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Self-tuning Continuous-time Generalized Minimum Variance Control

Ryota Hoshino * **Yasuchika Mori** *

* *Graduate School of System Design, Tokyo Metropolitan University, Asahigaoka6-6, Hino, Tokyo, Japan (E-mail: hoshino@kisl.tmit.ac.jp), (E-mail: ymori@cc.tmit.ac.jp)*

Abstract: There is Generalized Minimum Variance Control(GMVC) in one of the design methods of Self Tuning Control(STC). In general, STC is applied discrete-time design. According to selection of sampling period, discrete-time design has possibilities to generate unstable zero and odd time-delay, and fail to get a clear grasp of the controlled object at all times. Then, we propose continuous-time design of GMVC. For this reason it is called CGMVC. Because continuous-time design need not choose sampling period, CGMVC is easire design method than GMVC. In this paper, we confirm some advantage of CGMVC, and denote numerical example.

1. INTRODUCTION

In the process control such as chemical plants that vary pressure and temperature momentarily, the closed-loop system performance is made worse by different time-delay exists in MIMO systems and property fluctuation of controlled object. In addition, high stability, simple structure of control system, and comprehensible physical meaning of design parameter are demanded in chemical plants because it has a number of chancy processes. Then, STC is applied on such as systems. GMVC of discrete-time design is given in one of the design methods of STC. See Clarke and Gawthrop [1975] and [1979]. Some properties of GMVC are simple structure and polynomial approach to designing time-delay predictors. See Astrom [1970]. But, GMVC has possibilities to generate unstable zero (for example Astrom et al. [1980]) and odd time-delay. Furthermore, if we choose long sapling period, it is difficult to get a clear grasp of the controlled object at all times. Then, we propose continuous-time design of GMVC.

2. CONTINUOUS-TIME DESIGN OF GMVC (CGMVC)

In this section, the design method of CGMVC is described. In particular, we consider servo-type CGMVC. See Mori et al. [1998]. Servo-type design approach has advantageous by applying to STC. This is considered in numerical example section.

2.1 About controlled object

The controlled object is shown in form that matched to Controlled Auto-Regressive and Moving Average (CARMA) model. The controlled object is written as

$$A(d)y(t) = B(d)u(t-L) + C(d)\xi(t) \qquad (1)$$

where $A(d)$, $B(d)$, and $C(d)$ are polynomials in the differential operator d as

$$A(d) = d^n + a_{n-1}d^{n-1} + \ldots + a_0 \qquad (2)$$

$$B(d) = b_m d^m + b_{m-1}d^{m-1} + \ldots + b_0 \qquad (3)$$

$$C(d) = c_l d^l + c_{l-1}d^{l-1} + \ldots + c_0. \qquad (4)$$

It is assumed $a_0 \neq 0$, $b_0 \neq 0$, $c_0 \neq 0$, and $n \geq m$. $y(t)$, $u(t)$, and $\xi(t)$ are the system output, control input and white-noise of average value 0 and variance is σ^2. And, time-delay is represented as e^{-Ld}.

2.2 Control system design of servo-type CGMVC

In here, the design method of servo-type CGMVC is described. The controlled object form that matched to CARMA model of appearance that contains integrator is written as

$$d \cdot A(d)y(t) = d \cdot B(d)u(t-L) + d \cdot C(d)\xi(t). \qquad (5)$$

First of all, generalized output is defined as

$$h(t+L) = P(d)y(t+L) + Q(d)u(t) - R(d)w(t+L) \qquad (6)$$

where $w(t)$ is desired value and $P(d)$, $Q(d)$, $R(d)$ are polynomial weight factor, and the property of closed-loop system is detuned by this polynomial weigt factor. Cost function is written as

$$J = E\{h(t+L)^2\}. \qquad (7)$$

Expectation is represented E of (7). (7) is obtained by using generalized output (6). The control law that minimizes evaluation function (7) is called CGMVC. But, in this time, futural output of time-delay $y(t+L)$ must be obtained to minimize (7) because generalized output (6) is included it. Then, Diophantine-equation is introduced, and predict futural output of time-delay $y(t+L)$. Diophantine-equation is implied that disturbance term is divided futural and past part. And, the control parameter can be obtained by Diophantine-equation. In descrete-time design of GMVC, unique solution can be obtained by deciding the degree of control paramete. However, in continuous-time design, time-delay is expressed by irrational function. So, Diophantine-equation can not be resolved. Then, time-delay e^{-Ld} is transformed into rational function by Laguerre approximation. Laguerre approximation written as

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$$e^{-Ld} = \left(\frac{1 - \frac{Ld}{2j}}{1 + \frac{Ld}{2j}} \right)^j . \quad (8)$$

And, by expressing controlled object as $\frac{\left(1+\frac{Ld}{2j}\right)^{-j} B(d)}{\left(1+\frac{Ld}{2j}\right)^{-j} A(d)}$, Diophantine equation is defined as

$$P_1(d)C(d) = d \cdot \left(1 + \frac{Ld}{2j}\right)^{-j} A(d)P_2(d)E(d) + \left(\frac{1 - \frac{Ld}{2j}}{1 + \frac{Ld}{2j}} \right)^j F(d) \quad (9)$$

where

$$E(d) = e_{n_e} d^{n_e} + e_{n_e - 1} d^{n_e - 1} + \ldots + e_0 \quad (10)$$

$$F(d) = f_{n_f} d^{n_f} + f_{n_f - 1} d^{n_f - 1} + \ldots + f_0 \quad (11)$$

are control parameters, respectively. A unique solution can be obtained by deciding control paramete degree as

$$n_e = j - 1, \quad n_f = max\{n + n_p, n_p + l\}. \quad (12)$$

Using (6) and (9), the control low can be obtained as

$$u(t) = \frac{1}{d} \frac{C(d)R(d)w(t+L) - \frac{F(d)}{P_2(d)} y(t)}{\left(1 + \frac{Ld}{2j}\right)^{-j} B(d)E(d) + C(d)Q(d)} . \quad (13)$$

When closed-loop system is composed, it is written as

$$y(t) = \frac{B(d)R(d)}{T'(d)} w(t) + \frac{d \cdot \left(1 + \frac{Ld}{2j}\right)^{-j} B(d)E(d) + C(d)Q(d)}{T'(d)} \xi(t) \quad (14)$$

where

$$T'(d) = P(d)B(d) + d \cdot A(d)Q(d) \quad (15)$$

is the closed-loop characteristic equation. Block diagram of continuous-time design of GMVC is shown in Fig.1.

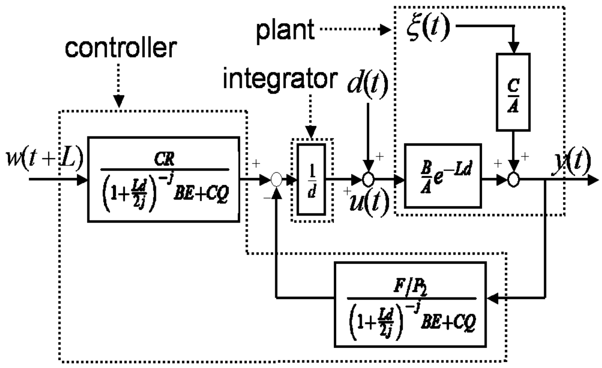

Fig. 1. Block Diagram of Continuous-time Design of GMVC

2.3 About the requirement of control degree

In discrete-time desining, using shift-operator q^{-1}. On the other hand, in continuous-time designing, using differential operator d. Therefore, we must note that the controller become non-proper-transfer-function. First of all, the prepositive-amends-controller $G_{CR}(d)$ is confirmed. The prepositive-amends-controller is written as

$$G_{CR}(d) = \frac{C(d)R(d)}{(1 + \frac{Ld}{2j})^{-j} B(d)E(d) + C(d)Q(d)} = \frac{C(d)R(d)(1 + \frac{Ld}{2j})^j}{B(d)E(d) + (1 + \frac{Ld}{2j})^j C(d)Q(d)} \quad (16)$$

where

$$deg \left[C(d)R(d)(1 + \frac{Ld}{2j})^j \right] = n_c + j \quad (17)$$

$$deg \left[B(d)E(d) \right] = n_b + j - 1 \quad (18)$$

$$deg \left[C(d)Q(d)(1 + \frac{Ld}{2j})^j \right] = n_c + j. \quad (19)$$

As a result, the numerator-degree (17) is equal to second-term-degree of denominator (19). In other words, the prepositive-amends-controller (16) is not composed non-proper-transfer-function.

Next, the feedback-controller $G_{CF}(d)$ is confirmed. The feedback-controller is written as

$$G_{CF}(d) = \frac{F(d)/P_2(d)}{(1 + \frac{Ld}{2j})^{-j} B(d)E(d) + C(d)Q(d)} = \frac{F(d)(1 + \frac{Ld}{2j})^j}{P_2(d)B(d)E(d) + (1 + \frac{Ld}{2j})^j C(d)P_2(d)Q(d)} \quad (20)$$

where

$$deg \left[F(d)(1 + \frac{Ld}{2j})^j \right] = j + max\{n_a + n_p - 1, n_c + n_p\} \quad (21)$$

$$deg \left[P_2(d)B(d)E(d) \right] = n_b + n_p + j - 1 \quad (22)$$

$$deg \left[C(d)P_2(d)Q(d)(1 + \frac{Ld}{2j})^j \right] = n_c + n_p + j. \quad (23)$$

The controlled object assumed to be intensity proper transfer function, and degree condition is

$$n_a > n_b. \quad (24)$$

In here, integrator is introduced and servo-type control system is considered. As a result, the requirment of degree is

$$n_a \geq n_c. \quad (25)$$

The degree is chosen like (25), the feedback controller does not become proper transfer fanction. But, when STC is applied, system identification accuracy of noise-term $C(d)$ is wrong. In fact, applying degree-requirment (25) is unsuitable. And so, pole is supplemented to feedback-controller based on the idea of Inxact-differential. Inxact-differential is written as

$$F(d) = \frac{f_{n_f} d^{n_f}}{\left(1 + \gamma f_{n_f}^{\frac{1}{n_f}} d\right)^{n_f}} + \frac{f_{n_f - 1} d^{n_f - 1}}{\left(1 + \gamma f_{n_f - 1}^{\frac{1}{n_f - 1}} d\right)^{n_f - 1}} + \ldots + f_0 \quad (26)$$

where γ is differential-gain. In general, the value of differential-gain is used about 0.1.

3. ABOUT THE ADVANTAGE OF CGMVC

In this section, about the advantage of CGMVC is described.

Remarks.

(1) Control parameter degree
In discrete-time designing, sampling period is important design parameter. Selecting long sampling period, it is difficult to get a clear grasp of the controlled object at all times. On the other hand, because of relations of time-delay-step and time-delay-approximation-degree, selecting short sampling period, control parameter degree is increased. In other words, complex calculation is demanded and we must deal with complex algorithm. In continuous-time designing, however, the controller degree is constant even if time-delay changes. Moreover, because sampling period need not be selected in continuous-time-designing-approach, cotrol system algorithm can be desingned simply and easily .

(2) Property fluctuation of controlled object
In discrete-time designing, remainder time-delay is generated by changing time-delay gradually, where the remainder time-delay that it is cannot be divided by sampling period. In other words, remainder time-delay cannot be considered in control-system-designing. In continuous-time designing, houever, remainder time-delay is not generated even if time-delay changes gradually.

(3) Unstable zero
In this time, system identification is used in discrete-time domain. So this control system is hybrid: discrete-time system identification and continuous-time designing. Discretization in short sampling period to identifying controlled object may generate unstable zero(for example Astrom et al. [1980]). In discrete-time designing, the control performance is declined by this unstable zero. In continuous-time designing, because discrete-time identification result is transformed into continuous-time transfer function, unstable zero is not necessary to treat. This is considered in later section.

4. ABOUT THE SYSTEM IDENTIFICATION

In this section, estimation algorithm of mathematical model is described. When the model of controlled object is inaccurate, it is necessary that the mathematical model is estimated based on the input-output data. Especially, when dynamic characteristic of control systems are changed, the controlled object might not be able to be kept steady. In this case, it is necessary that the controlled object that changes the dynamic characteristic hourly is estimated online. A lot of discrete-time system identification method is used in STC. See for example Astrom and Wittenmark [1973]. Moreover, continuou-time system identification is reported too. See for example Young [1966]. In this paper, the former method is applied: using discrete-time identification. Then, recursive least-squares method is used in this time.

- Recursive least-squares method
The algorithm of recursive least-squares method is given as

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{P(k-1)\varphi(k)}{\lambda + \varphi^T(k)P(k+1)\varphi(k)}\epsilon(k) \tag{27}$$

$$\epsilon(k) = y(k) - \varphi^T(k)\hat{\theta}(k-1) \tag{28}$$

$$P(k) = \frac{P(k-1)}{\lambda} - \frac{1}{\lambda}\left\{\frac{P(k-1)\varphi(k)\varphi^T(k)P(k-1)}{\lambda + \varphi^T(k)P(k-1)\varphi(k)}\right\} \tag{29}$$

where $\theta(k)$,$\phi(k)$,and $\epsilon(k)$ are controlled object parameter vector, input and output data, and prediction error. Parameter λ is called forgetting factor.

5. ABOUT THE DELETION OF UNNECESSARY ZERO

In this time, control system is composed of the hybrid system. Against controlled object with the second degree or more differences between denominator and numerator, choosing short sampling period causes unstable zero generation that did not exist in continuous-time domain. In discrete-time designing, the control performance is declined by this unstable zero. To deleting unstable and unnecessary zero, using approximation that used δ-operator in one of techniques. See for example Goodwin et al. [1986]. The unnecessary zero is generated by discretization and this is not existed in continuous-time transfer function. The unstable zero can be deleted by using δ-operator, however, short sampling period must be selected in this method. As a result, control-parameter-solution of discrete-time GMVC becomes complication by increasing degree. Then, discrete-time-mathmatical-model(DT-model) is transformed into continuous-time-mathmatical-model(CT-model) directly, and cntrol system is designed by using this CT-model. This method has the advantage that computational complexities are fewer than δ-operator approximation approach. The numerical example of deleting unnecessary zero is shown as follows.

- Example
This example was using MATLAB. First of all, controlled object is assumed as

$$G(s) = \frac{31}{300s^3 + 340s^2 + 41s + 10}. \tag{30}$$

And, continuous-time pole-zero configuration of (30) is shown in Fig.2.

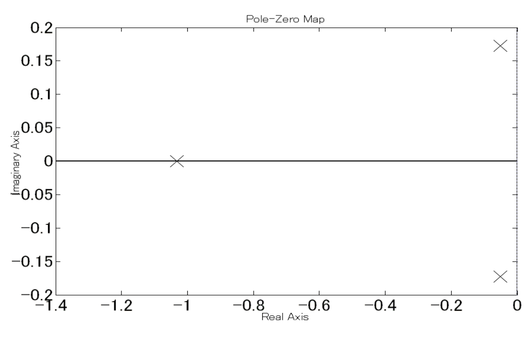

Fig. 2. Continuous-time Pole-Zero Configuration

In this time, controlled object is identified in discrete-time domain. By identifying controlled object (30) in discrete-time domain, DT-model as

$$G(q) = \frac{10^{-8} \times (1.717q^2 + 6.85q + 1.708)}{q^3 - 2.989q^2 + 2.977q - 0.9887} \quad (31)$$

can be obtained. (31) is identified by sampling period 0.01 second. And, discrete-time pole-zero configuration of (31) is shown in Fig.3.

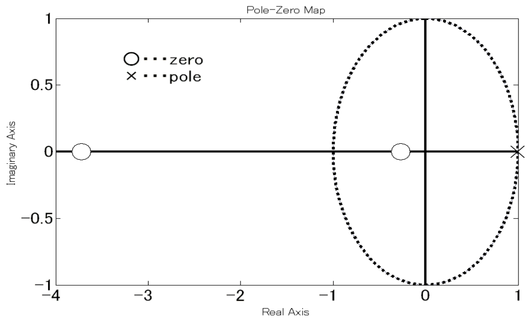

Fig. 3. Discrete-time Pole-Zero Configuration

The generation of unstable zero can be confirmed. This unstable zero is not exist in continuous-time domain. The DT-model (31) is transformed into CT-model directly, and 300 is multiplied to denominator and numerator, respectively. As a result, CT-model can be obtained as

$$G(s) = \frac{(-2.362 \times 10^{-16})s^2 - (8.961 \times 10^{-14})s + 31}{300s^3 + 340s^2 + 41s + 10}. \quad (32)$$

The coefficient of s^2 and s of numerator of (31) are very small scale. Then, by approximating this small-scale-cofficient is zero, we can obtained (30).

Thus CT-model that does not have unstable and unnecessary zero can be obtained. In other words, control performance is not declined even if discrete-time identification is applied to continuous-time designing.

6. NUMERICAL EXAMPLE

In this section, numerical example of STC with CGMVC is described. The controlled object is assumed as

$$G(s) = \frac{31.5}{1.28s^3 + 8.16s^2 + 9s + 1} e^{-5s}. \quad (33)$$

This controlled object is referred to water level control model that introduced by *Modern Control Systems*. See Richard et al. [2001].

6.1 Assumption of the numerical example

- Property fluctuation
  The controlled-object-gain is assumed as decreasing from 31.5 to 26.625 between 400 seconds and 500 seconds. At the same time, controlled-object-time-delay is assumed as increasing from 5 seconds to 10 seconds. In other words, the controlled object after property fluctuation is written as
  $$G(s) = \frac{23.625}{1.28s^3 + 8.16s^2 + 9s + 1} e^{-10s}. \quad (34)$$
- Desired value
  The desired value is input at the time of starting simulation. The desired value step size is 1.
- Disturbance
  The disturbance is input at the time of 250 seconds and 900 seconds. The disturbance step size is 0.01.

6.2 Parameter setting value of the numerical example

- The observation noise $\cdots$ White-noise of average value 0 and variance is 0.01^2.
- The Amplitude of M-sequence signal $\cdots$ 0.009.
- The time cycle of M-sequence signal $\cdots$ 0.1 second.
- The degree of time-delay-approximation $\cdots$ 4.
- The weighting factor $\cdots$ $P(d) = 1, R(d) = 1, Q(d) = 450$.
- The noise-term $C(d)$ is simulated as $C(d) = 1$.
- Initial value of controlled object $\cdots$ The accurate control parameter of system is known for 0.5 seconds beforehand. Because, this is treatment to prevent the initial excessive response in this simulation.

6.3 Result of the numerical example

The gain-identification-result at the time of property fluctuation is showed in Fig.4. The numerical-example-result of STC is showed in Fig.5.

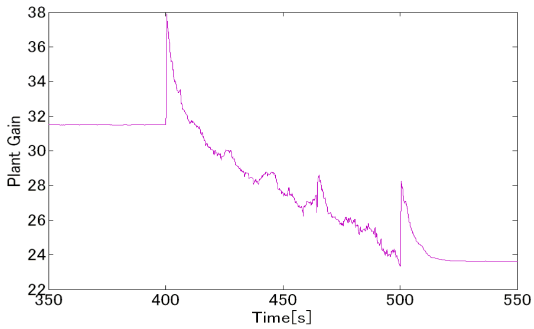

Fig. 4. Gain-Identification-Result at the time of property fluctuation

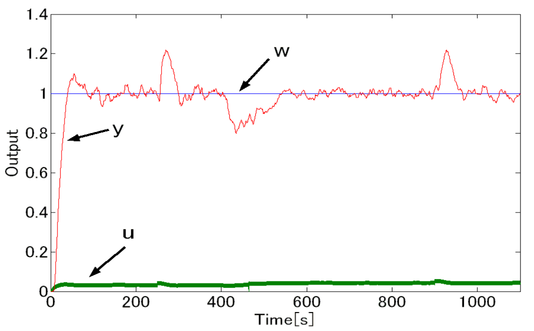

Fig. 5. Numerical-Example-Result of STC

6.4 Discussion of the numerical example

The error between actual-gain-decreasing and identification-result can be confirmed at the time of property fluctuation. As a result, the response is away from the desired value. This will be influence of time-delay-fluctuation. In this numerical example, time-delay was fluctuated gradually. So in control system designing, there is a difference between actual-time-delay and control-system-designing-time-delay at all times. The meaning of control-system-designing-time-delay is time-delay-value used for control-system-designing. However, even if such a few differences exist, big-disorder-response is not generated. This is because integrator is introduced in control system. This is an advantage to design servo-type CGMVC. Moreover, in continuous-time designing, the remainder time-delay that cannot be divided by sampling period is not generated even if time-delay changes gradually. So the response is not turbulence.

7. CONCLUSION

In this paper, STC that used CGMVC was simulated, and the effectiveness was shown. First of all, the effectiveness of degree was shown. When control system is designed with CGMVC, constant-degree-controllers can be designed at any time regardless of the sampling period. Therefore, controller can be simply calcurated. Second, the effectiveness of unnecessary-zero-influence was shown. Unnecessary zero can be easily deleted by continuous-time designing. Third, the effectiveness of time-delay-fluctuation is shown. It is possible to control without generating remainder time-delay even if time-delay changes gradually. Finaly, the effectiveness of sampling period is shown. Because it is not necessary to consider about sampling period, the controller can be simply designed in simple algorithm. Thus, it is shown that control system can be designed more easily, when it is designed with CGMVC.

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