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Operatorial Parametrizing of Controlled Dynamic Systems - Application to the Fed-Batch Bioreactor Control Problem

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Abstract: Dynamic models most of time involve differential equations, which are "time-local". Such models can also be considered "globally", that is in the sense of "trajectories" in the state "space-time". Up to adapted concepts, such a different interpretation reveals itself more flexible, namely because it allows to use various operatorial transformations whose time-local equivalent in general cannot exist and from which can result some remarkable properties. Namely, we introduce a principle of parametrizing for dynamic equations by means of such transformations. We then consider an example of bioreactor model for which we highlight how suitable time-nonlocal transformations can sometimes be used to efficiently solve some nonlinear control problems.

Keywords: Dynamic system, Parametrizing, Operator, Time Scale Transformation, Control, Fed-batch Bioreactor

1. INTRODUCTION

Most of controlled dynamic models are constituted by explicit differential equations of the standard abstract form $\dot{X} = f(u, X)$, that is:

$$\forall t, \quad \dot{X}(t) = f(u(t), X(t)). \tag{1}$$

So, for any t, as soon as $\dot{X}(t)$ is determined, equality (1) is valid in the state-space in which evolves $X(t)$ (in general $\mathbb{R}^n$). In that sense and thanks to the *local* [1] nature of the derivative operator, such an equation is said "time-local". In other words, all the dynamic features of model (1) are in some sense included in the function $f : \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}^n$: this last point is at the origin of various theoretical works based on finitedimensional algebra or differential geometry and devoted to control problems relating to (1). In particular, suitable diffeomorphic transformations of the state-space have sometimes been used in order to simplify the problem, for example in such a manner that in the new coordinates, the dynamic system becomes linear (see for example Bamieh [2007], Fujimoto [1996], Pavlov [2004]).

On the other hand, we can remark that any equation of the previous form can also be considered in the (weaker) sense of "trajectories": in that case, u and X are *globally* considered as functions (of t) and the differential equation as a *functional* one. Then, the time derivative and the function f become *operators* [2], acting on fitted *functional* spaces. According to this point of view, by denoting ∂_t the time derivative operator, we preferably write (1) under the forum:

$$\partial_t X = f(u, X). \tag{2}$$

Note that, according to its functional nature, this last expression can be equivalently written: $\forall t, \ (\partial_t X)(t) = (f(u, X))(t)$. Contrary to (1), the static nature of operator f, that is: $(f(u, X))(t) = f(u(t), X(t))$, is not necessary under the functional formulation (2) and thus must be explicitly supposed.

The functional interpretation of differential equations can of course appear more complex from the mathematical point of view. But up to suitable concepts (which are to be judiciously precised in concrete situations), it revels itself more flexible, namely because it suggests the possibility of using various operatorial transformations whose time-local equivalent cannot exist and from which can result some remarkable properties, as shown later on a concrete example.

We can also notice that more general dynamic models such as Volterra equations could be similarly considered by simply replacing the operator ∂_t in (2) by a convolution one, denoted [3] $H(\partial_t)$. We then get the model: $H(\partial_t)X = f(u, X)$ in which the left-hand side is dynamic but linear while the right-hand side is static and (in general) nonlinear.

This introductive paper is organized as follows. In the next section, we briefly present a few basic notions which permit to correctly define the so-called operatorial parametrizing of controlled dynamic systems. The statement is formal and reduced to the essential parts. In the third section, we describe some basic concrete operators possibly involved

[1] The determination of $\dot{X}(t)$ only involves values $X(\tau)$ with τ *arbitrarily close* to t.

[2] Note that the operator induced by the function f is a different mathematical object: from a rigorous point of view, it should be denoted with a different symbol.

[3] In this symbolic notation, H is the Laplace transform of the impulse response of the operator under consideration.

when parametrizing dynamic systems. In the fourth section, we consider an example of bioreactor model by means of which we highlight how such time-nonlocal transformations can be used to solve nonlinear control problems equivalently to simpler ones (a linear one in the presented case).

2. OPERATORIAL PARAMETRIZING OF DYNAMIC SYSTEMS

2.1 Graph parametrizing of an abstract equation

Given two manifolds $\mathcal{X}$ and $\mathcal{U}$, we consider the following abstract equation of unknown X, depending on data u:

$$\mathbf{\Phi}(u, X) = 0,\ X \in \mathcal{X},\ u \in \mathcal{U}, \tag{3}$$

and supposed to be well-posed[4]. Consequently, there exists a continuous application $\mathbf{F} : \mathcal{U} \to \mathcal{X}$ such that the solution X is expressed by:

$$X = \mathbf{F}(u) \in \mathbf{F}(\mathcal{U}) \subset \mathcal{X},$$

that is $(u, x) \in \text{graph}(\mathbf{F})$. In many cases, the application $\mathbf{F}$ cannot be explicited or is of too great complexity to be used by itself.

Let now consider a manifold $\mathcal{Y}$ and a continuous operator $\mathbf{A} : \mathcal{U} \times \mathcal{X} \to \mathcal{Y}$ such that $\mathbf{A}_{|\,\text{graph}(\mathbf{F})}$ is an homeomorphism; we denote:

$$y := \mathbf{A}(u, X) \tag{4}$$

and

$$(\mathbf{B}, \mathbf{C}) := (\mathbf{A}_{|\,\text{graph}(\mathbf{F})})^{-1} : \mathcal{Y} \to \mathcal{U} \times \mathcal{X}. \tag{5}$$

Then, for any solution (u, X) of (3), u and X are both parametrized by $y \in \mathcal{Y}$. If in addition $\mathcal{Y}$ and $(\mathbf{B}, \mathbf{C})$ can be explicitly described, the solutions (u, X) are directly accessible without resolving (3).

Remark 1. We have, from (3):

$$\mathbf{\Phi}(\mathbf{B}(y), \mathbf{C}(y)) = 0, \tag{6}$$

or, in a more condensed way $\mathbf{H}(y) = 0$ with $\mathbf{H} := \mathbf{\Phi} \circ (\mathbf{B}, \mathbf{C})$. The equation (3) is then transformed into (6), which implicitly defines the manifold $\mathcal{Y} = \mathbf{A}(\mathcal{U}, \mathbf{F}(\mathcal{U}))$.

Example 1. Consider the system of equations with unknown $X = (x_1, x_2)^T \in \mathbb{R}^2$:

$$\begin{cases} x_2 - u\cos(x_1) = 0 \\ e^{x_2} - x_1 x_2 = 0 \end{cases},$$

With:

$$y = u\cos(x_1) := \mathbf{A}(u, X),$$

we have the following parametrized expression of any couple (u, X) solution of (3):

$$\begin{cases} u = \dfrac{y}{\cos(\frac{e^y}{y})} := \mathbf{B}(y) \\ X = \begin{pmatrix} y \\ \frac{e^y}{y} \end{pmatrix} := \mathbf{C}(y). \end{cases}$$

The parametrizing of graph($\mathbf{F}$) can present some interest for problems defined on solutions (u, X) of (3). Consider for example an optimisation problem under the constraint (3):

$$\min_{u \in \mathcal{U}} \{J(u, X); \mathbf{\Phi}(u, X) = 0\}. \tag{7}$$

From (5) and by denoting $\tilde{J}(y) := J(\mathbf{B}(y), \mathbf{C}(y))$, problem (7) is equivalent to the problem:

$$\min_{y \in \mathcal{Y}} \tilde{J}(y). \tag{8}$$

As the solutions (u, X) of (3) are parametrized by $y \in \mathcal{Y}$, the solution u^* of (7) is then deduced from the solution y^* of (8) by $u^* = \mathbf{B}(y^*)$. So, if applications $(\mathbf{B}, \mathbf{C})$ are concretely tractable, this approach can result in a simplified resolution of (7). In some cases, the simplification can be highly significant, as shown in section 4.

Remark 2. Instead of a constrained optimization problem, we can more generally consider any problem of the form:

$$\begin{cases} \text{find } u \in \mathcal{U} \text{ such that } J(u, X) = 0 \\ \text{with } \mathbf{\Phi}(u, X) = 0, \end{cases}$$

which, after graph parametrizing, becomes:

$$\text{find } y \in \mathcal{Y} \text{ such that } \tilde{J}(y) = 0.$$

2.2 Cases of dynamic problems

The abstract notion presented above can be specified for dynamic equations which, to be coherent with the formalism previously introduced, must necessarily be taken in the sense of trajectories. Consequently, applications $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are operators and the problem of their choice is posed in an operatorial framework.

In the sequel, (3) is a dynamic system of the form:

$$\mathbf{\Phi}(u, X) = \mathcal{H}X - F(u, X) = 0, \tag{9}$$

with $\mathcal{H}$ a linear dynamic operator, F a static (nonlinear) operator (that is: $(F(u, X))(t) = F(u(t), X(t))$), and $\mathcal{U}$, $\mathcal{X}$, $\mathcal{Y}$ are suitable trajectory manifolds.

Remark 3. Possible conditions insuring uniqueness of the solution (initial condition, past evolution etc.) are included in (9) (cf example 2).

Example 2. Classical differential systems:

$$\begin{cases} \partial_t X = f(u, X),\ t \in]0, T[\\ X(0) = X_0, \end{cases} \tag{10}$$

are particular cases of (9) with:

$$\mathcal{H} = \begin{pmatrix} \partial_t \\ < \delta, \cdot > \end{pmatrix},\ F(u, X_0, X) = \begin{pmatrix} f(u, X) \\ X_0 \end{pmatrix}, \tag{11}$$

where $< \delta, \cdot >$ is the Dirac operator $v \mapsto v(0)$. Manifolds $\mathcal{U}$ and $\mathcal{X}$ must be adapted to the problem, for example $\mathcal{U} \subset L^\infty(0, T; \mathbf{U})$, $\mathcal{X} \subset C^0([0, T]; \mathbf{X})$, with $\mathbf{U}$, $\mathbf{X}$ Banach spaces.

Formulation (9) allows to consider a larger class of nonlinear dynamic systems than classical differential (and consequently time-local) one: instead of the derivative operator ∂_t, we can consider any causal convolution operator $H(\partial_t)$. Namely, Volterra equations, PDE's, hybrid systems etc. belong to this class of dynamic systems.

Remark 4. Equation (6), which characterizes the manifold $\mathcal{Y}$, can be useful when $\mathcal{Y}$ is not explicitly known. This equation is of functional type and, in general, not reducible to a differential or even integral equation.

[4] i.e. we have existence and uniqueness in $\mathcal{X}$, and continuous dependence on u of the solution X

As the resolution of (3) can be difficult (especially for nonlinear problems), this approach can result in significant simplifications, for example when (3) is a nonlinear dynamic constraint of an optimization problem. Then, we must define judicious operatorial transformations usable for parametrizing of dynamic systems.

3. OPERATORS FOR DYNAMIC SYSTEM PARAMETRIZING

The choice of the parametrizing operator $\mathbf{A}$ is crucial; indeed, a parametrizing will simplify the problem only if the following points are satisfied:

- the problem of unknown y (e.g. (8) in the case of optimal control) is concretely tractable;
- u and X are concretely deductible from y via (5).

Thus, the operators $(\mathbf{B}, \mathbf{C})$ must belong to a class which we can numerically deal with, taking into account possible computational cost contraints for real-time applications. We briefly describe in this section some useful categories of such operators.

3.1 Static operators

It is the simplest class of operators. An operator $\mathbf{G}$ is said static if $(\mathbf{G}(f))(t) = G(t, f(t))$ with G a classical function. In other words, the evaluation of $\mathbf{G}(f)$ at time t only requires the punctual value $f(t)$.

In the sequel, for simplicity of notations, if G is a function we will also denote by G the associated static operator, that is:

$$G(v) : t \mapsto G(v(t)).$$

3.2 Convolution operators, linear dynamic operators

Convolution operators are symbolically denoted $H(\partial_t)$ where H is the associated transfer function. They are particularly interesting in the context of parametrizing of dynamic systems. From the numerical point of view, such linear operators can be associated to matrices and, up to suitable representations, their action on functions can be evaluated with arbitrary precision and reasonable cost (compatible with real-time applications when suitable input-output state realizations are elaborated Montseny [2004]).

Furthermore, in the causal case, the well-known Titchmarch theorem insures that there is no zero divider in algebras of convolution operators, and so, in suitable distributional frameworks Schwartz [1966], the inversion of $H(\partial_t)$ is in general a well posed problem, in such a way that all the standard calculus rules are available on convolution operators. We refer to Montseny [2004] for a complete statement of such questions about convolution (ore more generally integral) operators.

In the sequel, we will consider causal convolution operators $H(\partial_t)$ acting on *fitted functional spaces* in such a way that $H(\partial_t)$ admits a unique causal inverse $H(\partial_t)^{-1}$. As an example, the inverse of ∂_t (in the algebra of operators on the space of continuous functions with support in $\mathbb{R}^+$) is the integrator:

$$\partial_t^{-1} : u \mapsto \partial_t^{-1} u, \quad (\partial_t^{-1} u)(t) = \int_0^t u(s)\, ds. \tag{12}$$

Many other linear dynamic operators and their inverse can then easily be defined from convolution operators and other linear ones by means of various mathematical combinations. For example, the inverse of the dynamic linear operator $\mathcal{H}$ used in (11) is given by:

$$\mathcal{H}^{-1} \begin{pmatrix} u \\ u_0 \end{pmatrix} = \partial_t^{-1} u + u_0. \tag{13}$$

3.3 Time-scale transformation operators

There are many examples of physical dynamic systems that admit an intrinsic time, under which equations are simplified Visintin [1994], Fangtand [2003]. Classically, a time-scale transformation (TST) of the trajectory is an operator of the form:

$$x \mapsto x \circ \varphi$$

where $\varphi(t)$ is the new time, defined by φ an invertible increasing function. As the times t and $\varphi(t)$ are different, the causality of a time-scale operator cannot satisfy the standard definitions. However, useful TST are in general defined by means of an operatorial transformation of a function v that pilot the clock φ, that is $\varphi = \boldsymbol{\varphi}(v)$ with $\boldsymbol{\varphi}$ an operator.

Definition 1. The TST operator

$$S : (v, x) \mapsto x \circ \boldsymbol{\varphi}(v)$$

is said causal if the operator $\boldsymbol{\varphi}$ is causal.
In this case, the clock $\boldsymbol{\varphi}(v)$ is computed in a causal way from the function v; such a TST operator can be implemented in real-time.

Remark 5. Time-scale transformations can depend on the variables u and/or X of the problem.

Remark 6. The inverse of operator $z \mapsto \widetilde{z} := z \circ \varphi$ is $\widetilde{z} \mapsto z = \widetilde{z} \circ \varphi^{-1}$.

3.4 Other operators

Of course, any finite combinaison of the above-mentioned operators can be used. More generally, we only impose that involved operators are concretely usable. Namely, from the numerical point of view, they must be approximable with an arbitrary accuracy and reasonable computational costs.

4. APPLICATION OF OPERATORIAL PARAMETRIZING TO FED-BATCH BIOREACTOR CONTROL PROBLEMS

Fermentation is a critical process of production of substances from organic molecules. The high cost associated to many fermentation processes makes optimization of bioreactor performance very desirable. Unfortunately, the bioreactor dynamic models are highly nonlinear, which makes optimal control a difficult problem studied in many works Rani [1999], Peroni [2005], Zhihua [2002],Moya [2002] and with many industrial applications (maximisation of bioethanol production etc.).

We show in the sequel how operatorial parametrizing significantly simplifies control problems relating to fed-batch bioreactor equations.

4.1 The model under consideration

We consider the following model of fed-batch bioreactors Wang [2001]:

$$\begin{cases} \partial_t x = \mu(X)\, x - x\, u \\ \partial_t s = -a_1 \mu(X)\, x + (s_i - s)\, u \\ \partial_t p = a_2 \mu(X)\, x - p\, u \\ X(0) = X_0, \end{cases} \tag{14}$$

where x, s, p are the respective concentrations of biomass, substrate and product, $X = (x, s, p)^T$, μ is the growth rate, s_i the substrate concentration in feed, u (the control) is the dilution of feed and X_0 the initial conditions.

4.2 Operatorial parametrizing of (14)

Time-scale transformation We consider the time-scale transformation $z \mapsto \widetilde{z} := z \circ \varphi^{-1}$, with $\varphi : t \mapsto \tau$ such that $\varphi' = u > 0$ and $\varphi(0) = 0$, that is[5] :

$$\varphi = \partial_t^{-1} u. \tag{15}$$

The associated TST has the following expression:

$$S : (u, z) \mapsto \widetilde{z} = z \circ \left(\partial_t^{-1} u\right)^{-1},$$

where $\widetilde{z}$ denotes the trajectory z in time $\tau = \varphi(t)$.
For convenience, we denote:

$$S_u := S(u, \cdot).$$

Remark 7. In the same way, we denote

$$\widetilde{u} := S_u(u) = u \circ \left(\partial_t^{-1} u\right)^{-1}.$$

Then, we know that:

$$S_u^{-1} : \widetilde{z} \mapsto z = \widetilde{z} \circ \left(\partial_t^{-1} u\right).$$

Moreover, as $d\tau = u\, dt$, we have:

$$\left(\varphi^{-1}\right)' = \frac{1}{\widetilde{u}}$$

and then:

$$\varphi^{-1} = \partial_\tau^{-1} \frac{1}{\widetilde{u}},$$

from which we deduce different expressions of operators $z \mapsto \widetilde{z}$ and $\widetilde{z} \mapsto z$:

$$S_{\frac{1}{\widetilde{u}}}^{-1} : z \mapsto \widetilde{z} = z \circ \partial_\tau^{-1} \frac{1}{\widetilde{u}}$$
$$S_{\frac{1}{\widetilde{u}}} : \widetilde{z} \mapsto z = \widetilde{z} \circ \left(\partial_\tau^{-1} \frac{1}{\widetilde{u}}\right)^{-1}.$$

By denoting $T_{\tilde{u}} := S_{\frac{1}{\widetilde{u}}}^{-1}$, we then have two expressions of the time-scaling transformation (and its inverse) depending respectively on u and $\tilde{u}$, as summarized in the following scheme:

$$z \underset{S_u^{-1}, T_{\tilde{u}}^{-1}}{\overset{S_u, T_{\tilde{u}}}{\longrightarrow \atop \longleftarrow}} \widetilde{z}.$$

As $\partial_t \varphi = u$, we can remark that the proposed time-scale transformation is governed by the feed dilution of the bioreactor, which is rather natural. Moreover, it is causal and then can be used in real time applications.

[5] We denote ∂_t^{-1} the operator $u \mapsto \int_0^t u\, ds$

We will denote by $\mathbf{S}$ the operator $z \mapsto \widetilde{z}$, indifferently referring to its expression depending on u (that is S_u) or $\widetilde{u}$ (that is $T_{\tilde{u}}$). For simplicity of notations, we consider that $\mathbf{S}$ can be applied either to scalar or vectorial trajectories by using the convention:

$$\mathbf{S}(z_1, ..., z_n) := (\mathbf{S}(z_1), ..., \mathbf{S}(z_n)).$$

Parametrizing operators Consider now the following operator:

$$\mathbf{A} : C^\infty(\mathbb{R}_t^+; \mathbb{R}) \times C^\infty(\mathbb{R}_t^+; \mathbb{R}^3) \to C^\infty(\mathbb{R}_t^+; \mathbb{R}) \times \mathbb{R}^3$$
$$(u, X) \mapsto y = (\widetilde{\mathbf{A}} \circ \mathbf{S})(u, X)$$

where

$$\widetilde{\mathbf{A}} : C^\infty(\mathbb{R}_\tau^+) \times C^\infty(\mathbb{R}_\tau^+; \mathbb{R}^3) \to C^\infty(\mathbb{R}_\tau^+; \mathbb{R}) \times \mathbb{R}^3$$
$$(\widetilde{u}, \widetilde{X}) \mapsto \left(\frac{\mu(\widetilde{X})\widetilde{x}}{\widetilde{u}}, \langle \delta, \widetilde{x} \rangle, \langle \delta, \widetilde{s} \rangle, \langle \delta, \widetilde{p} \rangle \right)^T$$

Note that the terms $\langle \delta, \widetilde{x} \rangle$ relate to initial conditions of (14).

By applying the operator $\mathbf{S}$ on (14), simple computations based on $y = \mathbf{A}(u, X)$ and the property $\frac{dx}{dt} = \frac{dx}{d\tau}\frac{d\tau}{dt}$, lead to the following linear system:

$$\begin{cases} \partial_\tau \widetilde{x} = -\widetilde{x} + y_1 \\ \partial_\tau \widetilde{s} = -\widetilde{s} + s_i - a_1 y_1 \\ \partial_\tau \widetilde{p} = -\widetilde{p} + a_2 y_1, \end{cases} \tag{16}$$

where y_i, $i = 1:4$ denotes the ith component of y. Furthermore, we deduce from (16) the expression of the associated operators $(\mathbf{B}, \mathbf{C})$:

$$u = \mathbf{B}(y) = (\mathbf{S}^{-1} \circ \widetilde{\mathbf{B}})(y) \tag{17}$$
$$X = \mathbf{C}(y) = (\mathbf{S}^{-1} \circ \widetilde{\mathbf{C}})(y),$$

with:

$$\widetilde{\mathbf{C}}(y) = \begin{pmatrix} (\partial_\tau + 1)^{-1}(y_1) + y_2\, e^{-(\cdot)} \\ (\partial_\tau + 1)^{-1}(s_i - a_1\, y_1) + y_3\, e^{-(\cdot)} \\ (\partial_\tau + 1)^{-1}(a_2\, y_1) + y_4\, e^{-(\cdot)} \end{pmatrix}$$

$$\widetilde{\mathbf{B}}(y) = \frac{\mu(\mathbf{C}(y))\, \mathbf{C}_1(y)}{y_1}.$$

We can remark that those operators are finite combinaison of static/linear dynamic/TST operators. Up to the above operatorial transformations, the system (16) is equivalent to (14).

Then, classical control methods of linear systems can be investigated on (16) (optimal control, linear feedback stabilization, predictive control and so on), and straightforward solutions relating to (14) are directly deduced from the ones of (16) by means of (17).

Figures 1,2,3 highlight how such solutions can be implemented on the physical process.

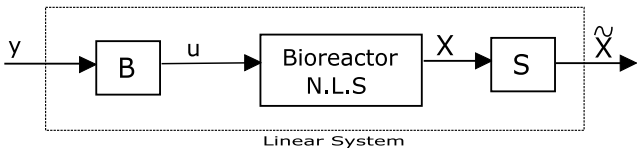

Fig. 1. The linear system (16)

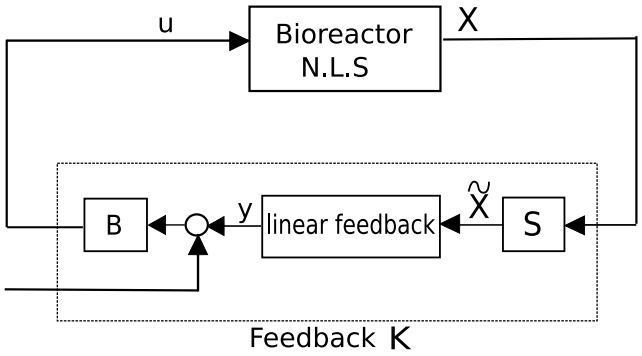

Fig. 2. Stabilizing feedback control

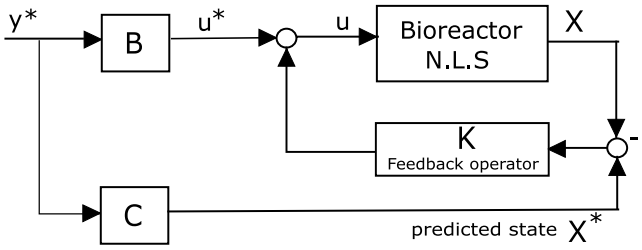

Fig. 3. Predictive control

5. CONCLUSION

The present paper must be seen as a brief introduction to a methodology based on operatorial transformations and devoted to nonlinear dynamic problems. Through a concrete example, it has been shown that in spite of some apparent complexity, efficient solutions can be found when an explicit operatorial parametrizing can be exhibited. The obtained linear formulation allows to envisage the practical implementation of optimal controls on fed-batch bioractors.

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