Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Adaptive Backstepping Control for Vibration Reduction in a Structure with Frictional and Hysteretic Actuators

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Abstract: This paper presents an alternative solution to the vibration problem in civil engineering structures. This kind of structures are characterized by the uncertainties of the parameters that describe their dynamics, such as stiffness and damping coefficients. Moreover, the actuators used to mitigate the vibrations caused by earthquakes are usually nonlinear devices with frictional or hysteretic dynamics. In order to account for the uncertainties and the nonlinearities, an adaptive backstepping controller is proposed. Semiactive control force is applied to the structure through a magnetorheological damper together with a passive frictional actuator that isolates the base of a 10-story building. The effectiveness of the controller is shown by comparing the simulation results of the cases when the structure is isolated and when it is not.

1. INTRODUCTION

The protection of structures such as buildings, bridges and towers against the hazardous vibrations caused by earthquakes and strong winds has been a major concern whose solution has become an active research field. This problem has been approached in several ways. One of the most usual solutions consists in placing actuators in the structure so as to vary its dynamics (Carlson [1999], Zapateiro and Luo [2007]). Passive control devices, such as elastomeric bearings, were first used and have proven to be a good solution. However, once tuned, they are not able to adapt to the changes in external loading conditions (Johnson et al. [1998]). Active control devices, such as the active tuned mass dampers (TMD), solve the adaptation problem of passive ones at the cost of high energy requirements and the risk of instability. The dependency of the active dampers on the electrical network can be problematic because the control device is intended to operate during events such as seismic motions when electricity breaks are prone to occur. Semiactive control strategies are particularly promising in solving the adaptation and energy problems (Dyke et al. [1998]).

Semiactive devices combine the features of passive and active ones: their properties can be adapted in real time, are inherently stable and do not require large power sources to operate (Yang et al. [2002]). Among semiactive control devices, magnetorheological (MR) dampers are particularly interesting because of the high damping force they can produce with low energy requirements (being possible to operate with batteries), simple mechanical design and low production costs. The damping force of MR dampers is produced when the MR fluid inside the device changes its rheological properties in the presence of a magnetic field. In other words, by varying the magnitude of an external magnetic field, the MR fluid can reversibly go from a solid state to a semisolid one or viceversa (Zapateiro et al. [2007]). Despite the above advantages, MR dampers exhibit a complex nonlinear behavior that makes modeling and control a challenging task. The force response of the MR damper to a velocity input is a hysteretic loop whose shape depends on the magnitude of the magnetic field. Several MR damper models have been proposed; see for example the study by (Butz and von Stryk [2002]).

Diverse control theories have been applied to mitigate vibrations in structures equipped with MR dampers. Some of them include the clipped optimal control (Dyke et al. [1996]), control based on Lyapunov's stability theory (Luo et al. [2003]), neural network and fuzzy logic control (Kim and Roschke [2006], Schurter and Roschke [2001]). In this paper, an adaptive backstepping control is presented as an extension to the work by (Villamizar [2005]) and (Luo et al. [2007]). Adaptation of some uncertain structure parameters is now considered. The objective is to design a controller for an MR damper that together with a frictional actuator isolates the structure of a 10-story building from the ground movement. This paper is organized as follows: Section 2 explains the mathematical model of the base isolated 10-story building. Next, the adaptive backstepping controller is formulated in Section 3. A numerical example is shown in Section 4. Finally, conclusions and future work are drawn in Section 5.

2. SYSTEM DESCRIPTION

Consider an uncertain 10-story building whose base is isolated by means of a passive frictional actuator and an MR damper, as shown in Fig. 1. Consider also that the system is perturbed by an incoming earthquake. The system dynamics can be divided into two subsystems, namely, the main structure (S_r) and the base (S_c).

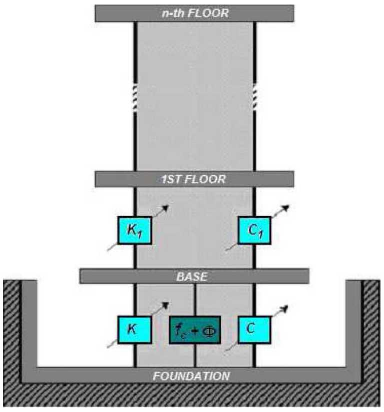

Fig. 1. Base-isolated 10-story building.

$$S_r : \mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = [c_1, ..., 0]^T \dot{y} + [k_1, ..., 0]^T y \quad (1)$$

$$S_c : m\ddot{y} + c\dot{y} + ky + f_{bf} = \Phi\left(\dot{y}, \dot{d}\right) + f_g + f_c \quad (2)$$

$$\Phi\left(\dot{y}, \dot{d}\right) = -sgn\left(\dot{y} - \dot{d}\right)\left[\mu_{max} - \Delta\mu e^{-\nu\left|\dot{y}-\dot{d}\right|}\right] Q \quad (3)$$

$$f_g = -c\dot{d} - kd \quad (4)$$

$$f_{bf} = c_{bf}(\dot{y} - \dot{x}_1) + k_{bf}(y - x_1) \quad (5)$$

In (1), $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K} \in \mathbb{R}$ are positive definite matrices representing the mass, damping coefficients and stiffness of the structure, respectively. The structure of those matrices is shown in (6). The structure remains in the linear region due to the effect of the passive frictional isolator.

$$\begin{aligned} \mathbf{M} &= diag\,(m_i)\,,\ i = 1, 2, ..., n \\ \mathbf{C} &= \begin{bmatrix} c_1 + c_2 & -c_2 & 0 & 0 \\ -c_2 & c_2 + c_3 & -c_3 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & -c_n & c_n \end{bmatrix} \\ \mathbf{K} &= \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & -k_n & k_n \end{bmatrix} \end{aligned} \quad (6)$$

$\mathbf{x} = [x_1, x_2, ..., x_n]^T \in \mathbb{R}^n$ is the structure horizontal absolute displacement vector (measured with respect to an inertial frame), $y \in \mathbb{R}$ is the horizontal base absolute displacement and d is the displacement of the seismic excitation. Equation (2) consists of a linear part, described by the mass m, damping coefficient c and stiffness k of the base, plus a nonlinear one, characterized by the dynamics of the actuators, Φ and f_c. Equation (3) describes the passive actuator dynamics. μ is the friction coefficient, ν is a constant, Q is the force normal to the surface, μ_{max} is the the coefficient for high sliding velocity and $\Delta\mu$ is the difference between μ_{max} and the friction coefficient for low sliding velocity. Equation (5) represents the linear force caused by the coupling of the base and the main structure. This force is represented by the damping coefficient $c_{bf} = c_1$, the stiffness $k_{bf} = k_1$ and the relative velocity $\dot{y} - \dot{x}_1$ between the base and the first floor of the structure.

The term f_c in (2) accounts for the dynamics of the semiactive actuator (the MR damper). Such dynamics is given by the Bouc-Wen model, as stated in (7):

$$f_c = -\delta(v)\dot{y} - \alpha(v)z \quad (7)$$

$$\dot{z} = -\gamma|\dot{y}|z|z|^{n-1} - \beta\dot{y}|z|^n + A\dot{y} \quad (8)$$

$$\alpha(v) = \alpha_a + \alpha_b v; \quad \delta(v) = \delta_a + \delta_b v \quad (9)$$

where z is an unmeasurable evolutionary variable, the parameters γ, β, n and A are constant values that can be used to adjust the shape of the hysteresis loop. $\alpha(v)$ is related to the hysteretic behavior of the damper and $\delta(v)$ is a damping coefficient; they are both voltage dependent. The voltage is the control signal to be generated: it is the input to a PWM system that generates the current, which in turn creates the magnetic field used to control the MR damper.

The following propositions about the intrinsic stability of the structure will be used in formulating the control law.

Proposition 1. The unforced main structure subsystem (1) (i.e., with the coupling term $[c_1, 0, ..., 0]^T\dot{y} + [k_1, 0, ..., 0]^T y \equiv 0, t \geq 0$) is globally exponentially stable for any bounded initial conditions.

Proposition 2. If the coordinates $(y, \dot{y})$ of the base and the coupling term $[c_1, 0, ..., 0]^T\dot{y} + [k_1, 0, ..., 0]^T y$ are uniformly bounded, then the main structure subsystem is stable and the coordinates $(\mathbf{x}, \dot{\mathbf{x}})$ of the main structure are uniformly bounded for all $t \geq 0$ and any bounded initial conditions.

The proofs of these propositions are detailed in (Luo et al. [2000]).

3. ADAPTIVE BACKSTEPPING CONTROLLER

The control objective is to reduce the absolute response in the base level and thus to make the base isolator work in its elastic region and also to decouple asymptotically the dynamic motion of the main structure from the base motion. In the control design, it is taken into account the uncertainties in the stiffness and damping coefficients of the base and main structure. Denote $y_1 = y$ and $y_2 = \dot{y}$. In order to design the backstepping controller, (2) is rewritten in state space form:

$$\dot{y}_1 = y_2 \quad (10)$$

$$\dot{y}_2 = -\frac{1}{m}\left[cy_2 + ky_1 + f_{bf} - \Phi - f_g - f_c\right] \quad (11)$$

c, k, c_{bf} and k_{bf} are uncertain parameters representing the damping coefficients and stiffness. The following control law asymptotically attenuates the vibrations and stabilizes the main structure (Luo et al. [2007]):

$$v = -\frac{(\delta_a - mh_1 + \overline{c})y_2 + y_1\overline{k} + \alpha_a z - \Phi - f_g}{\alpha_b z + \delta_b y_2} + \frac{-\overline{c_{bf}}(y_2 - \dot{x}_1) - \overline{k_{bf}}(y_1 - x_1) + me_1 + mh_2 e_2}{\alpha_b z + \delta_b y_2} \tag{12}$$

for all $\alpha_b z + \delta_b y_2 \neq 0$, otherwise $v = 0$. h_1 and h_2 are positive constants. In order to estimate the uncertain structural stiffness and damping parameters $\overline{c}$, $\overline{k}$, $\overline{c_{bf}}$ and $\overline{k_{bf}}$, the following adaptation laws are proposed:

$$\dot{\overline{c}} = \frac{r_1}{m}e_2 y_2 \tag{13}$$
$$\dot{\overline{k}} = \frac{r_2}{m}e_2 y_1 \tag{14}$$
$$\dot{\overline{c_{bf}}} = \frac{r_3}{m}e_2(y_2 - \dot{x}_1) \tag{15}$$
$$\dot{\overline{k_{bf}}} = \frac{r_4}{m}e_2(y_1 - x_1) \tag{16}$$

with $r_i, i = 1, ..., 4$ being positive constants. e_1 and e_2 are standard backstepping variables given by:

$$e_1 = y_1 \tag{17}$$
$$e_2 = y_2 - \alpha_1 \tag{18}$$
$$\alpha_1 = -h_1 e_1 \tag{19}$$

Proof Consider the following Lyapunov function candidate:

$$V = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2r_1}\widetilde{c}^2 + \frac{1}{2r_2}\widetilde{k}^2 + \frac{1}{2r_3}\widetilde{c_{bf}}^2 + \frac{1}{2r_4}\widetilde{k_{bf}}^2 \tag{20}$$

where $\widetilde{c} = c - \overline{c}$, $\widetilde{k} = k - \overline{k}$, $\widetilde{c_{bf}} = c_{bf} - \overline{c_{bf}}$ and $\widetilde{k_{bf}} = k_{bf} - \overline{k_{bf}}$. The derivative of V is given by:

$$\begin{aligned} \dot{V} =& e_1\dot{e}_1 + e_2\dot{e}_2 + r_1^{-1}(c - \overline{c})\dot{\overline{c}} + r_2^{-1}(k - \overline{k})\dot{\overline{k}} \\ &+ r_3^{-1}(c_{bf} - \overline{c_{bf}})\dot{\overline{c_{bf}}} + r_4^{-1}(k_{bf} - \overline{k_{bf}})\dot{\overline{k_{bf}}} \end{aligned} \tag{21}$$

Deriving (17) - (19) and substituting (10) and (11) into the result yields:

$$e_1\dot{e}_1 = e_1 e_2 - h_1 e_1^2 \tag{22}$$

$$\begin{aligned} e_2\dot{e}_2 =& -\frac{e_2}{m}[-\Phi - f_g + \alpha_a z + \delta_a y_2] + h_1 y_2 e_2 \\ &- \frac{c}{m}e_2 y_2 - \frac{k}{m}e_2 y_1 - \frac{e_2(\alpha_b z + \delta_b y_2)}{m}v \\ &- \frac{c_{bf}}{m}e_2(y_2 - \dot{x}_1) - \frac{k_{bf}}{m}e_2(y_1 - x_1) \end{aligned} \tag{23}$$

Substitution of (12) into (23) yields:

$$\begin{aligned} e_2\dot{e}_2 =& -\frac{1}{m}e_2 y_2(c - \overline{c}) - \frac{1}{m}e_2 y_1(k - \overline{k}) - h_2 e_2^2 \\ &- e_1 e_2 - \frac{1}{m}e_2(y_2 - \dot{x}_1)(c_{bf} - \overline{c_{bf}}) \\ &- \frac{1}{m}e_2(y_1 - x_1)(k_{bf} - \overline{k_{bf}}) \end{aligned} \tag{24}$$

Substitution of (13) - (16), (22) and (24) into (21) yields:

$$\dot{V} = -h_1 e_1^2 - h_2 e_2^2 < 0 \tag{25}$$

According to Lyapunov's stability theory, $e_1 \to 0$ and $e_2 \to 0$. Consequently, $y = y_1 = e_1 \to 0$ and $\dot{y} = y_2 = e_2 + h_1 e_1 \to 0$. According to Propositions 1 and 2, the vibration of the base is asymptotically attenuated and the asymptotic stability of the main structure is guaranteed.$\diamond$

The controller proposed in (12) contains some unmeasurable variables (z, f_g, Φ). In order to address these problems, some assumptions and approximations are made. First, it is assumed that the unknown seismic excitation $d(t)$ and $\dot{d}(t)$ are bounded by $|d(t)| \leq D_d$ and $|\dot{d}(t)| \leq D_v$ and thus, the unknown disturbance force f_g in (4) is bounded by

$$|f_g(t)| \leq F \;\; \forall t \geq 0 \tag{26}$$

with D_d, D_v and F being known positive constants.

On the other hand, the passive control force generated by the frictional actuator can make small the relative movements of the structure during the seismic excitation. Thus, the following relationship holds:

$$\nu|y_2 - \dot{d}| < 1 \Rightarrow |y_2 - \dot{d}| < 1/\nu \tag{27}$$

This fact can be verified with some standard earthquakes, as shown in Figure 2. In order to kame the simple approximation of the exponential function in the dynamic equation (3) of the frictional base isolator, the following Euler approximation is used:

$$e^{-\nu|y_2 - \dot{d}|} \simeq \frac{1}{1 + \nu|y_2 - \dot{d}| + \frac{\nu^2}{2}|y_2 - \dot{d}|^2 + \frac{\nu^3}{6}|y_2 - \dot{d}|^3} \tag{28}$$

By denoting $|y_2 - \dot{d}|_0$ as the maximum value of $|y_2 - \dot{d}|$, the passive actuator force can be approximated as:

$$\overline{\Phi} \leq \Delta_0 + \Delta_1 D_v - \Delta_1 y_2 \tag{29}$$

$$\Delta_0 = \left(\frac{\mu_{max} - \Delta\mu}{1 + \nu|y_2 - \dot{d}|_0 + \frac{\nu^2}{2}|y_2 - \dot{d}|_0^2 + \frac{\nu^3}{6}|y_2 - \dot{d}|_0^3}\right) Q \tag{30}$$

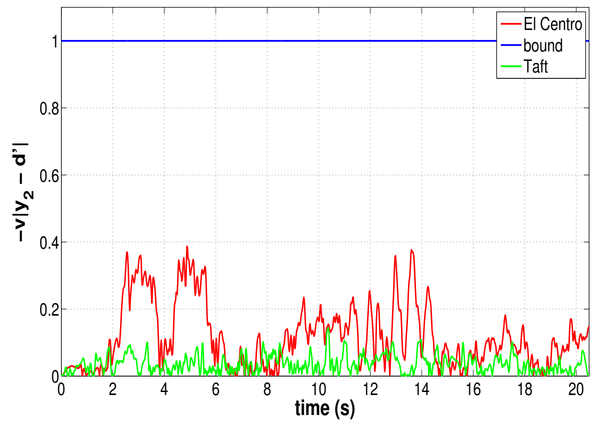

Fig. 2. Dynamics of the base isolator in the presence of some standard earthquakes.

$$\Delta_1 = \left(\frac{\mu_{max} \left[\nu + \frac{\nu^2}{2} |y_2 - \dot{d}|_0 + \frac{\nu^3}{6} |y_2 - \dot{d}|_0^2 \right]}{1 + \nu |y_2 - \dot{d}|_0 + \frac{\nu^2}{2} |y_2 - \dot{d}|_0^2 + \frac{\nu^3}{6} |y_2 - \dot{d}|_0^3} \right) Q \tag{31}$$

Finally, the evolutionary variable z is estimated using the following expression:

$$z = \hat{z} + \tilde{z}, \quad \dot{\hat{z}} = -\gamma |y_2| \hat{z} |\hat{z}|^{n-1} - \beta y_2 |\hat{z}|^n + A y_2 \tag{32}$$

Now with these ideas in mind, a modified controller is presented. Consider the assumptions made on the unknown disturbance force in (26), the approximation of the frictional actuator force in (29) and the estimation of the evolutionary variable in (32). Let $\tilde{z} = \lambda e_2$ and $h_2 = \lambda \alpha_a / m$ and consider the following characteristics of the shear-mode MR damper used: $n = 1$ and $\gamma \geq \beta > 0$, the following stabilizing control law is proposed:

$$\begin{aligned} v = & - \frac{(\overline{c} + \Delta_1 + \delta_a - m h_1) y_2 - (\Delta_0 + \Delta_1 D_v + F) sgn(e_2)}{\alpha_b \hat{z} + \delta_b y_2 + \alpha_b \lambda e_2} \\ & + \frac{-f_{bf} - \overline{k} y_1 - \alpha_a \hat{z} + m e_1 + m h_2 e_2}{\alpha_b \hat{z} + \delta_b y_2 + \alpha_b \lambda e_2} \end{aligned} \tag{33}$$

provided that $\alpha_b \hat{z} + \delta_b y_2 + \alpha_b \lambda e_2 \neq 0$; otherwise, $v = 0$.

Proof: Consider the following Lyapunov function candidate:

$$\begin{aligned} V = & \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2 + \frac{1}{2} \tilde{z}^2 + \frac{1}{2r_1} \tilde{c}^2 + \frac{1}{2r_2} \tilde{k}^2 \\ & + \frac{1}{2r_3} \widetilde{c_{bf}}^2 + \frac{1}{2r_4} \widetilde{k_{bf}}^2 \end{aligned} \tag{34}$$

The derivative of V is given by:

$$\begin{aligned} \dot{V} = & e_1 \dot{e}_1 + e_2 \dot{e}_2 + \tilde{z} \dot{\tilde{z}} + r_1^{-1} (c - \overline{c}) \dot{\overline{c}} + r_2^{-1} (k - \overline{k}) \dot{\overline{k}} \\ & + r_3^{-1} (c_{bf} - \overline{c_{bf}}) \dot{\overline{c_{bf}}} + r_4^{-1} (k_{bf} - \overline{k_{bf}}) \dot{\overline{k_{bf}}} \end{aligned} \tag{35}$$

In order to find the expression for $e_2 \dot{e}_2$, the result in (23) is used. Substitution of (33) into such result yields:

$$\begin{aligned} e_2 \dot{e}_2 = & - \frac{e_2}{m} [-\Phi - f_g - \Delta_1 y_2 + (c - \overline{c}) y_2 \\ & + (k - \overline{k}) y_1 + (c_{bf} - \overline{c_{bf}})(y_2 - \dot{x}_1) \\ & + (k_{bf} - \overline{k_{bf}})(y_1 - x_1) + m e_1 \\ & + (\Delta_0 + \Delta_1 D_v + F) sgn(e_2)] \end{aligned} \tag{36}$$

¿From (8) and (32), and taking into account $n = 1$ and $\gamma \geq \beta > 0$, the following expression for $\tilde{z}\dot{\tilde{z}}$ is obtained:

$$\begin{aligned} \tilde{z}\dot{\tilde{z}} = & - \tilde{z} [\gamma |y_2| (z |z|^{n-1} - \hat{z} |\hat{z}|^{n-1}) \\ & + \beta y_2 (|z|^n - |\hat{z}|^n)) \\ = & - \gamma |y_2| \tilde{z}^2 - \beta y_2 \tilde{z} (|z| - |\hat{z}|) \\ & \leq -(\gamma - |\beta|) |y_2| \tilde{z}^2 \leq 0 \end{aligned} \tag{37}$$

Substitution of (13) - (16), (22), (36) and (37) into (35) yields:

$$\begin{aligned} \dot{V} \leq & - \frac{1}{m} [(\overline{\Phi} + \Delta_1 y_2) |e_2| - (\Phi + \Delta_1 y_2) e_2 + F |e_2| \\ & - f_g e_2] - h_1 e_1^2 - (\gamma - |\beta|) |y_2| \tilde{z}^2 \leq 0 \end{aligned} \tag{38}$$

and therefore stability is ensured.◇

4. NUMERICAL EXAMPLE

The following example consists in controlling a seismically excited 10-story building. The horizontal seismic motion is a replica of that of Taft's earthquake. The mass of each floor, including that of the base is $m = 6 \times 10^5$ kg. The nominal value of the stiffness of the base is $k = 1.184 \times 10^7$ N/m. The stiffness of the structure varies linearly by story from 4.5×10^8 N/m to 9×10^8 N/m. The damping ratio of the base is $c = 0.1$ and that of the structure is $c_i = 0.05$. The parameters of the frictional actuator placed in the base along with the MR damper are: $Q = \sum_{i=1}^{10} m_i$, $\mu = 0.1$, $\nu = 2.0$, $\mu_{max} = 0.185$ and $\Delta\mu = 0.09$. The parameters of the MR damper are: $\gamma = \beta = 3 \times 10^2$ cm $^{-1}$, $n = 1$, $\alpha_a = 4.5 \times 10^4$, $\alpha_b = 3.6 \times 10^4$, $\delta_a = 3 \times 10^2$ kNs/m, $\delta_b = 1.8 \times 10^2$ kNs/m. The controller parameters are: $h_1 = 1.5$, $h_2 = 86.3$, $\lambda = 1$, $r_1 = 10$, $r_2 = 1$, $r_3 = 10$ and $r_4 = 1$.

Figure 3 shows the record of the Taft earthquake used to excite the structure in the example. Figure 4 compares the peak absolute displacement of each floor in 3 cases: (1) without any type of structural control; (2) with only the passive base isolator; and (3) with the hybrid scheme (passive base isolator plus semiactive MR damper). Figure 5 compares the peak absolute velocity of each in these same cases. Substantial reduction in displacement and velocity is observed under the action of the hybrid structural control system with respect to the uncontrolled and passively controlled cases. The control effort of the MR damper is shown in Figure 6.

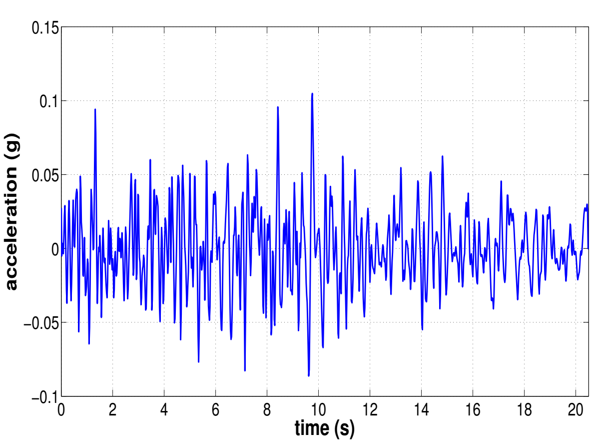

Fig. 3. The Taft earthquake.

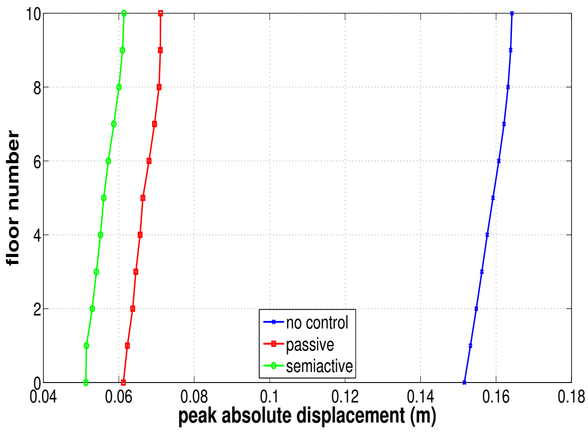

Fig. 4. Peak absolute displacement.

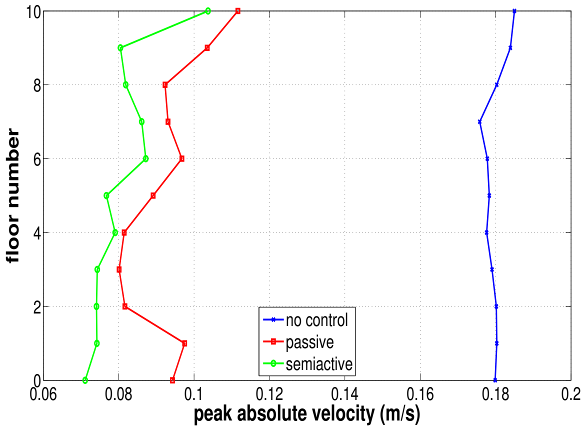

Fig. 5. Peak absolute velocity.

5. CONCLUSIONS

In this paper, an adaptive backstepping control scheme has been proposed to solve the vibration problem in a base-isolated building. The uncertainties that characterize the stiffness and damping coefficients have been approached by deriving adaptive laws that estimate their values. The control law also takes into account the nonlinearities of the frictional and hysteretic actuators as well as the unknown

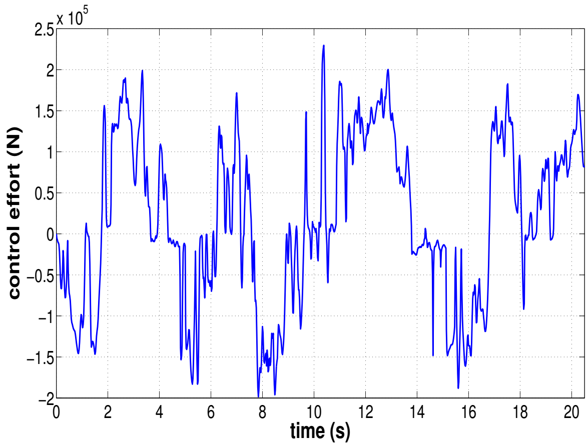

Fig. 6. MR damper control effort.

disturbances that the structure is subjected to. The simulations run for a base-isolated 10-story building show the effectiveness of the control law proposed for the hybrid isolation system by observing the reduction of the peak responses (absolute displacement and velocity).

ACKNOWLEDGEMENTS

This work has been partially funded by the European Union (European Regional Development Fund) and the Commission of Science and Technology of Spain (CICYT) through the coordinated research projects DPI-2005-08668-C03 and by the Government of Catalonia (Spain) through SGR00296. The first author acknowledges the FI Grant of the Department for Innovation, University and Enterprise of the Government of Catalonia.

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