Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Stabilizing Nonlinear Adaptive PID State Feedback Control for Spacecraft Capturing

Yuichi Ikeda * **Takashi Kida** ** **Tomoyuki Nagashio** **

* *Kushiro National College of Technology, Kushiro, Hokkaido, Japan (e-mail: yikeda@ mech.kushiro-ct.ac.jp).*
** *University of Electro-Communications, Chofu, Tokyo, Japan (e-mail: (kida,nagashio)@ mce.uec.ac.jp)*

Spacecraft; Tracking control; PID

Abstract: In the future space infrastructure, the missions of refueling and capturing of the inoperative spacecraft by the orbital servicing vehicle or the space robot are considered. To achieve them, the six degrees of freedom tracking control of the chaser spacecraft is required to approach the target spacecraft. Moreover, the stability of the connected system of the chaser and target must be ensured. In addition, it is also important to suppress the position and attitude error under the influence of the disturbance. In this paper, we derive the PID controller that satisfies the stability of the spacecraft system before and after capturing, and removes the states error caused by constant disturbance. The effectiveness of controller is verified by numerical simulations.

1. INTRODUCTION

In the future space infrastructure, the capturing of the inoperative spacecraft by the orbital servicing vehicle or the space robot is considered. To achieve the mission, the chaser spacecraft must fly around the target spacecraft so as to track its docking port whose position and attitude generally change with respect to the inertial frame, according to nutation or tumbling motion of the target spacecraft. When the relative errors of the position and attitude become sufficiently small, the chaser can safely capture the target. This is the first operation before capturing. Then, the connected spacecraft system of target and chaser must be stabilized by damping the energy, and carried to some place, say international space station (ISS), by tacking another given trajectory if necessary. After the second operation, the mission is completed. These control actions must be performed using only the controller of the chaser since that of the target has been out of order.

From the viewpoint of control problem, the following two issues are raised. First is six degrees-of-freedom (6 d.o.f.) tracking control of spacecraft under the influence of external disturbance. The controller must be designed for the nonlinear translation and rotation dynamics and kinematics coupled with each other. Second is the stabilizing control during above operation including the connecting instance.

On the nonlinear tracking control under the influence of external disturbance, almost all researchers have concentrated on the nonlinear $\mathcal{H}_\infty$ controller that makes $\mathcal{L}_2$ gain of closed-loop system from disturbance to controlled output less than $\gamma > 0$ (Dalsmo and Egeland [1996, 1997], Luo et al. [2004] and so on). They also employ PD type state feedback control. However, although they generally require higher feedback gains to achieve higher disturbance attenuation capability, it is not realizable since the maximum level of control input is practically constrained. Therefore the authors consider it is not necessarily only an approach to the control purpose. Moreover, they discuss only the problem before chaser connects to target. For this problem, authors have proposed the control method which ensures the stability before and after spacecraft connection without changing controller by passivity based control (Ito [2005], Ito et al. [2006]). However, it does not consider the influence of external disturbance.

Viewing this, we propose PID state feedback controller that guarantees the asymptotic stability and tracking capability before and after capturing without changing the controller, and can effectively attenuate the constant secular signal in disturbance by using backstepping approach (Kristi et al. [1995]). In addition, since it is difficult to know all physical parameters exactly, we extend it to the adaptive controller which can estimate the physical parameters. The proposed method has the advantage of ensuring the stability of the system before and after capturing without changing controller as the literatures (Ito [2005], Ito et al. [2006]). Finally, numerical simulation results are shown.

Following notations are used throughout this paper.

$\{o\}$: inertial frame,

$\{t\},\ \{c\}$: target and chaser body fiexd frame,

$\{s\},\ \{R\}$: connected system and reference trajectory fixed frame,

$m_i,\ J_i$: mass and inertia matrix,

$f_i,\ \tau_i$: control force and torque,

$w_f,\ w_\tau$: disturbance force and torque,

$r_j,\ [\varepsilon_j^T\ \eta_j]^T$: position and attitude vector of each frame,

$v_j,\ \omega_j$: linear and angular velocity vector of each frame,

$p_t,\ p_R$: constant vector fixed $\{t\}$ and $\{R\}$ frame

$\rho_{ii},\ \rho_{si}$: position vector from a nominal fixed point to force input point,

$l_{ii},\ l_{si}$: position vector from a nominal fixed point to center of mass of target and chaser,

l : distanece of center of mass between target and chaser,

I : 3×3 identity matrix,

$a^\times$: skey symmetric matrix determined by $a \in \mathsf{R}^3$,

$\mathcal{C}_X^Y$: direction cosine matrix from the frame X to Y,

$\|a\| = \sqrt{a^T a}$: the norm of vector a,

$\mathrm{diag}\{a, b, c, \ldots\}$: diagonal matrix

R^n : linear space of real vectors of dimension n,

S^q : hypersphere of dimension q,

where subscripts i and j represent $i = t, c$ and $j = t, c, s, R$.

2. MODELING AND PROBLEM DESCRIPTION

We consider the control problem that a nominal point A fixed at $\{c\}$ tracks a target point B fixed at $\{t\}$ before capturing as shown in Fig. 1 (Phase 1), and then a nominal point A fixed at $\{s\}$ tracks a target point B fixed at $\{R\}$ after connection as shown in Fig. 2 (Phase 2). In order to simplify the discussion on the stability for both cases, we only describe the Phase 2 problem, since they can be unified as described in Remark 3.

Here, the following assumptions are made.

Assumption 1. The chaser and the target are rigid, and they are connected rigidly by the docking port. Therefore, the spacecraft system is rigid before and after the connection.

Assumption 2. The control input is not applied to the target.

Then the translation and rotation dynamics equation of the rigid body around a nominal point A, that is not necessarily the center of mass, becomes in $\{s\}$ frame,

$$\mathcal{M}\dot{p}_s + \mathcal{C}p_s = u + w, \tag{1}$$

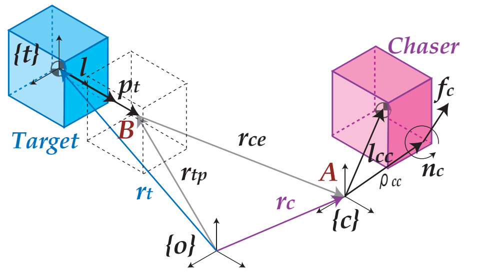

Fig. 1. Definition of target and chaser fixed frame, and the position vector (before capturing: Phase 1).

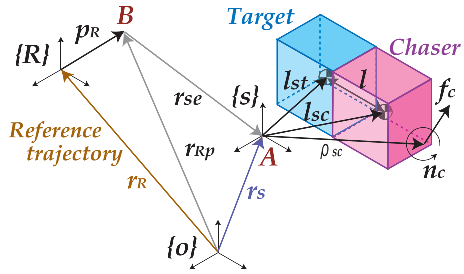

Fig. 2. Definition of connected system fixed frame and the position vector (after capturing: Phase 2).

where

$$\mathcal{M} = \sum \begin{bmatrix} m_i & -m_i l_{si}^\times \\ m_i l_{si}^\times & J_i - m_i l_{si}^\times l_{si}^\times \end{bmatrix},$$

$$\mathcal{C} = \sum \begin{bmatrix} m_i \omega_s^\times & -m_i \omega_s^\times l_{si}^\times \\ m_i l_{si}^\times \omega_s^\times & \omega_s^\times J_i - m_i l_{si}^\times \omega_s^\times l_{si}^\times \end{bmatrix},$$

$$q_s = [r_s^T\ \varepsilon_s^T]^T,\ p_s = [v_s^T\ \omega_s^T]^T,\ w = [w_f^T\ w_\tau^T]^T,$$

$$u = \mathcal{U}\begin{bmatrix} f_c \\ n_c \end{bmatrix} = \begin{bmatrix} u_f \\ u_\tau \end{bmatrix},\ \mathcal{U} = \begin{bmatrix} I & 0 \\ \rho_{sc}^\times & I \end{bmatrix}.$$

The inputs f_c and τ_c can be uniquely determined after u_f and u_τ are derived since the matrix $\mathcal{U}$ satisfies $\det \mathcal{U} \neq 0$. The position of A and the attitude of $\{s\}$ with respect to the inertial frame $\{o\}$ are given by following kinematics if quaternion is used for the attitude parameterization.

$$\dot{r}_s = v_s - \omega_s^\times r_s, \tag{2}$$

$$\dot{\theta}_s = \frac{1}{2}\begin{bmatrix} \eta_s I + \varepsilon_s^\times \\ -\varepsilon_s^T \end{bmatrix}\omega_s = \mathcal{E}(\theta_s)\omega_s, \tag{3}$$

where $\theta_s = [\varepsilon_s^T\ \eta_s]^T \in \mathsf{S}^3$ satisfies the constraint $\|\theta_s\| = 1$.

Our tracking control problem is to find a controller such that

$$r_s = r_{R_p},\ \varepsilon_s = \varepsilon_R,\ \eta_s = \eta_R,\ v_s = v_{R_p},\ \omega_s = \omega_R$$

when $t \to \infty$. The position and velocity of the point B fixed at $\{R\}$ are given by

$$r_{Rp} = r_R + p_R,\ v_{Rp} = v_R + \omega_R^\times p_R. \tag{4}$$

To this end, we describe an error system in $\{s\}$. Let the direction cosine matrix from $\{R\}$ to $\{s\}$

$$C_R^s = (\eta_{se}^2 - \varepsilon_{se}^T\varepsilon_{se})I + 2\varepsilon_{se}\varepsilon_{se}^T - 2\eta_{se}\varepsilon_{se}^\times \tag{5}$$

using the quaternion of relative attitude $q_e = [\varepsilon_e^T\ \eta_e]^T$, where ε_e and η_e are defined as

$$\varepsilon_{se} = \eta_R\varepsilon_s - \eta_s\varepsilon_R + \varepsilon_s^\times\varepsilon_R,\ \eta_{se} = \eta_s\eta_R + \varepsilon_s^T\varepsilon_R. \quad (6)$$

The relative position, linear velocity and angular velocity are given in the same $\{s\}$ frame as

$$\begin{aligned} r_{se} &= r_s - C_R^s r_{Rp},\ v_{se} = v_s - C_R^s v_{Rp}, \\ \omega_{se} &= \omega_s - C_R^s\omega_R. \end{aligned} \quad (7)$$

Hereafter, we simply represent C_R^s as C by dropping subscripts. Substitution of (7) into (1), (2) and (3), using the identity $\dot{C} = -\omega_e^\times C$, yields the relative equation of motion as

$$\mathcal{M}_e\dot{p}_{se} + \mathcal{C}_e p_{se} + \Delta_e = u + w, \quad (8)$$

$$\dot{r}_{se} = v_{se} - (\omega_{se} + C\omega_R)^\times r_{se}, \quad (9)$$

$$\dot{\theta}_{se} = \mathcal{E}(\theta_{se})\omega_{se}, \quad (10)$$

where

$$q_{se} = [r_{se}^T\ \varepsilon_{se}^T]^T,\ p_{se} = [v_{se}^T\ \omega_{se}^T]^T,\ \mathcal{M}_e = \mathcal{M},$$

$$\mathcal{C}_e = \sum \begin{bmatrix} c_{e11} & c_{e12} \\ c_{e21} & c_{e22} \end{bmatrix},\ \Delta_e = \sum[\Delta_{e1}^T\ \Delta_{e2}^T]^T,$$

$$c_{e11} = m_i(\omega_{se} + C\omega_R)^\times,\ c_{e12} = -m_i(\omega_{se} + C\omega_R)^\times l_{si}^\times,$$

$$c_{e21} = m_i l_{si}^\times(\omega_{se} + C\omega_R)^\times,\ c_{e22} = \omega_{se}^\times J_i - m_i l_{si}^\times\omega_{se}^\times l_{si}^\times,$$

$$\begin{aligned} \Delta_{e1} = m_i[&C(\dot{\omega}_R^\times p_R + \omega_R^{\times 2} p_R) - l_{si}^\times(C\dot{\omega}_R - \omega_{se}^\times C\omega_R) \\ &-(\omega_{se} + C\omega_R)^\times l_{si}^\times C\omega_R], \end{aligned}$$

$$\begin{aligned} \Delta_{e2} = m_i l_{si}^\times[&C(\dot{\omega}_R^\times p_R + \omega_R^{\times 2} p_R) - l_{si}^\times(C\dot{\omega}_R - \omega_{se}^\times C\omega_R) \\ &-\omega_{se}^\times l_{si}^\times C\omega_R - (C\omega_R)^\times l_{si}^\times(\omega_{se} + C\omega_R)] \\ &+J_i(C\dot{\omega}_R - \omega_{se}^\times C\omega_R) + \omega_{se}^\times J_i C\omega_R \\ &+(C\omega_R)^\times J_i(\omega_{se} + C\omega_R). \end{aligned}$$

By the transform, the tracking control problem is reduced to a regulation problem to design control input u_f and u_τ such that

$$r_{se} = 0,\ \varepsilon_{se} = 0,\ \eta_{se} = 1,\ v_{se} = 0,\ \omega_{se} = 0$$

when $t \to \infty$ according to (8)-(10).

Remark 3. By setting the parameters with respect to the target to zero, and the frames $\{s\} \to \{c\}$, $\{R\} \to \{t\}$ in relative equation of motion of the connected system (8)-(10), this can be described as the equation of motion of the chaser with respect to the target before capturing. Therefore, if the control law which makes the relative error asymptotically stable can be found, then this control law can stabilize both cases.

3. ADAPTIVE CONTROLLER DESIGN

In this section, we derive the adaptive PID control law which makes the relative error asymptotically stable under unknown physical parameters, i.e.

$$r_{se} = 0,\ \varepsilon_{se} = 0,\ \eta_{se} = 1,\ v_{se} = 0,\ \omega_{se} = 0,$$

by applying the backstepping approach when $w = 0$. On the reference signals, we assume the followings.

Assumption 4. The reference signals r_R, ε_R, η_R, v_R, ω_R and $\dot{\omega}_R$ are uniformly continuous, bounded and known for all $t \in [0, \infty)$.

The concrete design procedure is given as follows.

Step 1 :

We suppose that v_{se} and ω_{se} are the virtual inputs to the subsystem (9) and (10), and define the stabilizing functions such that

$$\alpha_1 = -K_{P1}r_{se} - K_{I1}\zeta_1,\ \alpha_2 = -K_{P2}\varepsilon_{se} - K_{I1}\zeta_2, \quad (11)$$

where K_{Pn} and K_{In} $(n = 1, 2)$ are the symmetric and positive definite matrices, ζ_n $(n = 1, 2)$ is the integral variable defined as

$$\zeta_1 = \int_0^t r_{se}\ dt,\ \zeta_2 = \int_0^t \varepsilon_{se}\ dt. \quad (12)$$

Now, we define the error variable between the state (v_{se}, ω_{se}) and the desired control (α_1, α_2) such as

$$z_1 = v_{se} - \alpha_1,\ z_2 = \omega_{se} - \alpha_2. \quad (13)$$

From (13), the subsystem (9) and (10) become

$$\dot{r}_{se} = (z_1 + \alpha_1) - (\omega_{se} + C\omega_R)^\times r_{se}, \quad (14)$$

$$\dot{\theta}_{se} = \mathcal{E}(\theta_{se})(z_2 + \alpha_2). \quad (15)$$

Define the following candidate Lyapunov function such as

$$\begin{aligned} V_1 = &\frac{1}{2}\|r_{se}\|^2 + \frac{1}{2}\zeta_1^T K_{I1}\zeta_1 \\ &+\|\varepsilon_{se}\|^2 + (\eta_{se} - 1)^2 + \frac{1}{2}\zeta_2^T K_{I2}\zeta_2. \end{aligned} \quad (16)$$

By utilizing the following skew symmetric matrix properties

$$a^T a^\times = 0,\ a^T b^\times a = 0,\ {}^\forall a, b \in \mathrm{R}^3,$$

the time derivative of (16) along the trajectories of the closed-loop system becomes

$$\begin{aligned} \dot{V}_1 = &r_{se}^T\{(z_1 + \alpha_1) - (\omega_{se} + C\omega_R)^\times r_{se}\} + r_{se}^T K_{I1}\zeta_1 \\ &+\varepsilon_{se}^T\{(\eta_{se}I + \varepsilon_{se}^\times)(z_2 + \alpha_2)\} - \varepsilon_{se}^T(\eta_{se} - 1)(z_2 + \alpha_2) \\ &+\varepsilon_{se}^T K_{I2}\zeta_2 \\ = &-r_{se}^T K_{P1}r_{se} - \varepsilon_{se}^T K_{P2}\varepsilon_{se} + r_{se}^T z_1 + \varepsilon_{se}^T z_2. \end{aligned} \quad (17)$$

From (17), obviously $r_{se} \to 0$, $\varepsilon_{se} \to 0$ and $\eta_{se} \to 1$ as $t \to \infty$ when $z_1 = z_2 = 0$.

Step 2 :

We derive the adaptive controller which makes the state $z = [z_1^T\ z_2^T]^T$ asymptotically stable, i.e. $z = 0$, when $w = 0$. From (8), (11) and (13), the dynamics with respect to z is as follows.

$$\mathcal{M}_z\dot{z} + \mathcal{C}_z z + \Delta_z = u, \quad (18)$$

where

$$\mathcal{M}_z = \mathcal{M}_e,\ \mathcal{C}_z = \mathcal{C}_{z1} + \mathcal{C}_{z2},$$

$$\mathcal{C}_{z1} = \begin{bmatrix} c_{e11} & c_{e12} \\ c_{e21} & 0 \end{bmatrix},\ \mathcal{C}_{z2} = \mathrm{diag}\{0, c_{e22}\},$$

$$\Delta_z = \begin{bmatrix} \sum m_i\phi_{i1} \\ \sum m_i\phi_{i2} + \psi\beta_s \end{bmatrix},$$

$$\begin{aligned}\phi_{i1} &= \dot\alpha_1 - l_{si}^\times\dot\alpha_2 + (\omega_{se} + C\omega_R)^\times\alpha_1 \\ &\quad -(\omega_{se} + C\omega_R)^\times l_{si}^\times\alpha_2 + C(\dot\omega_R^\times p_R + {\omega_R^\times}^2 p_R) \\ &\quad -l_{si}^\times(C\dot\omega_R - \omega_{se}^\times C\omega_R) - (\omega_{se} + C\omega_R)^\times l_{si}^\times C\omega_R, \\ \phi_{i2} &= l_{si}^\times[\dot\alpha_1 - l_{si}^\times\dot\alpha_2 + (\omega_{se} + C\omega_R)^\times\alpha_1 - \omega_{se}^\times l_{si}^\times\alpha_2 \\ &\quad +C(\dot\omega_R^\times p_R + {\omega_R^\times}^2 p_R) - l_{si}^\times(C\dot\omega_R - \omega_{se}^\times C\omega_R) \\ &\quad -\omega_{se}^\times l_{si}^\times C\omega_R - (C\omega_R)^\times l_{si}^\times(\omega_{se} + C\omega_R)], \\ \psi &= \psi_1(\dot\alpha_2 + C\dot\omega_R - \omega_{se}^\times C\omega_R) + \psi_2(\omega_{se}, \alpha_2 + C\omega_R) \\ &\quad +\psi_2(C\omega_R, \omega_{se} + C\omega_R), \\ \beta_s &= \sum\beta_i,\ \beta_i = [J_{i,11}\ J_{i,12}\ J_{i,13}\ J_{i,22}\ J_{i,23}\ J_{i,33}]^T, \\ &\psi_1(a)\beta_i = J_i a,\ \psi_2(a,b)\beta_i = a^\times J_i b,\end{aligned}$$

$J_{i,jk}$ $(i = t, c\ :\ j, k = 1, 2, 3,\ j \le k)$ is the (j,k) element of J_i. In addition, $\dot\alpha_1$ and $\dot\alpha_2$ can be easily calculated from (12), (14) and (15). Define the following candidate Lyapunov function such as

$$V_2 = V_1 + \frac{1}{2}z^T\mathcal{M}_z z + \sum\frac{1}{2\Gamma_{i1}}\tilde m_i^2 + \frac{1}{2}\tilde\beta_s^T\Gamma_2^{-1}\tilde\beta_s, \quad (19)$$

where $\Gamma_{i1},\ \Gamma_2$ are the scalar and symmetric positive definite matrices, $\tilde m_i = m_i - \hat m_i,\ \tilde\beta_s = \beta_s - \hat\beta_s$ are the estimate error, $\hat m_i, \hat\beta_s$ are the estimate variable, respectively. By utilizing the fact that $\mathcal{C}_{z1}$ is the skew symmetric matrix, the time derivative of (19) along the trajectories of the closed-loop system becomes

$$\begin{aligned}\dot V_2 &= \dot V_1 + z^T(-\mathcal{C}_z z - \Delta_z + u) - \sum\Gamma_{i1}^{-1}\tilde m\dot{\hat m} - \tilde\beta_s^T\Gamma_2^{-1}\dot{\hat\beta}_s \\ &= -r_{se}^T K_{P1} r_{se} - \varepsilon_{se}^T K_{P2}\varepsilon_{se} + r_{se}^T z_1 + \varepsilon_{se}^T z_2 \\ &\quad -z_1^T\sum m_i\phi_{i1} - z_2^T\sum m_i\phi_{i2} - z_2^T\bar\psi\beta_s \\ &\quad +z_1^T u_f + z_2^T u_\tau - \sum\Gamma_{i1}^{-1}\tilde m_i\dot{\hat m}_i - \tilde\beta_s^T\Gamma_2^{-1}\dot{\hat\beta}_s \\ &= -r_{se}^T K_{P1} r_{se} - \varepsilon_{se}^T K_{P2}\varepsilon_{se} + r_{se}^T z_1 + \varepsilon_{se}^T z_2 \\ &\quad -\sum m_i[\phi_{i1}^T\ \phi_{i2}^T]z - z_2^T\bar\psi\beta_s + z_1^T u_f + z_2^T u_\tau \\ &\quad -\sum\Gamma_{i1}^{-1}\tilde m_i\dot{\hat m}_i - \tilde\beta_s^T\Gamma_2^{-1}\dot{\hat\beta}_s, \end{aligned} \quad (20)$$

where

$$\begin{aligned}\bar\psi &= \psi_1(\dot\alpha_2 + C\dot\omega_R - \omega_{se}^\times C\omega_R) + \psi_2(\omega_{se}, z_2 + \alpha_2 + C\omega_R) \\ &\quad +\psi_2(C\omega_R, \omega_{se} + C\omega_R).\end{aligned}$$

Here, by selecting control inputs and estimate laws as

$$\begin{cases} u_f = -r_{se} - K_{D1}z_1 + \sum\hat m_i\phi_{i1} \\ u_\tau = -\varepsilon_{se} - K_{D2}z_2 + \sum\hat m_i\phi_{i2} + \bar\psi\hat\beta_s \end{cases}, \quad (21)$$

$$\begin{cases} \dot{\hat m}_t = -\Gamma_{t1}[\phi_{t1}^T\ \phi_{t2}^T]z \\ \dot{\hat m}_c = -\Gamma_{c1}[\phi_{c1}^T\ \phi_{c2}^T]z \\ \dot{\hat\beta}_s = -\Gamma_2\bar\psi^T z_2 \end{cases}, \quad (22)$$

where K_{Dn} $(n = 1, 2)$ is the symmetric and positive definite matrix, $\dot V_2$ becomes

$$\begin{aligned}\dot V_2 &= -r_{se}^T K_{P1} r_{se} - \varepsilon_{se}^T K_{P2}\varepsilon_{se} + r_{se}^T z_1 + \varepsilon_{se}^T z_2 \\ &\quad -\sum m_i[\phi_{i1}^T\ \phi_{i2}^T]z - z_2^T\bar\psi\beta_s \\ &\quad +z_1^T(-r_{se} - K_{D1}z_1 + \sum\hat m_i\phi_{i1}) \\ &\quad +z_2^T(-\varepsilon_{se} - K_{D2}z_2 + \sum\hat m_i\phi_{i2} + \bar\psi\hat\beta_s) \\ &\quad +\sum\tilde m_i[\phi_{i1}^T\ \phi_{i2}^T]z + \tilde\beta_s^T\bar\psi^T z_2 \\ &= -r_{se}^T K_{P1} r_{se} - \varepsilon_{se}^T K_{P2}\varepsilon_{se} - z_1^T K_{D1} z_1 - z_2^T K_{D2} z_2 \\ &\quad -\sum(m_i - \hat m_i)[\phi_{i1}^T\ \phi_{i2}^T]z + \sum\tilde m_i[\phi_{i1}^T\ \phi_{i2}^T]z \\ &\quad -(\beta_s^T - \hat\beta_s^T)\bar\psi^T z_2 + \tilde\beta_s^T\bar\psi^T z_2 \\ &= -r_{se}^T K_{P1} r_{se} - \varepsilon_{se}^T K_{P2}\varepsilon_{se} - z_1^T K_{D1} z_1 - z_2^T K_{D2} z_2 \\ &\quad -\sum\tilde m_i[\phi_{i1}^T\ \phi_{i2}^T]z + \sum\tilde m_i[\phi_{i1}^T\ \phi_{i2}^T]z \\ &\quad -\tilde\beta_s^T\bar\psi^T z_2 + \tilde\beta_s^T\bar\psi^T z_2 \\ &= -r_{se}^T K_{P1} r_{se} - \varepsilon_{se}^T K_{P2}\varepsilon_{se} - z_1^T K_{D1} z_1 - z_2^T K_{D2} z_2.\end{aligned} \quad (23)$$

Obviously, $\dot V_2 \le 0$ holds. Therefore, x is bounded since

$$V_2(x(t)) \le V_2(x(0)),\ \forall\ t \ge 0, \quad (24)$$

$$x = [\zeta_1^T\ \zeta_2^T\ r_{se}^T\ \varepsilon_{se}^T\ \eta_{se}\ v_{se}^T\ \omega_{se}^T\ \tilde m_t\ \tilde m_c\ \tilde\beta_s^T]^T,$$

and V_2 is radially unbounded in state space $\mathsf{R}^{23} \times \mathsf{S}^3$. Then $\dot x$ is also bounded since the control inputs (21) and the derivative of estimate variable are bounded by assumption 4. These follow that $\ddot V_2$ is bounded. Therefore, it is shown that

$$\dot V_2 \to 0 \iff r_{se} \to 0,\ \varepsilon_{se} \to 0,\ z_1 \to 0,\ z_2 \to 0$$

as $t \to \infty$ from Lyapunov-Like Lemma (Slotine and Li [1991]), and then

$$\zeta_1 \to 0,\ \zeta_2 \to 0,\ \eta_{se} \to 1,\ v_{se} \to 0,\ \omega_{se} \to 0$$

as $t \to \infty$ from (11)-(13) and $V_2 = 0$. Furthermore, the closed loop system becomes

$$\begin{cases} \sum\tilde m_i\phi_{i1} = 0 \\ \sum\tilde m_i\phi_{i2} + \bar\psi\hat\beta_s = 0 \end{cases} \quad (25)$$

when $t \to \infty$. It implies the estimate variables converging to some constant values, i.e. $\hat{m}_i \to c_{i1}$, $\hat{\beta}_s \to c_2$ as $t \to \infty$. Especially, from the first equation of (25)

$$\sum \tilde{m}_i \phi_{i1} = \tilde{m}_t \phi_{t1} + \tilde{m}_c \phi_{c1} = 0, \tag{26}$$

if $\|\phi_{i1}\| \neq 0$ for all $t \geq 0$ and ϕ_{i1} is linear independent, then $\tilde{m}_i$ obviously becomes $\tilde{m}_i \to 0$, i.e. $\hat{m}_i \to m$, as $t \to \infty$.

Remark 5. It is noted that the adaptive controller (21) and (22) can be stabilized before and after capturing by Remark 3.

4. NUMERICAL SIMULATION

4.1 Mission setup

As a numerical simulation, we suppose the situation where the chaser tracks the docking port fixed at the target that performs the tumbling motion. After capturing, as the problem of transportation to the space station, the connected system tracks the reference trajectory. Both cases are controlled by adaptive controller (21) and (22).

4.2 Capturing method of target spacecraft

In order to avoid the collision and excessive docking impact, the following scenario is considered in Phase 1. First, the chaser approach the target in a certain distance p_t. Then, during the tracking control, the target position is gradually decreased by δp_t. When $p_t = 1.75$ [m], the target is supposed to be captured by the chaser.

4.3 Physical model

The physical parameters used in the simulation are as follows.

$$m_t = 300 \text{ [kg]},\ m_c = 200 \text{ [kg]},$$

$$J_t = \text{diag}\{50, 275, 275\} \text{ [kgm}^2],$$

$$J_c = \begin{bmatrix} 75.0 & -28.125 & -28.125 \\ -28.125 & 75.0 & -28.125 \\ -28.125 & -28.125 & 75.0 \end{bmatrix} \text{ [kgm}^2],$$

$$l_{tt} = l_{cc} = l_{st} = [0\ 0\ 0]^T \text{ [m]},\ l_{sc} = [l\ 0\ 0]^T \text{ [m]},$$

$$\rho_{tt} = \rho_{cc} = \rho_{st} = [0\ 0\ 0]^T \text{ [m]},\ \rho_{sc} = [l\ 0\ 0]^T \text{ [m]},$$

where $l = 1.75$ [m]. A nominal point A is the mass center of the chaser before capturing, and it is set as the mass center of the target after capturing. In the latter case, it mean that $\{t\}$ and $\{s\}$ are the same frame.

4.4 Simulation results

Simulation results are shown in Figs. 3-6 where the initial values, the target constant vector fixed at $\{t\}$ and the controller gains are set as

$$r_t(0) = [20\ 20\ 5]^T \text{ [m]},\ \varepsilon_t(0) = [0\ 0\ 0]^T,\ \eta_t(0) = 1,$$

$$v_t(0) = [0\ 0\ 0]^T \text{ [m/s]},\ \omega_t(0) = [0.05\ 0\ -0.1]^T \text{ [rad/s]},$$

$$r_c(0) = [15\ 15\ 5]^T \text{ [m]},\ \varepsilon_c(0) = [0.18\ 0.31\ 0.18]^T,$$

$$\eta_c(0) = 0.92,\ v_c(0) = [0\ 0\ 0]^T \text{ [m/s]},$$

$$\omega_c(0) = [0\ 0\ 0]^T \text{ [rad/s]},$$

$$\hat{m}_t(0) = 180 \text{ [kg]},\ \hat{m}_c(0) = 120 \text{ [kg]},$$

$$\hat{\beta}_s(0) = [45.0\ -16.875\ -16.875\ 45.0\ -16.875\ 45.0]^T \text{ [kgm}^2],$$

$$p_t = [2.75\ \ 0\ \ 0]^T \text{ [m]},\ \delta p_t = 0.1 \text{ [m]},$$

$$K_{P1} = 0.2I,\ K_{D1} = 50I,\ K_{I1} = 0.04I,\ \Gamma_{t1} = \Gamma_{c1} = 10,$$

$$K_{P2} = 0.2I,\ K_{D2} = 250I,\ K_{I2} = 0.04I,\ \Gamma_2 = 1000.$$

The reference trajectory which the connected system tracks is assumed to be

$$p_R = [0\ \ 0\ \ 0]^T \text{ [m]},$$

$$v_R(t) = 30 C_o^R \nu [-\sin(\nu t)\ \ \cos(\nu t)\ \ 0]^T \text{ [m/s]},$$

$$\omega_R(t) = [0\ \ 0\ \ 0.05]^T \text{ [rad/s]},\ \nu = \frac{2\pi}{600}.$$

This reference trajectory is the circular orbit of radius of 30 [m] in $x - y$ plane at $\{i\}$. Furthermore, following constant external disturbance

$$w_f = [1\ \ 1\ \ 1]^T \text{ [N]},\ w_\tau = [0.5\ \ 0.5\ \ 0.5]^T \text{ [Nm]}$$

is added for all t. These values are ten times of the air drag disturbance at the lower earth orbit.

Figures 3 and 4 show time histories of the relative position, velocity, attitude and angular velocity. Fig. 5 shows trajectory of connected system (solid) and reference (dashed) at $\{i\}$ after capturing. It is observed that chaser arrives at the target vector p_t and gradually approaches to target by changing the reference signal under the constant external disturbance. After capturing is achieved at 203.1[s], the connected system tracks the reference trajectory under the constant external disturbance. Time responses of estimated parameters are shown Fig. 6. Although all estimated parameters not converge to real parameters, as described section 3, stability of the system is guaranteed under uncertain physical parameter.

5. CONCLUSION

In this paper, we have proposed the adaptive PID controller that ensures the stability before and after capturing an inoperative spacecraft in orbit. The effectiveness of controller is verified by numerical simulations. As the future works, collision avoidance of the chaser and the target before capturing, the extension to the case considering the dynamics of the docking port are considered.

REFERENCES

Dalsmo, M., and O. Egeland. Tracking of Rigid Body Motion via Nonlinear $\mathcal{H}_\infty$ control. *13th IFAC Triennial World Congress, San Francisco.* pages 395–400, 1996.

Dalsmo, M., and O. Egeland. State Feedback $\mathcal{H}_\infty$ Control of a Rigid Spacecraft. *IEEE Transactions on Automatic Control.* volume 42, pages 1186–1189, 1996.

Ito, A. *Stabilizing Nonlinear Control of Spacecraft at Capturing Phase.* Master Thesis, University of Electro-Communications, 2005.

Ito, A., Y. IKeda, T. Nagashio and T. Kida. Nonlinear Adaptive Control for Stabilizing Spacecraft System At Capturing Phase. *Proceedings of the 25th International Symposium on Space Technology and Science.* pages 625–630, 2006.

Kristi, M., I. Kanellakopoulos, and P. Kokotovi. *Nonlinear and Adaptive Control Design.* John Wiley & Sons Inc., 1995.

Luo, W., Y.C. Chu and K.V. Yang. $\mathcal{H}_\infty$ Tracking Control of a Rigid Body Spacecraft. *American Control Conference.* pages 2681–2686, 2004.

Slotine, J.E., and W. Li. *Applied Nonlinear Control.* Prentice Hall, 1991.

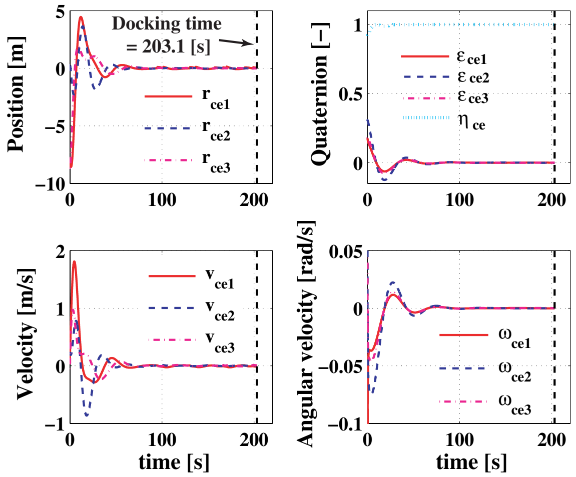

Fig. 3. Time response of relative position, velocity, attitude, and angular velocity before capturing.

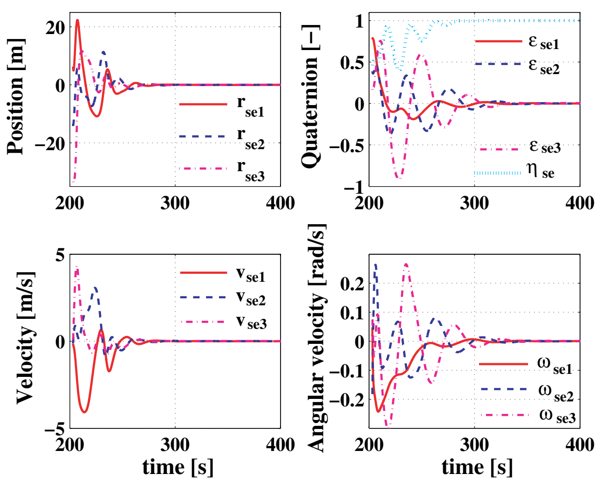

Fig. 4. Time response of relative position, velocity, attitude, and angular velocity after capturing.

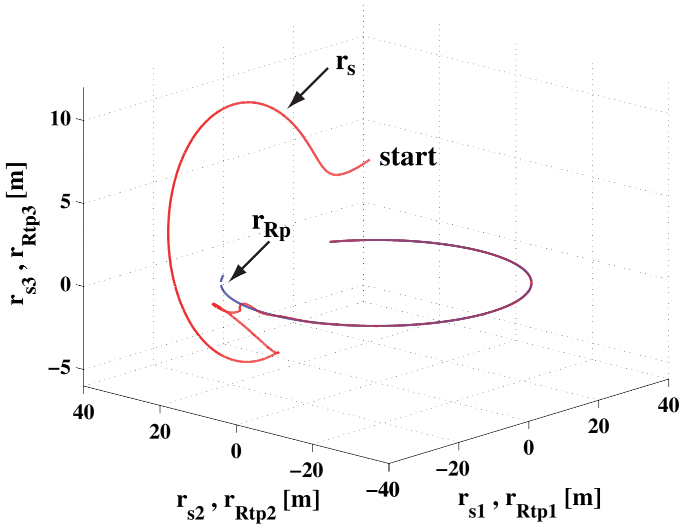

Fig. 5. Trajectory of connected system and reference at $\{i\}$ after capturing.

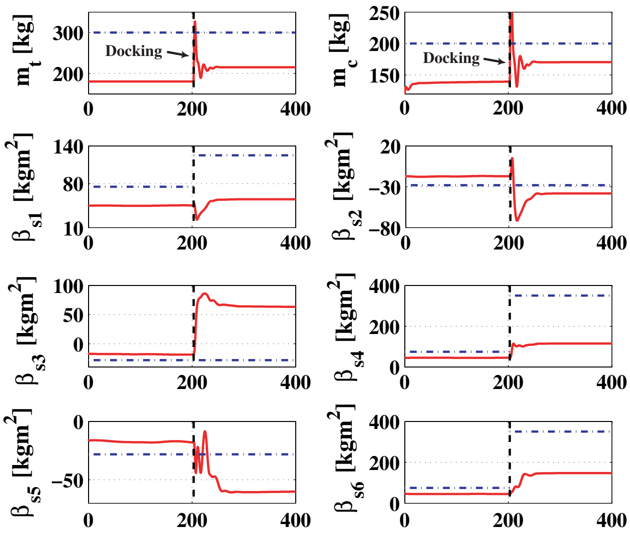

Fig. 6. Time response of estimated parameters $\hat{m}_t$, $\hat{m}_c$ and $\hat{\beta}_s$ (solid), and real parameters m_t, m_c and β_s (dashed-dotted).