**Proceedings of the 17th World Congress**
**The International Federation of Automatic Control**
**Seoul, Korea, July 6-11, 2008**

# A procedure to compute a probabilistic bound for the maximum tardiness using stochastic simulation

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**Abstract**: Dispatching rules are widely used to dynamically schedule the operations in a shop. Their efficiency depends on the performance criteria of interest. One of the most important objectives to deal with in a manufacturing system is the tardiness which can be measured through several performance measures. This paper proposes an effective procedure to estimate the first two central moments (*i.e.,* the mean and the variance) of the conditional tardiness and from this to compute a probabilistic bound for the maximum tardiness. These estimates are computed from the evaluation of the total tardiness, the number of tardy jobs and the root mean square tardiness obtained through a stochastic simulation. Different evaluations done by simulation show the effectiveness of the bound obtained.

Keywords : Simulation of stochastic systems; Discrete event systems in manufacturing.

## 1. INTRODUCTION

In a job shop dispatching decisions as to which job should be loaded on a machine when it becomes free are termed dynamic scheduling. One of the most common approaches to dynamically schedule the jobs is the use of dispatching rules (Blackstone et al., 1982). These are defined by Blackstone et al. (1982) as: "rules used to select the next job to process from jobs awaiting service". Their efficiency depends on the performance criteria of interest and on the operating conditions. The evaluation of their efficiency is usually done using simulation.

In a job shop, each job $i$ has associated with it a release date $r_i$ (*i.e.*, the earliest that the job can start executing), a due date $d_i$ (*i.e.*, the time by which the job must complete or else be considered late), and is composed by a list of operations. Each operation $j$ of a job $i$ necessitates at least one resource for its completion and is characterized by a processing time $p_{i,j}$. The completion time of the $j^{th}$ operation of job $i$ is noted as $C_{i,j}=t_{i,j}+p_{i,j}$ with $t_{i,j}$ as the starting time of the operation. The completion time of a job $i$ is noted as $C_i$. Performance measures can be classified into regular ones and those that are not. A *regular measure* is one that is non-decreasing in the completion times (French, 1982).

Literature specifies numerous performance measures, each focusing towards some particular objectives (*e.g.*, keep delivery due dates, optimize the productivity of the system or minimize the work-in-process and the stock). This paper focuses on performance measures related to the tardiness. The tardiness of a job is computed as $T_i=Max(0, C_i-d_i)$. To measure the effective performance of the system in regards with the tardiness, there are several performance measures of interest: the total tardiness noted as $\sum_i T_i$, the mean tardiness $\overline{T} = \sum_i T_i/n$, $n$ being the number of completed jobs, the maximum tardiness $T_{max} = \max_i T_i$, the number of tardy jobs noted as $n_T$ (*i.e.*, the number of completed jobs for which $T_i>0$), the conditional mean tardiness noted as $CMT = \sum_i T_i/n_T$, the root mean square tardiness noted as $RMST = \sqrt{\frac{1}{n}\sum_i T_i^2}$. All the performance measures cited above except *CMT* are regular performance measures. There are other performance measures as well related to the tardiness (French, 1982).

In this paper, we propose an effective procedure to estimate the first two central moments of the conditional tardiness (*i.e.,* the mean and the variance) and a probabilistic bound for the maximum tardiness. The conditional tardiness is the tardiness measured only over the tardy jobs. These estimates are computed from the evaluation of the total tardiness, the number of tardy jobs and the root mean square tardiness obtained through simulation. Different evaluations done by simulation show the effectiveness of the bound obtained.

## 2. PERFORMANCE MEASURES FOR THE TARDINESS

As already mentioned, there exist several performance measures to evaluate tardiness of an overall manufacturing system. The most used performance measure to evaluate the tardiness is the total tardiness (*i.e.*, $\sum_i T_i$). While using simulation to compare two different scheduling strategies, the number of completed jobs is usually fixed. Thus, in that case, the mean tardiness (*i.e.,* $\overline{T} = \sum_i T_i/n$, $n$ being the number of completed jobs) is equivalent to the total tardiness.

The maximum tardiness can be of great interest for the decision-maker in the shop. But, frequently, the evaluation of a dynamic scheduling strategy is made through random processes, especially using simulation. Thus, the maximum tardiness obtained from a simulation run is only a single estimate of the true value of this output (*i.e.,* the maximum tardiness). In practice, it would be more useful to have a bound of this value.

The conditional mean tardiness measures the average amount of tardiness for the completed jobs which are found to be tardy. But this measure is not a regular measure. It means that it can decrease while the completion times are not decreasing. For example, between two scheduling strategies with the same amount for the total tardiness, the one which has a greater number of tardy jobs will exhibit a conditional mean tardiness less than the other!

The root mean square tardiness permits to differentiate a system which presents a little number of tardy jobs with higher tardiness from a system which presents a lot of tardy jobs with low tardiness. This performance measure exhibits higher values for the first kind of systems **(**Moser and Engell, 1992**)**.

As said above, the evaluation of a dynamic scheduling strategy using simulation is frequently made through random processes and performance measures are consequently random variables. In that case, the simulation model is called a stochastic simulation model.

Let $X$ be a random variable, the distribution of $X$ is characterized by two values: the expectancy $E(X)$ which measures the mean $\mu$ and the variance $Var(X)$ measuring the statistical dispersion of $X$. For $X$ being a real-valued random variable, its expectancy is the first central moment and its variance is the second central moment. The variance can be computed as:

$Var(X) = E(X^2) - E(X)^2$.

Let $CT$ denote the conditional tardiness for a stochastic simulation, where $CT$ is a real-valued random variable. From a single simulation run, one can estimate the total tardiness (*i.e.,* $\sum_i T_i$), the number of tardy job $n_T$, and hence $E(CT)$ can be estimated as $E(CT) = \frac{\sum_i T_i}{n_T}$ which measures the Conditional Mean Tardiness ($CMT$). If $n$ is fixed, one can also estimate in the same single simulation run, the root mean square tardiness $RMST = \sqrt{\frac{1}{n}\sum_i T_i^2}$. From this, it is easy to compute the variance of the conditional tardiness as $Var(CT) = E(CT^2) - E(CT)^2$

$E(CT^2)$ can be estimated as $E(CT^2) = \frac{1}{n}\sum_i T_i^2 = RMST^2$

So, $Var(CT) = RMST^2 - CMT^2$

It means that with the estimations of

1. the total tardiness ($\sum_i T_i$),
2. the number of tardy job $n_T$ and
3. the root mean square tardiness $RMST$ ($\sqrt{\frac{1}{n}\sum_i T_i^2}$),

we can compute an estimation for the first two central moments of the conditional tardiness (*i.e.,* the mean and the variance).

## 3. A PROBABILISTIC BOUND FOR THE MAXIMUM TARDINESS

As already mentioned, in practice, a bound for the maximum tardiness would be helpful. In a stochastic simulation, the outputs are random variables, in which case, the bound for the maximum tardiness has to be a probabilistic one. In the previous section, we have proposed an effective way to compute estimates of the mean and the variance of the conditional tardiness. However with these first two central moments for this continuous random variable, we don't have the distribution density function and consequently it is not possible to compute directly any quantile.

Using only the first two central moments of any random variable, the Chebyshev's inequality permits to compute an interval where a given percentage of the observed values lie. Typically, the Chebyshev's inequality will provide rather loose bounds. However, these bounds cannot, in general be improved upon. The probabilistic formulation of the Chebyshev's inequality is as following: let $X$ be a random variable with expected value $\mu$ and finite variance $\sigma^2$. Then for any real number $k > 0$, $P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}$

The Chebyshev's inequality can be used to compute a probabilistic bound for the maximum tardiness. Indeed, for $CT$ to be a random variable for the conditional tardiness with expected value $\mu_{CT}$ and finite variance $\sigma_{CT}^2$, if $\frac{1}{k^2} = \alpha$ (i.e., a fixed probability), then it comes

$$P\left(|T - \mu_{CT}| \geq \frac{\sigma_{CT}}{\sqrt{\alpha}}\right) \leq \alpha \Leftrightarrow$$

$$P\left(\frac{-\sigma_{CT}}{\sqrt{\alpha}} \leq T - \mu_{CT} \geq \frac{\sigma_{CT}}{\sqrt{\alpha}}\right) \leq \alpha \Leftrightarrow$$

$$P\left(\mu_{CT} - \frac{\sigma_{CT}}{\sqrt{\alpha}} \leq T \geq \mu_{CT} + \frac{\sigma_{CT}}{\sqrt{\alpha}}\right) \leq \alpha, \text{ as } \mu_{CT} \geq 0,$$

And it comes that: $P\left(T \geq \mu_{CT} + \frac{\sigma_{CT}}{\sqrt{\alpha}}\right) \leq \alpha$ (1)

The probabilistic bound for the maximum tardiness is given by inequality (1). For a given probability $\alpha$, the bound for the maximum tardiness is computed as $\widehat{\mu_{CT}} + \frac{\widehat{\sigma_{CT}}}{\sqrt{\alpha}}$, with $\widehat{\mu_{CT}}$ the estimated conditional mean tardiness and $\widehat{\sigma_{CT}}$ the estimated standard-deviation of the conditional tardiness. The inequality (1) means that the probability for an observed job's tardiness to be greater than the bound (*i.e.,* $\widehat{\mu_{CT}} + \frac{\widehat{\sigma_{CT}}}{\sqrt{\alpha}}$) is less than or equal to $\alpha$.

## 4. A SIMULATION PROCEDURE TO EVALUATE TARDINESS PERFORMANCE MEASURES

In this section, we will describe the simulation procedure used to estimate the total tardiness, the number of tardy job $n_T$, and the root mean square tardiness $RMST$, in order to compute the conditional mean tardiness $CMT$, the variance of the conditional tardiness $\sigma_{CT}^2$ and a bound for the maximum tardiness. We will also present the experiments performed on a job shop model frequently used in the literature to evaluate dynamic scheduling strategies.

A job shop simulation can be considered as a non-terminating simulation (Law and Kelton, 2000). It means that the outputs have to be measured in a steady state. Let $X_1$, $X_2$, ...$X_m$ be $m$ random variables measuring the performance of $m$ entities through a single simulation run. These random variables define a stochastic process $\{X_i, i \geq 1\}$ and the mean performance $\mu$ of this process in a steady state, is defined by :

$$\mu = \lim_{m\to\propto} \frac{\sum_{i=1}^{m} x_i}{m}$$

whatever the initial conditions are (Law and Kelton, 2000).

It means that for a non-terminating simulation, whenever there is a steady state, the mean of an output can be estimated in a single run if it is long enough. In our study, the stochastic process of interest is the tardiness $\{T_i, i=1..n_T\}$, from where we can estimate the total tardiness (*i.e.,* $\sum_i T_i$), the *RMST* (*i.e., $RMST = \sqrt{\frac{1}{n}\sum_i T_i^2}$*, the total number of completed jobs $n$ being fixed). With a single simulation run, the number of tardy jobs $n_T$ can be considered as a constant. Thus, from this single simulation run, it is trivial to compute an estimate of the conditional mean tardiness, $\widehat{\mu_{CT}} = \frac{\sum_i T_i}{n_T}$, and an estimate of the standard-deviation of the conditional tardiness, $\widehat{\sigma_{CT}} = \sqrt{\frac{1}{n_i}\sum_i T_i^2 - (\frac{\sum_i T_i}{n_T})^2}$.

From these estimates and from the inequality (1), a bound for the maximum tardiness at a given probability $\alpha$ is equal to $\widehat{\mu_{CT}} + \frac{\widehat{\sigma_{CT}}}{\sqrt{\alpha}}$.

Using independent replications would have been an appropriate method to estimate total tardiness, *RMST* and the number of tardy jobs $n_T$. The problem is that using independent replications, each performance measure would have to be considered as a specific random variable, not independent from each other. Therefore their estimates cannot be used to compute $\widehat{\mu_{CT}}$ and $\widehat{\sigma_{CT}}$. In order to avoid this drawback, and because the system can be considered as a stochastic process, only one simulation run (long enough) is carried out in order to get estimates of the outputs.

## 5. EXPERIMENTS

In order to try to make a comparison as relevant as possible, our test model is a job shop model used by several researchers. For example, Eilon and Cotteril (1968) have used this model to test the effects of the $SI^x$ rule, Baker and Kanet (1983) to demonstrate the benefits of the MOD rule, Baker (1984) to examine the interaction between dispatching rules and due-dates assignment methods, Russel et al (1987) to analyze the effects of the CoverT rule comparing with several other dispatching rules, Schultz (1989) to demonstrate the benefits of the CEXSPT rule, Pierreval and Mebarki (1997) to evaluate the dynamic scheduling strategy they proposed.

The system is a four machine job-shop. Each machine can perform only one operation at a time. The number of operations of the jobs processed in the system is uniformly distributed, between 2 and 6. The routing of each job is random. More precisely, when a job leaves a machine and needs another operation, each machine has the same probability to be the next, except the one just released, which cannot be chosen. The processing times on machines are exponentially distributed with a mean of one.

The arrival of jobs in the system is modelled as a Poisson process, and due to the particularities of this test model, the shop utilization rate is equal to the mean arrival. Due dates of jobs are determined using the Total Work Content (TWK) method (Baker, 1984).

The dispatching rules used to evaluate the efficiency of our bound are FIFO, EDD, SLACK, CR/SI, CoverT, and MOD (see Annex 1 for a complete description of these rules). These rules have been chosen for their efficiency over a large variety of criteria, especially for performance measures related to tardiness.

**Table 1. Operating conditions tested.**

| Factor | Levels | Number of levels |
|---|---|---|
| Utilization rate of the resources | $\rho$=80%, $\rho$=90% | 2 |
| Due date | tight due dates<br>moderate due dates<br>loose due dates | 3 |

There are 2×3=6 configurations to simulate. Two levels of shop load were defined. A moderate shop load level, which corresponds to a utilization rate of 80% for the resources, and a high level of shop load, which corresponds to a utilization rate of 90%. For each load level, three different levels of due-date were established. Each given due date tightness (*e.g.*, tight, moderate or loose) is computed differently depending on the utilization rate of the resources (Schultz, 1989; Pierreval and Mebarki, 1997).

As indicated in the procedure described in section 4, one single simulation run was carried out for each configuration. The performance measures were collected for 500,000 jobs with a warm-up period of 1,000 time units (corresponding approximately to 500 jobs). The bound for the maximum tardiness is computed with $\alpha$=0.01. The results are given in Table 2.

**Table 2. Results obtained from a single simulation run.**

| | | | $n_T$ | *RMST* | *Total tardiness* | *CMT* | $\sigma^2_{CT}$ | Bound on $T_{max}$ ($\alpha$=0.01) |
|---|---|---|---|---|---|---|---|---|
| High load ($\rho$=90%) | Tight Due Dates | FIFO | 316019 | 28.32 | 5229798.43 | 26.14 | 583.64 | 267.72 |
| | | EDD | 235756 | 15.84 | 1908822.03 | 17.14 | 237.26 | 171.17 |
| | | SLACK | 254385 | 16.63 | 2252680.93 | 17.38 | 241.03 | 172.63 |
| | | CR/SI | 70407 | 14.10 | 94817.11 | 9.55 | 1318.69 | 372.68 |
| | | CoverT | 194118 | 17.44 | 488070.89 | 6.46 | 740.60 | 278.60 |
| | | MOD | 119328 | 22.92 | 339380.76 | 11.90 | 2055.65 | 465.29 |
| | Moderate Due Dates | FIFO | 189634 | 19.99 | 1684424.01 | 23.38 | 505.46 | 248.20 |
| | | EDD | 57407 | 5.55 | 80203.32 | 12.15 | 119.82 | 121.61 |
| | | SLACK | 61706 | 6.40 | 99994.57 | 13.11 | 159.69 | 139.48 |
| | | CR/SI | 8851 | 3.32 | 1057.69 | 6.74 | 576.11 | 246.76 |
| | | CoverT | 99133 | 6.83 | 59400.49 | 3.02 | 225.97 | 153.34 |
| | | MOD | 67335 | 15.48 | 90632.91 | 9.98 | 1675.63 | 419.32 |
| | Loose Due Dates | FIFO | 147757 | 17.07 | 985376.66 | 22.53 | 477.23 | 240.98 |
| | | EDD | 19939 | 2.96 | 8388.34 | 10.53 | 108.28 | 114.59 |
| | | SLACK | 20763 | 3.26 | 9889.42 | 11.45 | 124.99 | 123.25 |
| | | CR/SI | 3932 | 2.39 | 198.57 | 6.41 | 683.02 | 267.76 |
| | | CoverT | 72045 | 3.33 | 20187.01 | 1.94 | 73.22 | 87.51 |
| | | MOD | 50188 | 11.29 | 42318.52 | 8.39 | 1198.10 | 354.52 |
| Moderate load ($\rho$=80%) | Tight Due Dates | FIFO | 235522 | 10.97 | 1283312.27 | 11.55 | 121.74 | 121.88 |
| | | EDD | 141916 | 5.33 | 285393.08 | 7.07 | 49.93 | 77.74 |
| | | SLACK | 149142 | 5.52 | 317612.80 | 7.13 | 51.19 | 78.68 |
| | | CR/SI | 52091 | 3.92 | 22451.22 | 4.13 | 129.84 | 118.08 |
| | | CoverT | 130056 | 4.86 | 106320.78 | 3.14 | 80.97 | 93.12 |
| | | MOD | 98580 | 6.98 | 99378.50 | 5.10 | 220.83 | 153.71 |
| | Moderate Due Dates | FIFO | 136923 | 7.78 | 398144.70 | 10.60 | 108.07 | 114.56 |
| | | EDD | 27447 | 1.86 | 8135.29 | 5.39 | 33.89 | 63.61 |
| | | SLACK | 29447 | 1.99 | 9605.61 | 5.53 | 36.85 | 66.24 |
| | | CR/SI | 10085 | 1.38 | 621.24 | 3.05 | 84.50 | 94.97 |
| | | CoverT | 68179 | 1.95 | 17297.01 | 1.86 | 24.38 | 51.23 |
| | | MOD | 49643 | 4.76 | 21574.85 | 4.37 | 209.15 | 148.99 |
| | Loose Due Dates | FIFO | 106588 | 6.67 | 233502.33 | 10.26 | 103.00 | 111.75 |
| | | EDD | 11614 | 1.17 | 1271.73 | 4.71 | 36.73 | 65.31 |
| | | SLACK | 13138 | 1.20 | 1667.21 | 4.82 | 31.60 | 61.03 |
| | | CR/SI | 5291 | 0.70 | 128.04 | 2.29 | 40.78 | 66.15 |
| | | CoverT | 52185 | 0.94 | 8245.23 | 1.51 | 6.12 | 26.25 |
| | | MOD | 36727 | 3.43 | 9890.58 | 3.66 | 146.65 | 124.76 |

To measure the effectiveness of the bound computed for the maximum tardiness, another series of experiments were conducted using the same configuration and the same rules. The performance measures however were collected over 100 replications with each replication having number of jobs equal to 5,000 with the same warm-up period of 1,000 time units. Then for each replication, the percentage of tardy jobs that are less than the bound is measured. For each configuration and each rule the average is given in Table 3.

### Table 3. Average percentage of tardy jobs under the bound over 100 replications.

| | High load ($\rho$=90%) | | | Moderate load ($\rho$=80%) | | |
|---|---|---|---|---|---|---|
| Due Dates | Tight | Mode-rate | Loose | Tight | Mode-rate | Loose |
| FIFO | 100% | 100% | 100% | 99.999 | 99.999 | 99.999 |
| EDD | 100% | 100% | 100% | 100% | 100% | 100% |
| SLACK | 100% | 100% | 100% | 100% | 100% | 100% |
| CR/SI | 99.86 | 99.89 | 99.97 | 99.83 | 99.89 | 99.88 |
| CoverT | 99.84 | 99.91 | 99.87 | 99.81 | 99.87 | 99.78 |
| MOD | 99.85 | 99.85 | 99.76 | 99.84 | 99.84 | 99.81 |

The percentages presented in Table 3 show that the bound computed for the maximum tardiness is quite effective, much more than 99% expected (the probability $\alpha$ has been set to 0.01). In average, 99.94% of the tardy jobs are under the bound. It is consistent with the fact that the Chebyshev's inequality provides loose bounds.

It is clear that the effectiveness of the bound doesn't depend on the operating conditions but rather on the respective rule. We can consider two sets of rules:

- {FIFO, EDD, SLACK} noted as set 1 for which the average percentage of tardy jobs under the bound is almost 100%
- {CR/SI, CoverT, MOD} noted as set 2 for which the average of percentage of tardy jobs under the bound is found to be comparatively less, though their average is greater than the expected probability (*i.e.,* 99.85% compared to 99%)

The effectiveness of the bound is related to only one parameter which is the coefficient of variation, which measures the dispersion of a probability distribution (here the tardiness distribution). It is defined as the ratio of the standard deviation $\sigma$ to the mean $\mu$:

$$c_v = \frac{\sigma}{\mu}$$

Table 4 presents the average coefficient of variation for rules of set 1 and set 2, computed over all the operating conditions, in regards to the average percentage of tardy jobs under the bound.

### Table 4. Coefficient of variation versus average percentage of tardy jobs under the bound.

| Rule | $\overline{c_v}$ | Average percentage of tardy jobs under the bound |
|---|---|---|
| FIFO | 0.96 | 99.999% |
| EDD | 1.05 | 100.00% |
| SLACK | 1.02 | 100.00% |
| CR/SI | 3.33 | 99.89% |
| CoverT | 3.46 | 99.85% |
| MOD | 3.59 | 99.83% |

The less the coefficient of variation is, the higher the percentage of tardy jobs under the bound, and vice-versa.

For rules of set 1, under high shop load, the coefficient of variation is always less than 1, with a remarkable stability for FIFO (*i.e.*, between 0.989 and 0.924). It means that for rules of set 1, under high shop load, the standard deviation is less than the mean whereas for rules of set 2, as the coefficient of variation is always greater than 1, their standard deviation is always greater than their mean.

For rules of set 1, the coefficient of variation under high shop load is less than under moderate shop load. It is the opposite for rules of set 2.

Table 5 presents the average absolute difference between the probabilistic bound and the maximum of the maximum tardiness observed over 100 replications. This average is computed over all the operating conditions.

### Table 5. Average absolute difference between the bound and Max($T_{max}$) for each rule of sets 1 and 2.

| Rule | $\overline{\left\lvert Bound - \max_{1..100} T_{max} \right\rvert}$ |
|---|---|
| FIFO | 7% |
| EDD | 28% |
| SLACK | 26% |
| CR/SI | 233% |
| CoverT | 633% |
| MOD | 216% |

For rules of set 1, the bound is quite close to the maximum value observed. Most of the time it is greater than the maximum value observed, especially for rules EDD and SLACK for which the bound is always greater than any observed value for the maximum tardiness.

For rules of set 2, there is a larger difference between the bound and the maximum value observed. But the bound is always smaller than the maximum value observed. So, despite the fact that this probabilistic bound has been computed using Chebyshev's inequality, which is known to give loose bounds, in our situation, it gives a probabilistic bound which is not so loose. Probably, this is due to the fact that in our test model not only the operating times are random but also the arrival times and the routings. Moreover, operating times follow an exponential distribution which presents a high variance (standard-deviation of an exponential distribution is equal to the mean) and in our test model, due dates greatly depend on the operating times. So, it seems that the less the rule depends on the operating times (such as FIFO), the more balanced is the rule in regards with the dispersion of the maximum tardiness.

## 6. CONCLUSION AND PERSPECTIVES

In this paper, we have discussed tardiness based performance measures in a stochastic context. We have proposed an effective procedure to compute estimates for the first two central moments (*i.e.,* the mean and the variance) of the conditional tardiness and a probabilistic bound for the maximum tardiness. This bound is computed using Chebyshev's inequality and necessitates no other information than estimates of the mean and the variance.

This procedure has been tested on a job shop model used extensively in the literature on dynamic scheduling. The results obtained show the effectiveness of the bound. Discussion on the results show also the importance of the coefficient of variation on the effectiveness of this bound and this parameter enables to discriminate two sets of dispatching rules used in our job shop model.

A future work will try to improve the computed bound by using more information about rules' behaviour.

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## Appendix A. DEFINITION OF THE DISPATCHING RULES

Definition of symbols

$d_i$ is the due date of job $i$

$p$ is the present date

$t_{i,j}$ is the operating time of the $j^{th}$ operation of job $i$

$S_i$ is the set of all operations through which job $i$ must still pass in

$s_i$ is the slack of job $i$ $\quad s_i = d_i - p - \sum_{j\in S_i} t_{i,j}$

$w_{i,j}$ is the expected waiting time of the $j^{th}$ operation of job $i$

$d_{i,j}$ is the operation due date of the $j^{th}$ operation of job $i$

FIFO First In First Out: the first job entered the queue is the first served

EDD Earliest Due Date: Min($d_i$)

Slack: Min($s_i$)

Critical Ratio Shortest Imminent (CR/SI): Min($Z_i$)
$Z_i$=Max($p+ t_{i,j}$, $p+ t_{i,j}*(d_i - p)/ \sum_{j\in S_i} t_{i,j}$)

Cost Over Time (CoverT): Max($c_i / t_{i,j}$)
$c_i$=1 if ($s_i$<0)
$c_i$=0 if (*Control Parameter*$*w_{i,j}<s_i$)
$c_i$=(*Control Parameter*$* w_{i,j} - s_i$)/*Control Parameter*$* w_{i,j}$ otherwise

Control Parameter is a parameter of the CoverT rule to be set by the user. In our test model, it has been set to 3.

Modified Operation Due Date (MOD): Min($Z_i$)
$Z_i$ =Max($d_{i,j}$, $p+t_{i,j}$)