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A fault-tolerant control strategy for Takagi-Sugeno fuzzy systems

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Abstract: In this paper, a new active FTC strategy is proposed. First, it is developed in the context of linear systems and then it is extended to Takagi-Sugeno fuzzy systems. The key contribution of the proposed approach is an integrated FTC design procedure of the fault identification and fault-tolerant control schemes. Fault identification is based on the use of an observer. While, the FTC controller is implemented as a state feedback controller. This controller is designed such that it can stabilize the faulty plant using Lyapunov theory and LMIs.

Keywords: fault-tolerant control, state estimation, observers, fault diagnosis, convergence

1. INTRODUCTION

Fault Tolerant Control (FTC) is a relatively new idea recently introduced in the research literature - Blanke et al. [2003] which allows to have a control loop that fulfills its objectives (maybe with a possible degradation) when faults in components of the system (instrumentation, actuators and/or plant) appear. A control loop could be considered fault tolerant if there exist adaptation strategies of the control law included in the closed-loop or mechanisms that introduce redundancy in sensors and/or actuators. From the point of view of the control strategies, the literature considers two main groups: the *active* and the *passive* techniques. The passive techniques are control laws that take into account the fault appearance as a system perturbation. Thus, within certain margins, the control law has inherent fault tolerant capabilities, allowing the system to cope with the fault presence. In the works of Chen et al. [1998], Liang et al. [2000], Qu et al. [2001], Liao et al. [2002] and Qu et al. [2003], among many others, complete descriptions of passive FTC techniques can be found. On the other hand, the active fault tolerant control techniques consist on adapting the control law using the information given by the FDI block (see Blanke et al. [2003]). With this information, some automatic adjustments are done trying to reach the control objectives. See the work of Zhang and Jiang [2003] for a recent review of active FTC. The whole active FTC scheme can be expressed using the three-layer architecture for FTC systems proposed by Blanke et al. [2003] where the first layer corresponds to the control loop, the second layer corresponds to the fault diagnosis (Korbicz et al. [2004], Patton and Korbicz [1999], Witczak [2006, 2004]) and accommodation modules while the third layer is the supervisor system.

In this paper, a new active FTC strategy is proposed. First, it is developed in the context of linear systems and then it is extended to Takagi-Sugeno fuzzy systems. The key contribution of the proposed approach is an integrated FTC design procedure of the fault identification and fault-tolerant control schemes. Fault identification is based on the use of an observer. Once the fault have been identified, the FTC controller is implemented as a state feedback controller. This controller is designed such that it can stabilize the faulty plant using Lyapunov theory and LMIs.

The paper is organised as follows. Section 2 presents the details regarding the proposed FTC strategy. For the sake of simplicity, the developed scheme is described for linear systems and then suitably extended for Takagi-Sugeno (T-S) systems (Section 3). The final part of the paper presents a numerical example which shows the performance of the proposed approach.

2. FTC STRATEGY

Let us consider the following reference model:

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{B}\boldsymbol{u}_k, \tag{1}$$

$$\boldsymbol{y}_{k+1} = \boldsymbol{C}\boldsymbol{x}_{k+1}, \tag{2}$$

where $\boldsymbol{x}_k \in \mathbb{R}^n$ stands for the reference state, $\boldsymbol{y}_k \in \mathbb{R}^m$ is the reference output, and $\boldsymbol{u}_k \in \mathbb{R}^r$ denotes the nominal control input.

Let us also consider a possibly faulty system described by the following equations:

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$$\boldsymbol{x}_{f,k+1} = \boldsymbol{A}\boldsymbol{x}_{f,k} + \boldsymbol{B}\boldsymbol{u}_{f,k} + \boldsymbol{L}\boldsymbol{f}_k, \quad (3)$$
$$\boldsymbol{y}_{f,k+1} = \boldsymbol{C}\boldsymbol{x}_{f,k+1}, \quad (4)$$

where $\boldsymbol{x}_{f,k} \in \mathbb{R}^n$ stands for the system state, $\boldsymbol{y}_{f,k} \in \mathbb{R}^m$ is the system output, $\boldsymbol{u}_{f,k} \in \mathbb{R}^r$ denotes the system input, $\boldsymbol{f}_k \in \mathbb{R}^s$, $(s \leq m)$ is the fault vector, and $\boldsymbol{L}$ stands for its distribution matrix which is assumed to be known.

The main objective of this paper is to propose a novel control strategy which can be used for determining the system input $\boldsymbol{u}_{f,k}$ such that:

- the control loop for the system (3)–(4) is stable,
- $\boldsymbol{x}_{f,k+1}$ converges asymptotically to $\boldsymbol{x}_{k+1}$ irrespective of the presence of the fault $\boldsymbol{f}_k$.

The subsequent part of this section shows the development details of the scheme that is able to settle such a challenging problem.

The crucial idea is to use the following control strategy:

$$\boldsymbol{u}_{f,k} = -\boldsymbol{S}\hat{\boldsymbol{f}}_k + \boldsymbol{K}_1(\boldsymbol{x}_k - \boldsymbol{x}_{f,k}) + \boldsymbol{u}_k \quad (5)$$

where $\hat{\boldsymbol{f}}_k$ is the fault estimate. Note that, due to the separation principle, it is not assumed that $\boldsymbol{x}_{f,k}$ is available, i.e. an estimate $\hat{\boldsymbol{x}}_{f,k}$ can be used instead. Thus, the following problems arise:

- to determine $\hat{\boldsymbol{f}}_k$,
- to design $\boldsymbol{K}_1$ in such a way that the control loop is stable, i.e. the stabilisation problem.

2.1 Fault identification

Let us assume that the following rank condition is satisfied

$$\mathrm{rank}(\boldsymbol{C}\boldsymbol{L}) = \mathrm{rank}(\boldsymbol{L}) = s. \quad (6)$$

This implies that it is possible to calculate $\boldsymbol{H} = (\boldsymbol{C}\boldsymbol{L})^+ = \left[(\boldsymbol{C}\boldsymbol{L})^T\boldsymbol{C}\boldsymbol{L}\right]^{-1}(\boldsymbol{C}\boldsymbol{L})^T$. By multiplying (4) by $\boldsymbol{H}$ and then substituting (3) it can be shown that

$$\boldsymbol{f}_k = \boldsymbol{H}(y_{f,k+1} - \boldsymbol{C}\boldsymbol{A}\boldsymbol{x}_{f,k} - \boldsymbol{C}\boldsymbol{B}\boldsymbol{u}_{f,k}). \quad (7)$$

Thus, if $\hat{\boldsymbol{x}}_{f,k}$ is used instead of $\boldsymbol{x}_{f,k}$ then the fault estimate is given as follows

$$\hat{\boldsymbol{f}}_k = \boldsymbol{H}(\boldsymbol{y}_{f,k+1} - \boldsymbol{C}\boldsymbol{A}\hat{\boldsymbol{x}}_{f,k} - \boldsymbol{C}\boldsymbol{B}\boldsymbol{u}_{f,k}), \quad (8)$$

and the associated fault estimation error is

$$\boldsymbol{f}_k - \hat{\boldsymbol{f}}_k = -\boldsymbol{H}\boldsymbol{C}\boldsymbol{A}(\boldsymbol{x}_{f,k} - \hat{\boldsymbol{x}}_{f,k}). \quad (9)$$

Unfortunately, the crucial problem with practical implementation of (8) is that it requires $\boldsymbol{y}_{f,k+1}$ and $\boldsymbol{u}_{f,k}$ to calculate $\hat{\boldsymbol{f}}_k$ and hence it cannot be directly used to obtain (5). To settle this problem, it is assumed that there exists a diagonal matrix $\boldsymbol{\alpha}_k$ such that $\hat{\boldsymbol{f}}_k = \boldsymbol{\alpha}_k\hat{\boldsymbol{f}}_{k-1}$ and hence the practical form of (5) boils down to

$$\boldsymbol{u}_{f,k} = -\boldsymbol{S}\boldsymbol{\alpha}_k\hat{\boldsymbol{f}}_{k-1} + \boldsymbol{K}_1(\boldsymbol{x}_k - \boldsymbol{x}_{f,k}) + \boldsymbol{u}_k \quad (10)$$

2.2 Stabilisation problem

By substituting (5) into (4) it can be shown that

$$\boldsymbol{x}_{f,k+1} = \boldsymbol{A}\boldsymbol{x}_{f,k} - \boldsymbol{B}\boldsymbol{S}\hat{\boldsymbol{f}}_k + \boldsymbol{B}\boldsymbol{K}_1\boldsymbol{e}_k + \boldsymbol{B}\boldsymbol{u}_k + \boldsymbol{L}\boldsymbol{f}_k, \quad (11)$$

where $\boldsymbol{e}_k = \boldsymbol{x}_k - \boldsymbol{x}_{f,k}$ stands for the tracking error. Let us assume that $\boldsymbol{S}$ satisfies the following equality $\boldsymbol{B}\boldsymbol{S} = \boldsymbol{L}$, e.g. $\boldsymbol{S} = \boldsymbol{I}$ for actuator faults. Thus, $\boldsymbol{B}\boldsymbol{S} = \boldsymbol{L}$ and hence

$$\boldsymbol{x}_{f,k+1} = \boldsymbol{A}\boldsymbol{x}_{f,k} + \boldsymbol{L}(\boldsymbol{f}_k - \hat{\boldsymbol{f}}_k) + \boldsymbol{B}\boldsymbol{K}_1\boldsymbol{e}_k + \boldsymbol{B}\boldsymbol{u}_k. \quad (12)$$

Finally, by substituting (9) into (12) and then applying the result into $\boldsymbol{e}_{k+1} = \boldsymbol{x}_{k+1} - \boldsymbol{x}_{f,k+1}$ yields

$$\boldsymbol{e}_{k+1} = (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}_1)\boldsymbol{e}_k + \boldsymbol{L}\boldsymbol{H}\boldsymbol{C}\boldsymbol{A}\boldsymbol{e}_{f,k}. \quad (13)$$

where $\boldsymbol{e}_{f,k} = \boldsymbol{x}_{f,k} - \hat{\boldsymbol{x}}_{f,k}$ stands for the state estimation error.

2.3 Observer design

As was already mentioned, the fault estimate (8) is obtained based on the state estimate $\hat{\boldsymbol{x}}_{f,k}$. This raises the necessity for an observer design. Consequently, by substituting (7) into (3) it is possible to show that

$$\boldsymbol{x}_{f,k+1} = \bar{\boldsymbol{A}}\boldsymbol{x}_{f,k} + \bar{\boldsymbol{B}}\boldsymbol{u}_{f,k} + \bar{\boldsymbol{L}}\boldsymbol{y}_{f,k+1}, \quad (14)$$

where

$$\bar{\boldsymbol{A}} = (\boldsymbol{I} - \boldsymbol{L}\boldsymbol{H}\boldsymbol{C})\boldsymbol{A}, \bar{\boldsymbol{B}} = (\boldsymbol{I} - \boldsymbol{L}\boldsymbol{H}\boldsymbol{C})\boldsymbol{B}, \bar{\boldsymbol{L}} = \boldsymbol{L}\boldsymbol{H}.$$

Thus, the observer structure, which can be perceived as an unknown input observer (see, e.g. Hui and Zak [2005], Witczak [2004]), is given by

$$\hat{\boldsymbol{x}}_{f,k+1} = \bar{\boldsymbol{A}}\hat{\boldsymbol{x}}_{f,k} + \bar{\boldsymbol{B}}\boldsymbol{u}_{f,k} + \bar{\boldsymbol{L}}\boldsymbol{y}_{f,k+1} + \\ + \boldsymbol{K}_2(\boldsymbol{y}_{f,k} - \boldsymbol{C}\hat{\boldsymbol{x}}_{f,k}). \quad (15)$$

Finally, the state estimation error can be written as follows:

$$\boldsymbol{e}_{f,k+1} = (\bar{\boldsymbol{A}} - \boldsymbol{K}_2\boldsymbol{C})\boldsymbol{e}_{f,k}. \quad (16)$$

2.4 Integrated design procedure

The main objective of this section is to sumarise the presented results within an integrated framework for the development of fault identification and fault-tolerant control scheme. First, let us start with two crucial assumptions:

- the pair $(\bar{\boldsymbol{A}}, \boldsymbol{C})$ is detectable,
- the pair $(\boldsymbol{A}, \boldsymbol{B})$ is stabilisable.

Under these assumptions, it is possible to design the matrices $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ in such a way that the extended error

$$\bar{\boldsymbol{e}}_k = \begin{bmatrix} \boldsymbol{e}_k \\ \boldsymbol{e}_{f,k} \end{bmatrix}, \quad (17)$$

described by

$$\bar{\boldsymbol{e}}_{k+1} = \begin{bmatrix} \boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}_1 & \boldsymbol{L}\boldsymbol{H}\boldsymbol{C}\boldsymbol{A} \\ \boldsymbol{0} & \bar{\boldsymbol{A}} - \boldsymbol{K}_2\boldsymbol{C} \end{bmatrix} \bar{\boldsymbol{e}}_k = \boldsymbol{A}_0\bar{\boldsymbol{e}}_k, \quad (18)$$

converges asymptotically to zero.

Theorem 1. The extended error $\bar{\boldsymbol{e}}_k$ converges asymptotically to zero iff there exist matrices $\boldsymbol{W} \succ \boldsymbol{0}$, $\boldsymbol{L}_1$ and $\boldsymbol{P}_2 \succ \boldsymbol{0}$, $\boldsymbol{L}_2$ such that

$$\begin{bmatrix} \boldsymbol{W} & \boldsymbol{A}\boldsymbol{W} - \boldsymbol{B}\boldsymbol{L}_1 \\ \boldsymbol{W}\boldsymbol{A}^T - \boldsymbol{L}_1^T\boldsymbol{B}^T & \boldsymbol{W} \end{bmatrix} \succ \boldsymbol{0}, \quad (19)$$

$$\begin{bmatrix} \boldsymbol{P}_2 & \boldsymbol{P}_2\bar{\boldsymbol{A}} - \boldsymbol{L}_2\boldsymbol{C} \\ \bar{\boldsymbol{A}}^T\boldsymbol{P}_2 - \boldsymbol{C}^T\boldsymbol{L}_2^T & \boldsymbol{P}_2 \end{bmatrix} \succ \boldsymbol{0}. \quad (20)$$

Proof. It can be observed from the structure of $\boldsymbol{A}_0$ in (18) that the eigenvalues of the matrix $\boldsymbol{A}_0$ are the union of those of $\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}_1$ and $\bar{\boldsymbol{A}} - \boldsymbol{K}_2\boldsymbol{C}$. This clearly indicates that the design of the state feedback and the observer can be carried out independently (separation principle). Thus, it is clear from the Lyapunov theorem that $\bar{\boldsymbol{e}}_k$ converges asymptotically to zero iff there exist matrices $\boldsymbol{P}_1 \succ 0$ and $\boldsymbol{P}_2 \succ 0$ such that following inequalities are satisfied:

$$(\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}_1)^T\boldsymbol{P}_1(\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}_1) - \boldsymbol{P}_1 \prec \boldsymbol{0}, \quad (21)$$

$$(\bar{\boldsymbol{A}} - \boldsymbol{K}_2\boldsymbol{C})^T\boldsymbol{P}_2(\bar{\boldsymbol{A}} - \boldsymbol{K}_2\boldsymbol{C}) - \boldsymbol{P}_2 \prec \boldsymbol{0}. \quad (22)$$

By applying the Schur complements, it is possible to show that (21)–(22) are equivalent to

$$\begin{bmatrix} \boldsymbol{P}_1^{-1} & \boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}_1 \\ \boldsymbol{A}^T - \boldsymbol{K}_1^T\boldsymbol{B}^T & \boldsymbol{P}_1 \end{bmatrix} \succ \boldsymbol{0}, \tag{23}$$

$$\begin{bmatrix} \boldsymbol{P}_2^{-1} & \bar{\boldsymbol{A}} - \boldsymbol{K}_2\boldsymbol{C} \\ \bar{\boldsymbol{A}}^T - \boldsymbol{C}^T\boldsymbol{K}_2^T & \boldsymbol{P}_2 \end{bmatrix} \succ \boldsymbol{0}. \tag{24}$$

By substituting $\boldsymbol{W} = \boldsymbol{P}_1^{-1}$ and then multiplying (23) from left and right by diag$(\boldsymbol{I}, \boldsymbol{W})$ and (24) from left and right by diag$(\boldsymbol{P}_2, \boldsymbol{I})$ it can be show that

$$\begin{bmatrix} \boldsymbol{W} & \boldsymbol{A}\boldsymbol{W} - \boldsymbol{B}\boldsymbol{K}_1\boldsymbol{W} \\ \boldsymbol{W}\boldsymbol{A}^T - \boldsymbol{W}\boldsymbol{K}_1^T\boldsymbol{B}^T & \boldsymbol{W} \end{bmatrix} \succ \boldsymbol{0}, \tag{25}$$

$$\begin{bmatrix} \boldsymbol{P}_2 & \boldsymbol{P}_2\bar{\boldsymbol{A}} - \boldsymbol{P}_2\boldsymbol{K}_2\boldsymbol{C} \\ \bar{\boldsymbol{A}}^T\boldsymbol{P}_2 - \boldsymbol{C}^T\boldsymbol{K}_2^T\boldsymbol{P}_2 & \boldsymbol{P}_2 \end{bmatrix} \succ \boldsymbol{0}. \tag{26}$$

Subsequently, by substituting $\boldsymbol{L}_1 = \boldsymbol{K}_1\boldsymbol{W}$ and $\boldsymbol{L}_2 = \boldsymbol{P}_2\boldsymbol{K}_2$ it is possible to transform (25) and (26) into (19)–(20), which completes the proof. □

Finally, the design procedure boils down to solving the LMIs (19) and (20), and then determining $\boldsymbol{K}_1 = \boldsymbol{L}_1\boldsymbol{W}^{-1}$ and $\boldsymbol{K}_2 = \boldsymbol{P}_2^{-1}\boldsymbol{L}_2$.

2.5 Illustrative example

Let us consider (1)–(2) and (3)–(4) described by the following matrices:

$$\boldsymbol{A} = \begin{bmatrix} 0.2225 & 0.2093 & 0.1013 \\ 0.4659 & 0.4231 & 0.3361 \\ 0.2330 & 0.2626 & 0.4191 \end{bmatrix} \boldsymbol{B} = \begin{bmatrix} 0.0196 & 0.8318 \\ 0.6813 & 0.5028 \\ 0.3795 & 0.7095 \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \boldsymbol{L} = -\boldsymbol{B}$$

The reference input is defined by

$$\boldsymbol{u}_k = [0.3\tanh(k/100), 0.1 + 0.2\cos(\pi k/100)]^T, \tag{27}$$

for $k = 0 \ldots 1000$. Similarly, the fault scenario is as follows

$$\boldsymbol{f}_{k,1} = \begin{cases} 0, & k < 400 \\ 0.4, & k \geqslant 400 \end{cases}$$

$$\boldsymbol{f}_{k,2} = \begin{cases} 0, & k < 200 \\ 0.5 + 0.3\sin(\pi k/100), & k \geqslant 200 \end{cases}$$

Figures 1–3 presents the results achieved for the pro-

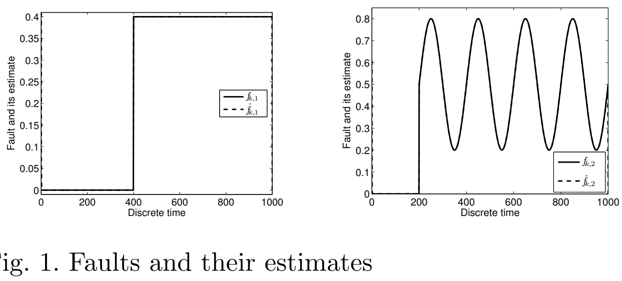

Fig. 1. Faults and their estimates

posed FTC strategy. In particular, the proposed design procedure was applied to obtain $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ and then (10) (with $\boldsymbol{\alpha}_k = \boldsymbol{I}$) was applied as a control strategy. As a result, Fig. 1 clearly shows that the faults can be estimated with a very high accuracy. Moreover, from Fig. 2 it can be observed that $\boldsymbol{u}_{f,k}$ is equal to $\boldsymbol{u}_k$ until the occurrence of the fault $\boldsymbol{f}_2$. After that time the control strategy $\boldsymbol{u}_{f,k}$ was changed. As can be easily observed, the control strategy was also changed when $\boldsymbol{f}_1$ occurred. The final conclusion

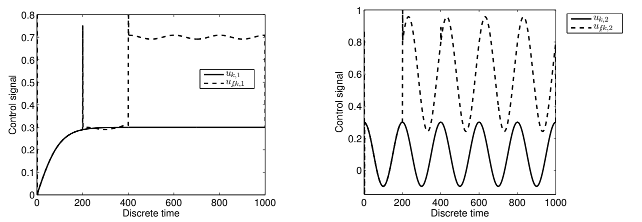

Fig. 2. Trajectories of $\boldsymbol{u}_k$ and $\boldsymbol{u}_{f,k}$

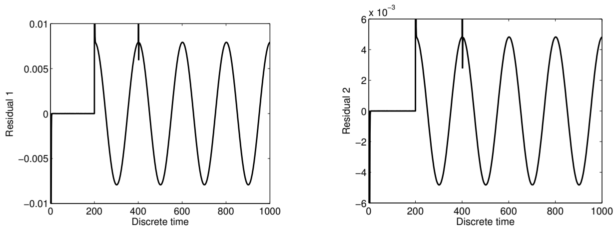

Fig. 3. Residual

is that the residual $\boldsymbol{z}_k = \boldsymbol{y}_k - \boldsymbol{C}\hat{\boldsymbol{x}}_{f,k}$ is very close to zero in the presence of faults (Fig. 3). This is because of the proposed control strategy for which $\boldsymbol{x}_{f,k}$ converges to $\boldsymbol{x}_k$ and consequently $\boldsymbol{z}_k$ converges to zero. On the other hand, the presence of faults can be easily determined from (8).

3. EXTENSION TO T-S FUZZY SYSTEMS

The main objective of this section is to extend the approach proposed in Section 2 to Takagi-Sugeno fuzzy systems (see Takagi and Sugeno [1985]). In order to make the paper self-contained let us start with a brief introduction to the T-S fuzzy systems.

3.1 Elementary background on T-S fuzzy systems

A non-linear dynamic system can be described in a simple way by a Takagi-Sugeno fuzzy model, which uses series of locally linearised non-linear models (see, e.g. Takagi and Sugeno [1985], Korbicz et al. [2004]). According to this model, a non-linear dynamic systems can be linearised around a number of operating points. Each of these linear models represents the local system behaviour around the operating point. Thus, a fuzzy fusion of all linear model outputs describes the global system behaviour. A T-S model is described by fuzzy IF-THEN rules which represent local linear I/O relations of the non-linear system. It has a rule base of M rules, each having p antecedents, where ith rule is expressed as

$$R^i: \text{ IF } w_k^1 \text{ is } F_1^i \text{ and } \ldots \text{ and } w_k^p \text{ is } F_p^i,$$
$$\text{THEN } \begin{cases} \boldsymbol{x}_{k+1} = \boldsymbol{A}_i\boldsymbol{x}_k + \boldsymbol{B}_i\boldsymbol{u}_k \\ \boldsymbol{y}_k = \boldsymbol{C}_i\boldsymbol{x}_k \end{cases}, \tag{28}$$

in which $i = 1, \ldots, M$, F_j^i $(j = 1, \ldots, p)$ are fuzzy sets and $\boldsymbol{w}_k = [w_k^1, w_k^2, \ldots, w_k^p]$ is a known vector of premise variables (Korbicz et al. [2004]) which may depend partially on the state $\boldsymbol{x}_k$.

Given a pair of $(\boldsymbol{w}_k, \boldsymbol{u}_k)$ and a product inference engine, the final output of the normalized T-S fuzzy model can be inferred as:

$$\begin{cases} \boldsymbol{x}_{k+1} = \sum_{i=1}^{M} h_i(\boldsymbol{w}_k)[\boldsymbol{A}_i \boldsymbol{x}_k + \boldsymbol{B}_i \boldsymbol{u}_k] \\ \boldsymbol{y}_{k+1} = \sum_{i=1}^{M} h_i(\boldsymbol{w}_{k+1}) \boldsymbol{C}_i \boldsymbol{x}_{k+1} \end{cases}, \tag{29}$$

where $h_i(\boldsymbol{w}_k)$ are normalized rule firing strengths defined as

$$h_i(\boldsymbol{w}_k) = \frac{\mathcal{T}_{j=1}^{p} \mu_{F_j^i}(w_k^j)}{\sum_{i=1}^{M} (\mathcal{T}_{j=1}^{p} \mu_{F_j^i}(w_k^j))} \tag{30}$$

and $\mathcal{T}$ denotes a t-norm (e.g., product). The term $\mu_{F_j^i}(w_k^j)$ is the grade of membership of the premise variable w_k^j. Moreover, the rule firing strengths $h_i(\boldsymbol{w}_k)$ $(i = 1, \ldots, M)$ satisfy the following constraints

$$\begin{cases} \sum_{i=1}^{M} h_i(\boldsymbol{w}_k) = 1 \\ 0 \leqslant h_i(\boldsymbol{w}_k) \leqslant 1, \quad \forall i = 1, \ldots, M \end{cases}. \tag{31}$$

3.2 FTC startegy

In the light of the approach presented in Section 2, the T-S reference model used in this paper is given by:

$$\boldsymbol{x}_{k+1} = \sum_{i=1}^{M} h_i(\boldsymbol{w}_k)[\boldsymbol{A}_i \boldsymbol{x}_k + \boldsymbol{B} \boldsymbol{u}_k], \tag{32}$$

$$\boldsymbol{y}_{k+1} = \sum_{i=1}^{M} h_i(\boldsymbol{w}_k) \boldsymbol{C} \boldsymbol{x}_{k+1}, \tag{33}$$

$$\tag{34}$$

or equivalently by

$$\boldsymbol{x}_{k+1} = \sum_{i=1}^{M} h_i(\boldsymbol{w}_k) \boldsymbol{A}_i \boldsymbol{x}_k + \boldsymbol{B} \boldsymbol{u}_k, \tag{35}$$

$$\boldsymbol{y}_{k+1} = \boldsymbol{C} \boldsymbol{x}_{k+1}. \tag{36}$$

Similarly, a possibly faulty T-S system is described by

$$\boldsymbol{x}_{f,k+1} = \sum_{i=1}^{M} h_i(\boldsymbol{w}_k) \boldsymbol{A}_i \boldsymbol{x}_{f,k} + \boldsymbol{B} \boldsymbol{u}_{f,k} + \boldsymbol{L} \boldsymbol{f}_k \tag{37}$$

$$\boldsymbol{y}_{f,k+1} = \boldsymbol{C} \boldsymbol{x}_{f,k+1}. \tag{38}$$

Following the same line of reasoning as in Section 2, it can be shown that the fault estimate is given by:

$$\hat{\boldsymbol{f}}_k = \boldsymbol{H}(\boldsymbol{y}_{f,k+1} - \boldsymbol{C} \sum_{i=1}^{M} h_i(\boldsymbol{w}_k) \boldsymbol{A}_i \hat{\boldsymbol{x}}_{f,k} - \boldsymbol{C}\boldsymbol{B} \boldsymbol{u}_{f,k}). \tag{39}$$

Similarly, the observer structure is

$$\hat{\boldsymbol{x}}_{f,k+1} = \sum_{i=1}^{M} h_i(\boldsymbol{w}_k) \bar{\boldsymbol{A}}_i \hat{\boldsymbol{x}}_{f,k} + \bar{\boldsymbol{B}} \boldsymbol{u}_{f,k} + \bar{\boldsymbol{L}} \boldsymbol{y}_{f,k+1} + \\ + \boldsymbol{K}_2(\boldsymbol{y}_{f,k} - \boldsymbol{C} \hat{\boldsymbol{x}}_{f,k}), \tag{40}$$

where $\bar{\boldsymbol{A}}_i = (\boldsymbol{I} - \boldsymbol{L}\boldsymbol{H}\boldsymbol{C})\boldsymbol{A}_i$ and $\bar{\boldsymbol{B}} = (\boldsymbol{I} - \boldsymbol{L}\boldsymbol{H}\boldsymbol{C})\boldsymbol{B}$. Finally, the T-S counterpart of the extended error (18) is

$$\bar{\boldsymbol{e}}_{k+1} = \sum_{i=1}^{M} h_i(\boldsymbol{w}_k) \begin{bmatrix} \boldsymbol{A}_i - \boldsymbol{B}\boldsymbol{K}_1 & \boldsymbol{L}\boldsymbol{H}\boldsymbol{C}\boldsymbol{A}_i \\ \boldsymbol{0} & \bar{\boldsymbol{A}}_i - \boldsymbol{K}_2 \boldsymbol{C} \end{bmatrix} \bar{\boldsymbol{e}}_k = \\ = \sum_{i=1}^{M} h_i(\boldsymbol{w}_k) \boldsymbol{A}_{0,i} \bar{\boldsymbol{e}}_k = \boldsymbol{A}_0(\boldsymbol{h}(\boldsymbol{w}_k)) \bar{\boldsymbol{e}}_k, \tag{41}$$

where the matrix $\boldsymbol{A}_0(\boldsymbol{h}(\boldsymbol{w}_k))$ belongs to a convex polytopic set defined as

$$\mathbb{A}_0 = \left\{ \boldsymbol{A}_0(\boldsymbol{h}(\boldsymbol{w}_k)) : \boldsymbol{A}_0(\boldsymbol{h}(\boldsymbol{w}_k)) = \sum_{i=1}^{M} h_i(\boldsymbol{w}_k) \boldsymbol{A}_{0,i}, \right. \\ \left. \sum_{i=1}^{M} h_i(\boldsymbol{w}_k) = 1, 0 \leqslant h_i(\boldsymbol{w}_k) \leqslant 1 \right\} \tag{42}$$

By adapting the general results of Oliveira et al. [1999], the following definition is introduced:

Definition 1. The extended error described by (41) is robustly convergent to zero in the uncertainty domain (42) iff all eigenvalues of $\boldsymbol{A}_0(\boldsymbol{h}(\boldsymbol{w}_k))$ have magnitude less than one for all values of $\boldsymbol{h}(\boldsymbol{w}_k)$ such that $\boldsymbol{A}_0(\boldsymbol{h}(\boldsymbol{w}_k)) \in \mathbb{A}_0$.

Theorem 2. The extended error described by (41) is robustly convergent to zero in the uncertainty domain (42) if there exist matrices $\boldsymbol{P}_{x_i} \succ \boldsymbol{0}$, $\boldsymbol{G}_1$, $\boldsymbol{L}_1$ and $\boldsymbol{P}_{y_i} \succ \boldsymbol{0}$, $\boldsymbol{G}_2$, $\boldsymbol{L}_2$ such

$$\begin{bmatrix} \boldsymbol{P}_{x_i} & \boldsymbol{A}_i \boldsymbol{G}_1 - \boldsymbol{B}\boldsymbol{L}_1 \\ \boldsymbol{G}_1^T \boldsymbol{A}_i^T - \boldsymbol{L}_1^T \boldsymbol{B}^T & \boldsymbol{G}_1 + \boldsymbol{G}_1^T - \boldsymbol{P}_{x_i} \end{bmatrix} \succ \boldsymbol{0}, \tag{43}$$

$$\begin{bmatrix} \boldsymbol{P}_{y_i} & \bar{\boldsymbol{A}}_i^T \boldsymbol{G}_2^T - \boldsymbol{C}^T \boldsymbol{L}_2^T \\ \boldsymbol{G}_2 \bar{\boldsymbol{A}}_i - \boldsymbol{L}_2 \boldsymbol{C} & \boldsymbol{G}_2 + \boldsymbol{G}_2^T - \boldsymbol{P}_{y_i} \end{bmatrix} \succ \boldsymbol{0}, \tag{44}$$

for all $i = 1, \ldots, M$.

Proof. Using Theorem 1 and then applying Theorem 1 and 2 from the work Oliveira et al. [1999] it is straightforward to complete the proof. □

Finally, the design procedure boils down to solving the set of M LMIs (43) and (44) and then determining $\boldsymbol{K}_1 = \boldsymbol{L}_1 \boldsymbol{G}_1^{-1}$ and then $\boldsymbol{K}_2 = \boldsymbol{G}_2^{-1} \boldsymbol{L}_2$.

4. ILLUSTRATIVE EXAMPLE

Let us consider the fault-free T-S fuzzy systems described by

R^1 : IF $y_{k,1}$ is F_1 THEN $\boldsymbol{x}_{k+1} = \boldsymbol{A}_2 \boldsymbol{x}_k + \boldsymbol{B}\boldsymbol{u}_k$
R^2 : IF $y_{k,1}$ is F_2 THEN $\boldsymbol{x}_{k+1} = \boldsymbol{A}_3 \boldsymbol{x}_k + \boldsymbol{B}\boldsymbol{u}_k$
R^3 : IF $y_{k,1}$ is F_3 THEN $\boldsymbol{x}_{k+1} = \boldsymbol{A}_1 \boldsymbol{x}_k + \boldsymbol{B}\boldsymbol{u}_k$

with

$$\boldsymbol{A}_1 = \begin{bmatrix} 0.2225 & 0.2093 & 0.1013 \\ 0.4659 & 0.4231 & 0.3361 \\ 0.2330 & 0.2626 & 0.4191 \end{bmatrix}, \tag{45}$$

$$\boldsymbol{A}_2 = \begin{bmatrix} 0.4751 & 0.2430 & 0.2282 \\ 0.1156 & 0.4456 & 0.0093 \\ 0.3034 & 0.3810 & 0.4107 \end{bmatrix}, \tag{46}$$

$$\boldsymbol{A}_3 = \begin{bmatrix} 0.2224 & 0.4609 & 0.2029 \\ 0.3077 & 0.3691 & 0.4677 \\ 0.3960 & 0.0881 & 0.4585 \end{bmatrix}, \tag{47}$$

where the membership functions are given in Fig. 4. The remaining parameters, signals and fault scenarios are the same as these used in the example presented Section 2.5. Figures 5–10 presents the results achieved for the proposed FTC strategy described in Section 3. In particular, the proposed design procedure was applied to obtain $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ and then (10) (with $\boldsymbol{\alpha}_k = \boldsymbol{I}$) was applied as a control strategy. As a result, Figs. 5–6 clearly show that the faults

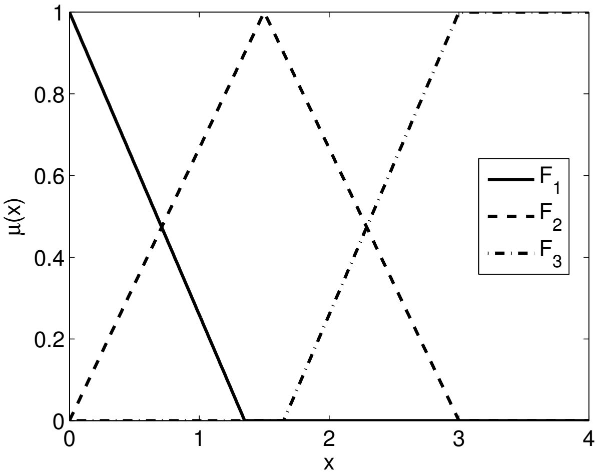

Fig. 4. Membership functions

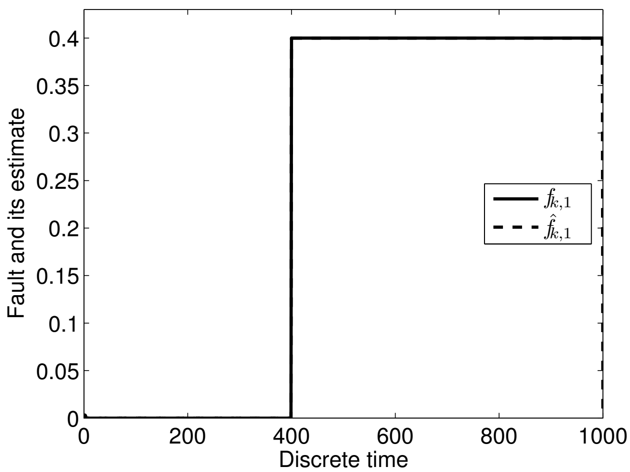

Fig. 5. Fault $\boldsymbol{f}_{k,1}$ and its estimate

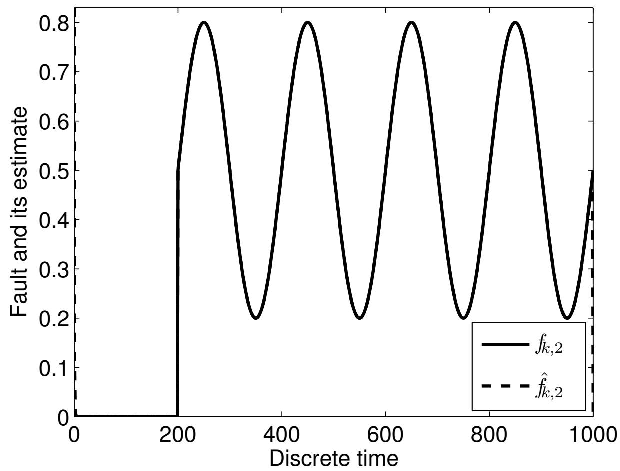

Fig. 6. Fault $\boldsymbol{f}_{k,2}$ and its estimate

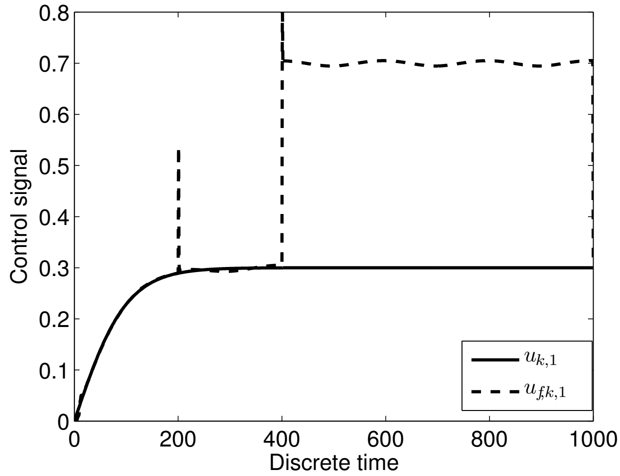

Fig. 7. Trajectory of $\boldsymbol{u}_{k,1}$ and $\boldsymbol{u}_{f,k,1}$

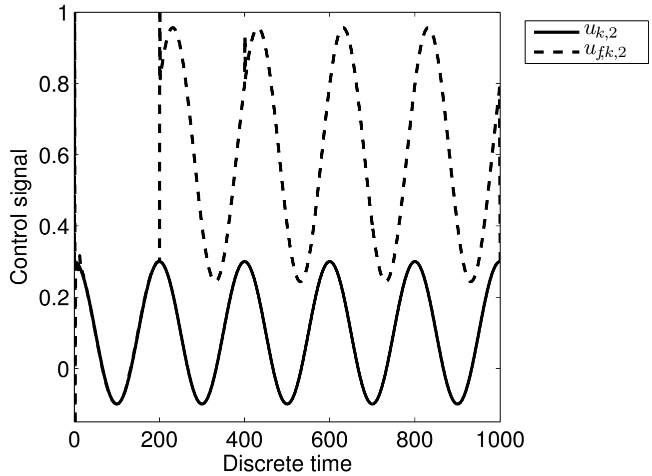

Fig. 8. Trajectory of $\boldsymbol{u}_{k,2}$ and $\boldsymbol{u}_{f,k,2}$

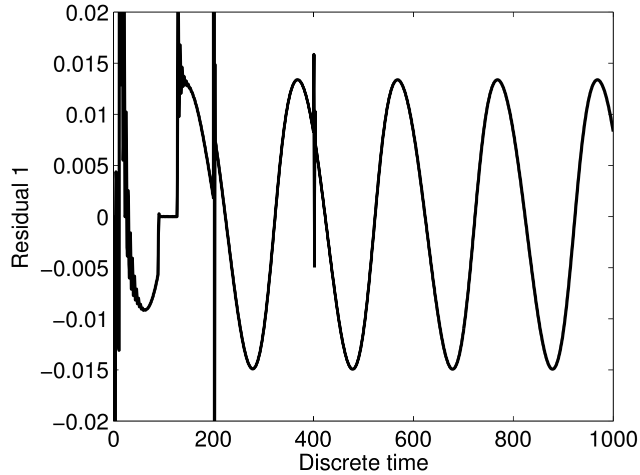

Fig. 9. Residual $z_{1,k}$

can be estimated with a very high accuracy. Similarly as in Section 2.5, from Figs. 7–8 it can be observed that $\boldsymbol{u}_{f,k}$ is equal to $\boldsymbol{u}_k$ until the occurrence of the fault $\boldsymbol{f}_2$. After that time the control strategy $\boldsymbol{u}_{f,k}$ was changed. The control strategy was also changed when $\boldsymbol{f}_1$ occurred. The final conclusion is that the residual (Figs. 9–10) is very close

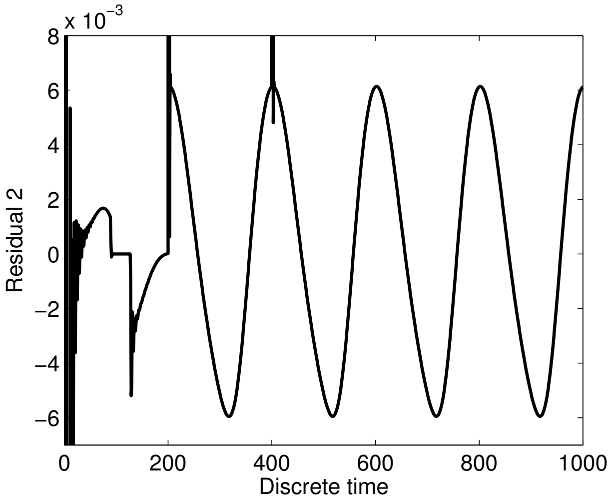

Fig. 10. Residual $z_{2,k}$

to zero in the presence of faults. This is because of the proposed control strategy for which $\boldsymbol{x}_{f,k}$ converges to $\boldsymbol{x}_k$ and consequently $\boldsymbol{z}_k$ converges to zero. On the other hand, the presence of faults can be easily determined from (8).

5. CONCLUSIONS

In this paper, a new active FTC strategy has been proposed. First, this new approach has been developed in the context of linear systems and then it was extended to Takagi-Sugeno fuzzy systems. The key contribution of the proposed approach is an integrated FTC design procedure of the fault identification and fault-tolerant control schemes. Fault identification is based on the use of an observer. Once the fault have been identified, the FTC controller is implemented as a state feedback controller. This controller is designed such that it can stabilize the faulty plant using Lyapunov theory and LMIs. Illustrative examples both for linear and non-linear systems described by T-S fuzzy models are provided that show the effectiveness of the proposed FTC approach.

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