Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Oil and Gas Production Optimization; Lost Potential due to Uncertainty ⋆

Steinar M. Elgsaeter * **Olav Slupphaug** ** **Tor Arne Johansen** ***

* *Norwegian University of Science and Technology, Trondheim, N-7491 Trondheim (Tel: 0047 91806886; e-mail: elgsater@ itk.ntnu.no)*
** *ABB, Oslo(e-mail: olav.slupphaug@no.abb.com)*
*** *Norwegian University of Science and Technology, Trondheim, N-7491 Trondheim (e-mail: torj@ itk.ntnu.no)*

Abstract:
The information content in measurements of offshore oil and gas production is often low, and when a production model is fitted to such data, uncertainty may result. If production is optimized with an uncertain model, some potential production profit may be lost. The costs and risks of introducing additional excitation are typically large and cannot be justified unless the potential increase in profits are quantified. In previous work it is discussed how bootstrapping can be used to estimate uncertainty resulting from fitting production models to data with low information content. In this paper we propose how lost potential resulting from estimated uncertainty can be estimated using Monte-Carlo analysis. Based on a conservative formulation of the production optimization problem, a potential for production optimization in excess of 2% and lost potential due to the form of uncertainty considered in excess of 0.5% was identified using field data from a North Sea field.

Keywords: Uncertainty, Monte Carlo simulation, Loss minimization, Process identification, Process control, Production systems

1. INTRODUCTION

Production in the context of offshore oil and gas fields, can be considered the total output of production wells, a mass flow with components including hydrocarbons, in addition to water, CO_2, H_2S, sand and possibly other components. Hydrocarbon production is for simplicity often lumped into oil and gas. Production travels as multiphase flow from wells through flow lines to a processing facility for separation, illustrated in Figure 1. Water and gas injection is used for optimizing hydrocarbon recovery of reservoirs. Gas lift can increase production to a certain extent by increasing the pressure difference between reservoir and well inlet.

Production is constrained by several factors, including: On the field level, the capacity of the facilities to separate components of production and the capacity of facilities to compress lift gas. The production of groups of wells may travel through shared flow lines or inlet separators which have a limited liquid handling capacity. The production of individual wells may be constrained due to slugging, other flow assurance issues or due to reservoir management constraints.

Multiphase flows are hard to measure and are usually not available for individual flow lines in real-time, however measurements of total single-phase produced oil and gas rates are usually available, and estimates of total water rates can often be found by adding different measured water rates after separation. To determine the rates of oil, gas and water produced from individual wells, the production of a single well is usually routed to a dedicated test separator where the rate of each separated single-phase component is measured. In *single-rate well tests*, only the rates of components with current production settings are measured, while in *multi-rate well tests* rates are measured for different settings. The total amount of gas and water which can be separated and processed is constrained by the capacity of facilities, these capacities are themselves uncertain. Normally production is at setpoints where some of these capacities are at their perceived constraints, therefore a multi-rate well test cannot be performed without simultaneously reducing production at some other well,

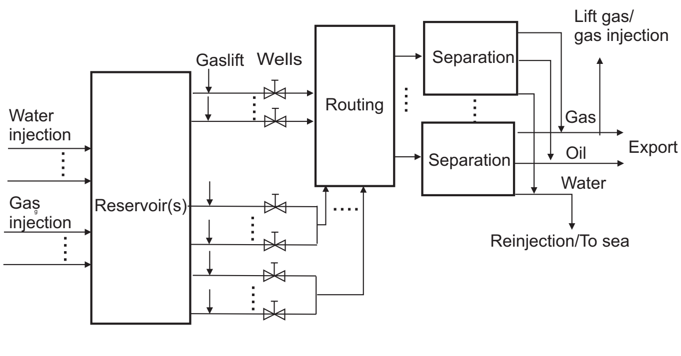

Fig. 1. A schematic model of offshore oil and gas production.

⋆ The authors would like to thank the Research Council of Norway, Norsk Hydro and ABB for funding this work.

978-1-1234-7890-2/08/$20.00 © 2008 IFAC

10.3182/20080706-5-KR-1001.2832

which may cause lost production and a cost. There is also a risk that changes in setpoints during testing may cause some part of the plant to exceed the limits of safe operation, which may force an expensive shutdown and re-start of production. Well tests are only performed when a need for tests has been identified, due to the costs and risks involved.

In the context of oil and gas producing systems, real-time optimization (RTO) has been defined in Saputelli et al. (2003) as a process of measure-calculate-control cycles at a frequency which maintains the system's optimal operating conditions within the time-constant constraints of the system. Saputelli et al. (2003) suggests dividing real-time optimization into subproblems on different time scales to limit complexity, and to consider separately reservoir management, optimization of injection and reservoir drainage on the time scales of months and years, and production optimization, maximization of value from the daily production of reservoir fluids. Reservoir management typically specifies constraints on production optimization to link these problems. We will refer to models for production optimization as production models.

The aim of production optimization is to determine setpoints for a set of chosen decision variables which are optimal by some criterion. These setpoints are implemented by altering the settings of production equipment. Decision variables may be any measured or computed variables associated with the production system which are influenced by changes in settings, but the number of decision variables is limited by the number of settings. We may for instance determine the settings of a gas lift choke by deciding a target lift gas rate, a target annulus pressure or a target gas lift choke opening. On short timescales the flow from individual wells can be manipulated by production choke settings, by gas lift choke settings and possibly by routing settings.

Parameters of production models should be fitted to production data through parameter estimation to compensate for un-modeled effects or disturbances and to set reasonable values for physical parameters which cannot be measured directly or determined in the laboratory. Erosion of production chokes is an example of an un-modeled disturbance.

Planned excitation is some planned variation in one or more decision variables designed to reveal information on production through measurements, for instance a multi-rate well test. The information flow in production optimization may be depicted as in Figure 2.

Lost profit can result if production models are fitted to production data with a low information content, as illustrated in Example 1. Throughout this paper variables with a hat ($\hat{}$) denote estimates, variables with bars ($\bar{}$) denote measurements, and variables with a star ($\star$) are true values in some sense.

Example 1. (Uncertainty in production optimization). This example illustrates a synthetic field which is producing at its processing constraints, where only single-rate well tests are available and where a production model matches historical production data with a low information content closely. The example is motivated by observations of pro-

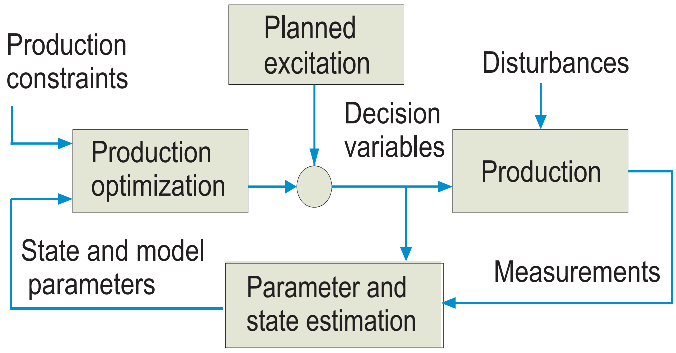

Fig. 2. Example 1: Relationship between subproblems in production optimization.

duction data from actual fields. The field produces oil q_o, water q_w and gas q_g from two wells, each with a gas lift rate q_{gl} according to equations

$$q_o^i = c_1^i + c_2^i(q_{gl}^i - q_{gl}^{l,i}) + c_3^i(q_{gl}^i - q_{gl}^{l,i})^2 \quad \forall i \in \{1,2\} \quad (1)$$

$$q_g^i = K_{GOR}^i q_O^i \quad (2)$$

$$q_w^i = \frac{K_{WC}^i}{1 - K_{WC}^i} q_o + w_1. \quad (3)$$

$q_{gl}^{l,i}$ are local operating points and $q_{gl}^{l,1} = q_{gl}^{l,2} = 1000$. Let $\theta^i \triangleq \begin{bmatrix} c_1^i & c_2^i & c_3^i & K_{WC}^i & K_{GOR}^i & w_1 \end{bmatrix}$ and

$$\begin{bmatrix} \theta^{\star,1} \\ \theta^{\star,2} \\ \hat{\theta}^1 \\ \hat{\theta}^2 \end{bmatrix} = \begin{bmatrix} 39.62 & 0.025 & -4 \cdot 10^{-6} & 0.173 & 30 & 0 \\ 76.62 & 0.030 & -4 \cdot 10^{-6} & 0.173 & 30 & 0 \\ 39.62 & 0.033 & -6 \cdot 10^{-6} & 0.185 & 30 & 0.85 \\ 76.62 & 0.032 & -12 \cdot 10^{-6} & 0.160 & 30 & 1.5 \end{bmatrix} \quad (4)$$

The ratio of water to produced liquid (watercut) and the gas-oil ratio are assumed rate-independent. True parameters $(\theta^{\star,1}, \theta^{\star,2})$ and fitted parameters $(\hat{\theta}^1, \hat{\theta}^2)$ produce predictions of measured total oil $q_o^{tot} = q_o^1 + q_O^2$, total gas $q_g^{tot} = q_g^1 + q_g^2$ and total water $q_w^{tot} = q_w^1 + q_w^2$ which match closely for a typical set of gas lift rates with little excitation, shown in Figure 3, and which match the single-rate well test available for $(q_{gl}^1, q_{gl}^2) = (1000, 1000)$.

Assume that at the current setpoint $(q_{gl}^1, q_{gl}^2) = (1000, 1000)$ with oil production $q_o^{tot} = 116.1$, all gas processing capacity (associated gas + gas lift) and water processing capacity is utilized. The current setpoint is optimal according to numerical optimization with parameters $(\hat{\theta}^1, \hat{\theta}^2)$, yet with the parameters $(\theta^{\star,1}, \theta^{\star,2})$ $(q_{gl}^1, q_{gl}^2) \approx (744, 1238)$ is optimal with oil production $q_o^{tot} = 116.4$. The two models and the setpoints are illustrated in Figure 4.

This example illustrates that lost profit can result even if production optimization is based on a model that fits production data well if production data has low information content. □

1.1 *Prior work*

In Elgsaeter et al. (2007) an analysis of production data from an oil and gas field determined that information content appeared low and significant uncertainty should be expected if production models are fitted to such data. Handling uncertainty has been identified as a key challenge

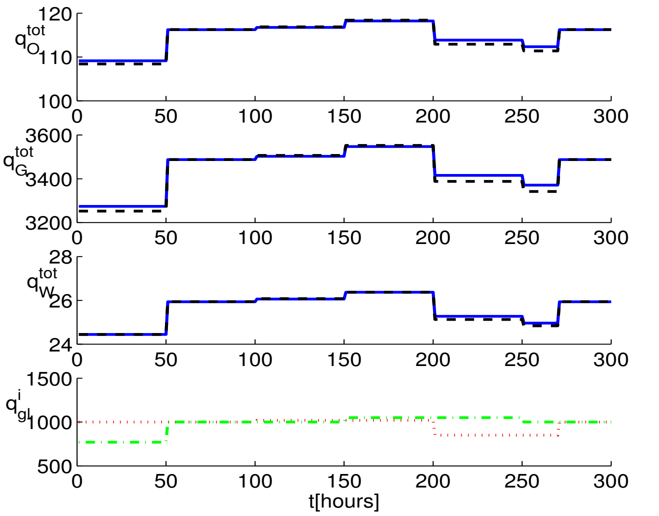

Fig. 3. Example 1: True (solid) and modeled (dashed) total production rates of oil, gas and water resulting from gas lift rates (lower graphs) q_{gl}^1 (dotted) and q_{gl}^2 (dashdot).

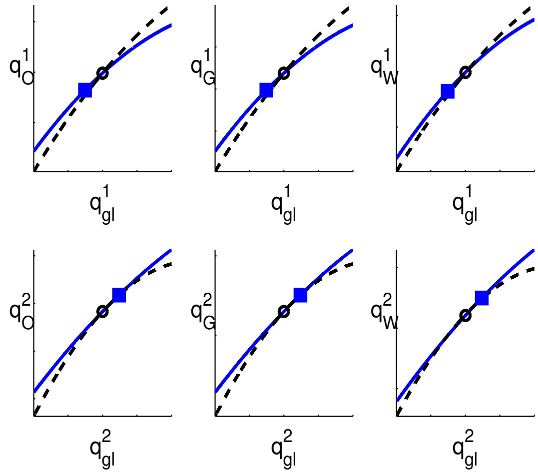

Fig. 4. Example 1: True (solid) and modeled (dashed) production of oil,gas and water, well 1 (upper graphs) and well 2 (lower graphs). Optimal operating point of the true production (square) and optimal operating point of modeled production (circle).

in real-time optimization of oil and gas production in Bieker et al. (2006a). The concept of a loss resulting from uncertainty is well established in control theory, it is for instance used to select controlled variables in Halvorsen et al. (2003). The concept of the value of information has been applied previously in reservoir management. In Branco et al. (2005) an analysis of what the authors called the value of the new information was used to justify investments to determine static and dynamic reservoir behavior. Monte Carlo simulations have been applied for analysis of uncertainty in production optimization previously in Bieker et al. (2006b), where it was applied to well test planning, and in Bieker et al. (2007), where it was used for optimal well management. In both cases, the focus was on uncertainty in gas-oil and water-oil ratios for wells where these ratios do not change when production change and where no gas-lift was applied and only test separator measurements were considered.

In Bieker et al. (2007) it has been proposed that uncertainty in production optimization should be related to production profit to allow structured business cases for uncertainty mitigation to be created. Elgsaeter et al. (2008) suggested how uncertainty due to low information content in production data can be quantified using bootstrapping. No references have been found which discuss the value of information in the context of daily oil and gas production optimization.

1.2 Problem formulation

The objective of this paper is to develop an analytical framework to quantify the lost potential that results from the low information content in production data used for production optimization, suitable to offshore oil and gas production.

2. ESTIMATING THE LOSS DUE TO UNCERTAINTY

2.1 Prerequisites

Let x be a vector of the flows of each modeled component of production for each modeled well and let u be a vector of decision variables.

In this paper we will consider production optimization problems on the form

$$\left[\hat{u}(\hat{\theta})\ \hat{x}(\hat{\theta})\right] = \arg\max_{u,x} M(x,u,d) \tag{5}$$

$$s.t \quad 0 = f(x,u,d,\hat{\theta}) \tag{6}$$

$$0 \leq c(x,u,d). \tag{7}$$

$\hat{\theta}$ is a vector of fitted parameters to be determined. $\hat{u}(\hat{\theta})$ and $\hat{x}(\hat{\theta})$ are the decision variable value and vector of flows, respectively, that are the optimal solution of (5)-(7) for a given $\hat{\theta}$. d is a vector of modeled and measured disturbances independent of u. $M(x,u,d)$ is a production profit measure which we seek to maximize subject to a production model (6) and production constraints (7) for a given $\hat{\theta}$. We only consider *instantaneous* optimization, determining $(\hat{u}(\hat{\theta}), \hat{x}(\hat{\theta}))$ for the current time.

When measurement uncertainty can be neglected, the relationship between the measured production $\bar{y}$ and x is given by the binary routing matrix $R[t]$:

$$\bar{y}[t] \triangleq R[t]x[t]. \tag{8}$$

Routing $R[t]$ may change with time, as wells are routed through different processing trains or to test separators. y may consist of test separator rate measurements, total rate measurements and in some cases multiphase rate measurements at some or all wells.

Let the modeled output for a given parameter $\hat{\theta}$ be

$$\hat{y}_{\hat{\theta}}[t] \triangleq R[t]\hat{x}(\hat{\theta})[t] + \hat{\beta}_y, \tag{9}$$

where $\hat{\beta}_y$ is the vector of measurement biases due to calibration inaccuracies to be determined. A set of production data spanning N time steps

$$Z^N = \begin{matrix}[\bar{y}[1] & \bar{d}[1] & \bar{u}[1] & \bar{y}[2] & \bar{d}[2] & \bar{u}[2] & \ldots \\ & & & \bar{y}[N] & \bar{d}[N] & \bar{u}[N]]\end{matrix} \tag{10}$$

has residuals

$$\epsilon[t] = \bar{y}[t] - \hat{y}_{\hat{\theta}}[t] \quad \forall t \in \{1, \ldots, N\} \quad . \tag{11}$$

Parameter estimation determines $\hat{\theta}$ by minimizing the sum of the squared residuals for the data set:

$$[\hat{\theta} \ \hat{\beta}_y] = \arg\min_{\theta, \beta_y} \sum_{t=1}^{N} w[t] \|\epsilon[t]\|_2^2, \tag{12}$$

where $w[t]$ is a user-specified weighting function. When Z^N has low information content, it may not be possible to determine $\hat{\theta}$ uniquely from (12), resulting in an uncertainty in $\hat{\theta}$ that can be quantified in terms of a probability density function $f_\theta(\hat{\theta})$ using *bootstrapping*, see Efron and Tibshirani (1993) or Elgsaeter et al. (2008). Bootstrapping is a computational approach which generates an estimate of uncertainty by re-solving (12) a number of times to fit the model to number of synthetically generated, or "re-sampled", data sets similar to (10). In Monte-Carlo simulations in subsequent sections we sample entire parameter vectors rather than sampling parameter values for each component of θ separately, ensuring that co-variance between terms in $\hat{\theta}$ are implicitly accounted for.

2.2 Definitions

Let the *potential for production optimization* $P_o^\star$ be the difference between the production profit at the optimal operating point $(x^\star, u^\star, d)$ and the current operating point (x, u, d):

$$P_o^\star \triangleq M(x^\star, u^\star, d) - M(x, u, d) \geq 0. \tag{13}$$

If there were no uncertainty, (5)–(7) would yield $\hat{u}(\theta) = u^\star$ and $\hat{x}(\theta) = x^\star$, $P_o^\star$ could be found from (13) and the entire potential could be realized by implementing $\hat{u}(\theta)$.

When $\hat{\theta}$ is uncertain, the solution to (5)–(7) will be uncertain. Uncertainty may cause estimates $\hat{u}(\theta)$ and $\hat{P}_o(\hat{\theta})$ to differ from $u^\star$ and $P_o^\star$, and when implementing $\hat{u}(\theta)$ we must expect some of the potential $P_o^\star$ to remain unrealized. This discussion motivates

$$\begin{aligned}\text{expected potential of production optimization}(\hat{P}_o) &\triangleq \\ \text{expected realizable potential } (\hat{P}_{o,r}) &+ \\ \text{expected lost potential due to uncertainty } (\hat{L}_u)&.\end{aligned} \tag{14}$$

Let

$$\hat{Q} \triangleq \frac{\hat{L}_u}{\hat{P}_o}, \tag{15}$$

be the quotient of potential that is lost due to uncertainty.

In this paper, we will determine probability density functions $f_{Po}(\hat{P}_o), f_{Lu}(\hat{L}_u)$, discussed below. In addition to depending on parameter uncertainty, these probability density functions will depend on production constraints and on the operational strategy, defined below.

$\hat{P}_o = 0$ and $\hat{L}_u = 0$ can be interpreted as production being optimal. The probability density functions $f_{L_u}(\hat{L}_u)$ and $f_{P_o}(\hat{P}_o)$ may consist of spikes near $\hat{P}_o = 0$ and $\hat{L}_u = 0$ and a distribution in interval away from zero. To allow these two parts of $f_{L_u}(\hat{L}_u)$ and $f_{P_o}(\hat{P}_o)$ to be studied separately, we define the two events

$$H_0 : \text{production is optimal} \tag{16}$$

$$H_1 : \text{production is suboptimal.} \tag{17}$$

Estimates $\hat{p}(H_0)$ and $\hat{p}(H_1)$ can be found from estimated probability density functions $f_{L_u}(\hat{L}_u)$ and $f_{P_o}(\hat{P}_o)$.

2.3 Estimating the potential of production optimization

An estimate $\hat{P}_o(\hat{\theta})$ of (13) for a given parameter estimate $\hat{\theta}$ when (x, u, d) is the current operating point is

$$\hat{P}_o(\hat{\theta}) = M(\hat{x}(\hat{\theta}), \hat{u}(\hat{\theta}), d) - M(x, u, d). \tag{18}$$

When $\hat{\theta}$ is uncertain, we may exploit an estimate of the probabilty density function $f_\theta(\hat{\theta})$ to estimate a probability density function for (18) using a *Monte Carlo* method, generating suitable random numbers and observing the fraction of the numbers obeying some property or properties, see Metropolis and Ulam (1949). The approach is outlined in Algorithm 1 in Appendix A.

2.4 Estimating lost potential due to uncertainty

A setpoint $\hat{u}(\hat{\theta})$ calculated with an uncertain $\hat{\theta}$ may be found to violate production constraints (7) when implemented. Therefore, a strategy for constraint handling is required. Lost potential due to uncertainty will depend on the strategy employed to deal with constraint handling, and to estimate lost potential due to uncertainty we must express this strategy as an algorithm. We will refer to such an algorithm as an *operational strategy function*.

Let u_0 be the setpoint implemented on the field prior to implementing $\hat{u}(\hat{\theta})$. When altering setpoints from u_0 along some path toward $\hat{u}(\hat{\theta})$, constraints may be encountered at an operating point (x^c, u^c) due to uncertainties. Lost potential due to uncertainty can be estimated in a Monte Carlo fashion by determining the profit that is lost when a setpoint $\hat{u}(\hat{\theta}_f)$ is implemented and the true parameter value is actually $\hat{\theta}_t$, drawing estimates $(\hat{\theta}_t, \hat{\theta}_f)$ from the estimated probability density function $f_\theta(\hat{\theta})$:

$$\begin{aligned}\hat{L}_u(\hat{\theta}_t, \hat{\theta}_f, d) = M(\hat{x}(\hat{\theta}_t), \hat{u}(\hat{\theta}_t), d) - \\ M(x^c(\hat{\theta}_f, \hat{\theta}_t), u^c(\hat{\theta}_f, \hat{\theta}_t), d).\end{aligned} \tag{19}$$

The first term in (19) is the optimal profit with the true parameter value $\hat{\theta}_t$, while the second term is an estimate of the profit obtained when $\hat{\theta}_f$ is erroneously assumed to be the true parameter. $(x^c(\hat{\theta}_f, \hat{\theta}_t), u^c(\hat{\theta}_f, \hat{\theta}_t))$ is determined by an operational strategy function. The proposed algorithm for determining $f_{L_u}(\hat{L}_u)$ is summarized in Algorithm 2 in Appendix A.

2.5 Discussion

In Algorithms 1 and 2 samples are drawn at random, Monte-Carlo approach in this paper. It may be possible to improve accuracy and reduce computational burden through the use of sampling techniques based on Hammersley-sequences, see Diwekar and Kalagnanam

(1997). Algorithms 1 and 2 can also be combined with more efficient bootstrap methods, see for instance Gigli (1996). To limit the scope and complexity of this paper, we leave such extensions for further work.

3. APPLICATIONS

In this section we will study applications of the suggested approach, first on a set of synthetic examples, then on data from a real field.

3.1 Introduction

We will consider oil, gas and water rates for each well i as elements in $\hat{x}$, that is q_o^i, q_g^i and q_w^i, respectively. The production of individual wells are assumed independent of each other, or *decoupled*, as will typically be the case for so-called platform wells. We will consider two alternative production models, (20) and (21),

$$\hat{q}_p^i = \max\{0, q_p^{l,i} f_z(z^i, z^{i,l}) \left(1 + \alpha_p^i \Delta q_{gl}^i\right)\} \tag{20}$$

$$\hat{q}_p^i = \max\{0, q_p^{l,i} f_z(z^i, z^{i,l}) \left(1 + \alpha_p^i \Delta q_{gl}^i + \kappa_p^i (\Delta q_{gl}^i)^2\right)\}, \tag{21}$$

for all wells i and for oil, gas and water $p \in \{o, g, w\}$, based on Elgsaeter et al. (2008). q_{gl}^i is gas lift rate and $z^i \in [0, 1]$ is the relative production valve opening of well i. $\Delta q_{gl}^i \triangleq \frac{q_{gl}^i}{q_{gl}^{l,i}} - 1$, where $q_{gl}^{l,i}$ is the gas lift rate measured at the time of optimization for well i, and $q_p^{l,i}$ is the measured rate of component p of well i at the most recent well test, respectively. $f_z(z^i, z^{l,i})$ is a nonlinear kernel which expresses the nonlinear relationship between valve opening and production, which obeys $f_z(0, z^{l,i}) = 0$ and $f_z(z^i, z^{l,i}) = 1$. $z^{l,i}$ is the relative valve opening at the most recent well test. In this paper we choose

$$f_z(z^i, z^{l,i}) = \frac{1 - (1 - z^i)^k}{1 - (1 - z^{l,i})^k}, \tag{22}$$

and choose $k = 5$. During optimization, we consider $u = \Delta q_{gl}$, $d = z$. We will consider $\theta = [\alpha_o \ \alpha_g \ \alpha_w]$ for (20) and $\theta = [\alpha_o \ \alpha_g \ \alpha_w \ \kappa_o \ \kappa_g \ \kappa_w]$ for (21).

Let

$$q_o^{tot} \triangleq \sum_{i=1}^{n_w} q_o^i \tag{23}$$

$$q_g^{tot} \triangleq \sum_{i=1}^{n_w} q_g^i + q_{gl}^i \tag{24}$$

$$q_w^{tot} \triangleq \sum_{i=1}^{n_w} q_o^i \tag{25}$$

where n_w is the number of wells. Let $\hat{q}_o^{tot}(\hat{\theta}), \hat{q}_g^{tot}(\hat{\theta})$ and $\hat{q}_w^{tot}(\hat{\theta})$ be estimates derived by combining (23)–(25) with (20) or (21) for a given $\hat{\theta}$.

In this paper we will consider the following measurement vector when solving (12)

$$y = \left[q_o^{tot} \ q_g^{tot} \ q_w^{tot}\right]^T. \tag{26}$$

The objective function is chosen as (27) and production constraints as (28)-(31):

$$\hat{u}(\hat{\theta}) = \arg\max_u \hat{q}_o^{tot}(\hat{\theta}) + b_o(\hat{\theta}) \tag{27}$$

$$\text{s.t.} \quad \hat{q}_g^{tot}(\hat{\theta}) + b_g(\hat{\theta}) \le q_g^{max} \tag{28}$$

$$\hat{q}_w^{tot}(\hat{\theta}) + b_w(\hat{\theta}) \le q_w^{max} \tag{29}$$

$$-U_{prc} \le u \le U_{prc} \tag{30}$$

$$u^i > u^{l,i} \quad \forall i \in W_c. \tag{31}$$

(28) is a constraint on gas processing capacity and q_g^{max} is the processing capacity for gas. (29) is a constraint on water processing capacity and q_w^{max} the processing capacity for water. (31) constrains the gas lift rate of wells with indices in the set W_c which may not have their gas-lift rates decreased due to flow assurance issues. $b_o(\hat{\theta}), b_g(\hat{\theta})$ and $b_w(\hat{\theta})$ are biases which may be calculated separately for each $\hat{\theta}$. All constraints are considered hard in this paper.

What model structure describes the relation between u and y is itself uncertain. It is reasonable to expect a fitted model to be a valid description locally around setpoints (u, d) similar to those observed in the tuning set. The mismatch between modeled outputs $\hat{y}$ and measurements $\bar{y}$ will grow when setpoints outside the range of setpoints observed in (10) are considered. If production optimization considers large changes in u, structural uncertainties may dominate and this observation motivates (30). U_{prc} limits u to within $U_{prc} \cdot 100\%$ of its current value and is a design paramter that must be chosen sufficiently small.

q_g^{max} and q_w^{max} are themselves uncertain in practice, but identifying or mitigating this uncertainty is beyond the scope of this paper. To avoid overstating q_g^{max} and q_w^{max}, and thereby overestimating $\hat{L}_u$ and $\hat{P}_o$, we assume conservatively that the field is producing at its constraints in water and gas capacity at the time of optimization. We wish to solve (27)–(31) at time $t = N$, the end of the tuning set (10). Measurements $\bar{y}$ have a high-frequency, low-amplitude, seemingly random component which is not modeled. If we choose $q_g^{max} = \bar{q}_g^{tot}[N]$ and $q_w^{max} = \bar{q}_w^{tot}[N]$, solutions of (27)–(31) will depend to some degree on these high-frequency disturbances. For this reason, we choose to consider tuning sets where no significant changes are visible in $\bar{u}[t]$ and no large disturbances visible in $\bar{y}[t], \bar{d}[t]$ for the interval $t \in [N - L, N]$ where $L << N$. Capacities are estimated as average measured rates during this interval, to reduce dependency on high-frequency disturbances:

$$q_g^{max} = \text{avg}(\bar{q}_g^{tot}[t]) \quad t \in [N - L, N] \tag{32}$$

$$q_w^{max} = \text{avg}(\bar{q}_w^{tot}[t]) \quad t \in [N - L, N]. \tag{33}$$

Similarly, we desire $\hat{q}_o^{tot}(\hat{\theta}), \hat{q}_g^{tot}(\hat{\theta})$ and $\hat{q}_w^{tot}(\hat{\theta})$ to equal averaged observations over $t \in [N - L, N]$ which motivates

$$\begin{bmatrix} b_o(\hat{\theta}) \\ b_g(\hat{\theta}) \\ b_w(\hat{\theta}) \end{bmatrix} = \text{avg}(\begin{bmatrix} \bar{q}_o^{tot}[t] - \hat{q}_o^{tot}(\hat{\theta})[t] \\ \bar{q}_g^{tot}[t] - \hat{q}_g^{tot}(\hat{\theta})[t] \\ \bar{q}_w^{tot}[t] - \hat{q}_w^{tot}(\hat{\theta})[t] \end{bmatrix}) \quad t \in [N - L, N]. \tag{34}$$

$b_o(\hat{\theta})$ is included in the objective function (27) so that (18) yields $\hat{P}_o = 0$ if (27)–(31) returns $\hat{u}(\theta)$ equal to the implemented u at the time of optimization. $(b_o(\hat{\theta}), b_g(\hat{\theta}), b_w(\hat{\theta}))$ may differ from elements of $\hat{\beta}_y$, as $\hat{\beta}_y$ is an estimate of the bias over in the tuning set, while $b_o(\hat{\theta}), b_g(\hat{\theta})$ and $b_w(\hat{\theta})$ are biases calculated for a short time interval at the end of the tuning set.

We will consider the operational strategy function

$$u^c(\hat{\theta}_f, \hat{\theta}_t) = u_0 + s(\hat{u}(\hat{\theta}_f) - u_0) \quad (35)$$

$$0 = f(x^c(\hat{\theta}_f, \hat{\theta}_t), u^c(\hat{\theta}_f, \hat{\theta}_t), d, \hat{\theta}_t) \quad (36)$$

$$\text{where} \quad s = \max \hat{s} \quad (37)$$

$$\text{s.t} \quad 0 \le \hat{s} \le 1 \quad (38)$$

$$u = u_0 + \hat{s}(\hat{u}(\hat{\theta}_f) - u_0) \quad (39)$$

$$0 \le c(x, u, d). \quad (40)$$

$$0 = f(x, u, d, \hat{\theta}_t). \quad (41)$$

(35)-(41) simulates moving the setpoint from u_0 toward $\hat{u}(\hat{\theta}_f)$ in a straight line until a constraint is met, when the system is described by $\hat{\theta}_t$. If implementing $(x^c(\hat{\theta}_f, \hat{\theta}_t), u^c(\hat{\theta}_f, \hat{\theta}_t))$ results in a decline in production profits, we will assume that the operating point is returned to is value prior to optimization when solving (19).

The set of operating points which are feasible is nonconvex in general, and the conservative statement of gas and water capacities q_g^{max} and q_w^{max} means that the choice $u_0 = 0$ may cause (35)–(41) to meet a constraint close to u_0, producing a large estimate of loss due to uncertainty $\hat{L}_u$. Instead, we choose $u_0 = -U_{prc}$, as simulations have shown this estimate to produce far more conservative estimates of $\hat{L}_u$. The real-world analogy to $u_0 = -U_{prc}$ is a field where production is moved toward a calculated setpoint $\hat{u}(\hat{\theta})$ as production is re-started after a shutdown.

We will consider two different formulations of the problem (27)-(31):

Formulation A: production model (20), a linear programming problem

Formulation B: production model (21), a nonlinear programming problem

The applications in the following section were implemented in *MATLAB* [1] and all optimization problems were solved with the *TOMLAB* [2] toolbox. All nonlinear programming problems were solved sequentially with global solvers Jones et al. (1993) and local solvers based on sequential quadratic programming, for parameter estimation based on Huschens (1994), and for production optimization based on Schittkowski (1982). In all cases analytical Jacobians and Hessians were supplied.

3.2 Synthetic example

We will revisit Example 1 in the following, to illustrate the the suggested methodology on a set of data where the true model is known.

Example 2. (Example 1 revisited: $f_{P_o}(\hat{P}_o)$ and $f_{L_u}(\hat{L}_u)$). We apply Algorithms 1 and 2 to the synthetic production data described in Example 1 with $N_t = N_f = 100$ for different U_{prc}. Results are shown in Figure 5. For this example, all estimates produced by Algorithm 1 gave $\hat{P}_o > 0$ using both formulations A and B, or $\hat{p}(H_1) = 100\%$.

[1] The Mathworks,Inc., version 7.0.4.365

[2] TOMLAB Optimization Inc., version 5.5

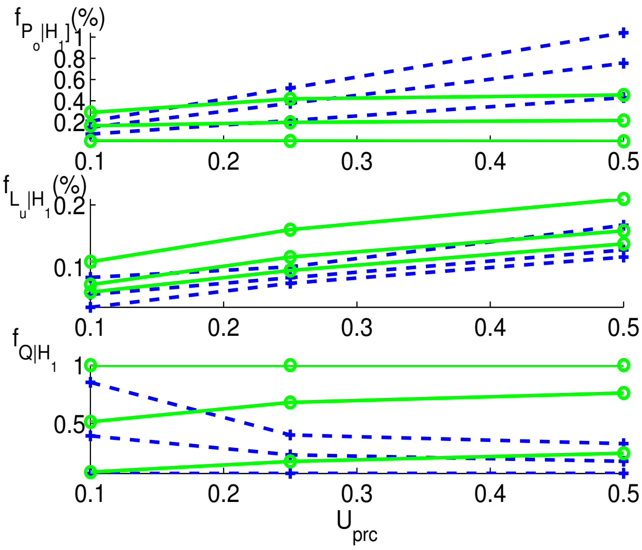

Fig. 5. Example 2: Estimated potential and lost potential for formulation A (plus) and B(circles). Center graphs are expected values, upper and lower graphs denote 95% confidence intervals.

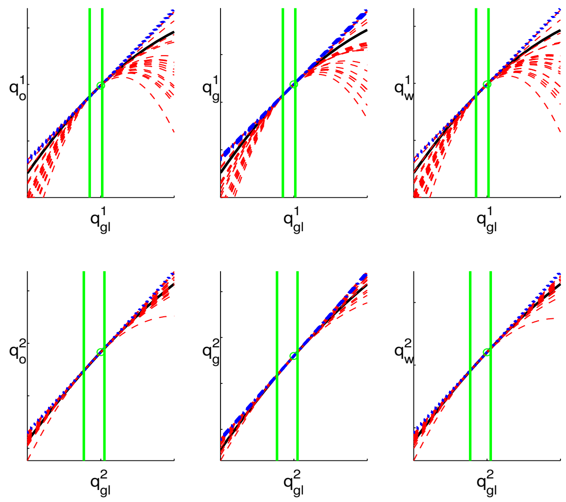

Fig. 6. Example 2: Production models (20) (dotted) and (21)(dashed) based on parameters derived using bootstrapping, operating point (x^l, u^l) (circle). Vertical line illustrates span of decision variable values in tuning set. The solid line illustrates the true model. To reduce clutter, only every fifth model based on resampled parameters are plotted.

The use of a nonlinear production model in formulation B resulted in estimates $\hat{P}_o$ and $\hat{L}_u$ that were less dependent of U_{prc} than those found with formulation A. While estimates of $\hat{P}_o$ remained in the region of $P_o^\star = 0.29\%$ with formulation B as U_{prc} grows, estimates $\hat{P}_o$ found with formulation A grow without bounds as U_{prc} grows. A closer analysis reveals that formulation A tends to find estimates $\hat{u}(\theta)$ at the constraint (30), while formulation B finds estimates $\hat{u}(\theta)$ within this constraint, and this may explain the difference observed.

As seen in Figure 6, the linear models used in formulation A predicts oil, gas and water rates within a narrow span as q^i_{gl} change compared with the nonlinear models used in formulation B. The narrow span in predicted rates with formulation A causes comparatively small estimates $(\hat{L}_u, \hat{Q})$ compared with formulation B. Estimates of $\hat{Q}$ found with formulation A decreases as U_{prc} increases, which seems counter-intuitive.

This example indicates that nonlinear models on the form (21) may be more suitable than linear models (20) in conjunction with the methods suggested in this paper. □

3.3 Case study: North Sea field

The set of data we will study is from a North Sea field with 20 gas lifted platform wells, producing predominantly oil, gas and water. Sporadic single-rate well tests are available, and no routing options are considered in u. The operator of the field requested that all data be kept anonymous, therefore all variables will be represented in normalized form. We choose a tuning set of 180 days. The sampling time was one hour, and production was not significantly changed for the last $L = 10$ hours prior to $t = N$. $f_{P_o}(\hat{P}_o)$ was estimated with Algorithm 1 and $f_{L_u}(\hat{L}_u)$ was estimated with Algorithm 2, both with $N_f = N_t = 100$. When estimating parameters, bounds $0 < \alpha < 1$ and $-1 < \kappa < 0$ based on physical knowledge were applied. Based on the knowledge that the watercut is rate-independent, soft constraints which penalize deviation from $\alpha_o = \alpha_w$ and $\kappa_o = \kappa_w$ were added to the objective function. A decline in $\bar{q}^{tot}_o$ was visible in the tuning set, and we chose to detrend $\bar{q}^{tot}_o$ and chose $w[t]$ to weigh older measurements less than newer measurements when solving (12). For further details on the modeling, refer to Elgsaeter et al. (2008). The set of indices of wells with flow assurance issues was $W_c = \{1, 4, 10, 13, 14, 16\}$.

Results The distribution of estimates $\hat{P}_o$, $\hat{L}_u$ and $\hat{Q}$ for varying U_{prc} are shown in Figure 7. The models used are shown in Figure 8. For formulation A $p(H_1) = 100\%$ while $p(H_1) = 95\%$ for formulation B.

Discussion U_{prc} limits change in decision variables to a certain range for which structural uncertainties can be neglected, and $\hat{P}_o$ and $\hat{L}_u$ depend on this design parameter. As we do not know with certainty when structural uncertainties become significant, we cannot determine a single value for U_{prc} and so when estimating $\hat{P}_o$ and $\hat{L}_u$ we have considered a range of U_{prc}. A conservative assumption of $U_{prc} = 0.1$ indicates $P_o \approx 2\%$. The quotient Q is relatively invariant of U_{prc}, so from the analysis it seems fair to conclude that *on average*, we should expect to loose about 25% of the $\hat{P}_o$ due to uncertainty, or $\hat{L}_u \approx 0.5\%$ when $U_{prc} = 0.1$. Unfortunately, we are unable to verify estimates $\hat{P}_o$ and $\hat{L}_u$ made in this paper without further experiments, and such experiments are associated with great cost and risk. However, the methods presented in this paper may be used motivate such experiments in further work, as the presented methods allow the associated benefits to be estimated. In this paper we have not considered *constraint uncertainty*. It is possible that at the time of the analysis the field was producing below its actual processing capacities. In this case, the potential for production optimization would be larger than our estimates.

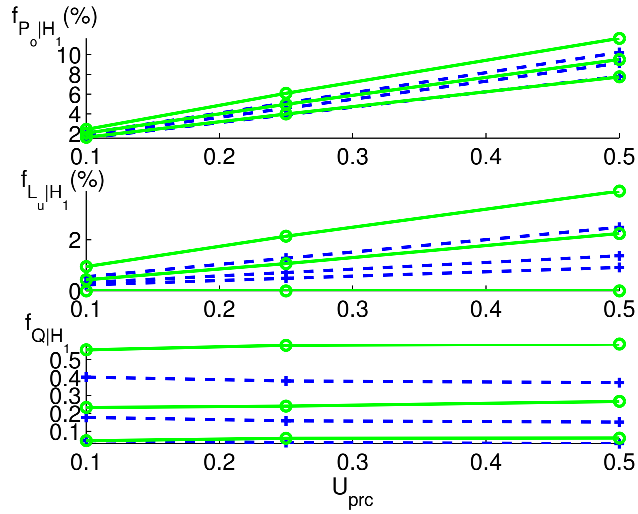

Fig. 7. Case study, North Sea field data: estimated potential and lost potential for formulation A (plus) and B (circles). Center graphs are expected values, upper and lower graphs denote 95% confidence intervals.

4. CONCLUSIONS

Based on a conservative formulation of the production optimization problem, we could identify potential for production optimization in excess of 2%, and lost potential due to the form of uncertainty considered in excess of 0.5%. As sales revenues from an oil and gas field may be very large and the costs of mitigating uncertainty are independent of revenue, we believe that further work may prove that these potentials are sufficient to justify the costs of mitigating uncertainty and optimizing production.

The methodology presented may have extensions to other processes optimized using models fitted to process measurements with low information content.

Appendix A. ALGORITHMS

Algorithm 1. (Estimating $f_{P_o}(\hat{P}_o)$). Given a dataset Z^N on the form (10)

- determine $\hat{\theta}$ using (12),
- determine $f_\theta(\hat{\theta})$, for instance by bootstrapping as described in Elgsaeter et al. (2007),
- sample $f_\theta(\hat{\theta})$ N_t times,
- determine $(\hat{x}(\theta), \hat{u}(\theta))$ using (5)-(7) for each of the N_t samples, and
- insert all determined $(\hat{x}(\theta), \hat{u}(\theta))$ in (18) to obtain the probability density function $f_{P_o}(\hat{P}_o)$.

Algorithm 2. (Estimating $f_{L_u}(\hat{L}_u)$). Given a dataset Z^N on the form (10), an operational strategy function, the current operating point (x, u, d) and a probability density function $f_\theta(\hat{\theta})$.

- do N_t times
  - · draw a sample $\hat{\theta}_t$ from $f_\theta(\hat{\theta})$.
  - · draw N_f samples $\hat{\theta}_f$ from $f_\theta(\hat{\theta})$.

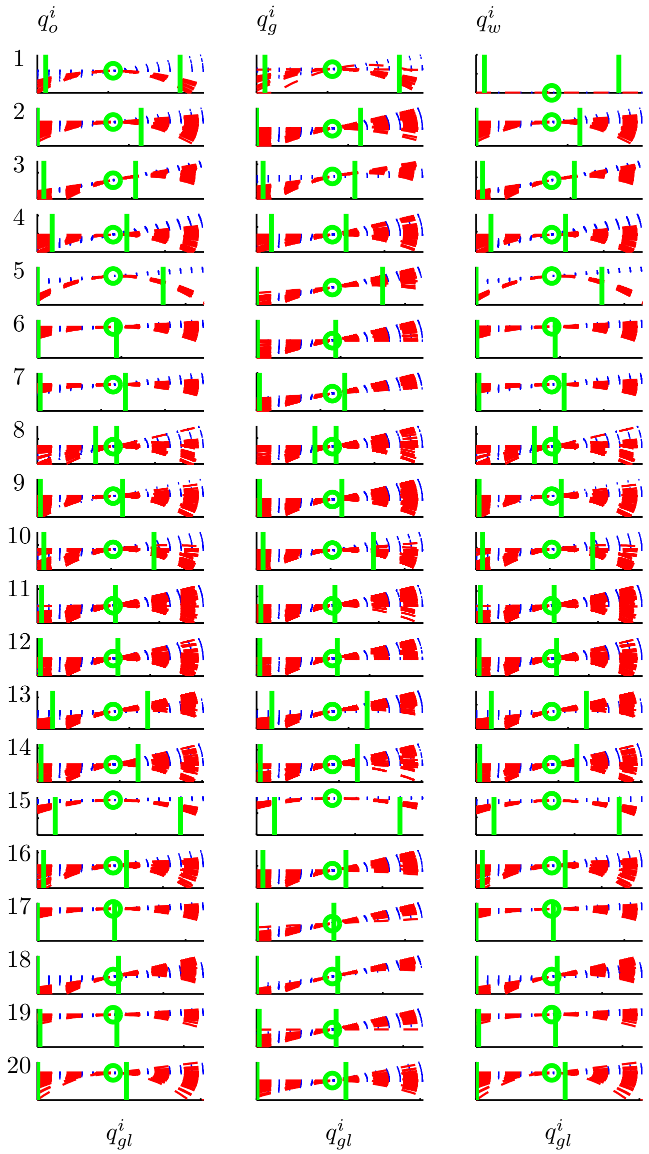

Fig. 8. Case study, North Sea field data: Production models (20) (dotted) and (21)(dashed) based on parameters derived by bootstrapping, operating point (x^l, u^l) (circle). Vertical lines illustrate the range of decision variable values in the tuning set.Lower left bound of all plots is (0,0).Well indices along left margin.

  · for each of the N_f samples of $\hat{\theta}_f$ and the sample $\hat{\theta}_t$ obtain a point estimate of L_u by solving (19)
- the distribution of $N_t \times N_f$ point estimates of $\hat{L}_u$ is an estimate of $f_{L_u}(\hat{L}_u)$

REFERENCES

H.P Bieker, O. Slupphaug, and T.A Johansen. Real-time production optimization of offshore oil and gas production systems: Technology survey. In *SPE Intelligent Energy Conference and Exhibition*, 2006a. SPE 99446.

H.P. Bieker, O. Slupphaug, and T.A. Johansen. Optimal well-testing strategy for production optimization: A monte carlo simulation approach. In *SPE Eastern Regional Meeting*, 2006b. SPE 104535.

H.P. Bieker, O. Slupphaug, and T.A. Johansen. Well management under uncertain gas or water oil ratios. In *SPE Digital Energy Conference and Exhibition*, 2007. SPE 106959.

C.C.M Branco, A.C.C. Pinto, P.M.B. Tinoco, P.M.F. Vieira, A.M. Sayd, R.L.A. Santos, and F. Prais. The role of the value of information and long horizontal wells in the appraisal and development studies of a brazilian offshore heavy-oil reservoir. In *SPE/PS-CIM/CHOA International Thermal Operations and Heavy Oil Symposium*, 2005. SPE 97846.

U.M. Diwekar and J.R. Kalagnanam. Efficient sampling technique for optimization under uncertainty. *AIChE Journal*, 43(2):440–447, 1997.

B Efron and R.J. Tibshirani. *An Introduction to the Bootstrap*. Chapman & Hall, 1993.

S.M. Elgsaeter, O. Slupphaug, and T.A. Johansen. Uncertainty in model-based production optimization; challenges. In *Proc. SPE International Petroleum Technology Conference 2007*, 2007.

S.M. Elgsaeter, O. Slupphaug, and T.A. Johansen. Production optimization; system identification and uncertainty estimation. In *Proc. SPE Intelligent Energy Conference 2008*, 2008.

A. Gigli. Efficient bootstrap methods; a review. *Statistical Methods and Applications*, 5(1):99–127, April 1996.

I.J Halvorsen, S. Skogestad, J. Morud, and V. Alstad. Optimal selection of controlled variables. *Ind. Eng. Chem. Res.*, 42(14):3273–3284, 2003.

J. Huschens. On the use of product structure in secant methods for nonlinear least squares problems. *SIAM Journal of Optimization*, 4(1):108–129, Jan 1994.

D.R. Jones, C.D. Perttunen, and B.E. Stuckman. Lipschitzian optimization without the lipschitz constant. *Journal of Optimization Theory and Applications*, 79(1): 157–181, Oct 1993.

N. Metropolis and S. Ulam. The monte carlo method. *Journal of the American Statistical Association*, 44 (247):335–341, September 1949.

L.A. Saputelli, S. Mochizuki, L. Hutchins, R. Cramer, M.B. Anderson, J.B. Mueller, A. Escorcia, A.L. Harms, C.D. Sisk, S. Pennebaker, J.T. Han, C.S. Brown, A. Kabir, R.D. Reese, G.J. Nuez, K.M. Landgren, C.J. McKie, and C. Airlie. Promoting real-time optimization of hydrocarbon producing systems. In *2003 SPE Offshore Europe Aberdeen*, 2003. SPE 839781.

K. Schittkowski. On the convergence of a sequential quadratic programming method with an augmented lagrangian line search function. Technical Report, System Optimization Laboratory, Stanford University, Stanford,CA, 1982.