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Quick Kelly Control: A Nonlinear Congestion Control Method for High Speed Networks

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Abstract: The traditional congestion control algorithms exhibit low convergence rate to equilibrium when the network capacity is very large. In this paper, we present a new group of algorithms called Quick Kelly Control (QKC) to accelerate the convergence rate. QKC is scalable to networks operating at very high speeds. The link utilization ratio function is used as feedback signal and a group of novel nonlinear update laws is constructed. We prove the stability of a primal-dual form of QKC (PDQKC) without considering delay. We also compare this algorithm with two classic algorithms and give simulation results. It is shown that PDQKC has powerful bandwidth scalability and offers fast convergence rate without sacrificing proportional fairness.

1. INTRODUCTION

Network congestion control is a distributed method to share network resources among competing sources. It consists of two components: a source algorithm that dynamically adjusts sending rate according to congestion in its path, and a link algorithm that updates congestion information and sends it back to sources using that link.

The traditional algorithms were designed during a time when the Internet was a relatively small network compared to its size today. These algorithms are inefficient when the network capacity is very large. Therefore, researchers are forced to design new congestion control algorithms with the goal enhancing TCP to make it scalable to high-speed networks. A large amount of theoretical and experimental work has been done to design stable congestion control for high-speed networks. Such examples include Fast TCP (David *et al*., 2006), Scalable TCP (Kelly, 2003a), and HSTCP (Floyd, 2003). All of these methods aim to get quick convergence to efficiency, stable rate trajectories, fair bandwidth sharing, and low packet loss. Another different direction in congestion control is to model the network from an optimization or game theoretic point of view (Kar *et al*., 2001) (Low *et al*., 1999) (Kunniyur *et al*., 2001) (Kunniyur *et al*., 2002) (Kunniyur *et al*., 2003).The original work is done by (Kelly, 1997).

In this paper, we aim to propose a new group of congestion control algorithms which can achieve quick convergence rate, proportional fairness. In this new group of algorithms, a link utilization ratio function, which is always positive, is used as network feedback signal. We construct a utilization ratio based nonlinear source controller to accelerate the convergence rate. A dynamic link control law is also proposed to give corresponding link prices. The source sending rate is then dynamically updated based on these link prices which are computed by link utilization ratios. The stability and performance of these algorithms are proved by theory and simulation results.

The rest of this paper is organized as follows. In section II, we give the basic network model and review related works. In section III, we present QKC and prove its stability and proportional fairness. In section IV, simulation results are given. In section V, we conclude our work and suggest directions of future research.

2. BASIC NETWORK FLOW CONTROL MODEL AND RELATED WORK

Network flows are modelled as the interconnection of information sources and communication links through the routing matrices as shown in (Kelly *et al*., 1998) (Wen *et al*., 2004). Suppose we have a set of users, R , and a set of links, L . For each user $r \in R$, its route involves a set of links, which is a subset of L , denoted L_r . For each link $l \in L$, it has a fixed capacity c_l . Based on its congestion and queue size, a link price p_l is computed. Associate each user r with a sending rate x_r . Thus, each link $l \in L_r$ has an associated link aggregate rate y_l . Suppose all links only feed back price information to the sources that utilize them. Set $A_{lr} = 1$, if $l \in L_r$ and set $A_{lr} = 0$ otherwise. So we have the following relationship (Wen *et al*., 2004):

$$y = Ax \quad q = A^T p$$

where A is a routing matrix, $x \in R^N$ is the source rate vector, $y \in R^L$ is the aggregate rate vector, $p \in R^L$ is the link price vector and $q \in R^N$ is the aggregate price vector. In this paper, we make an assumption: There is no delay in the loop. The network flow control problem can be decomposed into a static resource allocation optimization problem and a dynamic stabilization problem (Wen *et al*., 2004).

2.1 Static optimization problem

The static resource allocation optimization problem is to maximize the whole networks' performances. Its solution provides the desired steady state equilibrium: x^* , y^* , p^* , and q^* . Each user *r* is associated with a utility function $U_r(x_r)$ which indicates the utility to the user *r*. Then the static resource allocation optimization problem (Kelly *et al*., 1998) is

$$\max \sum_r U_r(x_r) \tag{1}$$

subject to $Ax \le C$ over $0 < x_r \quad r \in R$.

where C is a vector of link capacities c_l , and $U_r(x_r)$ is an increasing, strictly concave and continuous differentiable function. The equilibrium condition for problem (1) is (Kelly, 1997):

$$U'_r(x_r) - q_r = 0, \qquad r \in R. \tag{2}$$

$$p_l \begin{cases} = 0, & if \quad y_l < c_l \\ \ge 0, & if \quad y_l = c_l \end{cases}, \qquad l \in L. \tag{3}$$

2.2 Dynamic optimization problem

The dynamic stabilization problem is to design source rate and link price dynamic update laws which guarantee stability and robustness of the equilibrium. In (Kelly *et al*., 1998), two complementary congestion control algorithms are proposed: primal algorithm and dual algorithm. The primal algorithm can be expressed as follow. For each user *r*,

$$\dot{x}_r = \kappa_r(x_r)(U'_r(x_r) - q_r) \tag{4}$$

where $k_r(x_r)$ is an appropriately chosen scaling function. Each link *l* computes its price as

$$p_l = f_l(y_l) = \frac{(y_l - c_l + \varepsilon)^+}{\varepsilon^2} \ . \tag{5}$$

Kelly (Kelly *et al*., 1998) has shown that this primal algorithm globally converges to a unique equilibrium point.

The dual algorithm can be expressed as follow. Each link *l* updates p_l (Wen *et al*., 2004) (Kelly, 2003b) (Liu *et al*., 2003) by a dynamic equation:

$$\dot{p}_l = g_l(p_l)\,(y_l - c_l)^+ \tag{6}$$

The source update law is directly given by the primal solution:

$$x_r = U_r^{'-1}(q_r) \tag{7}$$

For this dual algorithm, global stability has proved in (Kelly *et al*., 1998) (Liu, 2003).

Therefore, (4) and (6) can be regarded as the primal-dual algorithm. In (Wen *et al*., 2004) (Liu *et al*., 2003), its global stability in the absence of feedback delay is proved.

In (Kelly *et al*., 1998), $k_r(x_r) = \kappa\, x_r$ and $U'_r(x_r) = w_r / x_r$. Substituting these into (4), we can get

$$\dot{x}_r = \kappa\, x_r\,(\frac{w_r}{x_r} - q_r) \tag{8}$$

In general, no price should be charged at the links which are not fully utilized. Under these circumstances, the sources increase their rates by κw_r per unit time before they reach full link utilization at the slowest link. This results in linear AIMD-like probing for new bandwidth. Thus the link utilization is very low in high-speed networks. That is un-scalable to large link capacity.

3. QUICK KELLEY CONTROL (QKC)

3.1 Motivation

We start our discussion with the following observations. To overcome the drawback of classic Kelly control, a variant version of Kelly's algorithms which is called Max-min Kelly Control (MKC) is proposed in (Zhang *et al*., 2004). MKC abandons proportional fairness and utilizes negative network feedback which signals the sources to increase their sending rates when $y_l < c_l$. Its price function can be expressed as follow:

$$p_l = \frac{y_l - c_l}{y_l} \tag{9}$$

$$q_r = \max_{l:\, l \in Lr} p_l \tag{10}$$

MKC achieves max-min fairness and exponential convergence to efficiency. However, when the link capacity is very large and the number of flows is unknown, MKC flows need a long time to converge to max-min fairness.

In fact, negative price is only one of the possible choices and any price form which gives sufficient link state information is feasible. Links can feed back their utilization ratios to the sources that utilize them. Obviously, the sum of the utilization ratios is always positive and those large links with low utilization ratios have small weights. This idea provides credible link prices even though certain sources use those links with huge difference. To accelerate the convergence rate, we give a new form of the source update law:

$$\dot{x}_r = \kappa w_r \cdot S(q_r) - \kappa x_r \cdot E(q_r) \tag{11}$$

where $S(q_r)$ is a positive decreasing function and $E(q_r)$ is a positive increasing function. Following these, a group of algorithms called Quick Kelly Control (QKC) is proposed in the following part of this section.

3.2 Primal Quick Kelly Control

Set $S(q_r) = \frac{1}{\sqrt{q_r}}$ and $E(q_r) = \sqrt{q_r}$. We have

$$\dot{x}_r = k_r\left(\frac{w_r}{\sqrt{q_r}} - x_r\sqrt{q_r}\right) \quad (12)$$

$$q_r = \sum_{l:\, l \in Lr} p_l \quad (13)$$

where k_r is a gain constant and w_r is a positive constant. Let

$$p_l = f_l\left(\frac{y_l}{c_l}\right). \quad (14)$$

We assume that each penalty function $f_l(\bullet)$ is positive and monotonically increasing, such as the follow form:

$$f_l\left(\frac{y_l}{c_l}\right) = \omega^2 + \exp\left[\frac{y_l}{c_l} - 1\right] \cdot \frac{y_l}{c_l} \quad (15)$$

where ω is a small positive constant. We call the resulting controller (12)-(15) Primal Quick Kelly control (PQKC).

Let $U_r'(x_r) = w_r / x_r$. Seting $\dot{x}_r(t)$ =0, the equilibrium of system (12)-(15) can be computed as:

$$x_r^* = U_r'^{-1}(q_r^*) \ , \qquad p_l^* = f\left(\frac{y_l^*}{c_l}\right). \quad (16)$$

PQKC and Kelly controller have an identical form of the equilibrium. However, this equilibrium only approximately satisfies the desired condition (2)-(3) and does not solve the system problem (1) exactly.

3.3 Dual Quick Kelly Control

Although PQKC gives a dynamic source update law, it does not take the link dynamics explicitly into account. Similar to (Kelly *et al.*, 1998), we give a dual form of QKC. The dynamic link control law is given as follow:

$$\dot{\delta}_l = \theta \cdot \left(\frac{y_l}{c_l} - 1\right) \quad (17)$$

$$p_l = g_l\left(\frac{y_l}{c_l}, \delta_l\right) \quad (18)$$

where θ is a positive constant and $p_l \geq \omega^2$. ω is an arbitrarily small positive constant. Here, we introduce a positive function g_l which is strictly increasing in both variables δ_l and y_l . When $\delta_l \to -\infty$, $p_l \to \omega^2$. The source update law is directly given by the primal solution:

$$x_r = U_r'^{-1}(q_r) = \frac{w_r}{q_r} \quad (19)$$

We call the resulting controller (17)-(19) Dual Quick Kelly control (DQKC).

3.4 Primal-Dual Quick Kelly Control

To achieve a more exact solution and better dynamic performances, we combine the above two algorithms and get a primal-dual form of QKC. Then (12) (13) (17) and (18) is called Primal-Dual Quick Kelly Control (PDQKC). The interconnection system of PDQKC is shown in Fig 1. For this algorithm, at equilibrium, we have

$$U_r'(x_r^*) = \frac{w_r}{x_r^*} = q_r^*, \qquad r \in R, \quad (20)$$

$$p_l \begin{cases} = \omega^2, & if \ \ \frac{y_l}{c_l} < 1, \\ \geq 0, & if \ \ \frac{y_l}{c_l} = 1, \end{cases} \qquad l \in L. \quad (21)$$

Since ω can be chosen arbitrarily small, the equilibrium approximates arbitrarily closely the desired condition (2)-(3) as $\omega \to 0$. So PDQKC converges to the solution of the optimization problem (1) as $\omega \to 0$.

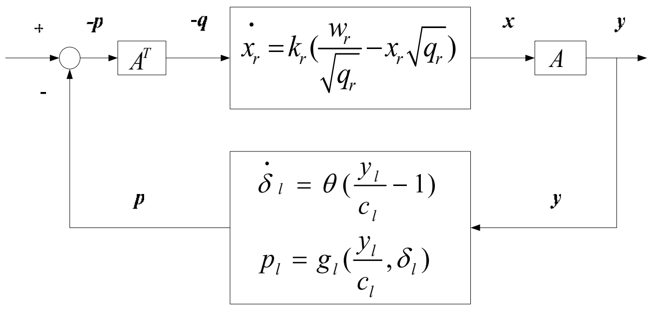

Fig. 1. Primal-Dual Quick Kelly Controller.

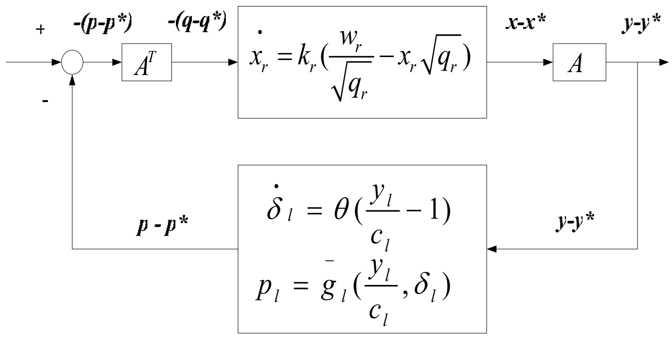

Fig. 2. Equivalent representation of Primal-Dual Quick Kelly Controller.

3.5 Stability analysis

In this part, we prove the stability of PDQKC similar to (Wen *et al.*, 2004). We first express the PDQKC system in an equivalent form in Fig. 2 based on the deviation from the equilibrium condition, and rewrite the link update law as:

$$\bar{g}_l\left(\frac{y_l}{c_l}-\frac{y_l^*}{c_l},\delta_l-\delta_l^*\right):=g_l\left(\frac{y_l}{c_l},\delta_l\right)-g_l\left(\frac{y_l^*}{c_l},\delta_l^*\right) \quad (22)$$

where δ^* is the equilibrium value of δ. In practice, source rate x_r is always limited by the network hardware. Suppose $0<x_r<M<\infty$, where M is an arbitrarily large positive constant. The following theorem shows that: Starting from any initial state, the sources rates x_r will converge to x^*_r as $t\to\infty$.

Theorem 1 Consider the feedback interconnection shown in Fig. 2. The equilibrium $x=x^*$ is asymptotically stable for the state space $\chi:=\{0<x_r<M,\quad r\in R\}$.

Proof:

Consider

$$V_1(x-x^*)=\frac{1}{2}\frac{\omega}{M}\sum_r\frac{(x_r-x_r^*)^2}{k}.$$

Obviously, V_1 is a positive-definite function. The deviation along the solution is

$$\dot{V}_1=\sum_r\frac{\omega}{M}(x_r-x_r^*)\left(\frac{w_r}{\sqrt{q_r}}-x_r\sqrt{q_r}\right)$$

$$=\sum_r\frac{\omega}{M}\frac{x_r}{\sqrt{q_r}}(x_r-x_r^*)\left(\frac{w_r}{x_r}-q_r\right).$$

Because $p_l\ge\omega^2$ and $q_r=\sum_{l:\,l\in Lr}p_l$, we have $q_r\ge\omega^2$. Since $0<x_r<M$, we can get $\frac{x_r}{\sqrt{q_r}}<\frac{M}{\omega}$.

Consequently,

$$\dot{V}_1\le\sum_r(x_r-x_r^*)\left(\frac{w_r}{x_r}-q_r\right).$$

By adding and subtracting q^* from q, we have

$$\dot{V}_1\le\sum_r(x_r-x_r^*)\left(\frac{w_r}{x_r}-q_r^*+q_r^*-q_r\right)$$

$$=\sum_r(x_r-x_r^*)\left(\frac{w_r}{x_r}-\frac{w_r}{x_r^*}+q_r^*-q_r\right)$$

$$=\sum_r(x_r-x_r^*)\left(\frac{w_r}{x_r}-\frac{w_r}{x_r^*}\right)+\sum_r(x_r-x_r^*)(q_r^*-q_r)$$

Since

$$\sum_r(x_r-x_r^*)(q_r^*-q_r)=(x-x^*)^T(q^*-q)$$

$$=(x-x^*)^TR^T(p^*-p)$$

$$=-(y-y^*)(p-p^*)$$

we can obtain

$$\dot{V}_1\le\sum_r(x_r-x_r^*)\left(\frac{w_r}{x_r}-\frac{w_r}{x_r^*}\right)-(y-y^*)(p-p^*)$$

Consider the storage function for the l th link

$$V_{2l}(\delta_l-\delta_l^*)=\frac{c_l}{\theta}\int_{\delta_l^*}^{\delta_l}\left(g_l\left(\frac{y_l^*}{c_l},z\right)-g_l\left(\frac{y_l^*}{c_l},\delta_l^*\right)\right)dz$$

Since g_l is strictly increasing with respect to δ_l, V_{2l} is a positive-definite function. Taking the deviation of V_{2l} along the solution, we have

$$\dot{V}_{2l}=\frac{c_l}{\theta}\left(g_l\left(\frac{y_l^*}{c_l},\delta_l\right)-g_l\left(\frac{y_l^*}{c_l},\delta_l^*\right)\right)\theta\left(\frac{y_l}{c_l}-1\right)$$

Rewriting the right side of the above equation, we can observe that

$$(y_l-c_l)\left(g_l\left(\frac{y_l^*}{c_l},\delta_l\right)-g_l\left(\frac{y_l^*}{c_l},\delta_l^*\right)\right)$$

$$\le(y_l-y_l^*)\left(g_l\left(\frac{y_l^*}{c_l},\delta_l\right)-g_l\left(\frac{y_l^*}{c_l},\delta_l^*\right)\right). \quad (23)$$

If $y_l^*=c_l$, then both sides are identical. If $y_l^*<c_l$, then $\delta_l^*\to-\infty$. Since g_l is strictly increasing with respect to δ_l, we have

$$g_l\left(\frac{y_l^*}{c_l},\ \delta_l\right)-g_l\left(\frac{y_l^*}{c_l},\ \delta_l^*\right)>0.$$

So the inequality holds. By adding and subtracting $g_l(\frac{y_l}{c_l},\ \delta_l)$ to the right side of (23), we have

$$\dot{V}_{2l}\le(y_l-y_l^*)\Big[-\Big(g_l\Big(\frac{y_l}{c_l},\ \delta_l\Big)-g_l\Big(\frac{y_l^*}{c_l},\delta_l\Big)\Big)+g_l\Big(\frac{y_l}{c_l},\delta_l\Big)-g_l\Big(\frac{y_l^*}{c_l},\delta_l^*\Big)\Big]$$

$$=-(y_l-y_l^*)\Big[g_l\Big(\frac{y_l}{c_l},\ \delta_l\Big)-g_l\Big(\frac{y_l^*}{c_l},\ \delta_l\Big)\Big]+(y_l-y_l^*)\Big[g_l\Big(\frac{y_l}{c_l},\ \delta_l\Big)-g_l\Big(\frac{y_l^*}{c_l},\ \delta_l^*\Big)\Big]$$

Since g_l is strictly increasing with respect to y_l, $(y_l-y_l^*)$ has the same sign as

$$g_l\left(\frac{y_l}{c_l},\ \delta_l\right)-g_l\left(\frac{y_l^*}{c_l},\delta_l\right).$$

We can get that

$$\dot{V}_{2l} \le (y_l - y_l^*)(g_l(\frac{y_l}{c_l}, \delta_l) - g_l(\frac{y_l^*}{c_l}, \delta_l^*))$$

$$= (y_l - y_l^*)(p_l - p_l^*)$$

Let $V_2 = \sum_l V_{2l}$. Then we can obtain

$$\dot{V}_2 \le \sum_l (y_l - y_l^*)(p_l - p_l^*) = (y - y^*)(p - p^*)$$

Now, we can use $V = V_1 + V_2$ as a Lyapunov function and obtain

$$\dot{V} \le \sum_r (x_r - x_r^*)(\frac{w_r}{x_r} - \frac{w_r}{x_r^*}) \le 0$$

Note that $\dot{V} < 0$ for $x \ne x^*$ and $\dot{V} < 0$ for $x = x^*$. Thus V_1 is strictly decreasing with t, unless $x = x^*$. Since M can be chosen arbitrarily large to include any initial state in $\chi := \{0 < x_r < M, \quad r \in R\}$. Hence, the equilibrium $x = x^*$ is globally asymptotically stable. The theorem follows. □

3.6 An example of Primal-Dual Quick Kelly Control

Now, we give an example of PDQKC. The source control law (12) is kept without change. Then the link control law of PDQKC is presented as follow:

$$\dot{\delta}_l = \begin{cases} \theta \cdot (\frac{y_l}{c_l} - 1), & \frac{y_l}{c_l} < 1.2 \\ 0.1|\delta_l|, & \frac{y_l}{c_l} > 1.2 \end{cases} \tag{24}$$

$$p_l = \omega^2 + \exp(\delta_l) \ \exp[\lambda(\frac{y_l}{c_l} - 1)] \ \frac{y_l}{c_l} \tag{25}$$

where λ is a positive constant. This control law consists of a link price discounter update law and a static price function.

4. SIMULATION RESULTS

Consider a simple four-source/three-link example which is presented in (Wen *et al.*, 2004). The corresponding routing matrix is

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}.$$

Its topology structure is shown in Fig.3. We assume that the link capacities are all c_0. For algorithms (KC, PDQKC) that follow proportional fairness criterion, suppose that all source utility functions are $U_r(x_r) = \log(x_r)$. The solution to the optimization problem (1) is (Wen *et al.*, 2004)

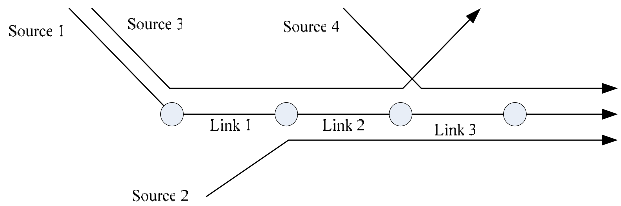

Fig. 3. Network example

$$x^* = [0.25c_0 \quad 0.25c_0 \quad 0.5c_0 \quad 0.5c_0]^T$$
$$y^* = [0.75c_0 \quad c_0 \quad c_0]^T.$$

For algorithms (MKC, PDQKC) that follow Max-min fairness criterion, the solution to the optimization problem (1) is

$$x^* = [\frac{1}{3}c_0 \quad \frac{1}{3}c_0 \quad \frac{1}{3}c_0 \quad \frac{1}{3}c_0]^T, \qquad y^* = \begin{bmatrix} \frac{2}{3}c_0 & c_0 & c_0 \end{bmatrix}^T.$$

The initial source rate is set to

$$x(0) = [0.1c_0 \quad 0.2c_0 \quad 0.25c_0 \quad 0.15c_0]^T.$$

The time step is set to 0.2. Consider the following controller:

KC: (9), (11) and (16). Set: $\kappa_r = 0.5$, $w_r = 1$ and $\varepsilon = 0.02c_0$.

MKC: (11) (12) and (13). Set: $\kappa_r = 0.5$, $w_r = 1$

PDQKC: (12) (13) (24) and (25). Set: $\kappa_r = 0.5$, $w_r = 1$, $\theta = 0.05$, $\lambda = 2.0$, $\omega = 0.000001$, $\delta(0) = -0.8\ln c_0$.

To compare the bandwidth scalability of these controllers, a Euclidean distance function, which describe the relative distance between the current state and the equilibrium point, is defined by (26).

$$d = \sqrt{\sum_r (\frac{x_r - x_r^*}{x_r^*})^2 + \sum_l (\frac{y_l - y_l^*}{y_l^*})^2}. \tag{26}$$

It is clear that, $d \to 0$ as $x \to x^*$ and $y \to y^*$. Set $c_0 = 100$, $c_0 = 1000$ and $c_0 = 10000$ respectively. By logarithmizing the time steps, Fig.4-6 show the bandwidth scalability of these controllers. In figure 4, d approximately tend to zero as the time increases. The convergence time of d is proportional to the link capacities. In figure 5, d is sharply decreased at the early state and all the links tend to their equilibrium exponentially no matter how large the link capacities are. However, except for the early state, the process of d is similar to Fig. 4. In Fig. 6, PDQKC shows its powerful bandwidth scalability and fast convergence to efficiency and fairness. d converges to zero in a short time. As the link capacities grow from 100 to 10000, only small variation of the convergence time is observed. Although MKC controller offers exponential convergence to efficiency, its source convergence rate to the optimization equilibrium is much slower than PDQKC.

5. CONCLUSIONS

This paper has presented a group of novel nonlinear network congestion control algorithms called quick Kelly control (QKC). In QKC, link utilization ratio information is used as feedback signal to accelerate convergence rate and improve the bandwidth scalability. We prove the stability of the PDQKC algorithm. PDQKC has very powerful bandwidth scalability. Compare to the classical Kelly control and MKC, PDQKC can achieve fast convergences to the optimization equilibrium. All the analysis and simulation results based on an assumption that there is no delay in the loop. Our future work involves improvement of dynamic performance and delay stability.

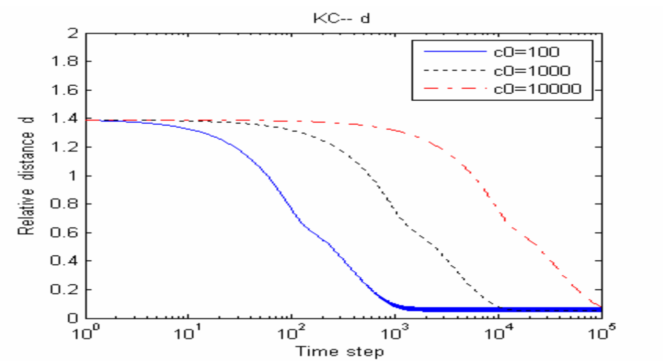

Fig. 4. d of KC

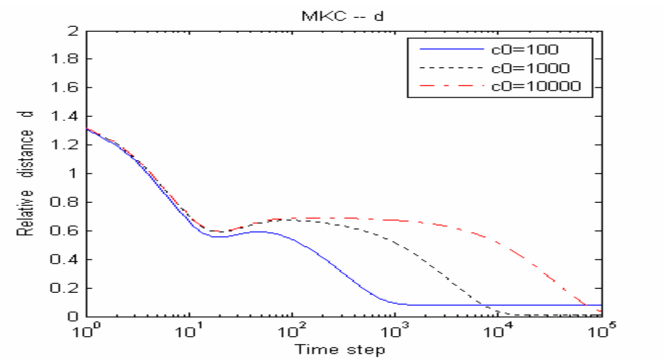

Fig. 5. d of MKC.

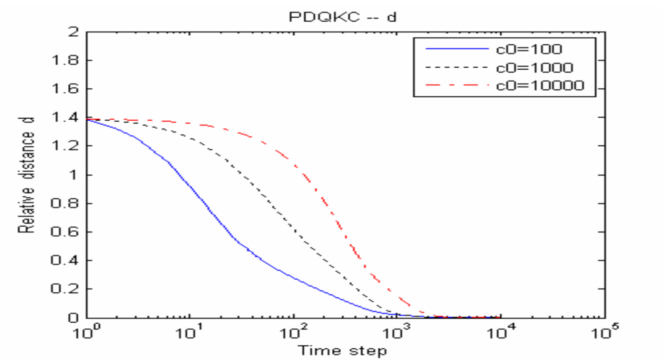

Fig. 6. d of PDQKC.

5. ACKNOWLEDGEMENT

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