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FUZZY MODEL BASED INDIRECT ADAPTIVE CONTROL DESIGN FOR NONLINEAR SYSTEMS WITH A DEAD-ZONE

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Abstract: In this paper, we present a fuzzy model based indirect adaptive control scheme for a class of nonlinear systems with a dead-zone. The Takagi-Sugeno (T-S) fuzzy model is used for representing a nonlinear system, where the parameters of the fuzzy model are updated online according to Lyapunov stability theorem. An inverse functions are cascaded with the plant to cancel the effects of dead-zone, and the dead-zone slopes in both positive and negative sides are assumed to be the same. In addition, the proposed adaptive fuzzy controller ensures the stability of the closed-loop system with dead-zone and the output is forced to follow the desired reference input. An inverted pendulum system is used to illustrate the effectiveness of the proposed method. The simulation can demonstrate the validity of the proposed scheme and achieve satisfy simulation results.

1. INTRODUCTION

There have been many research fields proposed that concerning the stability of the fuzzy control systems, online identification of the parameters and the design of the stabilizing fuzzy controller. The topics of the stabilization and tracking are two typical control problems in feedback control, sliding mode control, and adaptive control, etc.. General speaking, stabilization problems are simpler than tracking problems, especially for nonlinear systems. Many of the practical control systems have the structure of a dynamical system preceded by some nonsmooth nonlinearities in the actuator, such as dead-zone and saturation, etc. ([Zhou, Wen and Zhang (2006)]). Dead-zone is the one of the most important non-smooth nonlinearities in actuator, which can severely limit system performance and even cause undesirable inaccuracy or oscillations to instability. In [Ren et al. (2005)], since the dead-zone is an essentially nonlinear element, thhe authors make it possible to use earlier results on adaptive linear control, and the effects of an inaccurate dead-zone inverse are represented by a bounded disturbance, then the linear model reference controller is reparameterized with a filtered regressor signal to faciliate the adaptive control to achieve the tracking purpose. Online parameter estimations are important for indirect adaptive control systems, especially for a nonlinear system whose dynamics is approximated by a combination of linear models, since they can be computed in real time and easily be introduced in the on-line control strategies to produce adaptive control algorithms. Recently, the adaptive fuzzy control schemes using feedback control technique ([Chen et al. (2002)]) or using the Takagi-Sugeno fuzzy model based estimation ([Changand and Young (2002)]) have been introduced to deal with nonlinear system control problems. The Takagi-Sugeno fuzzy model can approximate a nonlinear system with a combination of several linear systems within a required accuracy. The basic idea of the adaptive fuzzy control is that given the weights form the fuzzy system and estimates from adaptation, one obtains the parameters in the control law. Most of the online parameter estimation schemes proposed in the indirect adaptive fuzzy control (see [Yu (2002)], [Giordano et al. (2002)], [Nounou and Passino (2002)]) can be only applied

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to the specified control structures such as mainly feedback control ([Huang and Lin (2002)]) and sliding mode control structures ([Tao et al. (2002)]). Moreover, most of them concentrated on systems with linear control inputs. However, nonlineraities in the control input, especially for dead zone nonlinearity, are generally not neglected. Thus, a parameters estimation scheme applicable to the general fuzzy models and controllers for stabilizing nonlinear systems with dead zone nonlinearity in the control input is needed. In this paper, we present a fuzzy model based indirect adaptive control scheme for a class of nonlinear systems with a dead-zone. The Takagi-Sugeno (T-S) fuzzy model is used for representing a nonlinear system, where the parameters of the fuzzy model are updated online according to Lyapunov stability theorem. An inverse functions are cascaded with the plant to cancel the effects of dead-zone, and the dead-zone slopes in both positive and negative sides are assumed to be the same. In addition, the proposed adaptive fuzzy controller ensures the stability of the closed-loop system with dead-zone and the output is forced to follow the desired reference input. An inverted pendulum system is used to illustrate the effectiveness of the proposed method. The simulation can demonstrate the validity of the proposed scheme and achieve satisfy simulation results.

2. PROBLEM FORMULATION AND FUZZY SYSTEMS

We use the fuzzy implications and the fuzzy reasoning methods suggested by Takgai and Sugeno to express a real plant model as follows:

The ith plant rule:

IF $x_1(t)$ is M_1^i and $\cdots$ and $x_n(t)$ is M_n^i

$$\text{THEN } \dot{\boldsymbol{x}} = \mathbf{A}_i\boldsymbol{x} + \mathbf{B}_i\Phi(u(t)),\ i = 1, 2, \cdots, \ell \quad (1)$$

where $\boldsymbol{x} = [x, \dot{x}, \cdots, x^{(n-2)}, x^{(n-1)}]^\top$ $= [x_1, \cdots, x_{n-1}, x_n]^\top$ denote the linguistic variables associated with the inputs of the fuzzy system, M_j^i, $j = 1, 2, \ldots, n$, are linguistic values of linguistic variables $\boldsymbol{x}$ in the universes of discourse $U \subset \mathbb{R}^n$, respectively, and

$$\mathbf{A}_i = \begin{bmatrix} 0 & 1 & \ldots & 0 & 0 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ a_n^i & a_{n-1}^i & \ldots & a_2^i & a_1^i \end{bmatrix},\ \mathbf{B}_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_i \end{bmatrix},$$

$\boldsymbol{a}_i^\top = [a_n^i, a_{n-1}^i, \cdots, a_1^i] \in \mathbb{R}^n$, $b^i \in \mathbb{R}$, $i = 1, \ldots, \ell$, are unknow parameters. By using a center-average defuzzifier, product inference and singleton fuzzifier, the final output of the fuzzy system is inferred as follows:

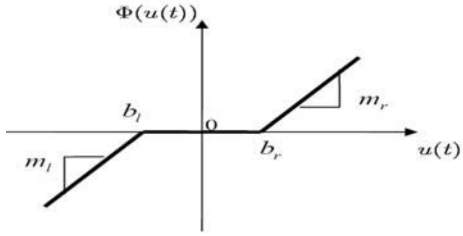

Fig. 1. The dead-zone nonlinearity.

$$\begin{aligned}\dot{\boldsymbol{x}} &= \frac{\sum_{i=1}^{\ell} w_i(\boldsymbol{x})\Big\{\mathbf{A}_i\boldsymbol{x} + \mathbf{B}_i\Phi(u(t))\Big\}}{\sum_{i=1}^{\ell} w_i(\boldsymbol{x})} \\ &= \sum_{i=1}^{\ell} h_i(\boldsymbol{x})\Big\{\mathbf{A}_i\boldsymbol{x} + \mathbf{B}_i\Phi(u(t))\Big\} \\ &= \sum_{i=1}^{\ell} h_i(\boldsymbol{x})\Big\{\mathbf{A}_i\boldsymbol{x} + m\mathbf{B}_i(u(t) + \psi(u(t)))\Big\} \quad (2)\end{aligned}$$

where $w_i(\boldsymbol{x}) = \Pi_{j=1}^{p} M_j^i(x_j(t))$, $h_i(\boldsymbol{x}) = w_i(\boldsymbol{x}) / \sum_{i=1}^{\ell} w_i(\boldsymbol{x})$, $M_j^i(x_j(t))$ is the grade of membership of $x_j(t)$ in M_j^i. It is assumed that $\sum_{i=1}^{\ell} w_i(\boldsymbol{x}) > 0$ and $w_i(\boldsymbol{x}) \geq 0$, for $i = 1, 2, \cdots, n$. To clarify the dead-zone nonlinearity $\Phi(\cdot)$, the dead-zone with input $u(t)$ and output $\Phi(u(t))$ shown in Fig. 1 is described by

$$\Phi(u(t)) = \begin{cases} m_r(u(t) - b_r), & u(t) \geq b_r, \\ 0, & b_l < u(t) < b_r, \\ m_l(u(t) - b_l), & u(t) \leq b_l, \end{cases} \quad (3)$$

where $b_r > 0$, $b_l > 0$ and $m_r > 0$, $m_l > 0$ are parameters and slopes of the dead-zone, respectively. In order to investigate the key features of the dead-zone in the control problems, it is assumed that the slopes of the dead-zone are the same, i.e., $m_r = m_l = m$. In addition, there exist known constants $b_{r_{min}}$, $b_{r_{max}}$, $b_{l_{min}}$, $b_{l_{max}}$, m_{min}, and m_{max} such that the unknown dead-zone parameters b_r, b_l, and m are bounded, i.e. $b_r \in [b_{r_{min}}, b_{r_{max}}]$, $b_l \in [b_{l_{min}}, b_{l_{max}}]$, and $m \in [m_{min}, m_{max}]$. From these assumptions, the expression (3) can be represented as

$$\Phi(u(t)) = mu(t) + \phi(u(t)), \quad (4)$$

where

$$\phi(u(t)) = \begin{cases} -mb_r, & for\ u(t) \geq b_r \\ -mu(t), & for\ b_l < u(t) < b_r \\ -mb_l, & for\ u(t) \leq b_l \end{cases} \quad (5)$$

It is seen that $\phi(u(t))$ is bounded and satisfies $|\phi(u(t))| \leq \rho$, and ρ is the upper-bound and can be chosen as $\rho = max\{m_{max}b_{r_{max}}, -m_{max}b_{l_{min}}\}$ for which b_{lmin} is a negative value. Hence, $\psi(u(t))$ in (2) can be given by

$$\psi(u(t)) = \begin{cases} -b_r, & for\ u(t) \geq b_r, \\ -u(t), & for\ b_l < u(t) < b_r, \\ -b_l, & for\ u(t) \leq b_l. \end{cases} \quad (6)$$

Let $\mathbf{A}_s$ be an arbitrary stable matrix having the same structure as $\mathbf{A}_i$ and all its eigenvalues are in the left half plane, and $\boldsymbol{a}_d^\top$ is the vector of the last row of $\mathbf{A}_s$. To develop the parameter estimator for the Takagi-Sugeno fuzzy modeled plant, we start with the plant parameterization as

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \mathbf{A}_s\boldsymbol{x} + \frac{\sum_{i=1}^{\ell} w_i(\boldsymbol{x})\Big\{(\mathbf{A}_i - \mathbf{A}_s)\boldsymbol{x} + \mathbf{B}_i\Phi(u(t))\Big\}}{\sum_{i=1}^{\ell} w_i(\boldsymbol{x})}\\ &= \mathbf{A}_s\boldsymbol{x} + \sum_{i=1}^{\ell} h_i(\boldsymbol{x})\Big\{(\mathbf{A}_i - \mathbf{A}_s)\boldsymbol{x} + \mathbf{B}_i\Phi(u(t))\Big\}\\ &= \mathbf{A}_s\boldsymbol{x} + \sum_{i=1}^{\ell} h_i(\boldsymbol{x})\Big\{(\mathbf{A}_i - \mathbf{A}_s)\boldsymbol{x} + m\mathbf{B}_i(u(t)\\ &\quad +\psi(u(t)))\Big\} \qquad (7)\end{aligned}$$

Since $\mathbf{A}_i$ and $\mathbf{B}_i$ are unknown, we define the estimation dynamic equation as

$$\begin{aligned}\dot{\hat{\boldsymbol{x}}} &= \mathbf{A}_s\hat{\boldsymbol{x}} + \frac{\sum_{i=1}^{\ell} w_i(\boldsymbol{x})\Big\{(\hat{\mathbf{A}}_i - \mathbf{A}_s)\boldsymbol{x} + \hat{\mathbf{B}}_i\Phi(u(t))\Big\}}{\sum_{i=1}^{\ell} w_i(\boldsymbol{x})}\\ &= \mathbf{A}_s\hat{\boldsymbol{x}} + \sum_{i=1}^{\ell} h_i(\boldsymbol{x})\Big\{(\hat{\mathbf{A}}_i - \mathbf{A}_s)\boldsymbol{x} + \hat{\mathbf{B}}_i\Phi(u(t))\Big\}\\ &= \mathbf{A}_s\hat{\boldsymbol{x}} + \sum_{i=1}^{\ell} h_i(\boldsymbol{x})\Big\{(\hat{\mathbf{A}}_i - \mathbf{A}_s)\boldsymbol{x} + m\hat{\mathbf{B}}_i(u(t)\\ &\quad +\psi(u(t)))\Big\} \qquad (8)\end{aligned}$$

where $\hat{\mathbf{A}}_i(t)$, $\hat{\mathbf{B}}_i(t)$ are the estimates of $\mathbf{A}_i(t)$, $\mathbf{B}_i(t)$ generated by the adaptive law, respectively. By defining the estimation error vector as $\boldsymbol{e} \equiv \boldsymbol{x} - \hat{\boldsymbol{x}} = [e_1\ e_2\ \cdots\ e_n]^\top = [x_1 - \hat{x}_1\ x_2 - \hat{x}_2\ \cdots\ x_n - \hat{x}_n]^\top$. By using (7) and (8), the error dynamic equation can be obtained as:

$$\begin{aligned}\dot{\mathbf{e}} &= \dot{\boldsymbol{x}} - \dot{\hat{\boldsymbol{x}}}\\ &= \mathbf{A}_s\boldsymbol{e} - \sum_{i=1}^{\ell} h_i(\boldsymbol{x})\tilde{\mathbf{A}}_i\boldsymbol{x} - m\sum_{i=1}^{\ell} h_i(\boldsymbol{x})\tilde{\mathbf{B}}_i(u(t)\\ &\quad +\psi(u(t))) \qquad (9)\end{aligned}$$

where $\tilde{\mathbf{A}}_i = \hat{\mathbf{A}}_i - \mathbf{A}_i$ and $\tilde{\mathbf{B}}_i = \hat{\mathbf{B}}_i - \mathbf{B}_i$.

3. ANALYSIS OF SYSTEM STABILITY

In this section, it is shown that the proposed fuzzy adaptive controller will achieve stability based on the Lyapunov stability theory.

Theorem 1. Consider the nonlinear system (1). There exists a positive definite symmetric matrix $\mathbf{P}$, which satisfies the following Lyapunov equation

$$\mathbf{A}_s^\top\mathbf{P} + \mathbf{P}\mathbf{A}_s = -\mathbf{Q} \qquad (10)$$

with $\mathbf{Q}$ being arbitrary $n \times n$ positive definite matrix, and the parameter adaptive laws are given as follows:

$$\begin{cases}\dot{\tilde{\boldsymbol{a}}}_i^\top = \dot{\hat{\boldsymbol{a}}}_i^\top = \gamma_{1i}h_i(\boldsymbol{x})\boldsymbol{p}_1^\top\boldsymbol{e}\boldsymbol{x}^\top(t),\\ \dot{\tilde{\boldsymbol{b}}}_i = \dot{\hat{\boldsymbol{b}}}_i = \gamma_{2i}h_i(\boldsymbol{x})\boldsymbol{p}_1^\top\boldsymbol{e}(u(t)+\psi(u(t))),\\ \text{if } |\hat{\boldsymbol{b}}_i| > \boldsymbol{b}_{i0} \text{ or if } |\hat{\boldsymbol{b}}_i| = \boldsymbol{b}_{i0} \text{ and } \boldsymbol{p}_1^\top\boldsymbol{e}usgn(\boldsymbol{b}_i) \geq 0,\end{cases} \qquad (11)$$

where $\hat{\boldsymbol{a}}_i$, $\hat{\boldsymbol{b}}_i$, and $\boldsymbol{p}_1$ are the last rows of $\hat{\mathbf{A}}_i$, $\hat{\mathbf{B}}_i$, and $\mathbf{P}$, respectively. Furthermore, let the control law be given by

$$u(t) = \sum_{i=1}^{\ell} h_i(\boldsymbol{x})\frac{(\boldsymbol{a}_d^\top - \hat{\boldsymbol{a}}_i^\top)\boldsymbol{x}}{m\hat{b}_i} - \psi(u(t)) \qquad (12)$$

Then, $\boldsymbol{e}$ and $\hat{\boldsymbol{x}}$ as well as the parameter estimation errors $\hat{\boldsymbol{a}}_i$ and $\hat{b}_i$ are guaranteed to be uniformly ultimately bounded (UUB) for systems with dead zone control inputs.

Proof: Choose the Lyapunov functional candidate as

$$\begin{aligned}V &= \frac{1}{m}\boldsymbol{e}^\top\mathbf{P}\boldsymbol{e} + \sum_{i=1}^{\ell}\frac{1}{m\gamma_{1i}}\tilde{\boldsymbol{a}}_i^\top\tilde{\boldsymbol{a}}_i + \sum_{i=1}^{\ell}\frac{1}{\gamma_{2i}}\tilde{\boldsymbol{b}}_i^2\\ &\quad +\hat{\boldsymbol{x}}^\top\mathbf{P}\hat{\boldsymbol{x}},\end{aligned}$$

where γ_{1i} and $\gamma_{2i} > 0$ are constant. Then, the time derivative of V along the trajectory of (9) is given by:

$$\begin{aligned}\dot{V} &= \frac{1}{m}\boldsymbol{e}^\top(\mathbf{A}_s^\top\mathbf{P} + \mathbf{P}\mathbf{A}_s)\boldsymbol{e}\\ &\quad -\frac{2}{m}\boldsymbol{e}^\top\mathbf{P}\Big\{\sum_{i=1}^{\ell} h_i(\boldsymbol{x})\tilde{\mathbf{A}}_i\boldsymbol{x} + m\sum_{i=1}^{\ell} h_i(\boldsymbol{x})\tilde{\mathbf{B}}_i(u(t)\\ &\quad +\psi(u(t)))\Big\} + \frac{2}{m}\sum_{i=1}^{\ell}(\frac{1}{\gamma_{1i}}\dot{\tilde{\boldsymbol{a}}}_i^\top\tilde{\boldsymbol{a}}_i)\\ &\quad +2\sum_{i=1}^{\ell}(\frac{1}{\gamma_{2i}}\tilde{\boldsymbol{b}}_i^\top\dot{\tilde{\boldsymbol{b}}}_i) + 2\dot{\hat{\boldsymbol{x}}}^\top\mathbf{P}\hat{\boldsymbol{x}}\\ &= -\frac{1}{m}\boldsymbol{e}^\top\mathbf{Q}\boldsymbol{e} - \frac{2}{m}\boldsymbol{e}^\top\mathbf{P}\Big\{\sum_{i=1}^{\ell} h_i(\boldsymbol{x})\tilde{\mathbf{A}}_i\boldsymbol{x}\\ &\quad +m\sum_{i=1}^{\ell} h_i(\boldsymbol{x})\tilde{\boldsymbol{b}}_i(u(t) + \psi(u(t)))\Big\}\\ &\quad +\frac{2}{m}\sum_{i=1}^{\ell}(\frac{1}{\gamma_{1i}}\dot{\tilde{\boldsymbol{a}}}_i^\top\tilde{\boldsymbol{a}}_i) + 2\sum_{i=1}^{\ell}(\frac{1}{\gamma_{2i}}\tilde{\boldsymbol{b}}_i^\top\dot{\tilde{\boldsymbol{b}}}_i) + 2\dot{\hat{\boldsymbol{x}}}^\top\mathbf{P}\hat{\boldsymbol{x}}\\ &= -\frac{1}{m}\boldsymbol{e}^\top\mathbf{Q}\boldsymbol{e} - \frac{2}{m}\boldsymbol{e}^\top\boldsymbol{p}_1\sum_{i=1}^{\ell} h_i(\boldsymbol{x})\tilde{\boldsymbol{a}}_i\boldsymbol{x}\\ &\quad -2\boldsymbol{e}^\top\boldsymbol{p}_1\sum_{i=1}^{\ell} h_i(\boldsymbol{x})\tilde{\boldsymbol{b}}_i(u(t) + \psi(u(t)))\\ &\quad +\frac{2}{m}\sum_{i=1}^{\ell}(\frac{1}{\gamma_{1i}}\dot{\tilde{\boldsymbol{a}}}_i^\top\tilde{\boldsymbol{a}}_i) + 2\sum_{i=1}^{\ell}(\frac{1}{\gamma_{2i}}\tilde{\boldsymbol{b}}_i^\top\dot{\tilde{\boldsymbol{b}}}_i) + 2\dot{\hat{\boldsymbol{x}}}^\top\mathbf{P}\hat{\boldsymbol{x}}\\ &= -\frac{1}{m}\boldsymbol{e}^\top\mathbf{Q}\boldsymbol{e} + \Big\{\frac{2}{m}(\sum_{i=1}^{\ell}(\frac{1}{\gamma_{1i}}\dot{\tilde{\boldsymbol{a}}}_i^\top\tilde{\boldsymbol{a}}_i)\\ &\quad -\sum_{i=1}^{\ell} h_i(\boldsymbol{x})\boldsymbol{p}_1^\top\boldsymbol{e}\boldsymbol{x}\tilde{\boldsymbol{a}}_i) + 2(\sum_{i=1}^{\ell}(\frac{1}{\gamma_{2i}}\tilde{\boldsymbol{b}}_i^\top\dot{\tilde{\boldsymbol{b}}}_i)\\ &\quad -\sum_{i=1}^{\ell} h_i(\boldsymbol{x})\tilde{\boldsymbol{b}}_i\boldsymbol{p}_1^\top\boldsymbol{e}(u(t) + \psi(u(t))))\Big\}\\ &\quad +2\Big\{a_d\boldsymbol{x} + \sum_{i=1}^{\ell} h_i(\boldsymbol{x})\Big\{(\hat{a}_i - a_d)\boldsymbol{x} + m\hat{b}_i(u(t)\\ &\quad +\psi(u(t)))\Big\}\Big\}^\top\boldsymbol{p}_1\hat{\boldsymbol{x}}.\end{aligned}$$

Let

$$\sum_{i=1}^{\ell}\left(\frac{1}{\gamma_{1i}}\dot{\hat{\boldsymbol{a}}}_i^{\top}\right) = \sum_{i=1}^{\ell} h_i(\boldsymbol{x})\boldsymbol{p}_1^{\top}\boldsymbol{e}\boldsymbol{x}^{\top}$$

$$\sum_{i=1}^{\ell}\left(\frac{1}{\gamma_{2i}}\dot{\hat{\boldsymbol{b}}}_i\right) = \sum_{i=1}^{\ell} h_i(\boldsymbol{x})\boldsymbol{p}_1^{\top}\boldsymbol{e}(u(t)+\psi(u(t)))$$

Then $\dot{\hat{\boldsymbol{a}}}_i^{\top}$ and $\dot{\hat{\boldsymbol{b}}}_i$ can be chosen, respectively, as follows:

$$\dot{\tilde{\boldsymbol{a}}}_i^{\top} = \dot{\hat{\boldsymbol{a}}}_i^{\top} = \gamma_{1i}h_i(\boldsymbol{x})\boldsymbol{p}_1^{\top}\boldsymbol{e}\boldsymbol{x}^{\top}, \tag{13}$$

$$\dot{\tilde{\boldsymbol{b}}}_i = \dot{\hat{\boldsymbol{b}}}_i = \gamma_{2i}h_i(\boldsymbol{x})\boldsymbol{p}_1^{\top}\boldsymbol{e}(u(t)+\psi(u(t))). \tag{14}$$

According to (11), (12), (13), and (14), we have $\dot{V} < 0$. In light of Lyapunov stability theory to the retarded functional differential equation ([Hale and Lunel (2002)]), $\boldsymbol{e}$ and $\hat{\boldsymbol{x}}$ as well as the parameter estimation errors $\hat{\boldsymbol{a}}_i$ and $\hat{b}_i$ are guaranteed to be uniformly ultimately bounded (UUB) for all realizations of uncertainties ([Khalil (2002)]). ■

4. SIMULATIONS

Consider a single link flexible joint manipulator whose dynamics has the following form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\dfrac{MgL}{I}sinx_1 - \dfrac{k}{I}(x_1 - x_3) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{K}{L}(x_1 - x_3) + \dfrac{1}{J}u \end{cases} \tag{15}$$

where I and J are the link and the rotor inertia moments, respectively, M is the link mass, k is the joint elastic constant, L is the distance from the axis of the rotation to the link center of mass, and $g = 9.8m/s^2$ is the gravitational acceleration. We first transform the nonlinear system (15) to the normal form as

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = z_3 \\ \dot{z}_3 = z_4 \\ \dot{z}_4 = a(\boldsymbol{z}) + b(\boldsymbol{z})u \end{cases} \tag{16}$$

where
$a(\boldsymbol{z}) = (MgL/I)sin(z_1)(z_2^2 + (MgL/I)cos(z_1) + (k/I)) - (z_3 + (MgL/I)sin(z_1))((k/I) + (k/I) + (MgL/I)cos(z_1))$ and $b(\boldsymbol{z}) = (k/IJ)$ for which $\boldsymbol{z} = [z_1\ z_2\ z_3\ z_4]^{\top}$. The fuzzy rules for T-S fuzzy model can be obtained from linearizing the nonlinear system (16) for $z_1, z_2 \in \{-\pi, 0, \pi\}$. That is, the triangular type membership function given in Figs. 2 and 3 is used to represent the meaning of the premise part of fuzzy rules:

Plant rule i : IF z_1 is about M_{i1} and z_2 is about M_{i2}

$$\text{THEN } \dot{\boldsymbol{z}} = \mathbf{A}_i\boldsymbol{z} + \mathbf{B}_i u,\ i = 1, 2, \cdots, 9. \tag{17}$$

The membership functions for the states, z_1 and z_2 are presented in Figs. 2 and 3. In this experiments, it is assumed that the physical parameters in the dynamics (16) are not known exactly.

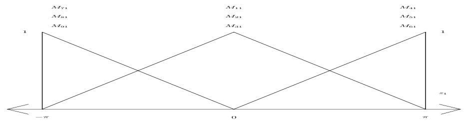

Fig. 2. Membership functions for z_1.

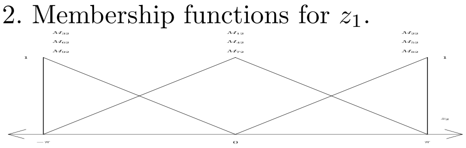

Fig. 3. Membership functions for z_2.

Hence, the parameters, $a_{i2}, a_{i4}, i = 1, 2, \cdots, 9$, are tuned by the estimator. From (12), the control law can be given by

$$u(t) = \sum_{i=1}^{4} h_i(\boldsymbol{x})\frac{(\boldsymbol{a}_d^{\top} - \hat{\boldsymbol{a}}_i^{\top})\boldsymbol{x}}{m\hat{b}_i} - \psi(u(t)), \tag{18}$$

where $\boldsymbol{a}_d^{\top} = [-15.5\ \ -15.5\ \ -15.5\ \ -15.5]$. By adaptive law (11), the adaptation rates are set as $\gamma_{11} = 75.5$, $\gamma_{12} = 750.5$, $\gamma_{13} = 503.0169$, $\gamma_{14} = 75.5$, $\gamma_{15} = 750.5$, $\gamma_{16} = 503.0169$, $\gamma_{17} = 75.5$, $\gamma_{18} = 750.5$, $\gamma_{19} = 1503.0169$, $\gamma_{21} = 1000.5$, $\gamma_{22} = 1000.5$, $\gamma_{23} = 1000.5$, $\gamma_{24} = 1000.5$, $\gamma_{25} = 1000.5$, $\gamma_{26} = 1000.5$, $\gamma_{27} = 1000.5$, $\gamma_{28} = 1000.5$, and $\gamma_{29} = 1000.5$. The plant parameters are adjusted online and the matrix $\mathbf{P}$ is obtained by solving the Lyapunov equation (11) with $\boldsymbol{a}_d = [-15.5\ -15.5\ -15.5\ -15.5]^{\top}$ and the last row of the positive definite matrix P is obtained as $\boldsymbol{p}_1 = [0.0062\ 0.00001\ 0.0062\ 0.00001]^{\top}$. Also, the values of b_{i0} are given by $b_{10} = 0.0051$, $b_{20} = 0.0051$, $b_{30} = 100.0051$, $b_{40} = 0.0051$, $b_{50} = 0.0051$, $b_{60} = 100.0051$, $b_{70} = 0.0051$, $b_{80} = 0.0051$, $b_{90} = 100.0051$, $\boldsymbol{z}(0) = [z_1(0)\ z_2(0)\ z_3(0)\ z_4(0)]^{\top} = [\pi/5\ \pi/5\ \pi/5\ 0]^{\top}$ and $\hat{\mathbf{Z}}(0) = [\hat{z}_1(0)\ \hat{z}_2(0)\ \hat{z}_3(0)\ \hat{z}_4(0)]^{\top}$ $= [\pi/3\ \pi/3\ \pi/3\ 0]^{\top}$. In the fuzzy system, the fuzzy subsystems are as follows: $\boldsymbol{a}_1 = [0\ -10\ 0\ -10]^{\top}$, $\boldsymbol{a}_2 = [0\ \ -10.5\ 0\ 85]^{\top}$, $\boldsymbol{a}_3 = [0\ \ -10.5\ 0\ \ -85]^{\top}$, $\boldsymbol{a}_4 = [0\ 7\ 0\ \ -9]^{\top}$, $\boldsymbol{a}_5 = [0\ 7\ 0\ \ -85]^{\top}$, $\boldsymbol{a}_6 = [0\ 7\ 0\ \ -85]^{\top}$, $\boldsymbol{a}_7 = [0\ 7\ 0\ \ -9]^{\top}$, $\boldsymbol{a}_8 = [0\ 7\ 0\ \ -85]^{\top}$, $\boldsymbol{a}_9 = [0\ 7\ 0\ \ -85]^{\top}$, and $b_i(0) = 1$, $i = 1, 2, \cdots, 9$. Based on the linearization of the nonlinear plant dynamicequation, the fuzzy model is given by $\hat{\boldsymbol{a}}_i = [\hat{a}_{i1}\ \hat{a}_{i2}\ \hat{a}_{i3}\ \hat{a}_{i4}]^{\top}$ and $\hat{\boldsymbol{b}}_i = [0\ 0\ 0\ \hat{b}_i]^{\top}$, $i = 1, 2, \cdots, 9$, where we assumed that the initial conditions of the plant model in this simulation are as follows: $\hat{a}_{11}(0) = -0.3$, $\hat{a}_{12}(0) = -9.9$, $\hat{a}_{13}(0) = 0.1$, $\hat{a}_{14}(0) = -9.9$, $\hat{a}_{21}(0) = -0.075$, $\hat{a}_{22}(0) = -10.52$, $\hat{a}_{23}(0) = 0.05$, $\hat{a}_{24}(0) = -84.98$, $\hat{a}_{31}(0) = -0.136$, $\hat{a}_{32}(0) = -10.43$, $\hat{a}_{33}(0) = 0.02$, $\hat{a}_{34}(0) = -84.98$, $\hat{a}_{41}(0) = -0.1$, $\hat{a}_{42}(0) = 7.015$, $\hat{a}_{43}(0) = 0.045$, $\hat{a}_{44}(0) = -8.954$, $\hat{a}_{51}(0) = -0.025$, $\hat{a}_{52}(0) = 6.99$, $\hat{a}_{53}(0) = 0.01$, $\hat{a}_{54}(0) = -84.99$, $\hat{a}_{61}(0) = -0.04$, $\hat{a}_{62}(0) = 7.02$, $\hat{a}_{63}(0) = 0.011$, $\hat{a}_{64}(0) = -84.99$, $\hat{a}_{71}(0) = -0.004$, $\hat{a}_{72}(0) = 7.0015$, $\hat{a}_{73}(0) = 0.0016$, $\hat{a}_{74}(0) = -8.9984$, $\hat{a}_{81}(0) = -0.0012$,

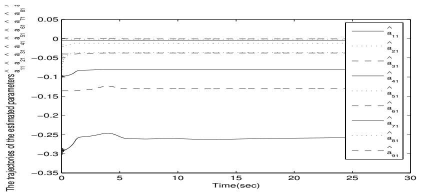

Fig. 4. The trajectories of estimated parameters $\hat{a}_{11}$, $\hat{a}_{21}$, $\hat{a}_{31}$, $\hat{a}_{41}$, $\hat{a}_{51}$, $\hat{a}_{61}$, $\hat{a}_{71}$, $\hat{a}_{81}$, and $\hat{a}_{91}$.

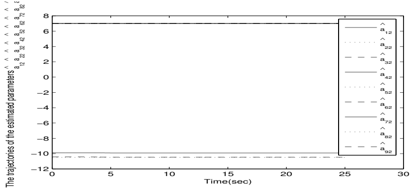

Fig. 5. The trajectories of estimated parameters $\hat{a}_{12}$, $\hat{a}_{22}$, $\hat{a}_{32}$, $\hat{a}_{42}$, $\hat{a}_{52}$, $\hat{a}_{62}$, $\hat{a}_{72}$, $\hat{a}_{82}$, and $\hat{a}_{92}$.

$\hat{a}_{82}(0) = 7.00057$, $\hat{a}_{83}(0) = 0.00026$, $\hat{a}_{84}(0) = -84.9996$, $\hat{a}_{91}(0) = 0.0008$, $\hat{a}_{92}(0) = 7.00001$, $\hat{a}_{93}(0) = -0.00058$, $\hat{a}_{94}(0) = -85.0001$, $\hat{b}_1(0) = 1.9$, $\hat{b}_2(0) = 1.9$, $\hat{b}_3(0) = 2.9$, $\hat{b}_4(0) = 1.9$, $\hat{b}_5(0) = 1.9$, $\hat{b}_6(0) = 1.9$, $\hat{b}_7(0) = 1.9$, $\hat{b}_8(0) = 1.9$, $\hat{b}_9(0) = 2.9$. Since the parameters of the nonlinear plant dynamic equation is unknown, or known partially, every parameters comprising the Takagi-Sugeno fuzzy model (17) should be tuned for constructing the control law. Let the true parameters of the dead-zone for simulations be set as $m = 1$, $b_r = 0.75$, $b_l = -0.75$, $b_{rmax} = 0.8$, $b_{rmin} = 0.1$, $b_{lmax} = -0.1$, $b_{lmin} = -0.8$, and $\psi = 0.6$. The simulation results presented are the trajectory of the tracking errors, the trajectories of estimated parameters, the trajectories of control input u with a dead-zone, the trajectories of the dead-zone, and the trajectories of $z_1(t)$, $\hat{z}_1(t)$, $z_2(t)$, $\hat{z}_2(t)$, $z_3(t)$, $\hat{z}_3(t)$, $z_4(t)$ and $\hat{z}_4(t)$ are shown in Figs. 4-14. In the simulations for each parameters can guarantee to converge, and tracking error converges to the neighborhood of zero. The trajectories of $z_1(t)$, $\hat{z}_1(t)$, $z_2(t)$, $\hat{z}_2(t)$, $z_3(t)$, $\hat{z}_3(t)$, $z_4(t)$ and $\hat{z}_4(t)$ are shown in Figs. 11, 12, 13, and 14, respectively. The trajectories of control input u with a dead-zone is shown in Fig. 10. From Figs. 9, the trajectory of the tracking errors e_1 e_2 e_3 and e_4 converge to the neighborhood of zero in the steady state.

5. CONCLUSION

The parameter estimator can be used in the plant parameters of the MIMO Takagi-Sugeno fuzzy model are varying or uncertain was proposed. In order to show the effectiveness of the control scheme, the fuzzy state feedback controllers with and without the estimator were respectively applied to the plant model. By using a description of the dead-zone and exploring the properties of this dead-zone model intuitively and mathematically, this adaptive fuzzy controller method is presented without constructing the dead-zone inverse. This

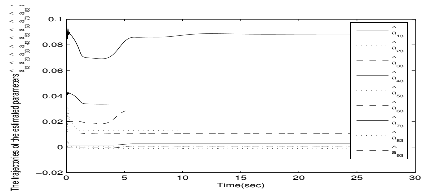

Fig. 6. The trajectories of estimated parameters $\hat{a}_{13}$, $\hat{a}_{23}$, $\hat{a}_{33}$, $\hat{a}_{43}$, $\hat{a}_{53}$, $\hat{a}_{63}$, $\hat{a}_{73}$, $\hat{a}_{83}$, and $\hat{a}_{93}$.

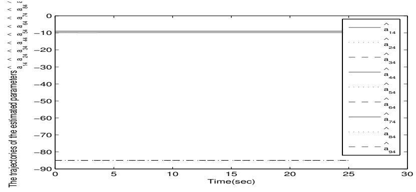

Fig. 7. The trajectories of estimated parameters $\hat{a}_{14}$, $\hat{a}_{24}$, $\hat{a}_{34}$, $\hat{a}_{44}$, $\hat{a}_{54}$, $\hat{a}_{64}$, $\hat{a}_{74}$, $\hat{a}_{84}$, and $\hat{a}_{94}$.

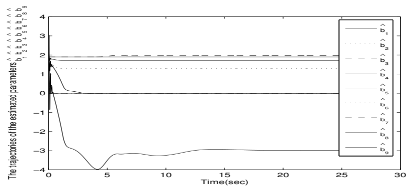

Fig. 8. The trajectories of estimated parameters $\hat{b}_1$, $\hat{b}_2$, $\hat{b}_3$, $\hat{b}_4$, $\hat{b}_5$, $\hat{b}_6$, $\hat{b}_7$, $\hat{b}_8$, and $\hat{b}_9$.

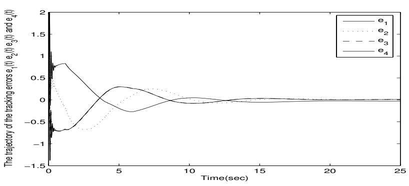

Fig. 9. The trajectory of the tracking errors e_1 e_2 e_3 and e_4.

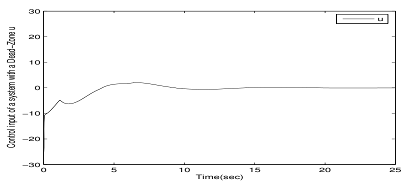

Fig. 10. The trajectories of control input u with a dead-zone.

paper deals with the adaptive fuzzy control of a class of uncertain nonlinear system preceded by a dead-zone. Based on Lyapunov stability theorem, the proposed adaptive fuzzy controller scheme can not only guarantee the stability of the whole closed-loop system with a dead-zone in the actuator, but also obtain the good tracking performance. Finally, some examples and simulation results are used to illustrate the effectiveness and performance of the proposed method.

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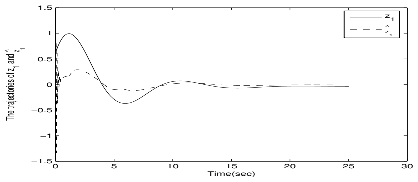

Fig. 11. The trajectories of $z_1(t)$ and $\hat{z}_1(t)$.

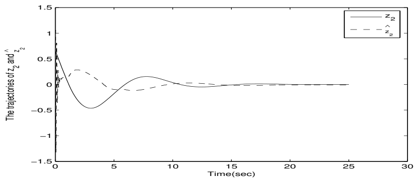

Fig. 12. The trajectories of $z_2(t)$ and $\hat{z}_2(t)$.

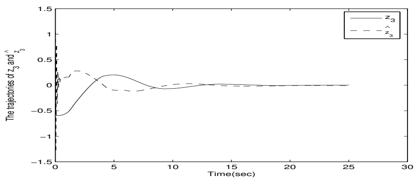

Fig. 13. The trajectories of $z_3(t)$ and $\hat{z}_3(t)$.

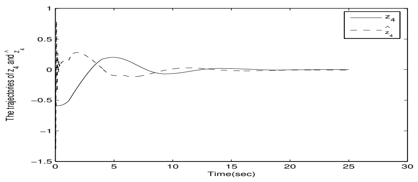

Fig. 14. The trajectories of $z_4(t)$ and $\hat{z}_4(t)$.

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