Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

IFAC

Stabilization of Lebesgue sampled systems with bounded controls: the chain of integrators case

Nicolas Marchand [*]

[*] *GIPSA-lab, Control Systems Departement (former Laboratoire d'Automatique de Grenoble), CNRS-INRIA-U. of Grenoble, ENSE[3] BP 46, Domaine Universitaire, 38400 Saint Martin d'Heres - France*

Abstract: In this paper, the stabilization of a chain of integrators in the Lebesgue sampling context is considered. Lebesgue sampling refers to a sampling scheme where measurements are not taken at periodic instants but when variables crosse a priori defined levels. The paper proposes a nonlinear control law that stabilizes the system in the sense that it renders asymptotically stable any a priori given hyper-rectangle strictly larger and encompassing the smallest set where the states fail to be detectable because of the quantization precision. The control law is a sum of saturated linear feedback computed with quantized measurements.

1. INTRODUCTION

The classical so-called discrete time framework of controlled systems consist in sampling the system uniformly in the time with some constant sampling period T and in computing and updating the control law (or observer) every time instants $t = kT$. This case will be denoted as the synchronous case in the sense that all signal measured are synchronous. This field has been widely investigated even in the case of sampling jitter or measurements loss that can be seen as some asynchronicity. More recently, some works address event based sampling. Many reasons motivate this growing interest and in particular because more and more systems with asynchronous needs are encountered. Important contributions come from the real-time control community where the control tasks are often considered as hard real time tasks requiring strong real time constraints. The main consequence is oversized computers entailing additional costs not very compatible with large scale production as for embedded systems. Efforts in this field are carried on the co-design between the controller and the task scheduler in order to soften the time constraint. The approach adopted in this field is often either to dynamically change the sampling period related to the load [Sename et al., 2003, Simon et al., 2005] or to use event driven control with events generated with a mix of level crossing and some maximal sampling period for stability reasons [Sandee et al., 2005, Arzen, 2005]. However, all these approaches are implicitly synchronous in the sense that time is used to determine if the control must be updated.

If this field is particularly active, an asynchronous control framework would be adequate for many other systems. For instance, biological systems are most of the time reacting to events, even if biological clocks exist. Decentralized systems, in particular networked control systems, are often problematic to synchronize. The need of low power electronic components in all embedded and miniaturized applications encourages companies to develop asynchronous versions of existing time triggered components with significant power consumption reduction: about four times less power than its synchronous counterpart for the 80C51 microcontroller of Philips Semiconductors [van Gageldonk et al., 1998]. Moreover, the absence of synchronization considerably reduces noise and electro-magnetic emissions by improving the time repartition of the events [Van Berkel et al., 1994, 1999]. Note that now sensors and actuators based on level crossing events exist rendering a complete asynchronous control loop possible.

Extending the analogy between Riemann and Lebesgue integral calculation (the first one sums the height at each instant whereas the second sums the instants at all heights), the notion of Lebesgue sampling was introduced to denote a scheme where measurements are taken only when variables cross specific levels by opposition to the Riemann sampling where measurements are taken at specific time instants. This notion is taking more and more importance in the signal processing community with now various publications on this subject (see for instance Aeschlimann et al. [2004] and references therein). In the control community, very little work has been done. In Astrom and Bernhardsson [2002], it is proved that such an approach reduces the number of sampling instants for the same final performance. However, the result is established in the context of impulse control that is not natural for most systems.

In this paper, only constant controls over sampling periods are considered. This is motivated by the fact that practically asynchronous constant control will reduce the network load by removing the control update and will enable the use of the above mentioned asynchronous electronic component. For the same reason, we do not consider the possibility to reconstruct the state using an observer that would require to update the control at least periodically.

By analogy to the periodic sampling scheme, the term sampling period denotes a time interval between two consecutive level-crossing of the measurement, that is two

successive sampling instants. The sampling periods are hence not constant in the Lebesgue sampling scheme. As shown in the next section, controlling a Lebesgue sampled system and a continuous time system with quantized measurements by means of a *control law constant over sampling periods* are equivalent problems. In the context of quantized measurements, is known that the classical notion of Lyapunov stabilization (asymptotic convergence with bounded trajectories) is not appropriate if infinitely precise sampling near the origin is not used. Indeed, in the neighborhood of the origin, non zero states can not be distinguished from zero and hence the trajectories may go close to an equilibrium but there is no hope to have asymptotic convergence to the origin with controls constant over sampling periods, at least for unstable systems. Hence, the aim is more to try to obtain some practical stability property of the closed-loop system. Quite in the spirit of the present paper, Kofman and Braslavsky [2006] showed that, on a continuous-time linear time invariant system, applying a continuous time stabilizing controller evaluated with values of the state of the system updated only when the state crosses a priori defined levels yield stability of a ball around the origin. The radius of the ball is related to the precision levels. This result obtained considering uniform sampling comes from the use of some kind of Taylor approximation along the state axis instead of along the time axis Kofman [2004] and follows from a naive use of a continuous-time controller in a Lebesgue sampling framework. However, one may hope to be able to drive the state of the system in the neighborhood of the origin where nonzero and zero states can not be distinguished which would be a stronger result than in Kofman and Braslavsky [2006]. Controlled systems with quantized measurements are actively studied by the control community. However, the two directions taken by the community concern Riemann sampled systems Delchamps [1990], Fagnani and Zampieri [2003, 2004], Brockett and Liberzon [2000] or continuous time systems with controls evolving with the time between two quantization levels crossing Brockett and Liberzon [2000], Liberzon and Nesic [2005], Liberzon [2006] sometimes coupled with a statistical or ergodic analysis. None addresses the kind of sampling considered in this paper. Extending results of Riemann sampled systems to Lebesgue sampled systems seems not to be a good strategy since, as underlined in Fagnani and Zampieri [2003], stabilization under periodic sampling requires hybrid tools where, as done in the present paper, level crossing sampling can be addressed with "classical" Lyapunov theory yet with some precautions due to the discontinuous right hand-side of the ODE involved.

The aim of this paper is to propose control laws for a Lebesgue sampled chain of integrators. The proposed control laws consist in a sum of saturated linear controllers computed with the quantized measurements of the state. It is inspired by Teel [1992], Sussmann et al. [1994], Marchand and Hably [2005] where saturated control laws for linear systems are given in a quantization free context. The control strategy proposed in these works relies on a coordinate transformation that puts the system in a feedforward from. The approach was therefore modified in order to enable quantization that necessarily happens in the original coordinate set. The performance goal for the controller is to be able render asymptotically stable any

a priori given rectangle neighborhood strictly larger than the "*minimal neighborhood*", that is the set where zero and non-zero states can not be distinguished. The next section is dedicated to the problem statement and various preliminaries. In section 3, the double integrator is given as a didactic example, the result being generalized to the general chain of integrator in section 4.

Notations: Let $\mathbb{R}_{>0}$ denote the set of strictly positive real numbers. For any $k > 0$ and $r = (r_1, \ldots, r_k) \in \mathbb{R}_{>0}^k$, let $\mathcal{R}(r)$ or $\mathcal{R}(r_1, \ldots, r_k)$ denote the hyper rectangle $\times_{i=1,\ldots,k}[-r_i, r_i] \subset \mathbb{R}^k$. If for all $i = 1, \ldots, k$, $r_i = d$ for some d, then $\mathcal{R}(d)$ will denote the hyper-rectangle $\times_{i=1,\ldots,k}[-d, d] \subset \mathbb{R}^k$ (more precisely, it is an hypercube). Finally, let $\mathrm{sat}_M(\cdot) := \max(\min(\cdot, M), -M)$.

2. PROBLEM STATEMENT

In this paper, we consider a general linear chain of integrators:

$$\dot{x} = Ax + Bu \tag{1}$$

where A is such that $a_{ij} = 1$ if $j = i + 1$ and $a_{ij} = 0$ otherwise, B is such that $b_i = 0$ for $i = 1, \ldots, n-1$ and $b_n = 1$, n being the dimension of the system. The topic of this paper is to find a stabilizing control law u for system (1) such that:

$$-\bar{u} \leq u \leq \bar{u} \tag{2}$$

where $\bar{u}$ is the control bound resulting either from a control saturation or from a maximal quantization level in the measure. The system is assumed to be Lebesgue sampled. As shown on figure 1, it means that a measure $\mathcal{Q}(x_j)$ of a state variable $x_j \in \mathbb{R}$ ($j \in \{1, \ldots, n\}$) must belong to an a priori defined set of quantized values q_i. This set is supposed to be such that $q_0 = 0$, $q_i = -q_{-i}$ and every finite interval $[a, b] \subset \mathbb{R}$ contains none or a finite number of quantization level q_i. Logarithmic sampling is thus not considered in the paper. For simplicity, we assume the same quantization for all the states of the system. The relation between the state variable x_j and its Lebesgue sampled measure $\mathcal{Q}(x_j)$ is given by:

$$\mathcal{Q}(x_j) = q_i \begin{cases} \text{if } x_j \in \left] \dfrac{q_{i-1} + q_i}{2}, \dfrac{q_i + q_{i+1}}{2} \right[\\ \text{or if } x_j = \dfrac{q_i + q_{i+1}}{2} \text{ and } x_j > 0 \\ \text{or if } x_j = \dfrac{q_{i-1} + q_i}{2} \text{ and } x_j < 0 \end{cases} \tag{3}$$

$\mathcal{Q}(x_j)$ is hence simply the closest q_i to x_j and the three cases introduced in (3) only assures symmetry between negative and positive half axes. The measure $\mathcal{Q}(x_j)$ of the state variable x_j is hence "updated" when x_j crosses *detection levels* fixed at the middle of two successive quantized values, justifying the terminology of "level crossing sampling". Practically, in order to avoid infinitely fast sampling of constant signals, the level detection is made using hysteresis. However, from a pure theoretical point of view, the solutions can be intended in the Filippov sense Filippov [1988]. The time instants when such a level is crossed will be denoted $t_k^{x_j}$ with $k \in \mathbb{N}$. Note that all the state variables do not necessarily cross levels at
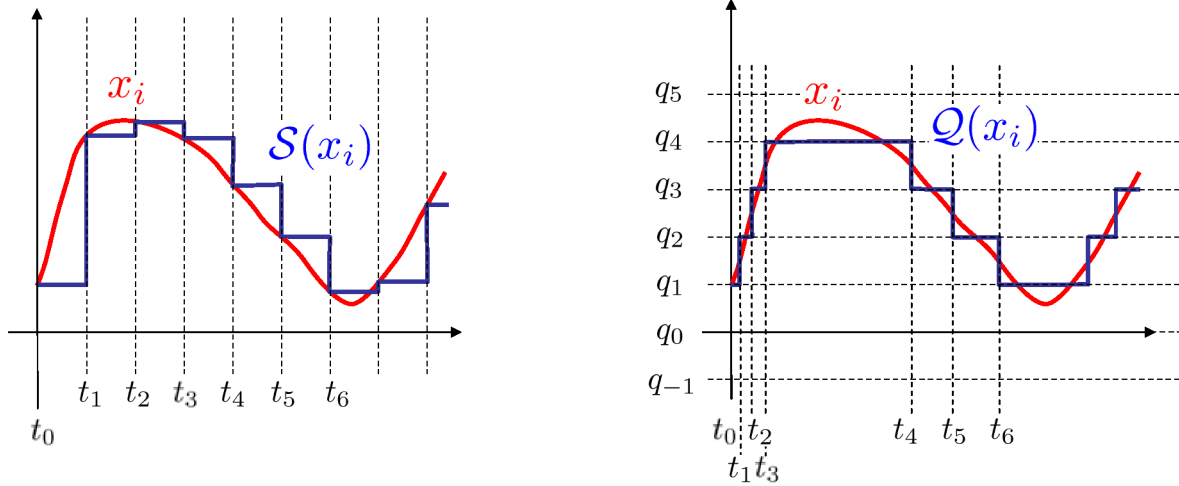
2

Fig. 1. Relation between a state variable x_i, its Riemann sampling $\mathcal{S}(x_i)$ (left) and Lebesgue uniform sampling $\mathcal{Q}(x_i)$ (right)

the same time and hence this instant has to characterize a specific state variable. $\{t_k\}_{k \in \mathbb{N}}$ will denote the set of chronologically ordered time instants (sampling instants) when at least one state of the system crosses a detection level. By analogy to the synchronous framework, the time intervals $[t_k, t_{k+1}]$ will be called sampling periods. In fact, controlling a Lebesgue sampled system and controlling a continuous-time system with quantized measurements and a control constant over sampling periods are equivalent problems since (3) is a quantization function. This is the approach adopted here. We propose here a control law, constant over the non uniform sampling periods $[t_i, t_{i+1}[$ with a simplicity close to the unconstrained and non quantized linear case. For a given $\delta > 0$, the aim will be to bring x into the hyper-rectangle $\mathcal{R}(\frac{q_1}{2} + \delta)$ and then to remain there. Note that $\mathcal{R}(\frac{q_1}{2})$ is the minimal neighborhood in the sense that in it, zero and non zero states can not be distinguished: there is hence no hope to get a better stability result.

3. THE DOUBLE INTEGRATOR

We consider here the second order integrator:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u \end{aligned} \qquad (4)$$

and take as control:

$$u = - a_1 \operatorname{sat}_{M_1}(\beta_1 \mathcal{Q}(x_1)) - a_2 \operatorname{sat}_{M_2}(\beta_2 \mathcal{Q}(x_2)) \qquad (5)$$

where a_1, a_2, M_1 and M_2 are positive parameters. Let us first take $a_1 M_1 + a_2 M_2 = \bar{u}$ where $\bar{u}$ denotes the upper control bounds. Take $V_2 = \frac{1}{2}x_2^2$, then its time derivative is:

$$\dot{V}_2 = x_2(-a_1 \operatorname{sat}_{M_1}(\beta_1 \mathcal{Q}(x_1)) - a_2 \operatorname{sat}_{M_2}(\beta_2 \mathcal{Q}(x_2)))$$

Assume first that $x_2 \notin [-\frac{q_1}{2}, \frac{q_1}{2}]$. Then, taking $a_1 M_1 < a_2 \min(\beta_2 \frac{q_1}{2}, M_2)$, it follows that x_2 and u will be of opposite sign ensuring the strict decrease of V_2 and thus the convergence of x_2 to the interval $[-\frac{q_1}{2}, \frac{q_1}{2}]$ in finite time. Once there, x_2 clearly remains in this interval. Moreover, during that time, thanks to Lemma 4 in Marchand and Hably [2005], x_1 can not diverge. Since in this set $\mathcal{Q}(x_2) = q_0 = 0$, the control law becomes (whatever the value of β_2):

$$u = -a_1 \operatorname{sat}_{M_1}(\beta_1 \mathcal{Q}(x_1))$$

Now take $V_1 := \frac{1}{2}(x_1 + \varepsilon x_2)^2$, then

$$\dot{V}_1 = (x_2 - \varepsilon a_1 \operatorname{sat}_{M_1}(\beta_1 \mathcal{Q}(x_1)))(x_1 + \varepsilon x_2)$$

Assuming that $x_1 \notin [-\frac{q_1}{2}, \frac{q_1}{2}]$, thanks to $x_2 \in [-\frac{q_1}{2}, \frac{q_1}{2}]$, the signs of $x_1 + \varepsilon x_2$ and x_1 will be identical as soon as $\varepsilon \leq 1$. Moreover, $|x_2| \leq \frac{q_1}{2}$ and $|\operatorname{sat}_{M_1}(\beta_1 \mathcal{Q}(x_1))| \geq \min(M_1, \beta_1 \frac{q_1}{2})$, giving that $\varepsilon a_1 \operatorname{sat}_{M_1}(\mathcal{Q}(x_1)) - x_2$ and x_1 also have the same sign as soon as $\varepsilon a_1 \min(M_1, \beta_1 \frac{q_1}{2}) > \frac{q_1}{2}$. This ensures the strict decrease of V_1 until $V_1 \leq \frac{1}{2}(\frac{q_1}{2} + \varepsilon \frac{q_1}{2})^2$, an inequality that remains valid for all future t. Using the triangle inequality, it follows that x_1 enters in finite time and remains in the interval $[-(1 + 2\varepsilon)\frac{q_1}{2}, (1 + 2\varepsilon)\frac{q_1}{2}]$. Therefore:

Theorem 1. For any $q_1 > \delta > 0$, let the parameters of control law (5) be such that:

- $a_1 M_1 < a_2 \min(M_2, \beta_2 \frac{q_1}{2})$
- $\frac{q_1}{2} < \varepsilon a_1 \min(M_1, \beta_1 \frac{q_1}{2})$ with $\varepsilon = \frac{\delta}{q_1}$

Then, system (4) with control (5) enters in finite time $\mathcal{R}(\frac{q_1}{2} + \delta, \frac{q_1}{2})$ where it remains thereafter. Also, the control law is bounded by $a_1 M_1 + a_2 M_2$.

Note that in fact one can take δ larger than q_1, the important thing being in that case to take $\varepsilon < 1$.

3.1 Simulations

Figure 2 shows the time response of the closed-loop double integrator. The quantization levels are $q_k = k$, $k \in \mathbb{Z}$. $\varepsilon = \delta = 0.1$ meaning that the rectangle $\mathcal{R}(0.5, 0.6)$ is asymptotically stable. The other parameters are: $\beta_1 = 6$, $\beta_2 = 2$, $a_1 = a_2 = 2$, $M_1 = 1$ and $M_2 = 2$. The initial condition is $(2, -5)$.

3.2 Discussion

The transient of the closed loop is related to (1) the closed loop poles (when the system is in the hyper-rectangle $\mathcal{R}(\frac{q_1}{2} + \delta, \frac{q_1}{2})$ where the saturation functions of the control law are not saturated) and (2) the choice of the saturation's levels M_1 and M_2. For (1), the closed loop
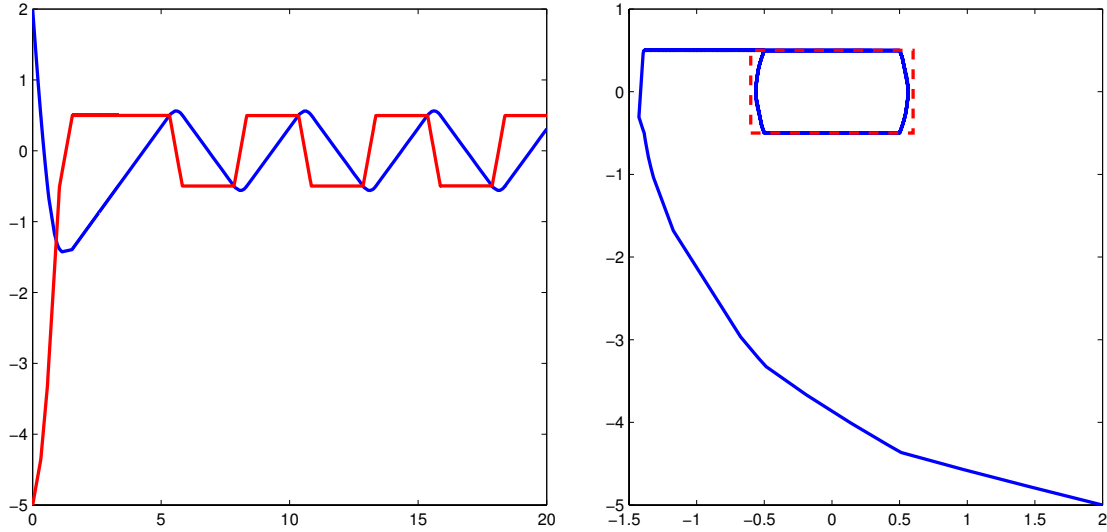
3

Fig. 2. Time response and phase portrait of the closed loop trajectory of the double integrator with $\delta = 0.1$, $q_1 = 1$, $\beta_1 = 6$, $\beta_2 = M_2 = a_1 = a_2 = 2$, $M_1 = 1$ and $x_0 = (2, -5)$. The dotted rectangle shows the final set $\mathcal{R}(\frac{q_1}{2} + \delta, \frac{q_1}{2})$ that can be proved to be asymptotically stable despite a unitary quantization precision.

poles can directly be tuned using the products $a_1\beta_1$ and $a_2\beta_2$. A discussion on how tuning these parameters can be found in [Johnson and Kannan, 2003] for a very similar control law. For (2), variable saturation levels proposed in [Marchand and Hably, 2005] could be adapted to improve the convergence of the system. With an adequate tuning, the performance of nested saturation control is known to be good.

Practically, the proposed control may yield very fast control change (chattering phenomena) due to very close successive level crossing. To avoid this, hysteresis is used in asynchronous electronic sensors. This would widen the frontier of the final hyper-rectangle. This is probably not the best solution since to reduce the average events frequency, it will be necessary to increase the final hyper-rectangle.

4. GENERAL CHAIN OF INTEGRATORS

We go back to the general chain of integrators (1) and choose a control law of the form:

$$u = -\sum_{i=1}^{n} a_i \, \mathrm{sat}_{M_i}(\beta_i \mathcal{Q}(x_i)) \qquad (6)$$

where the a_i's and M_i's are assumed to be strictly positive. Notice that the control law is bounded by $\sum_{i=1}^{n} a_i M_i$.

Step n: Take $V_n = \frac{1}{2}x_n^2$, then:

$$\dot{V}_n = -x_n \left(\sum_{i=1}^{n} a_i \, \mathrm{sat}_{M_i}(\beta_i \mathcal{Q}(x_i)) \right)$$

Taking $\sum_{i=1}^{n-1} a_i M_i < a_n \min(M_n, \beta_n \frac{q_1}{2})$ ensures that x_n and u are of opposite sign as long as $x_n \notin [-\frac{q_1}{2}, \frac{q_1}{2}]$. This clearly makes V_n decrease and hence forces x_n to join $[-\frac{q_1}{2}, \frac{q_1}{2}]$ in finite time and to remain there for all future time. Lemma 4 in Marchand and Hably [2005] proves then

that a finite time escape of $(x_1, \ldots, x_{n-1})$ is not possible during that time.

Step $n-1$: Take $V_{n-1} = \frac{1}{2}(x_{n-1} + \varepsilon_{n-1}x_n)^2$, then

$$\dot{V}_{n-1} = (x_{n-1} + \varepsilon_{n-1}x_n)(x_n - \varepsilon_{n-1}\sum_{i=1}^{n} a_i \, \mathrm{sat}_{M_i}(\beta_i \mathcal{Q}(x_i)))$$

Since $x_n \in [-\frac{q_1}{2}, \frac{q_1}{2}]$, $\mathcal{Q}(x_n) = 0$, therefore as long as $x_{n-1} \notin [-\frac{q_1}{2}, \frac{q_1}{2}]$:

- $x_{n-1} + \varepsilon_{n-1}x_n$ and x_{n-1} have the same sign as soon as $\varepsilon_{n-1} < 1$
- $\varepsilon_{n-1}\sum_{i=1}^{n} a_i \, \mathrm{sat}_{M_i}(\beta_i \mathcal{Q}(x_i)) - x_n$ and x_{n-1} have the same sign as soon as $\frac{q_1}{2} + \varepsilon_{n-1}\sum_{i=1}^{n-2} a_k M_k < \varepsilon_{n-1}a_{n-1}\min(M_{n-1}, \beta_{n-1}\frac{q_1}{2})$

Thus, V_{n-1} is strictly decreasing until $V_{n-1} \leq \frac{1}{2}(\frac{q_1}{2} + \varepsilon_{n-1}\frac{q_1}{2})^2$, an inequality that remains valid for all future t. Using the triangular inequality, it follows that x_{n-1} enters and remains in $[-(1 + 2\varepsilon_{n-1})\frac{q_1}{2}, (1 + 2\varepsilon_{n-1})\frac{q_1}{2}]$.

Step $n-2$: Take $V_{n-2} = \frac{1}{2}(x_{n-2} + \varepsilon_{n-2}x_n)^2$, then

$$\dot{V}_{n-2} = (x_{n-2} + \varepsilon_{n-2}x_n)(x_{n-1} - \varepsilon_{n-2}\sum_{i=1}^{n} a_i \, \mathrm{sat}_{M_i}(\beta_i \mathcal{Q}(x_i)))$$

Since $q_n \in [-\frac{q_1}{2}, \frac{q_1}{2}]$, as long as $x_{n-1} \notin [-\frac{q_1}{2}, \frac{q_1}{2}]$:

- $x_{n-2} + \varepsilon_{n-2}x_n$ and x_{n-2} have the same sign as soon as $\varepsilon_{n-2} < 1$

Moreover, since $(x_{n-1}, x_n) \in \mathcal{R}((1+2\varepsilon_{n-1})\frac{q_1}{2}, \frac{q_1}{2})$, one has $\mathcal{Q}(x_n) = 0$ and $\mathcal{Q}(x_{n-1}) = \pm\frac{q_1}{2}$ for ε_{n-1} sufficiently small. It follows:

- $\varepsilon_{n-2}\sum_{i=1}^{n} a_i \, \mathrm{sat}_{M_i}(\beta_i \mathcal{Q}(x_i)) - x_{n-1}$ and x_{n-2} have the same sign as soon as $(1 + 2\varepsilon_{n-1})\frac{q_1}{2} + \varepsilon_{n-2}a_{n-1}\min(M_{n-1}, \beta_{n-1}\frac{q_1}{2}) + \varepsilon_{n-2}\sum_{i=1}^{n-3} a_i M_i < \varepsilon_{n-2}a_{n-2}\min(M_{n-2}, \beta_{n-2}\frac{q_1}{2})$

4

Thus, V_{n-2} is strictly decreasing until $V_{n-2} \leq \frac{1}{2}(\frac{q_1}{2} + \varepsilon_{n-1}\frac{q_1}{2})^2$, inequality that remains valid for all future t. Here again, using the triangular inequality, it follows that x_{n-2} enters and remains in $[-(1+2\varepsilon_{n-2})\frac{q_1}{2}, (1+2\varepsilon_{n-2})\frac{q_1}{2}]$.

Going on with the same reasoning up to the first state, it follows:

Theorem 2. For any $\delta > 0$, let the parameters of control law (6) be such that:

$$\sum_{i=1}^{n-1} a_i M_i < a_n \min(M_n, \beta_n \frac{q_1}{2})$$

$$\frac{q_1}{2} + \varepsilon_{n-1} \sum_{i=1}^{n-2} a_i M_i < \varepsilon_{n-1} a_{n-1} \min(M_{n-1}, \beta_{n-1}\frac{q_1}{2})$$

$$(1 + 2\varepsilon_{n-1})\frac{q_1}{2} + \varepsilon_{n-2} a_{n-1} \min(M_{n-1}, \beta_{n-1}\frac{q_1}{2})$$
$$+ \varepsilon_{n-2} \sum_{i=1}^{n-3} a_i M_i < \varepsilon_{n-2} a_{n-2} \min(M_{n-2}, \beta_{n-2}\frac{q_1}{2})$$

$$\vdots$$

$$(1 + 2\varepsilon_{n-k+1})\frac{q_1}{2} + \varepsilon_{n-k} \sum_{i=n-k+1}^{n-1} a_i \min(M_i, \beta_i\frac{q_1}{2})$$
$$+ \varepsilon_{n-k} \sum_{i=1}^{n-k-1} a_i M_i < \varepsilon_{n-k} a_{n-k} \min(M_{n-k}, \beta_{n-k}\frac{q_1}{2})$$

$$\vdots$$

with for all $i \in \{1, \dots, n-1\}$, $\varepsilon_i < \min(1, \frac{\delta}{q_1})$. Then the control law (6) asymptotically stabilizes the hyper-rectangle $\mathcal{R}(\frac{q_1}{2} + \delta)$ of system (1). Moreover, the control law is bounded by $\sum_{i=1}^{n} a_i M_i$.

5. CONCLUSION

This paper proposes a stabilizing control law based on saturation functions for a Lebesgue sampled chain of integrators or equivalently for a continuous time chain of integrators with quantized measurements and piecewise constant control. The control scheme ensures by means of a bounded control, the stability of any a priori given hyper-rectangle strictly larger than some minimal set in the sense that inside it, zero and non-zero states can not be distinguished. This work is a very preliminary contribution and a lot of work remains to render the controller more easy to tune or to extend the approach to general linear systems. Maybe using a hybrid controller with some memory abilities would be a good idea to be able to extend to general linear systems.

REFERENCES

F. Aeschlimann, E. Allier, L. Fesquet, and M. Renaudin. Asynchronous fir filters: towards a new digital processing chain. In *Proceedings of the 10th International Symposium on Asynchronous Circuits and Systems*, pages 198–206, 19-23 April 2004.

K. E. Arzen. A simple event based PID controller. In *Proceedings of the 14th IFAC World Congress*, Prague, Czech republic, 2005.

K. J. Astrom and B. M. Bernhardsson. Comparison of Riemann and Lebesgue sampling for first order stochastic systems. In *Proceedings of the 41st IEEE Conference on Decision and Control*, volume 2, pages 2011–2016, 10-13 Dec. 2002.

R.W. Brockett and D. Liberzon. Quantized feedback stabilization of linear systems. *IEEE Trans. on Automatic Control*, 45(7):1279–1289, July 2000.

D. F. Delchamps. Stabilizing a linear system with quantized state feedback. *IEEE Trans. on Automatic Control*, 35(8):916–924, Aug. 1990.

F. Fagnani and S. Zampieri. Stability analysis and synthesis for scalar linear systems with a quantized feedback. *Automatic Control, IEEE Transactions on*, 48(9):1569–1584, Sept. 2003.

F. Fagnani and S. Zampieri. Quantized stabilization of linear systems: complexity versus performance. *Automatic Control, IEEE Transactions on*, 49(9):1534–1548, Sept. 2004.

A. F. Filippov. *Differential equations with discontinuous righthand sides.* Kluwer Academic Publishers, Dordrecht, Boston, London, 1988.

E. N. Johnson and S. K. Kannan. Nested saturation with guaranteed real poles. In *Proc. IEEE American Control Conf. (ACC)*, volume 1, pages 497–502, 2003.

E. Kofman. Discrete event simulation of hybrid systems. *SIAM Journal on Scientific Computing*, 25(5):1771–1797, 2004.

E. Kofman and J. Braslavsky. Level crossing sampling in feedback stabilization under data rate constraints. In *Proc. IEEE Int. Conf. on Decision and Control (CDC)*, San Diego, CA, USA, 2006.

D. Liberzon. Quantization, time delays, and nonlinear stabilization. *IEEE Trans. on Automatic Control*, 51 (7):1190–1195, July 2006.

D. Liberzon and D. Nesic. Input-to-state stabilization of linear systems with quantized feedback. In *joint IEEE Conf. on Decision and Control and European Control Conference*, pages 8197–8202, Seville, Spain, 12-15 Dec. 2005.

N. Marchand and A. Hably. Global stabilization of multiple integrators with bounded controls. *Automatica*, 41(12):2147–2152, 2005.

J. H. Sandee, W. Heemels, and P. P. J. van den Bosch. Event-driven control as an opportunity in the multidisciplinary development of embedded controllers. In *Proc. IEEE American Control Conf. (ACC)*, volume 3, pages 1776–1781, 8-10 June 2005.

O. Sename, D. Simon, and D. Robert. Feedback scheduling for real-time control of systems with communication delays. In *Emerging Technologies and Factory Automation, 2003. Proceedings. ETFA '03. IEEE Conference*, volume 2, pages 454–461, 16-19 Sept. 2003.

D. Simon, D. Robert, and O. Sename. Robust control/scheduling co-design: application to robot control. In *11th IEEE Real Time and Embedded Technology and Applications Symposium*, pages 118–127, 7-10 March 2005.

H. J. Sussmann, E. D. Sontag, and Y. Yang. A general result on the stabilization of linear systems using bounded controls. *IEEE Trans. on Automatic Control*, 39(12): 2411–2425, Dec. 1994.

A. R. Teel. Global stabilization and restricted tracking for multiple integrators with bounded controls. *Systems & Control Letters*, 18(3):165–171, March 1992.

C. H. Van Berkel, M. B. Josephs, and S. M. Nowick. Applications of asynchronous circuits. *Proceedings of the IEEE*, 87(2):223–233, Feb. 1999.

K. Van Berkel, R. Burgess, J. Kessels, M. Roncken, F. Schalij, and A. Peeters. Asynchronous circuits for low power: a dcc error corrector. *Design & Test of Computers, IEEE*, 11(2):22–32, Summer 1994.

H. van Gageldonk, K. van Berkel, A. Peeters, D. Baumann, D. Gloor, and G. Stegmann. An asynchronous low-power 80c51 microcontroller. In *Proceedings of the Fourth International Symposium on Advanced Research in Asynchronous Circuits and Systems*, pages 96–107, 30 March-2 April 1998.