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Decentralized Nonlinear Control Method of Components in Power Systems Based on Differential-Algebraic Sub-System Models

Kaifeng Zhang. Xianzhong Dai. Qiang Zang

Key Laboratory of Measurement and Control of CSE (School of Automation, Southeast University), Ministry of Education, Nanjing, China, 210096, (Tel: 86-25-83794165; e-mail: kaifengzhang@seu.edu.cn).

Abstract: Complete nonlinear differential-algebraic equation (DAE) sub-system models are considered in the paper when designing controllers of components in power systems. First-principle models are nonlinear DAE sub-system models, but they use non-local measurable variables to describe the mutual relation (interconnection) between component and the AC grid, and thus they are not suitable for designing decentralized controllers. In the paper, component structural models are constructed, in which the local-measurable interface variables are used to describe the mutual relation between component and the AC grid. Thus, the proposed models are equivalent to the first-principle models in essence, and have two characteristics: with local measurable interconnections and index 1. These two characteristics make it possible to transform the component structural models to nonlinear ordinary differential equation (ODE) sub-system models with measurable interconnections. Thus, traditional nonlinear control methods which are suitable for nonlinear ODE systems could be developed and expanded to be suitable for designing component controllers.

1. INTRODUCTION

In recent years, the application of nonlinear control methods to the components (such as synchronous generator sets, HVDC systems, FACTS apparatuses, etc.) in power systems has drawn much attention (Akhrif, *et al*, 1999, Dai, *et al*, 2004). Apparently, the models used in nonlinear control methods should fully describe the complex nonlinearities of the controlled objectives.

In the area of power systems, it is well-known that first-principle models, such as Structure Preserving Model (SPM) and Component Connection Model (CCM) (Wasysczuk, *et al*, 1981), can fully describe the complex nonlinearities of components. One typical expression of these models is (Kundur, 1994):

$$\begin{cases} \dot{\boldsymbol{x}}_i = \boldsymbol{f}_{Ui}(\boldsymbol{x}_i, \boldsymbol{U}_{Fi}, \boldsymbol{u}_i) \\ \boldsymbol{I}_{Fi} = \boldsymbol{f}_{Fi}(\boldsymbol{U}_{Fi}, \boldsymbol{x}_i) \end{cases} \quad (i = 1, \cdots, N) \tag{1}$$

$$\boldsymbol{I}_F = \boldsymbol{Y}_F \boldsymbol{U}_F \tag{2}$$

Equation (1) is the model of the i-th component ($i = 1, \cdots, N$). Equation (2) is the model of the AC grid. In (1) and (2), $\boldsymbol{x}_i \in \boldsymbol{R}^{X_i}$, $\boldsymbol{u}_i \in \boldsymbol{R}^{U_i}$ are the state and input variables, respectively; $\boldsymbol{I}_{Fi} \in \boldsymbol{R}^{2m_i}$, $\boldsymbol{U}_{Fi} \in \boldsymbol{R}^{2m_i}$ are the current and voltage vectors, where m_i is the number of the interconnection lines between the component and the AC grid when power systems are expressed by single-line mode; $\boldsymbol{I}_F = [\boldsymbol{I}_{F1}, \cdots, \boldsymbol{I}_{FN}]^T$, $\boldsymbol{U}_F = [\boldsymbol{U}_{F1}, \cdots, \boldsymbol{U}_{FN}]^T$; $\boldsymbol{Y}_F$ is the admittance matrix.

There are the following two features of the component model of (1):

- It is a differential-algebraic equation (DAE) system. In (1), $\dot{\boldsymbol{x}}_i = \boldsymbol{f}_{Ui}(\boldsymbol{x}_i, \boldsymbol{U}_{Fi}, \boldsymbol{u}_i)$ is the differential equation; $\boldsymbol{I}_{Fi} = \boldsymbol{f}_{Fi}(\boldsymbol{U}_{Fi}, \boldsymbol{x}_i)$ is the algebraic equation, and in power systems it is called current injection equation.
- It is a sub-system. In (1), $(\boldsymbol{I}_{Fi}, \boldsymbol{U}_{Fi})$ are the variables used to describe the relation between the component and the AC grid.

Compared with the nonlinear models used in present methods, the component model of (1) is very complex. As we know, to the present, although there have been some research results in the area of linear DAE system (Dai, 1989), there are still not many results in the area of nonlinear DAE system (Contou-Carrere, *et al*, 2005). For the nonlinear DAE sub-system, there are almost no results. Thus, although in essence the components in power systems are nonlinear DAE sub-system models, there is still lack of systematic nonlinear control methods based on nonlinear DAE sub-system models. Most of the present researches are based on relatively simple nonlinear ordinary differential equation (ODE) models (Akhrif, *et al*, 1999, Dai, *et al*, 2004). The DAE models considered in (Hill, *et al*, 1990) are DAE isolated system models, not sub-system models.

In the paper, the problem of designing component controllers based on nonlinear DAE sub-system models will be considered. However, it is difficult to directly design decentralized controllers based on the component model of (1):

- There is lack of ready control theory of nonlinear DAE sub-systems.
- In (1), there are some non-local measurable variables ($\boldsymbol{I}_{Fi}$ and $\boldsymbol{U}_{Fi}$). From the viewpoint of real power engineering, the component controller should be decentralized, or should only feedback local measurable variables.

In the paper, firstly, the interface variables, which are local-measurable, will be used to replace $(\boldsymbol{I}_{Fi},\boldsymbol{U}_{Fi})$ to describe the mutual relation between the component and the AC grid. Then, a new nonlinear DAE sub-system model of component, or the component structural model, will be constructed. The proposed structural model is equivalent to the model as shown in (1), and has two special characteristics: with local measurable interconnections and index 1. Because of these two features, it is possible to transform the component structural models to nonlinear ODE sub-system models with measurable interconnections, and then it would be relatively simple to design controllers based on component structural models. Or, traditional nonlinear control methods which are suitable for nonlinear ODE systems would be developed and expanded to be suitable for designing controllers of nonlinear DAE sub-systems.

2. COMPONENT STRUCTURAL MODEL

2.1 Interface Variables

As the component is a part (sub-system) of the large-scale power systems, when designing component controllers, the mutual relation between the *i*-th component and the rest of power systems (including the rest *N-1* components and the AC grid, see Fig.1) should be considered seriously. In the paper, this mutual relation is named "interface" (see Fig.1).

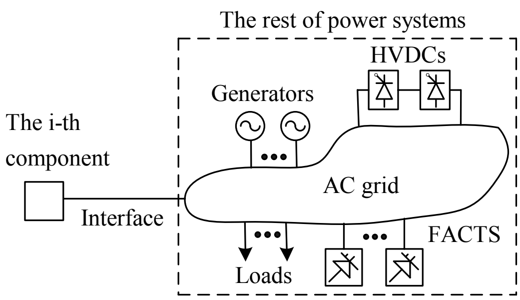

Fig. 1. The interface between the component and the rest of power systems

In (1), $(\boldsymbol{I}_{Fi},\boldsymbol{U}_{Fi})$ are used to describe the interface relation between component and the rest of power systems. As mentioned above, $(\boldsymbol{I}_{Fi},\boldsymbol{U}_{Fi})$ are all not local measurements, and thus the model of (1) is not suitable for designing decentralized controller. Meanwhile, in real power engineering, there are many local-measurable variables, such as V_{ti} (the amplitude of voltage), P_{ti} (the real power) and I_{ti} (the amplitude of current), etc., but they are not included in (1). In this section, a set of new variables, or interface variables (expressed by $\boldsymbol{v}_i$) will be defined to replace $(\boldsymbol{I}_{Fi},\boldsymbol{U}_{Fi})$ to describe the relation between the component and the AC grid. There should be the following relations between $\boldsymbol{v}_i$ and $(\boldsymbol{I}_{Fi},\boldsymbol{U}_{Fi})$:

$$\boldsymbol{v}_i = \boldsymbol{\Phi}(\boldsymbol{I}_{Fi},\boldsymbol{U}_{Fi}) \text{ and } (\boldsymbol{I}_{Fi},\boldsymbol{U}_{Fi})^T = \boldsymbol{\Phi}^{-1}(\boldsymbol{v}_i) \tag{3}$$

In (3), $\boldsymbol{\Phi}^{-1}(\bullet)$ is the inverse of $\boldsymbol{\Phi}(\bullet)$, or the interface variables are equivalent to $(\boldsymbol{I}_{Fi},\boldsymbol{U}_{Fi})$. Compared with $(\boldsymbol{I}_{Fi},\boldsymbol{U}_{Fi})$, the selection of $\boldsymbol{v}_i$ is very flexible, or some local-measurable variables could be chosen to describe the interface relation between the component and the rest of power systems. Choosing the generator set as an example, $(\boldsymbol{I}_{Fi},\boldsymbol{U}_{Fi})$ is $(I_{xi},I_{yi},U_{xi},U_{yi})$, and $(P_{ti},I_{ti},Q_{ti},\theta_{Ui})$ are reasonable interface variables. Meanwhile, the number of $\boldsymbol{v}_i$ is the same as that of $(\boldsymbol{I}_{Fi},\boldsymbol{U}_{Fi})$, or $4m_i$.

2.2 Component Structural Model

Substituting $(\boldsymbol{I}_{Fi},\boldsymbol{U}_{Fi})^T = \boldsymbol{\Phi}^{-1}(\boldsymbol{v}_i)$ (see (3)) into (1), one can get

$$\begin{cases} \dot{\boldsymbol{x}}_i = \boldsymbol{f}_i(\boldsymbol{x}_i,\boldsymbol{v}_i,\boldsymbol{u}_i) \\ \boldsymbol{g}_i(\boldsymbol{x}_i,\boldsymbol{v}_i) = \boldsymbol{0} \end{cases} \tag{4}$$

Apart from $\boldsymbol{x}_i$, $\boldsymbol{v}_i$ and $\boldsymbol{u}_i$,there may be other kind of variables in components. For example, in the model of generator, there are $[I_{di},I_{qi}]$. In the paper, they are named "affiliate variables" ($\boldsymbol{w}_i$). Thus, the following model is more universal for the components in power systems:

$$\begin{cases} \dot{\boldsymbol{x}}_i = \boldsymbol{f}_i^w(\boldsymbol{x}_i,\boldsymbol{w}_i,\boldsymbol{v}_i,\boldsymbol{u}_i) \\ \boldsymbol{g}_i^w(\boldsymbol{x}_i,\boldsymbol{w}_i,\boldsymbol{v}_i) = \boldsymbol{0} \end{cases} \tag{5}$$

Choosing the generator set as an example, when the generator adopts 3rd-order one-axis model without ignoring the transient saliency, only considering the governor control, and choosing $(P_{ti},I_{ti},Q_{ti},\theta_{Ui})$ as the interface variables, the structural model of generator set as shown in (5) is (For the explanation of the variables, please refer to (Kundur, 1994)):

$$\begin{cases} \dot{\delta}_i = \omega_i - \omega_0 \\ \dot{\omega}_i = (\omega_0 / H_i)\{P_{Hi} + C_{MLi}P_{mi0} - D_i(\omega_i - \omega_0)/\omega_0 \\ \qquad -[E'_{qi} + (x_{qi} - x'_{di})I_{di}]I_{qi}\} \\ \dot{P}_{Hi} = (-P_{Hi} + C_{Hi}P_{mi0} + C_{Hi}U_{ci})/T_{H\Sigma i} \end{cases} \tag{6}$$

$$\begin{cases} P_{ti} = [E'_{qi} + (x_{qi} - x'_{di})I_{di}]I_{qi} - r_{ai}(I_{di}^2 + I_{qi}^2) \\ I_{ti} = (I_{di}^2 + I_{qi}^2)^{0.5} \\ Q_{ti} = E'_{qi}I_{di} - x_{qi}I_{qi}^2 - x'_{di}I_{di}^2 \\ \theta_{Ui} = \delta_i - \arctan[(x_{qi}I_{qi} - r_{ai}I_{di})/(E'_{qi} - x'_{di}I_{di} - r_{ai}I_{qi})] \end{cases} \tag{7}$$

Furthermore, the interface variables $\boldsymbol{v}_i$ could be decomposed into two parts, or $\boldsymbol{v}_i = (\hat{\boldsymbol{v}}_i, \overline{\boldsymbol{v}}_i)^T$. Under this decomposition, equations (5) would be:

$$\begin{cases} \dot{\boldsymbol{x}}_i = \boldsymbol{f}_i^w(\boldsymbol{x}_i, \boldsymbol{w}_i, \boldsymbol{u}_i) \\ \boldsymbol{g}_i^w(\boldsymbol{x}_i, \boldsymbol{w}_i, \hat{\boldsymbol{v}}_i, \overline{\boldsymbol{v}}_i) = \boldsymbol{0} \end{cases} \quad (8)$$

And the following conditions should be satisfied (Zhang, *et al*, 2007):

(1) $\overline{\boldsymbol{v}}_i$ could "fully" describe the influence of the rest of power systems to the component, or $\overline{\boldsymbol{v}}_i$ are the interconnection inputs (disturbances) of the component;

(2) $\hat{\boldsymbol{v}}_i$ could "fully" describe the influence of the component to the rest of power systems.

Defining $\boldsymbol{z}_i = (\boldsymbol{w}_i, \hat{\boldsymbol{v}}_i)^T$, equation (8) would be (see Fig.2):

$$\begin{cases} \dot{\boldsymbol{x}}_i = \boldsymbol{f}_i^z(\boldsymbol{x}_i, \boldsymbol{z}_i, \boldsymbol{u}_i) \\ \boldsymbol{g}_i^z(\boldsymbol{x}_i, \boldsymbol{z}_i, \overline{\boldsymbol{v}}_i) = \boldsymbol{0} \end{cases} \quad (9)$$

Where, $\boldsymbol{z}_i \in R^{2m_i + W_i}$ are the algebraic variables (Dai, 1989) of the DAE sub-system; $\overline{\boldsymbol{v}}_i \in R^{2m_i}$ are the interconnection inputs.

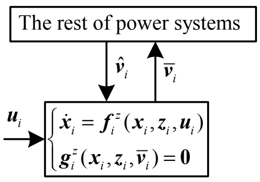

Fig. 2. The component structural model

In the paper, the component model of (9) is named "component structural model". This model is suitable for various components in power systems. For example, the structural model of TCSC is:

$$\dot{\alpha}_{TCSC} = \left(u - \alpha_{TCSC}\right)/T \quad (10)$$

$$\begin{cases} P_1 = -I_1^2 f_{TCSC}\left(\alpha_{TCSC}\right)\cos\theta_1 \cos\theta_2 / \sin\left(\theta_2 - \theta_1\right) \\ I_2 = I_1 \\ \theta_{I2} = \theta_{I1} - 180^0 \\ P_2 = -P_1 \end{cases} \quad (11)$$

Where: α_{TCSC} is the firing angle; X_{TCSC} is the equivalent reactance; $f_{TCSC}\left(\alpha_{TCSC}\right)$ is the complex relation between α_{TCSC} and X_{TCSC} . Please refer to (Jalali, *et al*, 1996) for the explanation of other variables.

For the model of (9), when discussing the control problem, the output equations should also be defined. The general expression of the output equation is:

$$\boldsymbol{y}_i = \boldsymbol{h}_i^z(\boldsymbol{x}_i, \boldsymbol{z}_i, \overline{\boldsymbol{v}}_i) \quad (12)$$

Finally, it should be noted that compared with the model of (1), one most important characteristic of the model of (9) is that the interconnections of (9), or $\overline{\boldsymbol{v}}_i$, are measurable. This characteristic will be very helpful for designing decentralized controller of component.

3. THE INDEX OF COMPONENT STRUCUTRAL MODEL

"Index" is an important and basic concept in the theory of DAE system. For a DAE system, index is the minimum derivative times of the algebraic equations that need to get the differential equation of the algebraic variables (Dai, 1989). This concept is also suitable for the DAE sub-system as shown in (9).

Derivate the algebraic equations in (9) to time t, one can get

$$(\partial \boldsymbol{g}_i^z / \partial \overline{\boldsymbol{v}}_i)\dot{\overline{\boldsymbol{v}}} + (\partial \boldsymbol{g}_i^z / \partial \boldsymbol{z}_i)\dot{\boldsymbol{z}}_i + (\partial \boldsymbol{g}_i^z / \partial \boldsymbol{x}_i)\dot{\boldsymbol{x}}_i = \boldsymbol{0} \quad (13)$$

If the condition of

$$rank(\partial \boldsymbol{g}_i^z(\boldsymbol{x}_i, \boldsymbol{z}_i, \overline{\boldsymbol{v}}_i)/\partial \boldsymbol{z}_i) = 2m_i + W_i \quad (14)$$

could be satisfied, one can get the differential equations of $\boldsymbol{z}_i$:

$$\begin{aligned} \dot{\boldsymbol{z}}_i = &-(\partial \boldsymbol{g}_i^z / \partial \boldsymbol{z}_i)^{-1}(\partial \boldsymbol{g}_i^z / \partial \boldsymbol{x}_i)\boldsymbol{f}_i^z(\boldsymbol{x}_i, \boldsymbol{z}_i, \boldsymbol{u}_i) \\ &-(\partial \boldsymbol{g}_i^z / \partial \boldsymbol{z}_i)^{-1}(\partial \boldsymbol{g}_i^z / \partial \overline{\boldsymbol{v}}_i)\dot{\overline{\boldsymbol{v}}}_i \end{aligned} \quad (15)$$

Then, the index of (9) is 1, or (15) is the index 1 condition of (9). Index 1 is a very good characteristic for DAE systems. Compared with the DAE systems with higher index, it is simpler to analyze the control problem of the system with index 1.

Fortunately, for power systems, the component structural model of (9) is just a DAE sub-system with index 1. The proof is in the following.

Firstly, the interface equations in (9) could be decomposed into two parts:

$$\begin{cases} \boldsymbol{g}_i^{z_1}(\boldsymbol{x}_i, \boldsymbol{w}_i, \hat{\boldsymbol{v}}_i, \overline{\boldsymbol{v}}_i) = \boldsymbol{w}_i - \boldsymbol{h}_i(\boldsymbol{x}_i, \hat{\boldsymbol{v}}_i, \overline{\boldsymbol{v}}_i) = \boldsymbol{0} \\ \boldsymbol{g}_i^{z_2}(\boldsymbol{x}_i, \hat{\boldsymbol{v}}_i, \overline{\boldsymbol{v}}_i) = \boldsymbol{g}_i(\boldsymbol{x}_i, \hat{\boldsymbol{v}}_i, \overline{\boldsymbol{v}}_i) = \boldsymbol{0} \end{cases} \quad (16)$$

For the first equation of (16), there is $rank(\partial \boldsymbol{g}_i^{z_1}(\boldsymbol{x}_i, \boldsymbol{w}_i, \hat{\boldsymbol{v}}_i, \overline{\boldsymbol{v}}_i)/\partial \boldsymbol{w}_i) = W_i$. For the last equation of (13), there is no $\boldsymbol{w}_i$. Thus, we should only prove the following proposition.

Proposition: For arbitrary interface variables $\boldsymbol{v}_i$ satisfying (3), and the corresponding interface equations in (9), in a certain neighbourhood, there must be a decomposition $\boldsymbol{v}_i = (\overline{\boldsymbol{v}}_i, \hat{\boldsymbol{v}}_i)^T$ (where: $\hat{\boldsymbol{v}}_i \in R^{2m_i}$, $\overline{\boldsymbol{v}}_i \in R^{2m_i}$) to ensure the existence of $rank(\partial \boldsymbol{g}_i(\boldsymbol{x}_i, \hat{\boldsymbol{v}}_i, \overline{\boldsymbol{v}}_i)/\partial \hat{\boldsymbol{v}}_i) = 2m_i$.

Prove:

Use the reduction to absurdity.

Assuming there is no decomposition $\boldsymbol{v}_i = (\bar{\boldsymbol{v}}_i, \hat{\boldsymbol{v}}_i)^T$ to ensure the existence of $rank(\partial \boldsymbol{g}_i(\boldsymbol{x}_i, \hat{\boldsymbol{v}}_i, \bar{\boldsymbol{v}}_i)/\partial \hat{\boldsymbol{v}}_i) = 2m_i$.

Substituting $(\boldsymbol{I}_{Fi}, \boldsymbol{U}_{Fi})^T = \boldsymbol{\Phi}^{-1}(\boldsymbol{v}_i)$ into $\boldsymbol{I}_{Fi} = \boldsymbol{f}_{Fi}(\boldsymbol{U}_{Fi}, \boldsymbol{x}_i)$ (see (1)), one can get the following interface equations:

$$\begin{aligned} &\boldsymbol{I}_{Fi} - \boldsymbol{f}_{Fi}(\boldsymbol{x}_i, \boldsymbol{U}_{Fi}) = \boldsymbol{F}_{Fi}(\boldsymbol{x}_i, \boldsymbol{\Phi}^{-1}(\boldsymbol{v}_i)) \\ &= \boldsymbol{g}_i(\boldsymbol{x}_i, \boldsymbol{v}_i) = \boldsymbol{g}_i(\boldsymbol{x}_i, \hat{\boldsymbol{v}}_i, \bar{\boldsymbol{v}}_i) = \boldsymbol{0} \end{aligned} \quad (17)$$

where: $\boldsymbol{g}_i(\cdot): R^{4m_i \times X_i} \to R^{2m_i}$, $\boldsymbol{g}_i(\cdot) = (g_{i1}(\cdot), \cdots, g_{i2m_i}(\cdot))^T$.

For it is arbitrary when decomposing the interface variables, according above assumption, there is

$rank(\partial \boldsymbol{g}_i(\boldsymbol{x}_i, \hat{\boldsymbol{v}}_i, \bar{\boldsymbol{v}}_i)/\partial \hat{\boldsymbol{v}}_i) = rank(\partial \boldsymbol{g}_i(\boldsymbol{x}_i, \hat{\boldsymbol{v}}_i, \bar{\boldsymbol{v}}_i)/\partial \boldsymbol{v}_i) < 2m_i$ (18).
Then, one can define

$$rank(\partial \boldsymbol{g}_i(\boldsymbol{x}_i, \hat{\boldsymbol{v}}_i, \bar{\boldsymbol{v}}_i)/\partial \boldsymbol{v}_i) = k_i < 2m_i \quad (19)$$

In the definition of interface equations, there is no special requirement for the order of the equations and variables, and thus one can assume $rank(\partial(g_{i1}, \cdots, g_{ik_i})/\partial(v_{i1}, \cdots, v_{ik_i})^T) = k_i$. Thus, for the first k_i equations of $\boldsymbol{g}_i(\boldsymbol{x}_i, \boldsymbol{v}_i) = \boldsymbol{0}$, according to the Implicit Function Theorem, in theory there are

$$\begin{cases} v_{i1} = \hat{g}_{i1}(\boldsymbol{x}_i, v_{i(k_i+1)}, \cdots, v_{i4m_i}) \\ \cdots \\ v_{ik_i} = \hat{g}_{ik_i}(\boldsymbol{x}_i, v_{i(k_i+1)}, \cdots, v_{i4m_i}) \end{cases} \quad (20)$$

Substituting (20) into the rest $2m_i - k_i$ equations of $\boldsymbol{g}_i(\boldsymbol{x}_i, \boldsymbol{v}_i) = \boldsymbol{0}$, $v_{i1}, \cdots, v_{ik_i}$ in $\boldsymbol{g}_i(\boldsymbol{x}_i, \boldsymbol{v}_i) = \boldsymbol{0}$ could be removed, and then one can get:

$$\begin{cases} \hat{g}_{i(k_i+1)}(\boldsymbol{x}_i, v_{i(k_i+1)}, \cdots, v_{i4m_i}) = 0 \\ \cdots \\ \hat{g}_{i2m_i}(\boldsymbol{x}_i, v_{i(k_i+1)}, \cdots, v_{i4m_i}) = 0 \end{cases} \quad (21)$$

Considering (19), one can see that there would be no variables $v_{i(k_i+1)}, \cdots, v_{i4m_i}$ in (21). Or, (21) are with the following expressions:

$$\begin{cases} \hat{g}_{i(k_i+1)}(\boldsymbol{x}_i) = 0 \\ \cdots \\ \hat{g}_{i2m_i}(\boldsymbol{x}_i) = 0 \end{cases} \quad (22)$$

Substituting $\boldsymbol{v}_i = \boldsymbol{\Phi}(\boldsymbol{I}_{Fi}, \boldsymbol{U}_{Fi})$ into the first k_i equations and the last $2m_i - k_i$ equations of $\boldsymbol{g}_i(\boldsymbol{x}_i, \boldsymbol{v}_i) = \boldsymbol{0}$, one can get

$$\begin{cases} g_{i1}(\boldsymbol{x}_i, \Phi_{i1}(\boldsymbol{I}_{Fi}, \boldsymbol{U}_{Fi}), \cdots, \Phi_{i4m_i}(\boldsymbol{I}_{Fi}, \boldsymbol{U}_{Fi})) = 0 \\ \cdots \\ g_{ik_i}(\boldsymbol{x}_i, \Phi_{i1}(\boldsymbol{I}_{Fi}, \boldsymbol{U}_{Fi}), \cdots, \Phi_{i4m_i}(\boldsymbol{I}_{Fi}, \boldsymbol{U}_{Fi})) = 0 \end{cases} \quad (23)$$

$$\begin{cases} g_{i(k_i+1)}(\boldsymbol{x}_i, \Phi_{i1}(\boldsymbol{I}_{Fi}, \boldsymbol{U}_{Fi}), \cdots, \Phi_{i4m_i}(\boldsymbol{I}_{Fi}, \boldsymbol{U}_{Fi})) = 0 \\ \cdots \\ g_{i2m_i}(\boldsymbol{x}_i, \Phi_{i1}(\boldsymbol{I}_{Fi}, \boldsymbol{U}_{Fi}), \cdots, \Phi_{i4m_i}(\boldsymbol{I}_{Fi}, \boldsymbol{U}_{Fi})) = 0 \end{cases} \quad (24)$$

According to the definition, one can see that equations (23) and (24) are just the current injection equations.

From above derivation of from (19) to (22), one can see that $\Phi_{i1}(\boldsymbol{I}_{Fi}, \boldsymbol{U}_{Fi}), \cdots, \Phi_{ik_i}(\boldsymbol{I}_{Fi}, \boldsymbol{U}_{Fi})$ (or $v_{i1}, \cdots, v_{ik_i}$) can also be solved based on (23). Substituting the result into (24), $\Phi_{i1}(\boldsymbol{I}_{Fi}, \boldsymbol{U}_{Fi}), \cdots, \Phi_{i4m_i}(\boldsymbol{I}_{Fi}, \boldsymbol{U}_{Fi})$ (or $v_{i1}, \cdots, v_{i4m_i}$) in (24) can also be omitted. Or $\boldsymbol{I}_{Fi}, \boldsymbol{U}_{Fi}$ in (24) can be omitted. Then, equation (24) could be converted to (22).

Yet we know, $I_{Fi1}, \cdots, I_{Fi2m_i}$ must exist in every equation of current injection equations, and thus it is impossible to omit $\boldsymbol{I}_{Fi}, \boldsymbol{U}_{Fi}$ in (24) based on (23).

Then, the assumption does not hold, or the original proposition holds.

□

Based on (14) and (16), according to the Implicit Function Theorem, there is:

$$\begin{cases} \boldsymbol{w}_i = \boldsymbol{h}_i(\boldsymbol{x}_i, \hat{\boldsymbol{v}}_i, \bar{\boldsymbol{v}}_i) \\ \hat{\boldsymbol{v}}_i = \hat{\boldsymbol{g}}_i(\boldsymbol{x}_i, \bar{\boldsymbol{v}}_i) \end{cases} \quad (25)$$

Furthermore, substituting the second equation of (25) into the first equation of (25), and considering the definition of $\boldsymbol{z}_i = (\boldsymbol{w}_i, \hat{\boldsymbol{v}}_i)^T$, there is

$$\boldsymbol{z}_i = \boldsymbol{p}_i(\boldsymbol{x}_i, \bar{\boldsymbol{v}}_i) \quad (26)$$

4. TRANSFORMATION OF DAE SUB-SYSTEM TO ODE SUB-SYSTEM

For the control problem of DAE system, one general approach is transforming the DAE system to traditional ODE system. As discussed above, the index of the component structural model, or (9), is 1, and for DAE system, the characteristic of index 1 is very helpful to transform the DAE system to ODE system. In this section, the problem of transforming the nonlinear DAE sub-system as shown in (9) to nonlinear ODE sub-system will be discussed.

Substituting (26) into (9) and (12), there is

$$\begin{cases} \dot{\boldsymbol{x}}_i = \boldsymbol{f}_i^p(\boldsymbol{x}_i, \boldsymbol{p}_i(\boldsymbol{x}_i, \bar{\boldsymbol{v}}_i), \boldsymbol{u}_i) = \boldsymbol{f}_i^{\bar{v}}(\boldsymbol{x}_i, \bar{\boldsymbol{v}}_i, \boldsymbol{u}_i) \\ \boldsymbol{y}_i = \boldsymbol{h}_i^p(\boldsymbol{x}_i, \boldsymbol{p}_i(\boldsymbol{x}_i, \bar{\boldsymbol{v}}_i), \bar{\boldsymbol{v}}_i) = \boldsymbol{h}_i^{\bar{v}}(\boldsymbol{x}_i, \bar{\boldsymbol{v}}_i) \end{cases} \quad (27)$$

Apparently, equation (27) is a standard nonlinear ODE sub-system model.

In (27), the state variables $\boldsymbol{x}_i$ are independent with each other, and thus the ODE sub-system as shown in (27) is the "minimum" state space realization of the original DAE sub-system.

It should be noted that, only when the analytic expressions of $\boldsymbol{p}_i(\boldsymbol{x}_i,\overline{\boldsymbol{v}}_i)$ in (26) exist, can one get the analytic expression of (27). Fortunately, for most kinds of components in power systems, we can all get the analytical expressions of (26).

However, in some special circumstances, the interface equations $\boldsymbol{g}_i^z(\boldsymbol{x}_i,\boldsymbol{z}_i,\overline{\boldsymbol{v}}_i)=\boldsymbol{0}$ may be very complex, and thus there may be very difficult or impossible to acquire the analytic expressions of $\boldsymbol{p}_i(\boldsymbol{x}_i,\overline{\boldsymbol{v}}_i)$. For example, when the generator adopts the 3rd-order one-axis model without ignoring the transient saliency, it is very difficult to acquire the analytic expression of $\boldsymbol{p}_i(\boldsymbol{x}_i,\overline{\boldsymbol{v}}_i)$. In this case, based on the characteristic of index 1, one can also get the differential equation of the algebraic variables $\boldsymbol{z}_i$, and then the nonlinear DAE sub-system as shown in (9) and (12) could be transformed to

$$\begin{cases}\dot{\boldsymbol{x}}_i=\boldsymbol{f}_i^z(\boldsymbol{x}_i,\boldsymbol{z}_i,\boldsymbol{u}_i)\\ \dot{\boldsymbol{z}}_i=-(\partial\boldsymbol{g}_i^z/\partial\boldsymbol{z}_i)^{-1}(\partial\boldsymbol{g}_i^z/\partial\boldsymbol{x}_i)\boldsymbol{f}_i^z(\boldsymbol{x}_i,\boldsymbol{z}_i,\boldsymbol{u}_i)\\ \quad-(\partial\boldsymbol{g}_i^z/\partial\boldsymbol{z}_i)^{-1}(\partial\boldsymbol{g}_i^z/\partial\overline{\boldsymbol{v}}_i)\dot{\overline{\boldsymbol{v}}}_i\\ \boldsymbol{g}_i^z(\boldsymbol{x}_i,\boldsymbol{z}_i,\overline{\boldsymbol{v}}_i)=\boldsymbol{0}\\ \boldsymbol{y}_i=\boldsymbol{h}_i^z(\boldsymbol{x}_i,\boldsymbol{z}_i,\overline{\boldsymbol{v}}_i)\end{cases} \tag{28}$$

In (28), the state variables have been expanded to $(\boldsymbol{x}_i,\boldsymbol{z}_i)^T$, and $\boldsymbol{x}_i$ and $\boldsymbol{z}_i$ are constrained with each other by the algebraic equation $\boldsymbol{g}_i^z(\boldsymbol{x}_i,\boldsymbol{z}_i,\overline{\boldsymbol{v}}_i)=\boldsymbol{0}$. Thus, the sub-system as shown in (28) is a constrained nonlinear ODE sub-system, or it is not the minimum state space realization of the original nonlinear DAE sub-system. Then, how to get the minimum state space realization of (28) is a question. The detailed discussion about this issue will be given in our next paper, and in the paper, the generator set will be chosen as an example to demonstrate it in Section 6.

5. DESIGN OF DECENTRALIZED NONLINEAR CONTROLLER OF COMPONENTS

For the nonlinear ODE sub-system models with measurable interconnections as shown in (27) or (28), traditional nonlinear control methods (e.g. differential geometric theory, direct feedback linearization (DFL) method, inversion control method, etc.) which are suitable for nonlinear ODE systems could be developed and expanded to be suitable for designing controllers. Detailed discussion about this issue will be given in our next paper. The general diagram is shown in Fig.3.

In Fig.3, there are three kinds of feedback, or state feedback, output feedback and interface feedback, in which interface feedback is not a traditional feedback kind. Here, one important reason that makes the interface feedback possible is that the interconnections (interface variables) of the component structural model are local measurable. By feed-backing interface variables, the component controller can "apperceive" the interconnections between the component and the rest of systems.

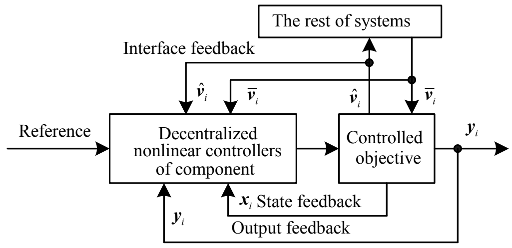

Fig. 3. Diagram of component nonlinear decentralized controller

6. EXAMPLES

In this section, synchronous generator set will be chosen as an example to illustrate the method of transforming DAE sub-system to ODE sub-system discussed above.

Example 1. In this example, the 3rd-order one-axis generator model without ignoring the transient saliency is considered to illustrate how to get the constrained nonlinear ODE sub-system with measurable interconnections.

The structural model of generator set is as shown in (6) and (7). The output equation could be defined as:

$$y_i=\omega_i \tag{29}$$

It could be proved that in the normal operating area of power systems, there is $\det(\partial g^z/\partial z)\neq 0$, or the index of the structural model of (6) and (7) is 1.

From the algebraic equations in (7), or $\boldsymbol{g}_i^z(\boldsymbol{x}_i,\boldsymbol{z}_i,\overline{\boldsymbol{v}}_i)=\boldsymbol{0}$, it is difficult to acquire the analytical expressions of $\boldsymbol{z}_i=(P_{ti},\theta_{Ui},I_{di},I_{qi})^T$ as shown in (26). Then, equations (6), (7) and (29) can be converted to the following constrained ODE sub-system:

$$\begin{cases}\dot{\delta}_i=\omega_i-\omega_0\\ \dot{\omega}_i=(\omega_0/H_i)\{P_{Hi}+C_{MLi}P_{mi0}-D_i(\omega_i-\omega_0)/\omega_0\\ \quad-[E_{qi}^{'}+(x_{qi}-x_{di}^{'})I_{di}]I_{qi}\}\\ \dot{P}_{Hi}=\dfrac{1}{T_{H\Sigma i}}(-P_{Hi}+C_{Hi}P_{mi0}+C_{Hi}U_{ci})\\ \dot{\boldsymbol{z}}_i=-(\partial\boldsymbol{g}_i^z/\partial\boldsymbol{z}_i)^{-1}(\partial\boldsymbol{g}_i^z/\partial\boldsymbol{x}_i)\boldsymbol{f}_i^z(\boldsymbol{x}_i,\boldsymbol{z}_i,\boldsymbol{u}_i)\\ \quad-(\partial\boldsymbol{g}_i^z/\partial\boldsymbol{z}_i)^{-1}(\partial\boldsymbol{g}_i^z/\partial\overline{\boldsymbol{v}}_i)\dot{\overline{\boldsymbol{v}}}_i\\ \boldsymbol{g}_i^z(\boldsymbol{x}_i,\boldsymbol{z}_i,\overline{\boldsymbol{v}}_i)=\boldsymbol{0}\\ y_i=\omega_i\end{cases} \tag{30}$$

In order to acquire the minimum state space realization, the following coordinate transform could be chosen.

$$\begin{cases} \xi_i^1 = T_i^1(\cdot) = \delta_i \\ \xi_i^2 = T_i^2(\cdot) = \omega_i - \omega_0 \\ \xi_i^3 = T_i^3(\cdot) = (\omega_0 / H_i)\{P_{Hi} + C_{MLi}P_{mi0} - \\ \quad (D_i/\omega_0)(\omega_i - \omega_0) - [E'_{qi} + (x_{qi} - x'_{di})I_{di}]I_{qi}\} \\ \chi_i = g_i^z(\cdot) \end{cases} \quad (31)$$

One can get

$$\begin{cases} \dot{\xi}_i^1 = \xi_i^2 \\ \dot{\xi}_i^2 = \xi_i^3 \\ \dot{\xi}_i^3 = (\partial T_{i3}/\partial P_{Hi})[(-P_{Hi} + C_{Hi}P_{mi0} + C_{Hi}U_{ci})/T_{H\Sigma i}] \\ \quad + (\partial T_{i3}/\partial \omega_i)\dot{\omega}_i + (\partial T_{i3}/\partial I_{di})\dot{I}_{di} + (\partial T_{i3}/\partial I_{qi})\dot{I}_{qi} \\ y_i = \xi_i^2 + \omega_0 \end{cases} \quad (32)$$

Then, substituting the expressions of $\dot{\omega}_i, \dot{I}_{di}, \dot{I}_{qi}$ (the expressions of $\dot{I}_{di}, \dot{I}_{qi}$ can be obtained in (30)), and substituting the new coordinate (ξ_i, χ_i) (where $\xi_i = (\xi_i^1, \xi_i^2, \xi_i^3)$) for the original coordinate (x_i, z_i), one can get the minimum realization of the generator set as following

$$\begin{cases} \dot{\xi}_i^1 = \xi_i^2 \\ \dot{\xi}_i^2 = \xi_i^3 \\ \dot{\xi}_i^3 = F_{i3}(\xi_i, U_{ci}, \overline{v}_i, \dot{\overline{v}}_i) \\ y_i = \xi_i^2 + \omega_0 \end{cases} \quad (33)$$

Example 2. In this example, the classical 3rd-order generator model is considered to illustrate how to get the nonlinear ODE sub-system with measurable interconnections.

Compared with the model of (6), (7) and (29), for the structural model of generator set adopting the classical 3rd-order generator model, there is only one difference, or $x_{qi} = x'_{di}$. Thus, one can directly get the nonlinear ODE sub-system with measurable interconnections as shown in (27).

$$\begin{cases} \dot{\delta}_i = \omega_i - \omega_0 \\ \dot{\omega}_i = (\omega_0 / H_i)\{P_{Hi} + C_{MLi}P_{mi0} - (D_i/\omega_0)(\omega_i - \omega_0) \\ \quad -[E'_{qi} + (x_{qi} - x'_{di})(Q_{ti} + x'_{di}I_{ti}^2)/E'_{qi}] \\ \quad \cdot[I_{ti}^2 - ((Q_{ti} + x'_{di}I_{ti}^2)/E'_{qi})^2]^{0.5}\} \\ \dot{P}_{Hi} = (-P_{Hi} + C_{Hi}P_{mi0} + C_{Hi}U_{ci})/T_{H\Sigma i} \\ y_i = \omega_i \end{cases} \quad (34)$$

7. CONCLUSIONS

This paper discussed the decentralized nonlinear control problem of components in power systems based on nonlinear DAE sub-system models. For traditional nonlinear DAE sub-system models (first principle models) of components are not suitable for designing decentralized nonlinear controllers, a new kind of nonlinear DAE sub-system model, or component structural model, is constructed, in which the interconnection is local measurable. Furthermore, based on its characteristic of index 1, this nonlinear DAE sub-system model is transformed to nonlinear ODE sub-system models with measurable interconnections, which is very helpful for designing decentralized nonlinear controllers of components.

Future works would be concerned with:

- For the constrained nonlinear ODE sub-system as shown in (28), it is need to research the general method to get its minimum state space realization.
- For the nonlinear ODE sub-system models with measurable interconnections as shown in (27) or (28), it is need to develop and expand traditional nonlinear control methods to design decentralized nonlinear controllers of components.

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REFERENCES

Akhrif, O., F.A. Okou, L.A. Dessaint, et al (1999). Application of a Multivariable Feedback Linearization Scheme for Rotor Angle Stability and Voltage Regulation of Power System. *IEEE Trans. Power Systems*, **vol. 14**, 620-628.

Contou-Carrere, M.N., P. Daoutidis (2005). *An Output Feedback Precompensator for Nonlinear DAE Systems with Control-Dependent State-Space*. IEEE Trans. Automatic Control, **vol. 50**, 1831-1835.

Dai, L. (1989). *Singular Control Systems*. Springer-Verlag, Berlin.

Dai, X., K. Zhang, T. Zhang, et al (2004). ANN Generalized Inversion Control of Turbo-Generator Governor. *IEE Proc. -Generation, Transmission and Distribution*, **vol. 151**, 327-333.

Hill, D.J., I. Mareels (1990). *Stability Theory for Differential/Algebraic Systems with Application to Power Systems*. IEEE Trans. Circuits and Systems, **vol. 37**, 1416-1423.

Jalali, S.G., R.A. Hedin, M. Pereira, et al (1996). "*A stability model for the advanced series compensator (ASC)* . IEEE Trans. Power Delivery, **vol. 11**, 1128-1137.

Kundur, P (1994). *Power system stability and control*. McGraw Hill, New York.

Wasysczuk, O., R.A. Decarlo (1981). *The Component Connection Model and Structure Preserving Model Order Reduction*. Automatica, v**ol. 17**, 619-626.

Zhang, K.F., X.Z. Dai, H. Qi, et al (2007). *Analysis and Application of Component Structural Model of Complex Power Systems*. Proceedings of the CSEE, **vol. 27**, 24-28 (in Chinese).