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Non-parametric H_∞ control synthesis with suboptimal controller sets.

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Abstract:
The effectiveness of norm-based control methodologies heavily relies on the quality of the model that describes the dynamic behavior of the plant. In practical applications, the requirement to accurately describe the system at hand often results in high-order plant-models. On the other hand, low-order models are desired to end-up with low-order controllers that reduce implementational costs. The resulting trade-off between dynamical order and closed-loop performance can not be handled in a straightforward manner since the closed-loop behavior is unknown at the moment of plant-parametrization.
This paper proposes a method to overcome this trade-off via non-parametric H_∞ control-synthesis, i.e. omitting parametrization of the plant. As a result, no data-reduction or data-interpolation is performed before synthesis. The resulting controller is represented as Frequency Response Sets for a given frequency grid. This data can be used as input for controller parametrization with explicit trade-off between closed-loop performance and controller order.
This is achieved by considering the mixed-sensitivity problem as a model-matching problem based on Youla-parametrization. Via a specific conceptual choice of the coprime-factorization for the Youla parametrization, it is proved that the SISO H_∞ control synthesis problem can be solved in a non-parametric way based on the plant zeros and frequency response coefficients of the system solely. A simulation study is performed on a fourth-order system to illustrate the main steps in the approach.

1. INTRODUCTION

The practical value of advanced optimal controller synthesis tools is heavily affected by the quality of the model that describes the dynamics of the plant. In practice, the plant model is commonly identified from experimental data. An attempt to model all phenomena in this data commonly results in high-order plant models. Unfortunately, the order of a controller synthesized by common optimal control synthesis techniques is generally equal to the number of states of the plant and the weighting filters (Skogestad and Postlethwaite [2005], Zhou et al. [1996]). This results in conflicting requirements with respect to closed-loop performance and implementational costs. On the one hand, a high-order plant model is needed to accurately describe the plant behavior and achieve closed-loop performance. On the other hand, a low-order controller is desired from an implementation point of view. To handle this trade-off, additional knowledge is required to determine which dynamics are relevant for closed-loop behavior. Unfortunately, this data is unknown during the parametrization-step of the plant since the controller is unknown.

One approach to handle this trade-off is to apply iterative approaches (de Callafon [1998], Hjalmarsson and M. Gevers [1996], den Hof and Schrama [1995]), which ensure that the resulting performance-loss due to model-mismatch is minimized and robust stability is maintained under the real plant behavior. To counteract the risks of stability and performance loss, one could also introduce uncertainty in the plant-model (Douma [1996]). This however results in a conservative design and does not reveal the performance that could be achieved if uncertainty is lowered in a certain frequency region.

An alternative for the iterative approach is high-order-plant-modeling followed by controller order-reduction techniques to obtain a controller of acceptable dynamical order. However, after time-consuming high-order plant modeling stability and performance is not necessarily guaranteed under balanced closed-loop order-reduction (Wortelboer [1994], Wortelboer et al. [1999]).

These arguments motivate the main contribution of this work which is a method to perform non-parametric controller synthesis for the SISO H_∞ mixed-sensitivity problem. The synthesis is non-parametric in the sense that the all frequency response data (FRD) of the plant is used for control synthesis.

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In fact, the FRD is used as a high-order model whereas neither data-interpolation nor data-reduction of the experimental data is applied before the synthesis. As a result, all the dynamics that may be relevant for closed-loop behavior are maintained during the controller synthesis step. The resulting controller is described as Frequency Response Sets on a given frequency grid. This data can be used to parameterize the controller with explicit knowledge of the trade-off between controller order and closed-loop performance. It has to be mentioned that the explicit parametrization of the controller is not considered in this paper but several approaches can be found in literature Y. Chait and Hollot [1999], Hansen and Walster [2004].

The main approach is the following: the H_∞ mixed sensitivity problem is considered as a model-matching problem. Under the condition of a stable plant, it can be guaranteed via the Youla-parametrization (Zames and Francis [1983], Vidyasagar [1985]) that closed-loop stability is achieved based on the non-parametric data and zero's of the plant. Additionally, an explicit description of the set of controller frequency response behavior is given that corresponds to a certain norm of the mixed sensitivity problem via Nevalinaa-Pick interpolation techniques.

The idea to use non-parametric measured plant data for synthesis closely relates to the line of reasoning proposed by Favoreel et al. [1999] and Woodley [2001]. Beside differences in the theoretical basis, the proposed approach is based on the frequency domain instead of the time-domain, it has the ability to give a set description of the controllers that satisfy a certain norm-constraint. Quantitative Feedback Theory (QFT) techniques, as proposed in Horowitz [1993] and Yaniv [1999], can also be regarded as closely related to the proposed approach. It seems however that the concept of model-matching, used in this paper, is more rooted into standard optimal control concepts as proposed in Doyle et al. [1990] and Francis [1987].

The outline of the paper is as follows. Section 2 introduces the frequency domain mixed sensitivity problem which is used as the theoretical framework of this paper. This section shortly describes the conversion of the H_∞ optimal control synthesis problem into a model-matching problem by introduction of the Youla parameter. Given a stable plant and the parametric zeros's, this model-matching appears to be solvable in a non-parametric manner as described in Section 4. The set description of suboptimal controller FRD's is formulated in Section 5 whereas Section 6 describes an example to illustrate the proposed approach.

2. FREQUENCY DOMAIN OPTIMAL CONTROL

The robust controller synthesis problem considered in this paper is defined as the design of a stabilizing controller with transfer function $C(s)$ for the plant $P(s)$ such that the following SISO mixed-sensitivity problem is solved:

$$\left\| \begin{bmatrix} W_1 S \\ W_2 T \end{bmatrix} \right\|_\infty \leq \gamma \tag{1}$$

whereas $S \hat{=} \frac{1}{1+PC}$, $T \hat{=} \frac{PC}{1+PC}$, W_1 and W_2 respectively represent the sensitivity function, the complementary sensitivity function, and the corresponding weighting filters which are chosen such that $W_1, W_2 \in \mathcal{RH}_\infty$. Furthermore, $\| \, . \, \|_\infty$ denotes the $\mathcal{H}_\infty$-norm.

The frequency domain approach described in Francis [1987] and Doyle et al. [1990] is used as basis to solve this problem and therefore is described briefly to introduce the main line of reasoning. Section 4 describes that the method proposed in Francis [1987] and Doyle et al. [1990]can be used to overcome plant parametrization for the case of stable SISO plants such that synthesis can be performed non-parametrically.

2.1 From Mixed-sensitivity to Model-matching Problem

For the purpose of easy reading, the main steps in the proposed approach are summarized.

(1) The set of all stabilizing controllers is parametrized in terms of the Youla parameter $Q \in \mathcal{RH}_\infty$.
(2) The control problem formulated in (1) is rewritten as a model-matching problem in terms of Q. The subsequent synthesis can be performed in this new design parameter Q due to bijection between Q and C.
(3) Classical Nevanlinna-Pick interpolation is applied to obtain a Q that solves (1) and guarantees stability of the closed-loop system. As a final step, C is computed from Q.
(4) If P is stable, the coprime factorizations of the plant and the controller can be chosen such that the model-matching problem can be solved without knowledge about the poles of the plant. The zeros of plant are the only parametric plant data needed to solve (1) and guarantee stability of the closed-loop system.

In order to describe the set of stabilizing controllers, the coprime-factorization of the plant and a stabilizing controller, denoted by C_0, are introduced (see Vidyasagar [1985]):

$$P = \frac{N}{M}, \qquad C_0 = \frac{X_0}{Y_0}, \qquad N, M, X_0, Y_0 \in \mathcal{RH}_\infty \tag{2}$$

Given one stabilizing controller C_0, the set of all stabilizing rational controllers is given by(Francis [1987], Zhou et al. [1996]):

$$\mathcal{C} := \{C = \frac{X_0 + MQ}{Y_0 - NQ} \mid Q \in \mathcal{RH}_\infty\} \tag{3}$$

Due to bijection between Q and C, Q can also be expressed in terms of C:

$$\mathcal{Q} := \{Q = \frac{CY_0 - X_0}{M + CN} \mid C \in \mathcal{C}\} = \mathcal{RH}_\infty \tag{4}$$

The sequel of this section is used to rewrite (1) as a matching problem in terms of Q which is easier to solve that the original problem due to convexity in the term Q (Francis [1987]):

$$\|T_1 - T_2 Q\|_\infty \leq 1, \; Q \in \mathcal{RH}_\infty \tag{5}$$

Proposition 1. There exist a $T_1, T_2 \in \mathcal{RH}_\infty$ such that $C \in \mathcal{C}$ satisfies (1) if and only if Q satisfies (5). The key observation is that (3) is a bijection between the set of stabilizing controllers $\mathcal{C}$ and $\mathcal{Q} = \mathcal{RH}_\infty$.

The remainder of this section is proof, with (14) and (15) as the most important transfer functions.

The mixed sensitivity problem formulated in (1) can be written as:

$$\| |W_1 S|^2 + |W_2 T|^2 \|_\infty < \gamma^2, \quad C \in \mathcal{C} \tag{6}$$

Substitution of (3) in (6) gives:

$$\| |W_1 M(Y - NQ)|^2 + |W_2 N(X + MQ)|^2 \|_\infty \leq \gamma^2 \tag{7}$$

This can be rewritten as (see Doyle et al. [1990]):

$$\| |U_1 - U_2 Q|^2 + U_3 \|_\infty \leq \gamma^2 \tag{8}$$

where U_1, U_2 and U_3 are defined as:

$$\bar{U}_2 U_1 = \overline{W_1 MN} W_1 MY - \overline{W_2 MN} W_2 NX \tag{9}$$

$$\bar{U}_2 U_2 = \overline{W_1 MN} W_1 MN + \overline{W_2 MN} W_2 MN \tag{10}$$

$$U_3 = \frac{\bar{W}_1 W_1 \bar{W}_2 W_2}{\bar{W}_1 W_1 + \bar{W}_2 W_2} \tag{11}$$

The notation $\bar{\cdot}$ is defined as the complex conjugate of $\{.\}$, i.e. $\bar{H}(s) = H(-s)$.

The structure of the transfer-functions described in (9), (10) and (11) makes the $\bar{U}_2 U_1$, $\bar{U}_2 U_2$ and U_3 exhibit a symmetrical pole-zero pattern in both the imaginary and real axis. Via the spectral factorization, denoted by $\mathcal{F}$, such functions can be decomposed into a part with poles and zeros in the right-half plane and a part with poles and zeros in the left-half plane.

The spectral factorization of the right-hand of (10) gives an expression for $\bar{U}_2$ which is substituted in (9):

$$U_1 = \frac{\overline{W_1 MN} W_1 MY - \overline{W_2 MN} W_2 NX}{\bar{U}_2} \tag{12}$$

Equation (8) can be cast into an inequality over all frequencies:

$$\begin{aligned} |U_1 - U_2 Q|^2 + U_3 &\leq \gamma^2 &&, \forall \omega \\ |U_1 - U_2 Q|^2 &\leq (\gamma^2 - U_3) &&, \forall \omega \\ |\mathcal{F}(\gamma^2 - U_3)^{-1} U_1 - \mathcal{F}(\gamma^2 - U_3)^{-1} U_2 Q|^2 &\leq 1 &&, \forall \omega \\ \|\mathcal{F}(\gamma^2 - U_3) U_1 - \mathcal{F}(\gamma^2 - U_3) U_2 Q\|_\infty &\leq 1 && \end{aligned} \tag{13}$$

This brings us to the model-matching problem formulated in (5):

$$T_1 = \mathcal{F}(\gamma^2 - U_3) U_1 \tag{14}$$

$$T_2 = \mathcal{F}(\gamma^2 - U_3) U_2 \tag{15}$$

Remark that $U_1 \notin \mathcal{RH}_\infty$ due to unstable poles generated by the terms $\bar{U}_2^{-1}$, $\overline{W_1 MN}$ and $\overline{W_2 MN}$. As described by Doyle et al. [1990](page 188), it is allowed to replace U_1 with $VU_1 \in \mathcal{RH}_\infty$ where V is chosen such that $V\bar{V}$ has unit-gain and V cancels the instable poles of T_1 by unstable zeros.

3. SOLVING THE MODEL-MATCHING PROBLEM

In this section, the matching problem formulated in (5) is written as an interpolation problem that can be solved by classical Nevanlinna-Pick interpolation.

The following filter $G \in \mathcal{RH}_\infty$ is introduced (see (5)):

$$G = T_1 - T_2 Q \leq 1 \tag{16}$$

Given the filter G, Q can be computed via:

$$Q = -T_2^{-1}(G - T_1) \tag{17}$$

In order to guarantee that $Q \in \mathcal{RH}_\infty$, the unstable poles of the term T_2^{-1} have to be canceled by the zero's of $(G - T_1)$.

The set of unstable zeros of T_2, i.e. the poles of T_2^{-1} are defined as:

$$\mathcal{Z}_{T_2} := \{z \in \mathbb{C}^+ \cup \infty \mid T_2(z) = 0\} \tag{18}$$

Now the aim is to find $G \in \mathcal{RH}_\infty$ such that:

$$\|G\|_\infty \leq 1 \tag{19}$$

$$G(a) = T_1(a) := b \;\forall\, a \in \mathcal{Z}_{T_2} \tag{20}$$

A filter G that satisfies these conditions can be found using classical Nevanlinna-Pick interpolation (Ball et al. [1990], Francis [1987]). Given the filter G, the filter Q can be acquired via (17).

The order of G is generally equal to the number of interpolation constraints. Furthermore, a solution that satisfies (20) can only be found if the Pick-matrix is non-singular. Else, the problem formulated in (6) is non-solvable given the current weighting filters and predefined γ. For more details on properties of the Pick-matrix is referred to Ball et al. [1990] and Francis [1987].

It will appear in Section 4 that the only plant data required the obtain the interpolation conditions are the zeros's of the plant.

4. THE NON-PARAMETRIC MODEL-MATCHING PROBLEM

The derivations given in Section 2 and Section 3 are used as a starting point to apply the matching interpolation approach for non-parametric controller synthesis. This is achieved using a certain conceptual choice of the coprime factorization of P and C_0 such that all the plant-data vanish out of U_1. As a result, the zeros of the plant are the only required parametric plant data to generate the matching conditions given in (20).

Since both parametric, e.g. weighting filters, and non-parametric data will be used, a different notation is introduced for a non-parametric description of a frequency response function:

$$H_i = H(j\omega_i) \tag{21}$$

where ω_i denotes a discrete frequency grid to evaluate the frequency response function (FRF) $H(j\omega)$.

Proposition 2. Given a stable plant P: W_1, W_2 and the zero's of P are the only required parametric data to generate the Nevanlinna-Pick interpolation conditions that solve (5).

Given $P \in \mathcal{RH}_\infty$, the following coprime factorizations are allowed:

$$\begin{aligned} N = P, \quad M = 1 \\ X_0 = 0, \quad Y_0 = 1 \end{aligned} \tag{22}$$

Substitution of (22) in (10) gives:

$$\bar{U}_2 U_2 = \bar{P} P (\bar{W}_1 W_1 + \bar{W}_2 W_2) \tag{23}$$

Evaluating the zeros of T_2 (see (15)) shows that $\mathcal{Z}_{T_2}$ equals the union of the zeros of P, $\bar{P}$ (the negative counterparts of the zero's of P), the zeros of $\mathcal{F}(\bar{W}_1 W_1 + \bar{W}_2 W_2)$ and the zero's of $\mathcal{F}(\gamma^2 - \frac{\bar{W}_1 W_1 \bar{W}_2 W_2}{\bar{W}_1 W_1 + \bar{W}_2 W_2})$.

Substitution of $\bar{U}_2$ into (12) gives:

$$U_1 = \frac{\overline{W_1 P} W_1}{\bar{P}\mathcal{F}(\bar{W}_1 W_1 + \bar{W}_2 W_2)} \quad (24)$$

$$= \frac{\bar{W}_1 W_1}{\mathcal{F}(\bar{W}_1 W_1 + \bar{W}_2 W_2)} \quad (25)$$

Substitution in (15) shows that T_1 is a function of the weighting filters only and therefore can be evaluated for arbitrary $s \in \mathbb{C}$, e.g. the zeros of T_2. As as result, the interpolation conditions $G(a) = T_1(a) \mid a \in \mathcal{Z}_{T_2}$ can be generated given W_1, W_2 and the zeros of P. This finishes the proof of Proposition 2.

Given the interpolation conditions (Proposition 2), G can be computed. Since T_1 and T_2 are known non-parametrically, the corresponding Q_i can be computed by non-parametric evaluation of (17). The non-parametric description of the controller, C_i, can be obtained by evaluating (3) for every frequency point.

5. SET DESCRIPTION OF SUBOPTIMAL CONTROLLERS

The infemum of γ in (6), defined as γ_{opt}, results in unique input-output behavior of the controller. Lowering the performance requirement, i.e. $\gamma > \gamma_{opt}$, results in a set of suboptimal controllers. This set-description is interesting for several reasons. First of all, the additional freedom can be used to parameterize low-order controllers. By lowering the desired performance specifications, deviations are allowed around the optimal controller behavior. In this manner, the trade-off between order and closed-loop performance can be handled in direct manner. Moreover, due to the equivalence of controller perturbations and plant perturbations in a SISO feedback configuration, the set-description of allowable perturbations in the open-loop behavior, can be used to find a robust controller over a set of plant FRF's.

Given the set of interpolation conditions $\{a_i, b_i\}$ derived in Section 4, the set of solutions for the model-matching problem is given by Ball et al. [1990]:

$$G(s) = [\Theta_{11}(s)R(s) + \Theta_{12}(s)][\Theta_{21}(s)R(s) + \Theta_{22}(s)]^{-1} \quad (26)$$

where $R(s)$ is an arbitrary filter contained in the set $\mathcal{R} := \{\|R\|_\infty \leq 1 \mid R \in \mathcal{RH}_\infty\}$. $\Theta(s)$ is defined as:

$$\Theta(s) = I + \begin{bmatrix} b_1 \dots b_n \\ 1 \dots 1 \end{bmatrix} \begin{bmatrix} (s-a_1)^{-1} & & 0 \\ & \ddots & \\ 0 & & (s-a_n)^{-1} \end{bmatrix} \Lambda(a,b)^{-1} \begin{bmatrix} -\bar{b}_1 & 1 \\ \vdots & \vdots \\ -\bar{b}_n & 1 \end{bmatrix} \quad (27)$$

The Pick-matrix $\lambda(a, b)$ is defined a:

$$\Lambda(a,b) = \left[\frac{1 - \bar{b}_i b_j}{\bar{a}_i + a_j}\right]_{1 \leq i,j \leq n} \quad (28)$$

The infemum of γ for which the matching problem is solvable (Pick-matrix non-singular), results in a unique optimal solution, i.e. R is not contributing to G. For suboptimal solutions, (27) describes the mapping from the unit circle, described by $\mathcal{R}$, onto the region of allowed perturbations around the optimal controller for every frequency.

Since it is known that conformal mapping (Rahman [1997]) holds for proper functions like (26), the outer boundary of $\mathcal{R}$ at frequency ω_i, defined as $\mathcal{R}'_i := \{\mathcal{R} : \|R(\omega_i)\|_\infty = 1\}$, is mapped onto the outer-boundary of the set of corresponding Youla parameters, described by Q'_i, via (27) and (17).

One element of the set Q'_i can be computed non-parametrically by direct substitution of the complex number $R_i \in \mathcal{R}'_i$ in (27). This can be understood as follows: consider one point on the unit disc $R_i \in \mathcal{R}'_i$ at frequency ω_i. Then there exist a filter $R(s) \in \mathcal{R}$ such that $R(j\omega_i) = R_i$. Substitution of this particular $R(s)$ in (26) gives a filter G which is in the set of G's that corresponds to stabilizing controllers that satisfy (6). This imaginary parametrization of R can be performed for every element of $\mathcal{R}'_i$ for every frequency point separately and hence results in a set description of all allowable controller FRD's.

To enable numerical implementation, $\mathcal{R}'_i$ is approximated by a discrete grid. The mapping of this grid via (26) results in a non-parametrical region of allowable perturbation of the controller FRD. These regions can be approximated by parameterizations, e.g. ellipsoids, for every frequency grid point. The appearance of ellipsoids can be explained from the fact that interconnection described in (26) can be regarded as a Mobius transform of $\mathcal{R}$.

6. SIMULATION EXAMPLE

The proposed approach is illustrated using a fourth order plant. Despite the choice for this low-order system, the approach is particulary very suited for high order systems where the trade-off between order and performance is non-trivial.

The following transfer-function is used to generate FRD of the plant:

$$P(s) = \frac{2s^2 + 4.5s + 1125}{s^4 + 4.5s^3 + 1130s^2 + 2025s + 202500} \quad (29)$$

Equation (29) describes a system with an eigenfrequency at $30[rad/s]$ and $30/4[rad/s]$ and a damping of 5% for both resonances as depicted in Fig. 1.

In order to mimic experimental data, the FRD of the plant is synthesized by substitution of $\{s = j\omega_i \mid \omega_i = 10^{x_i},\ \ x \in \{-4, -3.99, -3.98, \dots 4.98, 4.99\}\}$ into $P(s)$. This non-parametric data is used as input for the controller synthesis. The zeros, needed for synthesis, are directly extracted from (29):

$$z_{1,2} = -1.1250 \pm 23.6904j \quad (30)$$

In case of experimental data, the undamped zeros can be parametrized relatively easily since they appear as undamped anti-resonances. By taking the inverse of P_i, these zeros appear as undamped poles that can be fitted locally by standard tools. Acquiring the value of real-valued zeros however, e.g. due to amplifier dynamics, is more complicated and may require fitting routines. Section 6.1 shortly comments on this aspect.

The weighting filters W_1 and W_2 of (6) are chosen such that a low sensitivity function at low frequencies and low complementary function at high frequencies is enforced:

$$W_1(s) = \frac{37.9}{s + 0.01}, \quad W_2(s) = \frac{0.02}{0.01s + 1} \quad (31)$$

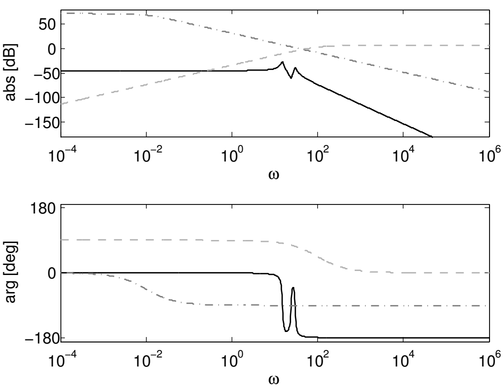

Fig. 1. P_i(-), $W_1(s)$(-.) and $W_2(s)$(- -)

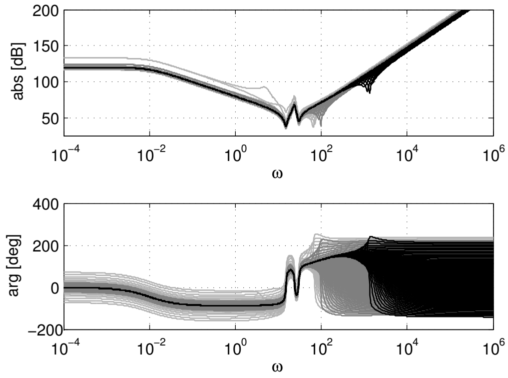

Fig. 2. Bode Controller for: $\gamma = 1$ (-), $\gamma = 1.5$ (-) and $\gamma = 2$ (-)

Fig.1 depicts the bode-plot of P_i, $W_1(s)$ and $W_2(s)$.

In order to generate the interpolation conditions, the filter T_1 and the unstable zeros of filter T_2 are computed as proposed in Section.4. Evaluating the filter T_1 at the zeros of T_2 gives the interpolation conditions:

$$\begin{aligned} a &= \{0.01,\ 32.21 \pm 29.29j\} \\ b &= \{8 \cdot 10^{-14},\ -0.1214 \pm 0.3759j\} \end{aligned} \quad (32)$$

To compute the set-description of G, the following discrete grid over the unit circle is used for $\mathcal{R}_i'$:

$$\mathcal{R}_i' = \{1 \cdot e^{j\phi} \mid \quad \phi \in \{0,\ 0.025, \ldots, 2\pi\}\} \quad (33)$$

For every element of $\mathcal{R}_i'$, the given interpolation conditions are substituted into (26) in order to compute $G(s)$. It has to be emphasized that R_i itself is chosen as a complex constant and therefore does not represent a proper filter. As a consequence, every mapping of R_i into $G(s)$ on itself results in a C_i which is not realizable as a analytical filter. However, either for the optimal solution, the term R_i vanishes such that $G(s)$ is analytical and therefore realizable, or within the set of suboptimal solutions an analytical parametrization can be found by combining several solutions $G(s)$ that correspond to different values of R_i.

Given the non-parametric expressions for T_1 and T_2, Q_i can be computed from $G(s)$ for every grid point of R_i' using (27). Given $G(s)$, Q_i can be obtained via (17). Substitution of (22) in (3) gives the FRD of the controller denoted by C_i.

Fig.2, Fig.3 and Fig.4 depicts the results for several values of γ. Fig.3 and 4 show the allowable deviations of the open-loop for $\gamma = 1$. It can be observed that even for a nearly optimal controller, a certain degree of freedom exists which can be exploited during parametrization of the controller.

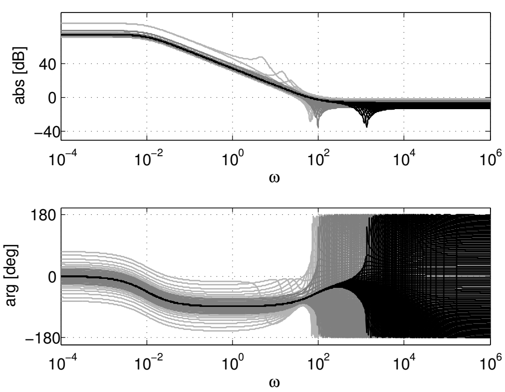

Fig. 3. Open-loop using the set of (sub)optimal controllers for: $\gamma = 1$ (-), $\gamma = 1.5$ (-) and $\gamma = 2$ (-)

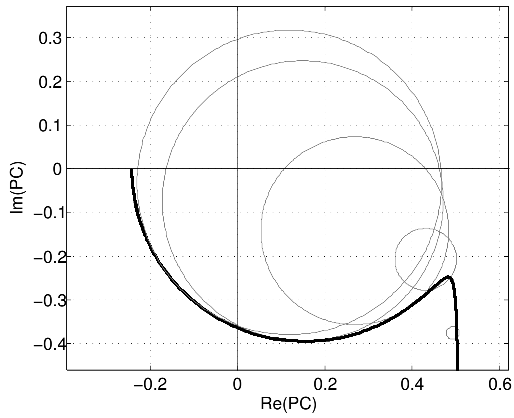

Fig. 4. Nyquist-plot of open-loop with circles of allowable perturbations for $\gamma = 1$ (suboptimal solution)

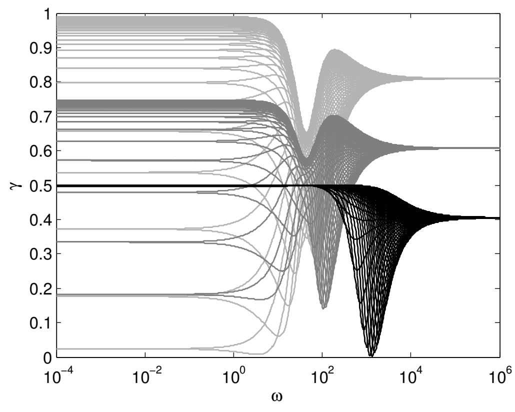

Fig. 5. Check on γ for: $\gamma = 1$ (-), $\gamma = 1.5$ (-), $\gamma = 2$ (-).

A check on the performance specification of the synthesized controller FRD is performed by evaluating (6). The results are depicted in Fig.5. Since the controller both satisfies performance and guarantees closed-loop stability (Nevanlinna-Pick interpolation guarantees that $Q \in \mathcal{RH}_\infty$), the problem formulated in (6) is solved in a non-parametric way by the proposed approach.

6.1 Discussion

Due to finite measurement time and limited computational power, plant FRD is only available on a limited number of frequency data-points. Between these data-points, dynamic effects could occur which are not captured by the FRD and hence can threat stability and performance of the closed-loop system. This is however does not differ parametrization of plant data. In fact, it can be observed from the Youla parametrization in (3) that both the interpolation of P and C have a dual role in the description of the set of stabilizing controllers. If the non-parametric controller FRD is interpolated, the interpolation function of P is determined and visa versa. It is expected that good engineering insight in the characteristics of the plant dynamics at hand is sufficient to acquire FRD which captures all dynamic effects relevant for control synthesis (Ljung [1999].

Parametrization of the controller FRD, i.e. approximation of the given sets C_i by a parametric transfer-function $C(s)$, is required to enable implementation. This raises the question whether a parametrization of acceptable order, i.e. lower or equal than the order of the plant and the weighting filters, can be fitted over C_i. This question can be answered positively. From (27) can be observed that the filter $G(s)$ is a proper analytical filter if $R(s) \in \mathcal{S}$. The order of $G(s)$ is generally equal to the number of interpolation points. This combined with the fact that T_1 and T_2 originate from physical LTI system behavior, Q_i also represent an analytical filter. Extending this line of reasoning proves that also C is analytical and therefore is implementable/realizable as a physical filter.

7. CONCLUSION

In practical design problems, a trade-off appears between closed-loop performance and the order of the controller. This trade-off can not be handled in a straightforward manner since the controller to be synthesized is unknown during plant parametrization.

The proposed approach supplies a method to perform synthesis of optimal controllers in a non-parametric manner for the class of SISO stable LTI plants. In this manner, no interpolation or data-reduction is applied such that all experimental plant data is maintained during controller synthesis. The actual parametrization is performed on the controller such that knowledge of the closed-loop can be exploited.

The approach is based on the frequency domain mixed-sensitivity H_∞ problem synthesis using model-matching interpolation techniques. By choosing a conceptual coprime factorization for the plant, the interpolation conditions can be obtain while omitting parametrization of the poles of the plant. Using this approach, the frequency response data of the controller can be synthesized. For suboptimal performance, a set description of the suboptimal controller frequency response data can be computed. This reveals the relation between between order and closed-loop performance which generates the ability to parameterize low-order controllers or generate robust controllers.

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