Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

A New Output Feedback Control for Nonlinear Differential-Algebraic-Equation Systems

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Abstract: This note considers the problem of stabilization by output feedback for a class of nonlinear Differential-Algebraic-Equation systems. The output feedback controller is constructed which ensures the closed-loop systems asymptotically stable. Not based on separation-principle that is commonly adopted in the literature, the output feedback controller design is coupled with that of the non-initialized linear high gain state observer. The numerical simulation results illustrate the effectiveness of the proposed scheme.

1. INTRODUCTION

Many physical problems yield mathematical descriptions that are a mixture of ordinary differential equations and algebraic equations (DAE) (Kumar, 1999). In order to provide a measure of the difference between DAE systems and ordinary-differential-equation (ODE) systems, the notion of differential index is commonly used, which corresponds to the minimum number of differentiations of the algebraic equations required to obtain an equivalent ODE systems (Petzold, 1982). Control of linear DAE systems has been studied extensively, either assuming arbitrary initial conditions with the objective of removing the impulsive behaviour through feedback, or yielding the smooth solution within the more conventional framework with consistent initial conditions (Kumar, 1996).

Among nonlinear DAE systems, the systems of index one represent an important class of physical systems such as power systems, electric circuits and so on. The Lyapunov method of nonlinear ODE systems is extended for nonlinear DAE systems of index one and the sufficient conditions of stability are presented (Hill, 1991). For affine nonlinear DAE systems of index one, the problem of exact linearization is considered (Wang, 2001). The problem of regularization of high index nonlinear DAE systems is considered through output feedback precompensator (Marie-Nathalie, 2005).

One of the important problems in the field of nonlinear control is stabilization by output feedback. A high gain observer, which is in fact a "Luenberger-like" nonlinear observer, is proposed for nonlinear DAE systems of index one (Yaagoubi, 2005 and Assoudi, 2005). In order to obtain the state estimation, two crucial restrictions, i.e., the Lipschitz condition and lower-triangular structure of the equivalent systems, are needed. While in many cases, the system under consideration cannot satisfy above two restrictions. Motivated by (Qian, 2003), in this paper we will present a new output feedback controller design for a class of nonlinear DAE systems which have been considered by Yaagoubi and Assoudi. Not based on separation principle, our state observer is not a copy of original systems but coupled with the controller design. By choosing the gain parameters of the observer and the virtual controllers step-by-step, a linear output dynamic compensator is obtained, making the closed-loop systems asymptotically stable.

2. PROBLEM DESCRIPTION

In this paper, we consider following single-input single-output (SISO) nonlinear DAE systems

$$\begin{aligned} \dot{x} &= f_1(x,z) + g(x,z)u \\ 0 &= f_2(x,z) \\ y &= h(x,z) \end{aligned} \tag{1}$$

where $x = (x_1, \cdots, x_n)^T \in R^n$, $z = (z_1, \cdots, z_m)^T \in R^m$, $u \in R$ and $y \in R$ are the vector of differential variables, vector of algebraic variables, input and output respectively. The mappings f_1, f_2, g and h are sufficiently smooth.

Denote Ω the set of zeros of $f_2 : \Omega = \left\{ (x,z) \in R^n \times R^m : f_2(x,z) = 0 \right\}$. Without loss of generality, we assume that (1) has an isolated equilibrium in Ω which we regard to be the origin.

Throughout this paper, the following assumptions are made for (1).

Assumption 1: The nonlinear DAE systems (1) are of index one, i.e., the Jacobian matrix of $f_2(x,z)$ with respect to z has constant full rank on Ω:

$$rank(\frac{\partial f_2}{\partial z}) = m \tag{2}$$

For convenience, we give following notation

$$F(x,z)=\begin{bmatrix} I_n \\ -\left(\frac{\partial f_2}{\partial z}\right)^{-1}\frac{\partial f_2}{\partial x}\end{bmatrix} \tag{3}$$

and define nonlinear transformation

$$\begin{bmatrix}\xi \\ \cdots \\ \chi\end{bmatrix}=\begin{bmatrix}T(x,z)\\ \cdots \\ f_2(x,z)\end{bmatrix}\triangleq \Psi(x,z) \tag{4}$$

where $\xi=(\xi_1,\cdots,\xi_n)^T\in R^n$ and $T=\begin{pmatrix}h(x,z)\\ \vdots \\ L_{Ff_1}^{n-1}h(x,z)\end{pmatrix}$ with

$L_{Ff_1}^{i}h\triangleq L_{Ff_1}\left(L_{Ff_1}^{i-1}h\right)\triangleq\left(\frac{\partial L_{Ff_1}^{i-1}h}{\partial x}\frac{\partial L_{Ff_1}^{i-1}h}{\partial z}\right)Ff_1,\ i=1,\cdots,n$.

Assumption 2: Ψ is a diffeomorphism from a tubular neighborhood $\Omega_\varepsilon=\{(x,z)\in\Omega \mid \|f_2(x,z)\|<\varepsilon\}$ of Ω.

Assumption 3: There is some constant $\sigma>0$ such that $\|\Psi(x_1,z_1)-\Psi(x_2,z_2)\|\ge\sigma\|(x_1-x_2,z_1-z_2)\|$.

Assumption 4: Ψ transforms the restriction of systems (1) to Ω into the following systems:

$$\begin{aligned}\dot\xi_1&=\xi_2+\phi_1(\xi,\chi,u)\\ &\vdots\\ \dot\xi_{n-1}&=\xi_n+\phi_{n-1}(\xi,\chi,u)\\ \dot\xi_n&=u+\phi_n(\xi,\chi,u)\\ \dot\chi&=0\\ y&=\xi_1\end{aligned} \tag{5}$$

restricted to the space $\chi=0$.

Assumption 5: For $i=1,\cdots,n$, there exists a constant $c\ge 0$ such that

$$|\phi_i(\xi,\chi,u)|\le c(|\xi_1|+\cdots+|\xi_i|) \tag{6}$$

The objective of this paper is to design an output dynamic compensator such that the closed-loop systems (1) are asymptotically stable at the zero equilibrium. It must be pointed out that systems (1) with Assumptions 1~5 cover a class of systems whose stabilization by output feedback does not seem to be solvable by existing design method. A nonlinear observer design scheme is proposed using the extension of high gain observer tech and separation-principle of nonlinear ODE systems (Yaagoubi, 2005 and Assoudi, 2005). In addition to Assumptions 1~4, their results also needs that the equivalent systems (5) satisfy following two crucial restrictions: 1): so-called lower triangular structure, i.e., $\phi_i(\xi,\chi,u)=\phi_i(\xi_1,\cdots,\xi_i), i=1,\cdots,n-1$ and $\phi_n(\xi,\chi,u)=\phi_n(\xi_1,\cdots,\xi_n)u$; 2) the nonlinear terms $\phi_i, i=1,\cdots,n$ satisfying Lipschitz condition, i.e., $\|\phi_i(\xi,\chi,u)-\phi_i(\bar\xi,\bar\chi,u)\|\le c\left(\|\xi-\bar\xi\|+\|\chi-\bar\chi\|\right)$ for some constant $c>0$. However many practical systems neither bear lower triangular structure nor satisfy the geometric conditions that can transform the controlled systems into lower triangular structure through some nonlinear coordinate transformation. On the other hand, the Lipschitz condition is sometimes been destroyed due to the disturbance and the noisy of measurements.

3. MAIN RESULTS

In this section, we prove that Assumptions 1~5 suffice to guarantee the existence of a stabilizing output feedback controller for systems (1).

Theorem 3.1: Under Assumptions 1~5, there exist a non initialized linear observer and a linear output feedback controller making the nonlinear DAE systems (1) asymptotically stable.

Proof: The proof consists of two parts. First of all we design a non-initialized high gain linear observer. Here "non initialized" means that the initial state of the observer satisfies $(\hat x(0),\hat z(0))\in\Omega_{\varepsilon_0}$ which will be defined later, instead of constrainting to $f_2(\hat x(0),\hat z(0))=0$. We then construct an output feedback controller based on a feedback domination design, making the closed-loop systems asymptotically stable.

3.1 Non-Initialized Linear High Gain State Observer

Our non-initialized observer takes the following form:

$$\begin{aligned}\dot{\hat\xi}_1&=\hat\xi_2+\theta k_1(\xi_1-\hat\xi_1)\\ &\vdots\\ \dot{\hat\xi}_{n-1}&=\hat\xi_n+\theta^{n-1}k_{n-1}(\xi_1-\hat\xi_1)\\ \dot{\hat\xi}_n&=u+\theta^n k_n(\xi_1-\hat\xi_1)\\ \dot{\hat\chi}&=-\Lambda\hat\chi\end{aligned} \tag{7}$$

where $\theta>1$ is the gain parameter to be determined later, $k_i>0, i=1,\cdots,n$ are coefficients of the Hurwitz polynominal $P(s)=s^n+k_1s^{n-1}+\cdots+k_{n-1}s+k_n$ and Λ is a $m\times m$ symmetric positive definite matrix.

We say (7) is a non-initialized linear state observer with the controller to be designed. This claim will be proved at the end of the proof. Set

$$\varepsilon = (\Delta_\theta)^{-1}(\xi - \hat{\xi}) \tag{8}$$

where $\Delta_\theta = \begin{pmatrix} \theta & & 0 \\ & \ddots & \\ 0 & & \theta^n \end{pmatrix}$. A simple calculation gives

$$\dot{\varepsilon} = \theta A \varepsilon + \begin{bmatrix} \phi_1(\xi,\chi,u) \\ \frac{1}{\theta}\phi_2(\xi,\chi,u) \\ \vdots \\ \frac{1}{\theta^{n-1}}\phi_n(\xi,\chi,u) \end{bmatrix} \tag{9}$$

where $\varepsilon = (\varepsilon_1,\cdots,\varepsilon_n)^T$, $A = \begin{bmatrix} -k_1 & 1\cdots 0 \\ \vdots & \vdots\ddots\vdots \\ -k_{n-1} & 0\cdots 1 \\ -k_n & 0\cdots 0 \end{bmatrix}$. From the choosing of $k_i > 0, i = 1,\cdots,n$, clearly A is a Hurwitz matrix. Therefore, there is a symmetric positive definite matrix P such that $A^T P + PA = -I$.

Define Lyapunov function $V_0(\varepsilon,\hat{\chi}) = V(\varepsilon) + W(\hat{\chi})$, where $V(\varepsilon) = (n+1)\varepsilon^T P \varepsilon$ and $W(\hat{\chi}) = \hat{\chi}^T \hat{\chi}$. Then

$$\dot{W}(\hat{\chi}) = -2\hat{\chi}^T \Lambda \hat{\chi} \le -2\lambda_{\min}(\Lambda) W(\hat{\chi}) \tag{10}$$

where $\lambda_{\min}(\Lambda)$ denotes the smallest eigenvalue of Λ. On the other hand, from the definition of proportion error (8) we have $\xi_i = \hat{\xi}_i + \theta^{i-1}\varepsilon_i$, with this in mind and Assumption 5, there exists a real constant $c_1 > 0$, which is independent of θ, such that the time derivative of $V_0(\varepsilon,\hat{\chi})$ is

$$\begin{aligned}
\dot{V}_0(\varepsilon,\hat{\chi}) &= -(n+1)\theta\|\varepsilon\|^2 - 2\lambda_{\min}(\Lambda)W(\hat{\chi}) \\
&\quad + 2(n+1)\varepsilon^T P \begin{bmatrix} \phi_1(\xi,\chi,u) \\ \frac{1}{\theta}\phi_2(\xi,\chi,u) \\ \vdots \\ \frac{1}{\theta^{n-1}}\phi_n(\xi,\chi,u) \end{bmatrix} \\
&\le -(n+1)\theta\|\varepsilon\|^2 - 2\lambda_{\min}(\Lambda)W(\hat{\chi}) \\
&\quad + c_1\|\varepsilon\|\left(|\xi_1| + \frac{1}{\theta}|\xi_2| + \cdots + \frac{1}{\theta^{n-1}}|\xi_n|\right) \\
&\le -\left((n+1)\theta - c_1\sqrt{n} - \frac{n}{2}c_1\right)\|\varepsilon\|^2 - 2\lambda_{\min}(\Lambda)W(\hat{\chi}) \\
&\quad + c_1\frac{1}{2}\left(\hat{\xi}_1^2 + \frac{1}{\theta^2}\hat{\xi}_2^2 + \cdots + \frac{1}{\theta^{2(n-1)}}\hat{\xi}_n^2\right)
\end{aligned} \tag{11}$$

3.2 Construction of an Output Feedback Controller

The procedure is similar to that of (Qian, 2003), there is no need to repeat it. We only give the inductive step and the last step at which step the output feedback controller will be given.

Inductive Step k: Suppose until to the kth step, we have defined error variables $e_1 = \hat{\xi}_1, e_{j+1} = \hat{\xi}_{j+1} - \alpha_{j+1}$ for $j = 1,\cdots,k$ where $\alpha_{j+1} = -\theta b_j e_j$ is virtual control with $b_1 = n + \frac{1}{4}k_1^2 + \frac{1}{2}$ and $b_2,\cdots,b_k > 0$ being constant independent of θ .and found a Lyapunov function $V_k(\varepsilon,\hat{\chi},e_1,\cdots,e_k)$ such that

$$\begin{aligned}
\dot{V}_k &\le -((n+1-k)\theta - (\sqrt{n} + \frac{n}{2})c_1)\|\varepsilon\|^2 \\
&\quad - \sum_{j=1}^{k}\frac{1}{\theta^{2j-2}}\left((n+1-k)\theta - c_1 a_j^2\right)e_j^2 \\
&\quad + \frac{1}{2}c_1\left(\frac{1}{\theta^{2k+2}}\hat{\xi}_{k+2}^2 + \cdots + \frac{1}{\theta^{2(n-1)}}\hat{\xi}_n^2\right) \\
&\quad + \frac{c_1}{\theta^{2k}}e_{k+1}^2 + \frac{1}{\theta^{2(k-1)}}e_k e_{k+1} - 2\lambda_{\min}(\Lambda)W(\hat{\chi})
\end{aligned} \tag{12}$$

where $V_1(\varepsilon,\hat{\chi},e_1) = V_0(\varepsilon,\hat{\chi}) + \frac{1}{2}e_1^2$ and $V_j = V_{j-1} + \frac{1}{2\theta^{2(j-1)}}e_j^2, j = 2,\cdots,k$. Now define $e_{k+2} = \hat{\xi}_{k+2} - \alpha_{k+2}$ with α_{k+2} being the virtual control to de designed. Consider the following Lyapunov function

$$V_{k+1} = V_k + \frac{1}{2\theta^{2k}}e_{k+1}^2 \tag{13}$$

It is not difficult to deduce that

$$\frac{d}{dt}\left(\frac{1}{2\theta^{2k}}e_{k+1}^2\right) = \frac{1}{\theta^{2k}}e_{k+1}\left(\hat{\xi}_{k+2} + \theta^{k+1}k_{k+1}\varepsilon_1 + \theta b_k \sum_{j=1}^{k}\frac{\partial e_k}{\partial \hat{\xi}_j}\left(\hat{\xi}_{j+1} + \theta^j k_j \varepsilon_1\right)\right)$$

$$
\begin{aligned}
&+\theta b_k \sum_{j=1}^{k} \frac{\partial e_k}{\partial \hat{\xi}_j}\left(\hat{\xi}_{j+1}+\theta^j k_j \varepsilon_1\right)\Bigg) \\
&= \frac{1}{\theta^{2k}} e_{k+1}\Big(\hat{\xi}_{k+2}+\theta^{k+1}k_{k+1}\varepsilon_1 \\
&\quad +\sum_{j=1}^{k}\theta^{k-j+1} b_k \cdots b_j \left(e_{j+1}-\theta b_j e_j+\theta^j k_j \varepsilon_1\right)\Big) \\
&= \frac{1}{\theta^{2k}} e_{k+1}\Big(\hat{\xi}_{k+2}+\theta^{k+1}d_{k,0}\varepsilon_1+\theta^{k+1}d_{k,1}e_1 \\
&\quad +\theta^k d_{k,2}e_2+\cdots+\theta d_{k,k+1}e_{k+1}\Big)
\end{aligned} \tag{14}
$$

where $d_{j,i}>0, i=0,\cdots,j+1$ are all constant that are independent of θ. From (12) and (14), we have

$$
\begin{aligned}
\dot{V}_{k+1} \le& -\left((n+1-k)\theta-(\sqrt{n}+\frac{n}{2})c_1\right)\|\varepsilon\|^2 \\
&-\sum_{j=1}^{k}\frac{1}{\theta^{2j-2}}\left((n+1-k)\theta-c_1 a_j{}^2\right)e_j{}^2 \\
&+\frac{1}{2}c_1\left(\frac{1}{\theta^{2k+4}}\hat{\xi}_{k+3}{}^2+\cdots+\frac{1}{\theta^{2(n-1)}}\hat{\xi}_n{}^2\right) \\
&+\frac{c_1}{\theta^{2k+2}}\xi_{k+2}{}^2+\frac{c_1}{\theta^{2k+2}}\alpha_{k+1}{}^2 \\
&+\frac{1}{\theta^{2k}}e_{k+1}e_{k+2}+\frac{1}{\theta^{2k}}e_{k+1}\alpha_{k+1}-2\lambda_{\min}(\Lambda)W(\hat{\chi}) \\
&+e_{k+1}\left(\frac{d_{k,0}}{\theta^{k-1}}\varepsilon_1+\frac{d_{k,1}}{\theta^{k-1}}e_1+\frac{d_{k,2}}{\theta^{k-2}}e_2\right. \\
&\left.+\cdots+\frac{d_{k,k-1}}{\theta^{2k-3}}e_{k-1}+\frac{1+d_{k,k}}{\theta^{2k-2}}e_k+\frac{\frac{c_1}{\theta}+d_{k,k+1}}{\theta^{2k-1}}e_{k+1}\right)
\end{aligned} \tag{15}
$$

From the well-known Young's Inequality and $\theta>1$, it is easy to check that the following inequalities hold:

$$
\begin{aligned}
\frac{d_{k,0}}{\theta^{k-1}}e_{k+1}\varepsilon_1 &= \frac{d_{k,0}e_{k+1}}{\theta^{k-1}\sqrt{\theta}}\sqrt{\theta}\varepsilon_1 \\
&\le \frac{d_{k,0}{}^2}{4\theta^{2k-1}}e_{k+1}{}^2+\theta\|\varepsilon\|^2 \\
\frac{d_{k,1}}{\theta^{k-1}}e_{k+1}e_1 &\le \frac{d_{k,1}{}^2}{4\theta^{2k-1}}e_{k+1}{}^2+\theta e_1^2 \\
&\vdots \\
\frac{1+d_{k,k}}{\theta^{2k-2}}e_{k+1}e_k &\le \frac{1+d_{k,k}}{\theta^{k-(1/2)}}e_{k+1}\frac{1}{\theta^{k-(3/2)}}e_k \\
&\le \frac{(1+d_{k,k})^2}{4\theta^{2k-1}}e_{k+1}{}^2+\frac{1}{\theta^{2k-3}}e_k^2
\end{aligned} \tag{16}
$$

Submit (16) into(15), we have

$$
\begin{aligned}
\dot{V}_{k+1} \le& -\left((n-k)\theta-(\sqrt{n}+\frac{n}{2})c_1\right)\|\varepsilon\|^2 \\
&-\sum_{j=1}^{k}\frac{1}{\theta^{2j-2}}\left((n-k)\theta-c_1 a_j{}^2\right)e_j{}^2 \\
&+\frac{1}{2}c_1\left(\frac{1}{\theta^{2k+4}}\hat{\xi}_{k+3}{}^2+\cdots+\frac{1}{\theta^{2(n-1)}}\hat{\xi}_n{}^2\right) \\
&+\frac{c_1}{\theta^{2k+2}}\xi_{k+2}{}^2+\frac{c_1}{\theta^{2k+2}}\alpha_{k+2}{}^2 \\
&+\frac{1}{\theta^{2k}}e_{k+1}e_{k+2}+\frac{1}{\theta^{2k}}e_{k+1}\alpha_{k+2}-2\lambda_{\min}(\Lambda)W(\hat{\chi}) \\
&+e_{k+1}{}^2\frac{1}{\theta^{2k-1}}\left(\frac{d_{k,0}{}^2}{4}+\frac{d_{k,1}{}^2}{4}+\cdots\right. \\
&\left.+\frac{d_{k,k-1}{}^2}{4}+\frac{\left(1+d_{k,k}\right)^2}{4}+d_{k,k+1}+1\right)
\end{aligned} \tag{17}
$$

Then the virtual control α_{k+2} can be chosen as

$$
\alpha_{k+2}=-\theta b_{k+1}e_{k+1} \tag{18}
$$

where

$$
\begin{aligned}
b_{k+1} =& n-k+\frac{d_{k,0}{}^2}{4}+\frac{d_{k,1}{}^2}{4}+\cdots \\
&+\frac{d_{k,k-1}{}^2}{4}+\frac{\left(1+d_{k,k}\right)^2}{4}+d_{k,k+1}+1>0
\end{aligned}
$$

is a constant that is independent of θ. Submit (18) into (17) we have

$$
\begin{aligned}
\dot{V}_{k+1} \le& -\left((n-k)\theta-(\sqrt{\theta}+\frac{n}{2})c_1\right)\|\varepsilon\|^2 \\
&-\sum_{j=1}^{k+1}\frac{1}{\theta^{2j-2}}\left((n-k)\theta-c_1 a_j{}^2\right)e_j{}^2 \\
&+\frac{1}{2}c_1\left(\frac{1}{\theta^{2k+4}}\hat{\xi}_{k+3}{}^2+\cdots+\frac{1}{\theta^{2(n-1)}}\hat{\xi}_n{}^2\right) \\
&+\frac{c_1}{\theta^{2k+2}}\xi_{k+2}{}^2+\frac{1}{\theta^{2k}}e_{k+1}e_{k+2}-2\lambda_{\min}(\Lambda)W(\hat{\chi})
\end{aligned} \tag{19}
$$

Similar to the inductive step, at the nth step we give the controller design.

Consider following Lyapunov function

$$V_n = V_{n-1} + \frac{1}{2\theta^{2(n-1)}} e_n^{\ 2} \tag{20}$$

and choose the controller as

$$\begin{aligned} u &= -\theta b_n e_n \\ &= -\theta b_n \left(\hat{\xi}_n + \theta b_{n-1} (\hat{\xi}_{n-1} + \cdots + \theta b_2 (\hat{\xi}_2 + \theta b_1 \hat{\xi}_1) \cdots \right) \end{aligned} \tag{21}$$

where

$$\begin{aligned} b_n = 1 + \frac{d_{n-1,0}^{\ \ 2}}{4} + \frac{d_{n-1,1}^{\ \ 2}}{4} + \cdots \\ + \frac{(1+d_{n-1,n-1})^2}{4} + 1 + d_{n-1,n} \end{aligned}$$

and $b_i > 0, i = 1, \cdots, n-1$ are constants that are independent of θ. Then we have

$$\begin{aligned} \dot{V}_n \le & -\left(\theta - (\sqrt{n} + \frac{n}{2}) c_1 \right) \|\varepsilon\|^2 - 2\lambda_{\min}(\Lambda) W(\hat{\chi}) \\ & - \sum_{j=1}^{n-1} \frac{1}{\theta^{2j-2}} \left(\theta - c_1 a_j^{\ 2} \right) e_j^{\ 2} - \frac{1}{\theta^{2n-2}} \theta e_n^{\ 2} \end{aligned} \tag{22}$$

It is clear that if we choose the gain constant

$$\theta > \max\{1, \sqrt{n} + \frac{n}{2} c_1, c_1 b_1^{\ 2}, \cdots, c_1 b_{n-1}^{\ \ 2}\} \tag{23}$$

then the right-hand side of (22) becomes negative definite. Therefore, the closed-loop system is asymptotically stable.

Now we will prove that with controller (21), system (7) forms an exponential observer for (1). More precisely, let $\Omega_{\varepsilon_0} = \left\{ f_2(x,z)^T \Lambda f_2(x,z) < \varepsilon_0 \right\}$ contained in the set Ω_ε defined in Assumption 3, then we say for $\forall \theta$ chosen as (23), there exist $\beta_1 > 0, \beta_2 > 0$ such that

$$\|(x,z) - (\hat{x}, \hat{z})\| \le \beta_1 e^{-\beta_2 t} \left\| \tilde{\xi}(0), \hat{\chi}(0) \right\| \tag{24}$$

where $\tilde{\xi}(0) = \hat{\xi}(0) - \xi(0)$ and $(\hat{x}, \hat{z})$ is the estimate of (x,z). This claim can be proved as following.

From (10) we have obtained $\dot{W}(\hat{\chi}) \le -2\lambda_{\min}(\Lambda) W(\hat{\chi})$, thus we can obtain

$$\|\hat{\chi}(t)\|^2 \le e^{-\gamma t} \|\hat{\chi}(0)\|^2 \tag{25}$$

where $\gamma = 2\lambda_{\min}(\Lambda)$.From (22) it is easy to show that there exist constants $\tau_1 > 0, \tau_2 > 0$ such that

$$\left\| \hat{\xi}(t) - \xi(t) \right\|^2 \le \tau_1 e^{-\tau_2 t} \left\| \hat{\xi}(0) - \xi(0) \right\|^2 \tag{26}$$

Define state estimate of (1) as $(\hat{x}, \hat{z}) = \Psi^{-1}(\hat{\xi}, \hat{\chi})$, we have

$$\begin{aligned} \|(x,z) - (\hat{x},\hat{z})\| &\le \left\| \Psi^{-1}(\xi,\chi) - \Psi^{-1}(\hat{\xi},\hat{\chi}) \right\| \\ &\le \frac{1}{\sigma} \left\| (\xi,\chi) - (\hat{\xi},\hat{\chi}) \right\| \le \beta_1 e^{-\beta_2 t} \left\| \tilde{\xi}(0), \hat{\chi}(0) \right\| \end{aligned} \tag{27}$$

where $\beta_1 > 0, \beta_2 > 0$ are constants that are determined by $\tau_1, \tau_2, \sigma, \gamma$ and θ. This ends the proof.

4. AN ILLUSTRATIVE EXAMPLE

In this section, we use an example to illustrate applications of Theorem 3.1. We examine the following seemingly simple but nontrivial example

$$\begin{aligned} \dot{x}_1 &= x_2 + x_1 \sin z \\ \dot{x}_2 &= u + x_2 \sin x_2 \\ 0 &= f_2(x,z) = z + x_2^2 \\ y &= x_1 \end{aligned} \tag{28}$$

By simple calculation, the transformation (4) that satisfies Assumption 4 can be chosen as

$$(\xi_1, \xi_2, \chi) = (x_1, x_2, z + x_2^2) \tag{29}$$

Then (28) can be equivalently transformed into

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 + \xi_1 \sin(\chi - \xi_2^2) \\ \dot{\xi}_2 &= u + \xi_2 \sin \xi_2 \\ \dot{\chi} &= 0 \\ y &= \xi_1 \end{aligned} \tag{30}$$

According to (5), here $\phi_1(\xi_1,\xi_2,\chi) = \xi_1 \sin(\chi - \xi_2^2)$ and $\phi_2(\xi_1,\xi_2,\chi) = \xi_2 \sin \xi_2$. Due to the presence of ϕ_1, system (30) is not in a lower triangular form. Moreover, ϕ_2 is a non Lipschtiz function and there is no constant $c \ge 0$ satisfying $\left| \phi_2(\xi_2) - \phi_2(\overline{\xi}_2) \right| \le c \left| \xi_2 - \overline{\xi}_2 \right|$. Therefore, the existing observer schemes (Yaagoubi, 2005 and Assoudi, 2005), can not be applied to (28).

On the other hand, it is easy to verify that Assumption 5 obviously holds:

$$|\phi_1(\xi_1,\xi_2,\chi)| \le |\xi_1|, |\phi_2(\xi_1,\xi_2,\chi)| \le |\xi_2| \tag{31}$$

By Theorem 3.1, the non-initialized linear high gain state observer is

$$\dot{\hat{\xi}}_1 = \hat{\xi}_2 + \theta(y - \hat{\xi}_1)$$
$$\dot{\hat{\xi}}_2 = u + \theta^2(y - \hat{\xi}_1) \qquad (32)$$
$$\dot{\hat{\chi}} = -\hat{\chi}$$

and the output feedback controller

$$u = -\theta b_2(\hat{\xi}_2 + \theta b_1 \hat{\xi}_1) \qquad (33)$$

with a suitable choice of the parameters θ, b_1 and b_2 (e.g., $b_1 = 11/4, b_2 = 15$ and $\theta = 10$). The simulation is shown in Fig.1.

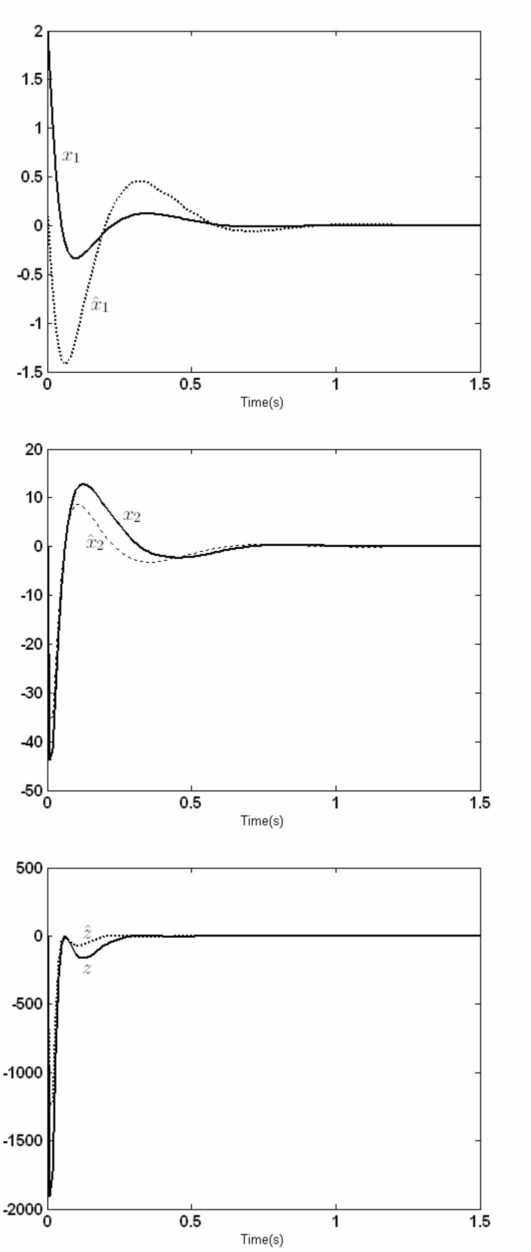

Fig.1. Transient responses of (28), (32) and (33) with $(x_1(0), x_2(0), z(0), \hat{x}_1(0), \hat{x}_2(0), \hat{z}(0)) = (0.1, 0, 0, 2, 10, -96)$

Apparently controller (33) asymptotically stabilizes (28).

5. CONCLUSIONS

A new output feedback controller design scheme is proposed for a class of nonlinear SISO DAE systems making the closed-loop system asymptotically stable. The proposed state observer is a linear non-initialized one. The observer and controller design are heavily coupled with each other. This output feedback synthesis can be used to solve the output feedback control problem for a class of DAE systems that cannot be handled by existing methods.

Acknowledgements: This work is supported by National Natural Science Foundation of China (No. 50507002).

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