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Initial State Iterative Learning For Final State Control In Motion Systems

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Abstract: In this work, an initial state iterative learning control (ILC) approach is proposed for final state control of motion systems. ILC is applied to learn the desired initial states in the presence of system uncertainties. Four cases are considered where the initial position or speed are manipulated variables and final displacement or speed are controlled variables. Since the control task is specified spatially in states, a state transformation is introduced such that the final state control problems are formulated in the phase plane to facilitate spatial ILC design and analysis. An illustrative example is provided to verify the validity of the proposed ILC algorithms.

1. INTRODUCTION

Motion control tasks can be classified into set-point control and tracking control. Set-point control problems arise because of two reasons – only the final states are of concern and specified, and/or the control system is constrained such that only the final states can be controlled. For instance, stopping a moving vehicle at a desired position is a set-point control task. Another example is to shoot a ball into basket, which can only be a set-point control task because it is unnecessary and impossible to specify and control the entire motion trajectory of the ball when only the initial shooting angle and speed are adjustable. Further from energy saving or ecological point of view, we may not want to continuously apply control signals if the desired states can be reached with appropriate initial state values. For instance we can let a train *slip* into and stop at a station with certain initial speed and initial distance. Even if a braking is applied to shorten the slipping time, we may not want to change the braking force so as to keep a smooth motion of the train. In such circumstances, it is imperative to start slipping from appropriate initial position and speed. In this work we focus on final state control of motion systems with initial state manipulation.

The final state control of motion systems can expressed as (1) achieving a desired displacement at a prespecified speed or (2) achieving a desired speed at a prespecified position. It is worth to note the difference between the above two cases. In the first case, we can image that an observer sits in a train and checks the displacement when the train speed drops to a prespecified value. In the second case, we can image that an observer stands in a station and checks the speed when the train enters. In the first case, the information used for control should be the position displacement, whereas in the second case the information used for control should be the speed.

In practice, it is not an easy task to find the appropriate initial states when the desired final states are given, due to two reasons. First, we do not know the exact model of a motion system due to the unknown friction coefficients, unknown load, or other unknown environmental factors such as slope. Thus it is impossible to compute the required initial states as the control inputs. Second, a motion system such as vehicle could be highly nonlinear due to its internal driving characteristics Uwe et al. (2000) and external interactions with environment in the air, water or on ground nonlinear frictions Brian et al. (1994). It is in general impossible to obtain an analytic solution trajectory for such a highly nonlinear dynamics.

On the other hand, many motion control tasks are frequently repeated under the same circumstances, for example the repeated basketball shot exercise, a train entering the same station regularly, an airplane landing on the same runway, etc. The performance of a motion system that executes the same tasks repeatedly can be improved by learning from previous executions (trials, iterations, passes). Iterative learning control is a suitable method to deal with repeated control tasks Bien et al. (1998); Xu et al. (2003). In this paper, we further demonstrate that ILC is also a suitable method to learn appropriate initial states as control inputs while only the final state information is available.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider a motion system

$$\begin{cases} \dfrac{dx}{dt} = v, \\ \dfrac{dv}{dt} = -f(x,v), \end{cases} \tag{1}$$

where f is continuous on the domain $\mathcal{R}_+^2 \stackrel{\triangle}{=} [0,\infty)\times[0,\infty)$, x is the displacement and v is the speed.

The control objective is to bring the system states (x, v) to an ε-neighbourhood of the desired final state $x_d > 0$ or $v_d \geq 0$ by means of adjusting initial state x_0 or v_0. Clearly the initial states are control inputs. The ε-neighbourhood is defined as $|x_d - x| \leq \varepsilon$ or $|v_d - v| \leq \varepsilon$, where ε is a positive constant. Consider two sets of initial states $x_0 = 0,\ v_0 = u_v$, or $x_0 = u_x,\ v_0 = \mathcal{A}$, where the control inputs u_x and u_v are respectively the initial position and speed, $\mathcal{A}$ is a fixed speed greater than v_d.

In real world most motion systems without control are stable or dissipative in nature. Therefore it is reasonable to assume that a motion system will stop when no exogenous driving control applies. In this work it is assumed that the position $x(t)$ is monotonically increasing, or equivalently

Assumption 1. $v \geq 0$.

In motion systems the desired final states may be defined in a very generic manner with the position and speed linked together, that is, defining the final states in the spatial domain. For instance in final position control, the desired final displacement shall be achieved at a prespecified speed, not necessarily at a zero speed. Analogously, in final speed control the desired final speed shall be achieved at a prespecified position.

Now, by eliminating the time t we convert the motion system into the phase plane (v, x). Dividing the first equation in (1) by the second equation yields

$$\frac{dx}{dv} = -g(v, x), \tag{2}$$

where $g = v/f$. According to (2), the state x is a function of the argument v and control inputs. For simplicity, we write $x(v, u_x)$ when the initial speed is fixed at $\mathcal{A}$ and the control input is the initial position, and write $x(v, u_v)$ when the initial position is fixed at zero and the control input is the initial speed. As far as g is well defined near $f = 0$, the existence and uniqueness of solution ensure that two solution trajectories of (1) and (2) describe the same physical motion for $v \in [0, \infty)$, one in the time domain and the other in the phase plane. As such, we can derive the same control property when the same control law is applied.

Note that $g(v, x)$ can be viewed as the inverse of generalized damping or friction coefficient. The characteristics of the motion system (2) is solely determined by $g(v, x)$.

Assumption 2. For $v,\ x_1,\ x_2 \in \mathcal{R}_+$, there exists a known integrable Lipschitz function $L(v)$ such that

$$|g(v, x_1) - g(v, x_2)| \leq L(v)|x_1 - x_2|. \tag{3}$$

Remark 1. Assumption 2 states that the inverse of generalized damping or friction coefficient should meet the Lipschitz continuity condition. In the theory of differential equation, Lipschitz continuity condition is necessary to ensure the existence and uniqueness of the solution trajectory. In motion systems, the solution trajectory should be existing and unique under the same dynamics and same initial condition.

In practice, many motion systems are discontinuous when speed is zero, due to the presence of static friction. Consider the Gaussian friction model Brian et al. (1994)

$$\begin{cases} \dfrac{dx}{dt} = v, \\ \dfrac{dv}{dt} = -\dfrac{1}{m}\left(\left(f_c + (f_s - f_c)e^{-(\frac{v}{v_s})^\delta}\right) sgn(v) + f_v v\right) \end{cases} \tag{4}$$

where f_c is the minimum level of kinetic friction, f_s is the level of static friction, f_v is the level of viscous friction, $v_s > 0$ and $\delta > 0$ are empirical parameters. The signum function from static friction represents a non-Lipschitzian term, and owing to this term a vehicle running on ground can always stop in a finite time interval instead of asymptotically stop. The choice of the dx/dv relationship enables the inclusion of the static friction because, according to definition in (2), g is continuous both in x and v.

Next define the final position and final speed in spatial domain. In position control, the final displacement, x_e is observed at a prespecified speed v_f. If the initial speed is lower than v_f, v_f cannot be reached. In such circumstance, the final displacement is defined to be

$$x_e(u) \triangleq \begin{cases} x(v, u), & \text{when } v = v_f \\ 0, & v_f \text{ cannot be reached} \end{cases} \tag{5}$$

where $x(v_f, u)$ is the position of the system (2) at the speed v_f with the control input u.

In speed control, the final speed, v_e, is observed at a prespecified position x_f. However, if the initial speed is low, the final position may not reach x_f while the final speed already drops to zero. In such circumstances, the final speed is defined to be zero. Therefore the final speed is defined in two cases

$$v_e(u) \triangleq \begin{cases} v(x, u), & \text{when } x = x_f \\ 0, & x_f \text{ cannot be reached} \\ & \text{when motion stops.} \end{cases} \tag{6}$$

In (5) and (6), the control input u is either initial position or speed.

From *Assumptions* 1 and 2, we can derive an important property summarized below.

Property 1. For any two initial quantities $u_j \neq u_j^*$ where (u_j, u_j^*) are either initial positions or speeds, we have $(u_j - u_j^*)[x_e(u_j) - x_e(u_j^*)] > 0$ in final position control and $(u_j - u_j^*)[v_e(u_j) - v_e(u_j^*)] > 0$ in final speed control.

Proof.

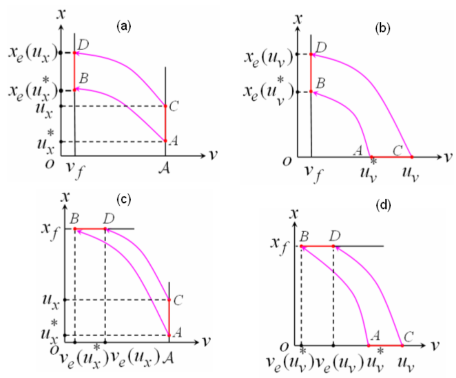

Fig. 1. Initial state tuning for final state control.

(1) Initial position tuning for final position control

Look into the phase plane in Fig.1 (a), two solution trajectories $\widehat{AB}$ and $\widehat{CD}$ represent solution trajectories of the dynamics (2) with different initial positions $u_x^* < u_x$. By virtue of the uniqueness of the solution, two trajectories do not intersect each other. As a result, $x(v, u_x^*) < x(v, u_x)$ and so is $x_e(u_x^*) < x_e(u_x)$. Therefore we have $(u_x - u_x^*)[x_e(u_x) - x_e(u_x^*)] > 0$.

(2) Initial speed tuning for final position control

In Fig.1 (b), the trajectory $\widehat{AB}$ starts from the initial speed u_v^* and the trajectory $\widehat{CD}$ starts from the initial speed u_v, while the initial displacements are zero. From Fig.1 (b) and the uniqueness of solution, $u_v > u_v^*$ leads to the positions $x_e(u_v) > x_e(u_v^*)$ at the points D and B corresponding to the prespecified speed v_f. As a result we have $(u_v - u_v^*)[x_e(u_v) - x_e(u_v^*)] > 0$.

(3) Initial position tuning for final speed control

When $u_x > u_x^*$, from phase plane Fig. 1 (c) we can see that the trajectory $\widehat{CD}$ is above the trajectory $\widehat{AB}$ because of the uniqueness of solution. When both positions drop to the same level at x_f, the speed D is obviously farther than the speed B. Therefore we have $(u_x - u_x^*)[v_e(u_x) - v_e(u_x^*)] > 0$.

(4) Initial speed tuning for final speed control

From Fig.1 (d) and the uniqueness of solution, we can see that trajectory $\widehat{AB}$ with initial speed u_v^* is always on the left of the trajectory $\widehat{CD}$ with the initial speed u_v, because $u_v > u_v^*$. Accordingly $v_e(u_v) > v_e(u_v^*)$, that is, the point D is on the right of the point B. As a result we have $(u_v - u_v^*)[v_e(u_v) - v_e(u_v^*)] > 0$. ∎

3. INITIAL STATE ITERATIVE LEARNING

With initial or final position and speed, we have four cases

- (i) initial position iterative learning for final position control;
- (ii) initial speed iterative learning for final position control;
- (iii) initial position iterative learning for final speed control;
- (iv) initial speed iterative learning for final speed control.

Denote $x_{i,e}$ and $v_{i,e}$ the final position and speed defined in (5) and (6) respectively at the ith iteration, where $i = 1, 2, \cdots$ denotes the iteration number. The ILC algorithms corresponding to the four cases are

$$\begin{array}{rl} \text{(i)} & u_{x,i+1} = u_{x,i} + \gamma(x_d - x_{i,e}) \\ \text{(ii)} & u_{v,i+1} = u_{v,i} + \gamma(x_d - x_{i,e}) \\ \text{(iii)} & u_{x,i+1} = u_{x,i} + \gamma(v_d - v_{i,e}) \\ \text{(iv)} & u_{v,i+1} = u_{v,i} + \gamma(v_d - v_{i,e}) \end{array} \tag{7}$$

where $\gamma > 0$ is a learning gain, $u_{x,i}$ is the initial position and $u_{v,i}$ is the initial speed at the ith iteration.

Let u denote either initial speed or position, and z either final speed or position, from *Property* 1 we have

$$\begin{aligned} |u_d - u_{i+1}| &= |(u_d - u_i) - \gamma(z_d - z_i)| \\ &= \left||u_d - u_i| - \gamma|z_d - z_i|\right|. \end{aligned} \tag{8}$$

To achieve learning convergence, a key issue is to determine the range of values for the learning gain γ, which is summarized in the following *Lemma*.

Lemma 1. Suppose there exists a constant λ such that $|z_d - z_i| \le \lambda|u_d - u_i|$, and there exists a $M < \infty$ such that $|u_d - u_1| = M$. For any given $\varepsilon > 0$, by applying the control law (7) and choosing the learning gain in the range

$$\frac{1-\rho}{\lambda} < \gamma < \frac{1+\rho}{\lambda}, \qquad 0 < \rho < 1, \tag{9}$$

the output z_i will converge to the ε-neighbourhood of the desired output z_d with a finite number of iterations no more than

$$N = \frac{\log \dfrac{\varepsilon}{M\lambda}}{\log\left(1 - (1-\rho)\dfrac{\varepsilon}{M\lambda}\right)} + 1.$$

Proof. Since $|z_d - z_i| \le \lambda|u_d - u_i|$, there exists a quantity $0 < \lambda_i \le \lambda$ such that

$$|z_d - z_i| = \lambda_i|u_d - u_i|. \tag{10}$$

Let $\gamma = r/\lambda$, from the constraint of γ we have $1 - \rho < r < 1 + \rho$. Substituting (10) into (8) yields

$$|u_d - u_{i+1}| = |1 - \gamma\lambda_i||u_d - u_i| = |1 - r\frac{\lambda_i}{\lambda}||u_d - u_i|.$$

The convergence of iteration learning is determined by the magnitude of the factor $|1 - r\frac{\lambda_i}{\lambda}|$. The upper bound for $|1 - r\frac{\lambda_i}{\lambda}|$ indicates the slowest convergence rate. Next we derive this upper bound with two cases.

Case 1. $\min\{\frac{\lambda}{\lambda_i}, 1+\rho\} = \frac{\lambda}{\lambda_i}$. When $1 - \rho < r \le \frac{\lambda}{\lambda_i}$,

$$|1 - r\frac{\lambda_i}{\lambda}| = 1 - r\frac{\lambda_i}{\lambda} < 1 - (1-\rho)\frac{\lambda_i}{\lambda} \triangleq \rho_i \le 1.$$

When $\frac{\lambda}{\lambda_i} < r < 1 + \rho$,

$$\begin{aligned} |1 - r\frac{\lambda_i}{\lambda}| &= r\frac{\lambda_i}{\lambda} - 1 < (1+\rho)\frac{\lambda_i}{\lambda} - 1 \\ &< \rho = 1 - (1-\rho) \le \rho_i. \end{aligned}$$

From $(1+\rho)\frac{\lambda_i}{\lambda} < 1 + \rho$ we conclude $(1+\rho)\frac{\lambda_i}{\lambda} - 1 < \rho$ and, thus,

$$(1+\rho)\frac{\lambda_i}{\lambda} - 1 < \rho = 1 - (1-\rho) \le \rho_i.$$

Case 2. $\min\{\frac{\lambda}{\lambda_i}, 1+\rho\} = 1 + \rho$. In this case, we have

$$|1 - r\frac{\lambda_i}{\lambda}| = 1 - r\frac{\lambda_i}{\lambda} < 1 - (1-\rho)\frac{\lambda_i}{\lambda} = \rho_i.$$

Thus the upper bound of the convergence factor is

$$\rho_i = 1 - (1-\rho)\frac{\lambda_i}{\lambda}. \tag{11}$$

for all iterations. Note that when $u_i \ne u_d$, $z_i \ne z_d$ by the uniqueness of solution, consequently $\lambda_i \ne 0$ by (10) and the upper bound ρ_i will be strictly less than 1 as far as u_i does not converge to u_d.

Let ε denote the desired ε-precision bound of learning, i.e. $|z_d - z_i| < \varepsilon$. Now we show that the sequence z_i can enter the prespecified ε-precision bound after a finite number of iterations. Let M denote the initial input error $|u_d - u_x| = M < \infty$.

First, considering the fact $\rho_i \leq 1$, using (11) repeatedly yields

$$|z_d - z_i| = \lambda_i |u_d - u_i| = \lambda_i \prod_{j=1}^{i-1} \rho_j |u_d - u_x| \leq \lambda_i M.$$

Before z_i enters the ε-bound, $\varepsilon < |z_d - z_i| \leq \lambda_i |u_d - u_x| \leq \lambda_i M$ which gives the lower bound of the coefficient λ_i, $\lambda_i \geq \epsilon / M$ for all iterations before learning terminates. Similarly by using the relationship (11) repeatedly, and substituting the lower bound of λ_i, we can derive

$$|z_d - z_i| \leq \lambda |u_d - u_i| \leq \lambda \prod_{j=1}^{i-1} \rho_j |u_d - u_x|$$
$$= \lambda \prod_{j=1}^{i-1} \left(1 - (1-\rho)\frac{\lambda_j}{\lambda}\right) M \leq M\lambda \left(1 - (1-\rho)\frac{\epsilon}{M\lambda}\right)^i$$

which gives the upper bound of $|z_d - z_i|$. Solving for $M\lambda \left(1 - (1-\rho)\frac{\epsilon}{M\lambda}\right)^{i-1} \leq \varepsilon$ with respect to i, the maximum number of iterations needed is

$$i \leq \frac{\log \dfrac{\varepsilon}{M\lambda}}{\log \left(1 - (1-\rho)\dfrac{\varepsilon}{M\lambda}\right)} + 1.$$

■

Remark 2. The existence of a finite M can be easily verified as u_d is finite, and u_1 is always chosen to be a finite initial state in practical motion control problems.

In terms of *Lemma 1*, all we need to do is to find λ from the motion system so that the range of the learning gain γ can be determined. In Theorem 1, we derive the value of λ for all four cases.

Theorem 1. The ILC convergence is guaranteed for cases (i) – (iv) when the learning gain is chosen to meet the condition (9), and the values of λ can be calculated respectively for four cases below.

(i) In the initial position iterative learning for final position control, choose $\lambda = \exp\left(\int_{v_f}^{\mathcal{A}} L(v)dv\right)$.

(ii) In the initial speed iterative learning for final position control, choose $\lambda = \max_{v \in [v_f, \mathcal{A}]} g_1(v) \exp\left(\int_{v_f}^{\mathcal{A}} L(v)dv\right)$, where g_1 is an upper bounding function satisfying $g(v,x) \leq g_1(v)$.

(iii) In the initial position iterative learning for final speed control, choose $\lambda = \frac{1}{c} \exp\left(\int_{v_d}^{\mathcal{A}} L(v)dv\right)$, where c is a lower bound satisfying $0 < c \leq g(v,x)$.

(iv) In the initial speed iterative learning for final speed control, choose $\lambda = \frac{1}{c} \max_{v \in [v_d, \mathcal{A}]} g_1(v) \exp\left(\int_{v_d}^{\mathcal{A}} L(v)dv\right)$.

Proof: For simplicity, in subsequent graphics we demonstrate $u_{x,i} > u_{x,d}$ or $u_{v,i} > u_{v,d}$ only. By following the same derivation procedure, we can easily prove learning convergence for opposite cases $u_{x,i} < u_{x,d}$ or $u_{v,i} < u_{v,d}$. Denote $\widehat{AB}$ the trajectories of (2) associated with the desired control inputs, and $\widehat{CD}$the trajectories associated with the actual control inputs at the ith iteration.

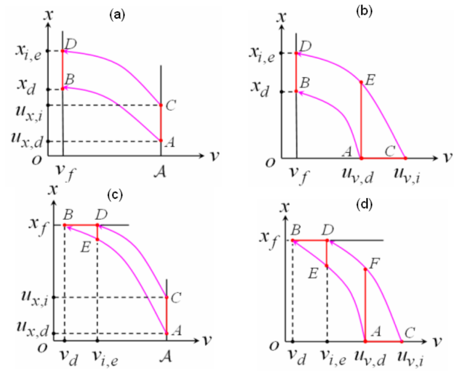

Fig. 2. Phase portraying of system (2) in v-x plane with initial state learning for final state control.

(i) Initial position iterative learning for final position control

The initial speed is fixed at $\mathcal{A}$. Denote $u_{x,d}$ the desired initial position that achieves the desired final position x_d at the prespecified speed v_f, that is, applying $u_{x,d}$ to the dynamics (2) yields $x_e = x_d$.

Integrating (2) yields

$$x_d - x_{i,e}$$
$$= u_{x,d} - u_{x,i} - \int_{\mathcal{A}}^{v_f} \left[g\left(v, x(v, u_{x,d})\right) - g\left(v, x(v, u_{x,i})\right)\right] dv.$$

Applying the Lipschitz continuity condition (3) yields

$$|x_d - x_{i,e}|$$
$$\leq |u_{x,d} - u_{x,i}| + \int_{v_f}^{\mathcal{A}} L(v) |x(v, u_{x,d}) - x(v, u_{x,i})| dv. \quad (12)$$

Define $\lambda = \exp\left(\int_{v_f}^{\mathcal{A}} L(v)dv\right)$. Applying the generalized Grownwall inequality to (12) we obtain $|x_d - x_{i,e}| \leq \lambda |u_{x,d} - u_{x,i}|$. As shown in Fig. 2 (a), $\overline{BD} = |x_d - x_{i,e}| \leq \lambda |u_{x,d} - u_{x,i}| = \lambda \overline{AC}$. Therefore, choose a $\rho < 1$ and the learning gain according to λ and (9), the learning convergence is obtained.

(ii) Initial speed iterative learning for final position control

As shown in Fig.2 (b), draw a line $\overline{AE}$ starting from A such that it parallels the x-axis, where E is the point intersected with $\widehat{CD}$. In order to find the relationship between the initial speed and final position, we first derive the relationship between $\overline{BD}$ and $\overline{AE}$, then derive the relationship between $\overline{AE}$ and $\overline{AC}$.

Using the result of case (i), we can obtain the relationship between the initial position difference $\overline{AE}$ and final position difference $\overline{BD}$

Next investigate the relationship between the position difference $\overline{AE}$ and initial speed difference $\overline{AC}$. Denote x^* the position at E. Integrating (2), the position difference $\overline{AE}$ at the ith iteration can be estimated using the mean value theorem

$$\begin{aligned}\overline{AE} &= x^* - 0 = -\int_{u_{v,i}}^{u_{v,d}} g\left(v, x(v, u_{v,i})\right) dv \\ &= g\left(v, x(v, u_{v,i})\right)(u_{v,i} - u_{v,d}) \qquad \exists v \in [u_{v,d}, u_{v,i}] \\ &\leq \max_{v\in[u_{v,d},u_{v,i}]} g\left(v, x(v, u_{v,i})\right) \cdot |u_{v,i} - u_{v,d}|. \qquad (13)\end{aligned}$$

Using this bounding condition $g(v, x) \leq g_1(v)$ and (13), we obtain

$$\begin{aligned}&\max_{v\in[u_{v,d},u_{v,i}]} g\left(v, x(v, u_{v,i})\right) \\ &\leq \max_{v\in[u_{v,d},u_{v,i}]} g_1(v) \leq \max_{v\in[v_f,\mathcal{A}]} g_1(v).\end{aligned}$$

Define $\lambda_2 = \max_{v\in[v_f,\mathcal{A}]} g_1(v)$, we have $\overline{AE} \leq \lambda_2\overline{AC}$, $\overline{BD} \leq \lambda_1\overline{AE} \leq \lambda\overline{AC}$ where $\lambda = \lambda_1\lambda_2$. Therefore, choose $\rho < 1$ and the learning gain according to λ and (9), the learning convergence is guaranteed.

(iii) Initial position iterative learning for final speed control

As shown in Fig.2 (c), draw a line through point D such that it parallels the x-axis and intersects the trajectory $\widehat{AB}$ at the point E. Denote x^* the position at E. In order to find the relationship between the initial position and final speed, we first derive the relationship between $\overline{AC}$ and $\overline{ED}$, then derive the relationship between $\overline{ED}$ and $\overline{BD}$.

Using the result of case (i), we can obtain the relationship between the initial position difference $\overline{AC}$ and final position difference $\overline{ED}$

$$\overline{ED} \leq \lambda_1\overline{AC}, \qquad \lambda_1 = \exp\left(\int_{v_d}^{\mathcal{A}} L(v)dv\right). \qquad (14)$$

Next investigate the relationship between the initial position difference $\overline{ED}$ and the final speed difference $\overline{BD}$. Integrating (2), the speed difference $\overline{ED}$ at the ith iteration can be estimated using the mean value theorem

$$\begin{aligned}\overline{ED} &= x_f - x^* = \int_{v_d}^{v_{i,e}} g\left(v, x(v, x^*)\right) dv \\ &= g\left(v, x(v, x^*)\right)(v_{i,e} - v_d) \qquad \exists v \in [v_d, v_{i,e}] \\ &\geq \min_{v\in[v_d,v_{i,e}]} g\left(v, x(v, x^*)\right)(v_{i,e} - v_d). \qquad (15)\end{aligned}$$

Substitute the relationship $\min_{v\in[v_d,v_{i,e}]} g\left(v, x(v, x^*)\right) \geq c$ into (15) and note $\overline{BD} = v_{i,e} - v_d$, we have $\overline{BD} \leq \lambda_2\overline{ED}$, $\lambda_2 = 1/c$. Finally using (14) it can be derived that $\overline{BD} \leq \lambda_2\overline{ED} \leq \lambda\overline{AC}$ where $\lambda = \lambda_1\lambda_2$. Therefore, choose $\rho < 1$ and the learning gain according to λ and (9), the learning convergence is guaranteed.

(iv) Initial speed iterative learning for final speed control

As shown in Fig.2 (d), the learning convergence in this case can be derived directly by using the results of cases (i), (ii) and (iii). Draw two straight lines $\overline{ED}$ and $\overline{AF}$. There exist three relations. The first relationship is between the initial speed difference $\overline{AC}$ and the final position difference $\overline{AF}$, which has been discussed in the second part of case (ii). The second relationship is between the initial position difference $\overline{AF}$ and the final position difference $\overline{ED}$, which has been explored in case (i). The third relationship is between the initial position difference $\overline{ED}$ and the final speed difference $\overline{BD}$, which was given in the second part of case (iii). Therefore the value of λ given in the theorem consists of three factors

$$\max_{v\in[v_f,\mathcal{A}]} g_1(v), \qquad \exp\left(\int_{v_f}^{\mathcal{A}} L(v)dv\right), \qquad \frac{1}{c}.$$

■

The prior knowledge required for four cases differs. The first case from position to position requires minimum prior knowledge from the motion system, the lower and upper bounds of $g(v, x)$ are not required. In the second case from speed to position, only the upper bounding function is required. In the third case from position to speed, only the lower bounding function is required. In the fourth case from speed to speed, however, both the lower and upper bounding functions are required.

Since g is the inverse of generalized damping or friction coefficient, the lower bound for g is to rule out the scenario where the generalized damping or friction coefficient would be infinity. Physically an overlarge damping or overlarge friction coefficient implies that an immediate stop-motion may occur, and we are unable to achieve the final speed control at a prespecified position x_f. Therefore the lower bound is required in cases (iii) and (iv) for final speed control.

The upper bound for g is required for initial speed learning to rule out the scenario where the generalized damping or friction coefficient would be too small. Look into the proof of case (ii), if the generalized damping or friction is too small, trajectories $\widehat{AB}$ and $\widehat{CD}$ will be very steep. As a result, a small change in the initial speed $\overline{CA}$ yields a significant position difference $\overline{AE}$. In other words, the system gain is extremely large and an extremely lower learning gain should be used. g_1 confines the system gain so that the lower bound of the learning gain can be determined.

4. A DUAL INITIAL STATE LEARNING

In (1), consider such a scenario where f may drop to zero due to environmental changes, such as extremely low surface friction at certain places, meanwhile f could remain continuous with $v > 0$, $v_f > 0$ and $v_e > 0$. In such circumstances, it is appropriate to consider dv/dx in the phase plane

$$\frac{dv}{dx} = -\frac{f}{v} = -g(x, v), \qquad (16)$$

where the generalized damping or friction coefficient is $g(x, v) = f/v$. Comparing with (2), in the dual problem (16) the positions of x and v are swopped, x is the argument and v is a function of x and the control inputs. The control tasks remain the same as the final position or speed control by means of the initial position or speed tuning. Thus the analysis in Theorem 1 can be directly extended to this dual scenario because *Assumption* 1 does not change and *Assumption* 2 holds with x and v swopped. Since the two control problems associated with (2) and (16) are the same except for the swopping between x and v, by employing the same ILC algorithms (7), the learning convergence properties for the four cases can be derived in a dual manner by swopping x_i with v_i, x_d with v_d, x_f with v_f, as summarized in Theorem 2.

Theorem 2. The ILC convergence is guaranteed for cases (i) − (iv) when the learning gain is chosen to meet the condition (9), where the value of λ can be calculated respectively for four cases.

(i) In initial position iterative learning for final position control, choose $\lambda = \frac{1}{c}\max_{x\in[0,x_d]} g_1(x)\exp\left(\int_0^{x_d} L(x)dx\right)$.

(ii) In initial speed iterative learning for final position control, choose $\lambda = \frac{1}{c}\exp\left(\int_0^{x_d} L(x)dx\right)$.

(iii) In initial position iterative learning for final speed control, choose $\lambda = \max_{x\in[0,x_f]} g_1(x)\exp\left(\int_0^{x_f} L(x)dx\right)$.

(iv) In initial speed iterative learning for final speed control, choose $\lambda = \exp\left(\int_0^{x_f} L(x)dx\right)$.

5. ILLUSTRATIVE EXAMPLE

Consider system (4) with parameters $m = 1$, $f_c = 3.5$, $f_s = 3.65$, $f_v = 1.06$, $v_s = 0.1$, $\delta = 0.05$. The target is to bring the motion system to a final state $(x_d, v_f) = (20, 0)$, i.e., let the motion system reach a displacement 20 m and stop. Since g is independent of x, Lipschitz function $L(v)$ is chosen to be zero.

Note that

$$g(v,x) = \frac{mv}{(f_c + (f_s - f_c)e^{-(\frac{v}{v_s})^\delta} + f_v v} < \frac{m}{f_v},$$

holds for any values of v, we can choose the upper bounding function $g_1(v) = \frac{m}{f_v} = 0.9434$. In terms of Theorem 1, when applying initial position learning which is case (i), $\lambda = 1$; and when applying initial speed learning which is case (ii), $\lambda = 0.9434$. The ILC law is given by (i) or (ii) in (7). In this example, choose the factor $\rho = 0.4$. According to Theorem 1, $0.6 < \gamma < 1.4$ for (i) initial position learning and $0.64 < \gamma < 1.48$ for (ii) initial speed learning. The ε-neighbourhood is chosen with $\varepsilon = 0.001$ m. Now, set a uniform learning gain $\gamma = 0.95$ and the learning results are shown in Fig.3 and Fig.4. In both cases, a quick learning convergence is achieved after repeating the learning process a few iterations.

6. CONCLUSION

In this work we addressed a class of final state control problems for motion systems where the manipulated variables are initial states. Through iterative learning with the final state information, the desired initial states can be generated despite the existence of unknown nonlinear uncertainties in the motion systems. Both theoretical analysis and numerical simulations verify the effectiveness of the proposed learning control schemes.

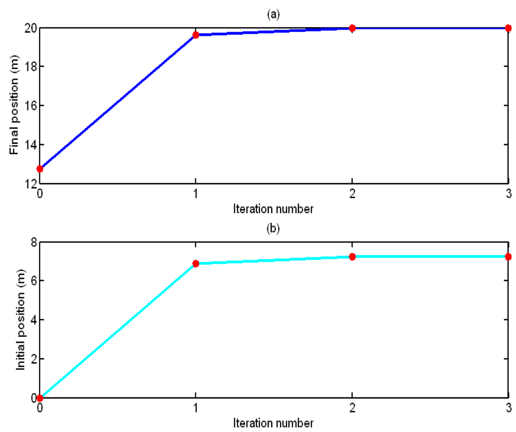

Fig. 3. Initial position learning for final position control: $u_{x,1} = 0.0$ m, $\mathcal{A} = 20.0$ m/s. (a) The observed final position; (b) The learning results of initial position.

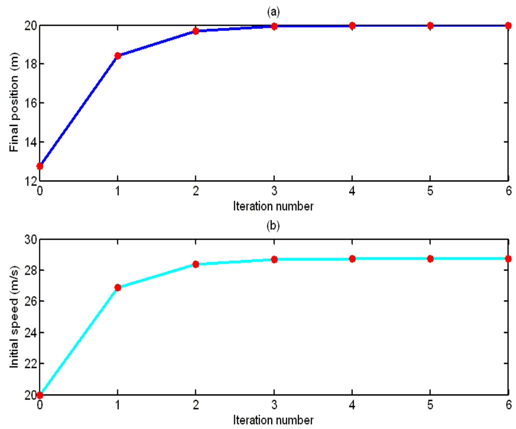

Fig. 4. Initial speed learning for final position control: $u_{v,1} = 20.0$ m/s. (a) The observed final position; (b) The learning results of initial speed.

Our next phase is to extend the iterative learning approach to more generic scenarios where multiple initial states can be adjusted simultaneously. Learning with optimality will be explored to address the control redundancy arising from multiple initial states.

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