s)\right)^{-1}K(s)\right\}W_s(s) \tag{20}$$

and

$$P_{22}(s) = -W_{ftc}(s)G_p(s)\left(I + K(s)G_p(s)\right)^{-1}K(s)G_p(s)$$
$$\ldots + W_{ftc}(s)G_p(s)$$
$$= W_{ftc}(s)\left\{I - G_p(s)\left(I + K(s)G_p(s)\right)^{-1}K(s)\right\}G_p(s) \tag{21}$$

With the conditions laid out the closed loop system shown above is guaranteed to be tolerant to sensor faults and modelling uncertainty, stable for any non-linear, time varying and stable *K(s)* and *Q(s)* due to the minimization of the transfer matrix between fault generating signal $f_s'(s)$ to the performance evaluation signal, *z(s)*.

3. A NUMERICAL EXAMPLE

A simulated example of the proposed FTCS implementation on a double inverted pendulum system for tolerance towards sensor faults is shown to demonstrate its feasibility. The implementation is tested for sensor fault tolerance in nominal and under plant uncertainty conditions.

3.1 The double inverted pendulum system

The double inverted pendulum system is an example of a fast, multivariable, nonlinear and unstable process. It is a standard classical control test rig for the verification of different control methods, and is among the most difficult system to control in the field of control engineering. Similar to the single inverted pendulum problem, the control task for the double inverted pendulum is to stabilise the two pendulums. The position of the carriage on the track is controlled quickly and accurately so that the pendulums are always erected in its inverted position during such movements.

The system is made up of two aluminium arms connected to each other, with the lower arm attached to a cart placed on a guiding rail, as illustrated in Fig. 3. Data used in this case study has been obtained from (Niemann and Stoustrup, 2005). The aluminium arms are constrained to rotate within a single plane; the axis of rotation and is perpendicular to the direction of the force acting on the cart motion, *f*. The cart can move along a linear low friction track and is moved by a belt driven by a servo motor system. Sensors providing measurements of cart position x_c, the pendulums' angles θ_1 and θ_2, controller output, *u* and motor current, *i*, are assumed available for the purpose of control. The control law has to regulate the lower arm angle and upper arm angle, θ_1 and θ_2, respectively from an initial condition, and the control of the position of the cart x_c from an initial position.

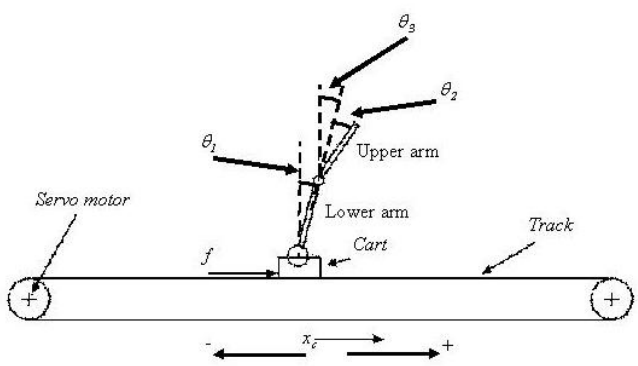

Fig. 3. The double inverted pendulum system

Any type of controller can be designed for use as the suitable nominal controller *K*. In this case an H_∞ loop shaping controller is chosen as the high performance nominal controller, *K* for the MIMO system. The controller is designed using the *MATLAB* instruction *ncfsyn.m* using the appropriate data available. Sensors for detecting e_x (cart positional error), θ_1 and θ_2 are fault prone sensors. Motor voltage and current are denoted by *u* and *i*, respectively.

The controller output variable is the corresponding motor voltage demand, *u*. The controller performance was simulated on the *SIMULINK* model of the double inverted pendulum system with initial conditions θ_1*=0.05rad* and θ_2*=-0.04rad*. The cart is required by the command signal, r_c to move, from initial location (*0.5m*) towards location (*-0.5m*) after *50s*, as shown in Fig. 4. Nominal system responses without faults or uncertainties are shown in Fig. 5. Output responses are observed to be within the limits of specifications, and the cart position set points have been achieved in a stable and smooth manner.

3.2 Implementing the sensor FTCS

Fault indicating residuals are denoted by $f_{\theta 1}$, $f_{\theta 2}$ and f_{ex} for faults in the corresponding sensors. The design problem is formulated in the form shown in (19) so that the solution of *Q(s)* can be found. Note that the performance weights $W_{ftc}(s)$ (shown in Appendix) to establish post fault performance requirements reuses the elements in the original specification function, W_p which are related to the fault prone sensors i.e. sensors providing measurements of cart position x_c, the pendulums' angles θ_1 and θ_2. The solution for *Q(s)* is obtaineable using *MATLAB's μ-Analysis and Synthesis Toolbox* (Balas *et al.*, 2001), which iteratively solves the optimisation criterion set out in (11). When *γ* value of below *1* is obtained, the solution of a satisfactory *Q(s)* is used.

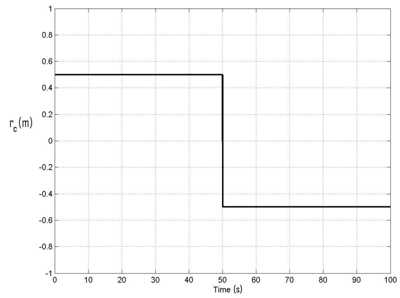

Fig. 4. Command signal requiring the cart to move from 0.5m to -0.5m

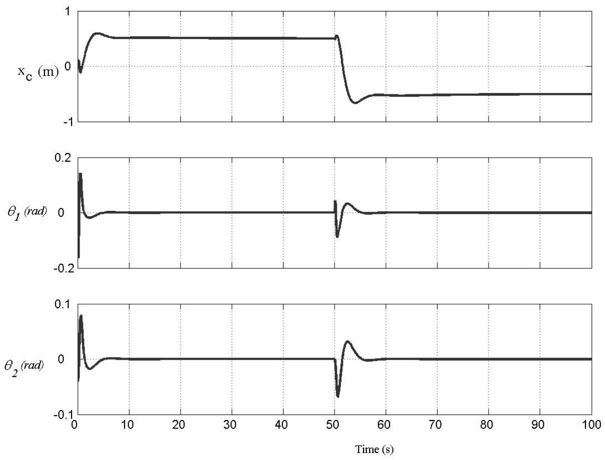

Fig. 5. System responses with nominal controller K (position of cart, x_c is shown instead of cart position error, e_x)

3.3 Results of sensor faults simulations

The performances of the controllers were evaluated by simulating the occurrence of faults in the relevant sensors. Sensor effectiveness indicating faults are simulated as deterioration of performance; *0%* for no fault, *100%* for total failure. Results are shown for conditions with and without modelling uncertainty. Responses of the inverted double pendulum system performances with the proposed FTCS and with only the H∞ controllers are recorded for comparison purposes. Fault indicating residuals are denoted by $f_{\theta 1}$, $f_{\theta 2}$ and f_{ex} for faults in the corresponding sensor

A. Without modelling uncertainties and sensor faults

Nominal performances of all controllers for healthy system are recorded in Fig. 6. Apparently the proposed FTCS produces faster cart positioning response compared to all other control system responses, initiating slightly higher overshoots in θ_1 and θ_2.

B. Multiple sensor faults, without plant uncertainty.

Multiple sensor faults are assumed to occur at 2, 4 and 6 seconds after the simulation has been initiated (e_x at *90%* deterioration, θ_1 at *20%* deterioration and θ_2 at *10%* deterioration, respectively, without any form of disturbance). The output responses are shown in the following Fig. 7. Observe that only the proposed FTCS managed to handle the faults and achieve satisfactory control responses. Stability could not be maintained by the H_∞ controller.

C. Multiple sensor faults, with plant uncertainty.

Tests for control systems to handle system uncertainty and multiple sensor faults were also performed. Conditions were made similar to the tests performed for the nominal system with multiple sensor faults. The supremacy of the proposed FTCS to accommodate for faults even under the influence of system uncertainties is seen in Fig. 8.

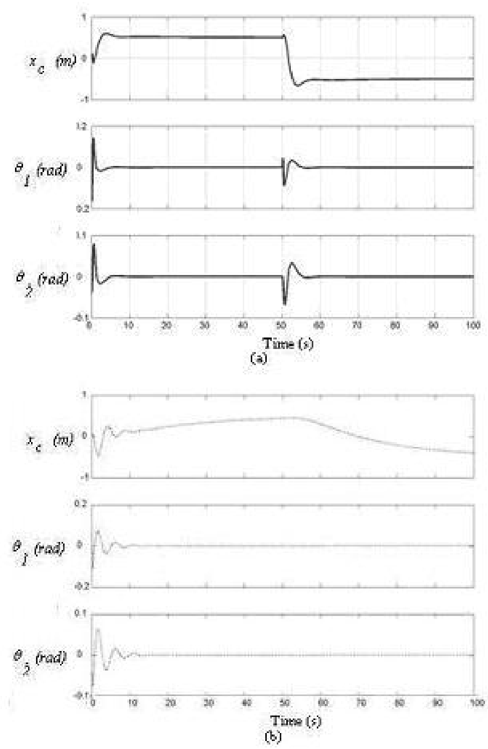

Fig. 6. Nominal double inverted pendulum system responses of all controllers under healthy conditions. (a) FTCS (b) *H∞* controller

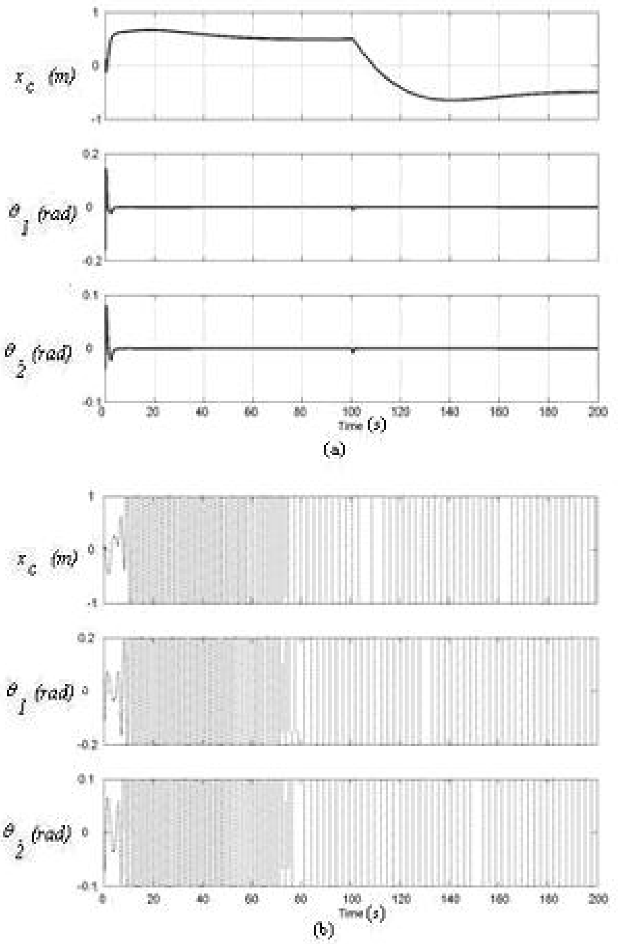

Fig. 7. System responses of all controllers under multiple sensor fault condition, without modelling uncertainty. (a) FTCS (b) *H*∞ controller

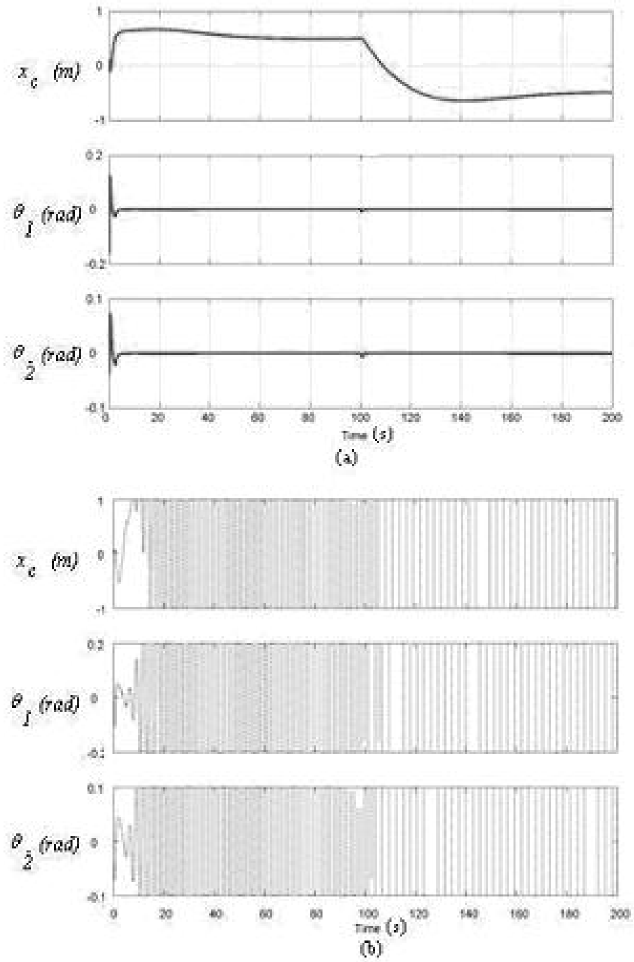

Fig. 8. System responses of all controllers under multiple sensor fault condition, with modelling uncertainty. (a) FTCS (b) *H*∞ controller

4. CONCLUSIONS

The proposed FTCS has been observed to have managed all faults simulated. Robust performance assessments showing the performance of the control systems when faced with system uncertainty in addition to sensor faults were also simulated. Again it is observed that fault tolerance capability of the proposed FTCS has been maintained. The proposed improvement to the model based FTCS structure provides a potential framework for the realisation of an implicit MIMO fault detection and controller based FTCS. This design framework is suitable as it inherently incorporates fault residuals as feedback and allows the application of established robust MIMO control design concept. The test results show the capability of the proposed FTCS to maintain availability and an acceptable level of performance for multiple deteriorated sensor conditions. Nevertheless it must be pointed out that the proposed method can only handle conditions whereby sensor components have not suffered total failure *i.e.* faults not amounting to failure. Conditions where measurements are totally unavailable could not be tolerated.

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Appendix:

POST FAULT PERFORMANCE WEIGHT MATRIX

$$W_{ftc} = \begin{bmatrix} W_e & 0 & 0 \\ 0 & W_{\theta 1} & 0 \\ 0 & 0 & W_{\theta 2} \end{bmatrix}$$

where,

$W_e = \dfrac{25}{50s+1}$ denotes performance weight related to e_x

$W_{\theta 1} = \dfrac{50}{s+10}$ denotes performance weight related to θ_1

$W_{\theta 2} = \dfrac{45}{s+10}$ denotes the performance weight related to θ_2

Note:

The performance function of the signals provided are weighted to characterise the following limits,

- limiting cart position tracking error, e_x at *0 m* at high frequency and relaxed for low frequency at a maximum error of *0.04 m*
- limiting the vertical to lower arm angle, θ_1 at *0 radians* at high frequency and relaxed for low frequency at a maximum angle of *0.20 radians*
- limiting the vertical to upper arm angle, θ_2 at *0 radians* at high frequency and relaxed for low frequency at a maximum angle of *0.22 radians*