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Fuzzy Model-based Model Following Control for a Class of Nonlinear Systems

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Abstract: This paper presents nonlinear model following control for a class of nonlinear systems using the fuzzy model-based control approach. We propose the construction method of augmented fuzzy control system for continuous-time nonlinear systems by differentiating the original nonlinear system. Moreover, we introduce the dynamic fuzzy controller which can make outputs of the nonlinear system converge to outputs of the reference nonlinear system, and derive the controller design conditions in terms of LMIs. A design example illustrates the utility of this approach.

1. INTRODUCTION

Recently, fuzzy model-based control has been discussed in a huge number of literatures [2]–[5]. Most of them deal with Takagi-Sugeno (T-S) fuzzy model [1] and LMI-based designs[11]. By employing the T-S fuzzy model, which utilizes local linear system description for each rule, we can devise a control methodology to fully take advantages of linear control theory. However, most of literatures have mainly dealt with the regulation problem to discuss stability or convergence to the origin. Unfortunately, theoretical controllability of model following control for nonlinear systems was not discussed in the literature. In [6], nonlinear model following control based on cancellation technique is discussed. The control approach is powerful. However, it is difficult to apply the control approach to systems which the cancellation technique cannot work well.

In this paper, we deal with model following control for a class of nonlinear systems using the fuzzy model-based control approach. We propose the construction method of augmented fuzzy control system for continuous-time nonlinear systems by differentiating the original nonlinear system. Moreover, we introduce the dynamic fuzzy controller which can make outputs of the nonlinear systems converge to the outputs of the reference nonlinear system, and derive the controller design conditions in terms of LMIs. A design example illustrates the utility of this approach.

2. PRELIMINARY RESULTS

In this section, we explain the basic procedures of fuzzy model-based control approach for nonlinear systems and model following control for linear systems.

2.1 Fuzzy Model-based Control [6]

Consider the following continuous-time nonlinear system.

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}_1(\boldsymbol{x}(t)) + \boldsymbol{f}_2(\boldsymbol{x}(t))\boldsymbol{u}(t) \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{g}(\boldsymbol{x}(t)) \tag{2}$$

where $\boldsymbol{x}(t) = [x_1(t)\ x_2(t)\ \cdots\ x_n(t)]^T$ is the state vector, $\boldsymbol{u}(t) = [u_1(t)\ u_2(t)\ \cdots\ u_m(t)]^T$ is the input vector, $\boldsymbol{y}(t) = [y_1(t)\ y_2(t)\cdots\ y_q(t)]$ is the output vector. For the above nonlinear system, by applying sector nonlinearity concept [6], we can obtain the following T-S fuzzy model.

$$\text{Rule } i: \text{ IF } z_1(t) \text{ is } M_{i1} \text{ and } \cdots \text{and } z_p(t) \text{ is } M_{ip}$$
$$\text{THEN} \begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{u}(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}_i\boldsymbol{x}(t) \end{cases} \tag{3}$$

where, $i = 1, 2, \cdots, r$ and r is the number of fuzzy model rules. M_{ij} is the fuzzy set. $z_j(t)$ is the known premise variable. The fuzzy reasoning process is defined as

$$\dot{\boldsymbol{x}}(t) = \frac{\sum_{i=1}^{r} w_i(\boldsymbol{z}(t))\left(\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{u}(t)\right)}{\sum_{i=1}^{r} w_i(\boldsymbol{z}(t))} = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))\left(\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{u}(t)\right) \tag{4}$$

$$\boldsymbol{y}(t) = \frac{\sum_{i=1}^{r} w_i(\boldsymbol{z}(t))\boldsymbol{C}_i\boldsymbol{x}(t)}{\sum_{i=1}^{r} w_i(\boldsymbol{z}(t))} = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))\boldsymbol{C}_i\boldsymbol{x}(t) \tag{5}$$

where

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$$\boldsymbol{z}(t) = [z_1(t)\; z_2(t)\; \cdots\; z_p(t)]$$
$$w_i(\boldsymbol{z}(t)) = \prod_{j=1}^{P} M_{ij}(z_j(t)),\; h_i(\boldsymbol{z}(t)) = \frac{w_i(\boldsymbol{z}(t))}{\sum_{i=1}^{r} w_i(\boldsymbol{z}(t))}$$

$M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}. $w_i(\boldsymbol{z}(t))$ and $h_i(\boldsymbol{z}(t))$ have the following properties.

$$\sum_{i=1}^{r} w_i(\boldsymbol{z}(t)) > 0,\;\; w_i(\boldsymbol{z}(t)) \geq 0,\; \forall i$$
$$\sum_{i=1}^{r} h_i(\boldsymbol{z}(t)) = 1,\;\; h_i(\boldsymbol{z}(t)) \geq 0,\; \forall i$$

To stabilize the T-S fuzzy model (4), we employ the so-called parallel distributed compensation (PDC) control approach [2, 3]. The PDC fuzzy controller is represented as

$$\boldsymbol{u}(t) = -\sum_{i=1}^{r} h(\boldsymbol{z}(t))\boldsymbol{K}_i\boldsymbol{x}(t) \tag{6}$$

where $\boldsymbol{K}_i$ is a feedback gain. The PDC fuzzy controller design is to determine the feedback gains $\boldsymbol{K}_i$. By substituting (6) into (4), the overall fuzzy control system is represented as follows:

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(t))h_j(\boldsymbol{z}(t))\left(\boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_j\right)\boldsymbol{x}(t) \tag{7}$$

The feedback gain $\boldsymbol{K}_i$ is determined by solving Theorem 1.

Theorem 1. [6] If there exist positive definite matrix $\boldsymbol{X}$ and $\boldsymbol{M}_i$ satisfying (8), (9) and (10), then the fuzzy model (4) can be stabilized by the fuzzy controller (6).

$$\boldsymbol{X} > \boldsymbol{0}, \tag{8}$$
$$\boldsymbol{A}_i\boldsymbol{X} + \boldsymbol{X}\boldsymbol{A}_i^T - \boldsymbol{B}_i\boldsymbol{M}_i - \boldsymbol{M}_i^T\boldsymbol{B}_i^T < \boldsymbol{0},\;\; \forall i, \tag{9}$$
$$\boldsymbol{A}_i\boldsymbol{X} + \boldsymbol{X}\boldsymbol{A}_i^T + \boldsymbol{A}_j\boldsymbol{X} + \boldsymbol{X}\boldsymbol{A}_j^T - \boldsymbol{B}_i\boldsymbol{M}_j - \boldsymbol{M}_j^T\boldsymbol{B}_i^T - \boldsymbol{B}_j\boldsymbol{M}_i - \boldsymbol{M}_i^T\boldsymbol{B}_j^T < \boldsymbol{0},\quad \forall i, i < j, \tag{10}$$

where $\boldsymbol{K}_i = \boldsymbol{M}_i\boldsymbol{X}^{-1}$.

2.2 Model Following Control for Linear Systems

In this section, we explain the model following control for continuous-time linear systems. Consider the following linear system.

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{11}$$
$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) \tag{12}$$

For the above linear system, we consider the model following control problem, that is, the control problem to make the output $\boldsymbol{y}(t)$ converge to the output of the following reference linear system.

$$\dot{\boldsymbol{x}}_r(t) = \boldsymbol{A}_r\boldsymbol{x}_r(t) + \boldsymbol{B}_r\boldsymbol{r} \tag{13}$$
$$\boldsymbol{y}_r(t) = \boldsymbol{C}_r\boldsymbol{x}_r(t) \tag{14}$$

where $\boldsymbol{r}$ is the constant vector. We assume that $\boldsymbol{x}(t)$ and $\boldsymbol{x}_r(t)$ are measurable.

Firstly, we define the error vector $\boldsymbol{e}(t)$ and its time derivative as follows:

$$\boldsymbol{e}(t) = \boldsymbol{y}(t) - \boldsymbol{y}_r(t) \tag{15}$$
$$\frac{d}{dt}\boldsymbol{e}(t) = \dot{\boldsymbol{y}}(t) - \dot{\boldsymbol{y}}_r(t) = \boldsymbol{C}\dot{\boldsymbol{x}}(t) - \boldsymbol{C}_r\dot{\boldsymbol{x}}_r(t) \tag{16}$$

Then, by differentiating the linear system (11) and the reference system (13) with respect to time t, we can obtain the following equations.

$$\frac{d}{dt}\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\dot{\boldsymbol{x}}(t) + \boldsymbol{B}\dot{\boldsymbol{u}}(t) \tag{17}$$
$$\frac{d}{dt}\dot{\boldsymbol{x}}_r(t) = \boldsymbol{A}_r\dot{\boldsymbol{x}}_r(t) \tag{18}$$

Next, we construct the following augmented system by adding (16), (17) and (18).

$$\frac{d}{dt}\begin{bmatrix}\dot{\boldsymbol{x}}(t)\\ \dot{\boldsymbol{x}}_r(t)\\ \boldsymbol{e}(t)\end{bmatrix} = \begin{bmatrix}\boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{A}_r & \boldsymbol{0}\\ \boldsymbol{C} & -\boldsymbol{C}_r & \boldsymbol{0}\end{bmatrix}\begin{bmatrix}\dot{\boldsymbol{x}}(t)\\ \dot{\boldsymbol{x}}_r(t)\\ \boldsymbol{e}(t)\end{bmatrix} + \begin{bmatrix}\boldsymbol{B}\\ \boldsymbol{0}\\ \boldsymbol{0}\end{bmatrix}\dot{\boldsymbol{u}}(t)$$
$$= \hat{\boldsymbol{A}}\begin{bmatrix}\dot{\boldsymbol{x}}(t)\\ \dot{\boldsymbol{x}}_r(t)\\ \boldsymbol{e}(t)\end{bmatrix} + \hat{\boldsymbol{B}}\dot{\boldsymbol{u}}(t) \tag{19}$$

Finally, we design the following dynamic controller to stabilize the augmented system (19).

$$\dot{\boldsymbol{u}}(t) = -\boldsymbol{K}\begin{bmatrix}\dot{\boldsymbol{x}}(t)\\ \dot{\boldsymbol{x}}_r(t)\\ \boldsymbol{e}(t)\end{bmatrix} \tag{20}$$

where $\boldsymbol{K}$ is a feedback gain. The controller design is to determine the feedback gain $\boldsymbol{K}$. By substituting (20) into (19), we can obtain the following linear control system.

$$\frac{d}{dt}\begin{bmatrix}\dot{\boldsymbol{x}}(t)\\ \dot{\boldsymbol{x}}_r(t)\\ \boldsymbol{e}(t)\end{bmatrix} = \left(\hat{\boldsymbol{A}} - \hat{\boldsymbol{B}}\boldsymbol{K}\right)\begin{bmatrix}\dot{\boldsymbol{x}}(t)\\ \dot{\boldsymbol{x}}_r(t)\\ \boldsymbol{e}(t)\end{bmatrix} \tag{21}$$

The feedback gain $\boldsymbol{K}$ is determined by solving Theorem 2.

Theorem 2. If there exist positive definite matrix $\boldsymbol{X}$ and $\boldsymbol{M}$ satisfying (22) and (23), then the augmented system (19) can be stabilized by the dynamic controller (20).

$$\boldsymbol{X} > \boldsymbol{0}, \tag{22}$$
$$\hat{\boldsymbol{A}}\boldsymbol{X} + \boldsymbol{X}\hat{\boldsymbol{A}}^T - \hat{\boldsymbol{B}}\boldsymbol{M} - \boldsymbol{M}^T\hat{\boldsymbol{B}}^T < \boldsymbol{0}, \tag{23}$$

where $\boldsymbol{K} = \boldsymbol{M}\boldsymbol{X}^{-1}$.

By using the designed controller, we can make the output $\boldsymbol{y}(t)$ of the linear system (11) converge to the output $\boldsymbol{y}_r(t)$ of the linear reference system (13).

3. FUZZY MODEL-BASED MODEL FOLLOWING CONTROL FOR NONLINEAR SYSTEMS

In this section, we propose fuzzy model based model following control. Consider the following continuous-time nonlinear system.

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t)) \tag{24}$$
$$\boldsymbol{y}(t) = \boldsymbol{g}(\boldsymbol{x}(t)) \tag{25}$$

For the above nonlinear system, we consider the control problem to make the output $\boldsymbol{y}(t)$ converge to the output of the following reference system.

$$\dot{\boldsymbol{x}}_r(t) = \boldsymbol{f}_r(\boldsymbol{x}_r(t)) \quad (26)$$

$$\boldsymbol{y}_r(t) = \boldsymbol{g}_r(\boldsymbol{x}_r(t)) \quad (27)$$

We assume that $\boldsymbol{f}$, $\boldsymbol{g}$, $\boldsymbol{f}_r$ and $\boldsymbol{g}_r$ are known. We define the error vector $\boldsymbol{e}(t)$ as follows:

$$\boldsymbol{e}(t) = \boldsymbol{y}(t) - \boldsymbol{y}_r(t) \quad (28)$$

3.1 Construction of Augmented Fuzzy System

Firstly, we construct the following time-derivative systems by differentiating the nonlinear system (24) and the reference system (26) with respect to time t.

$$\ddot{\boldsymbol{x}}(t) = \boldsymbol{F}(\boldsymbol{x}(t), \dot{\boldsymbol{x}}(t), \boldsymbol{u}(t), \dot{\boldsymbol{u}}(t)) \quad (29)$$

$$\ddot{\boldsymbol{x}}_r(t) = \boldsymbol{F}_r(\boldsymbol{x}_r(t), \dot{\boldsymbol{x}}_r(t)) \quad (30)$$

Then, by differentiating error vector (28) with respect to time t, we can obtain the following equation.

$$\begin{aligned} \dot{\boldsymbol{e}}(t) &= \dot{\boldsymbol{y}}(t) - \dot{\boldsymbol{y}}_r(t) \\ &= \boldsymbol{G}(\boldsymbol{x}(t), \dot{\boldsymbol{x}}(t)) - \boldsymbol{G}_r(\boldsymbol{x}_r(t), \dot{\boldsymbol{x}}_r(t)) \end{aligned} \quad (31)$$

Next, by adding (29), (30) and (31), the augmented system is constructed as follows:

$$\begin{bmatrix} \ddot{\boldsymbol{x}}(t) \\ \ddot{\boldsymbol{x}}_r(t) \\ \dot{\boldsymbol{e}}(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}(\boldsymbol{x}(t), \dot{\boldsymbol{x}}(t), \boldsymbol{u}(t), \dot{\boldsymbol{u}}(t)) \\ \boldsymbol{F}_r(\boldsymbol{x}_r(t), \dot{\boldsymbol{x}}_r(t)) \\ \boldsymbol{G}(\boldsymbol{x}(t), \dot{\boldsymbol{x}}(t)) - \boldsymbol{G}_r(\boldsymbol{x}_r(t), \dot{\boldsymbol{x}}_r(t)) \end{bmatrix} \quad (32)$$

By applying sector nonlinearity concept [6] to each nonlinear term in the augmented system (32), we can obtain the following augmented T-S fuzzy model.

$$\begin{aligned} \begin{bmatrix} \ddot{\boldsymbol{x}}(t) \\ \ddot{\boldsymbol{x}}_r(t) \\ \dot{\boldsymbol{e}}(t) \end{bmatrix} &= \sum_{i=1}^{r} h_i(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{x}_r(t)) \\ &\times \left(\begin{bmatrix} \boldsymbol{A}_i & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A}_{ri} & \boldsymbol{0} \\ \boldsymbol{C}_i & -\boldsymbol{C}_{ri} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{x}}_r(t) \\ \boldsymbol{e}(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_i \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix} \dot{\boldsymbol{u}}(t) \right) \end{aligned} \quad (33)$$

Remark 1. From the property of differentiation, note that total derivatives of (29), (30) and (31) are represented as the following forms.

$$\begin{aligned} \ddot{\boldsymbol{x}}(t) &= \frac{d}{dt}\dot{\boldsymbol{x}}(t) \\ &= \frac{\partial \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t))}{\partial \boldsymbol{x}(t)}\dot{\boldsymbol{x}}(t) + \frac{\partial \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t))}{\partial \boldsymbol{u}(t)}\dot{\boldsymbol{u}}(t) \\ \ddot{\boldsymbol{x}}_r(t) &= \frac{d}{dt}\dot{\boldsymbol{x}}_r(t) \\ &= \frac{\partial \boldsymbol{f}_r(\boldsymbol{x}_r(t)}{\partial \boldsymbol{x}_r(t)}\dot{\boldsymbol{x}}_r(t) \\ \dot{\boldsymbol{e}}(t) &= \frac{d}{dt}\boldsymbol{y}(t) - \frac{d}{dt}\boldsymbol{y}_r(t) \\ &= \frac{\partial \boldsymbol{g}(\boldsymbol{x}(t))}{\partial \boldsymbol{x}(t)}\dot{\boldsymbol{x}}(t) - \frac{\partial \boldsymbol{g}_r(\boldsymbol{x}_r(t))}{\partial \boldsymbol{x}_r(t)}\dot{\boldsymbol{x}}_r(t) \end{aligned}$$

This means that the time derivatives of the nonlinear system (24), the reference system (26) and the error vector (28) are linear with respect to $\dot{\boldsymbol{x}}(t)$, $\dot{\boldsymbol{x}}_r(t)$ and $\dot{\boldsymbol{u}}(t)$. Therefore, the augmented system (32) can be represented as the augmented T-S fuzzy model (33) which has linear consequent parts with respect to $\dot{\boldsymbol{x}}(t)$, $\dot{\boldsymbol{x}}_r(t)$ and $\dot{\boldsymbol{u}}(t)$.

3.2 Dynamic Fuzzy Controller Design

To stabilize the augmented T-S fuzzy model (33), we propose the following dynamic PDC controller.

$$\dot{\boldsymbol{u}}(t) = -\sum_{i=1}^{r} h_i(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{x}_r(t))\hat{\boldsymbol{K}}_i \begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{x}}_r(t) \\ \boldsymbol{e}(t) \end{bmatrix} \quad (34)$$

where $\hat{\boldsymbol{K}}_i$ is a feedback gain. By utilizing the dynamic controller, note that the membership function $h_i(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{x}_r(t))$ can be calculated although the membership function includes the control input $\boldsymbol{u}(t)$. Moreover, $\dot{\boldsymbol{x}}(t)$ and $\dot{\boldsymbol{x}}_r(t)$ can be calculated from Eqs. (24) and (26). By substituting the dynamic controller (34) into the augmented fuzzy model (33), we can obtain the following fuzzy control system.

$$\begin{aligned} \begin{bmatrix} \ddot{\boldsymbol{x}}(t) \\ \ddot{\boldsymbol{x}}_r(t) \\ \dot{\boldsymbol{e}}(t) \end{bmatrix} &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{x}_r(t))h_j(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{x}_r(t)) \\ &\times \left(\begin{bmatrix} \boldsymbol{A}_i & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A}_{ri} & \boldsymbol{0} \\ \boldsymbol{C}_i & -\boldsymbol{C}_{ri} & \boldsymbol{0} \end{bmatrix} - \begin{bmatrix} \boldsymbol{B}_i \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix} \hat{\boldsymbol{K}}_j \right) \begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{x}}_r(t) \\ \boldsymbol{e}(t) \end{bmatrix} \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{x}_r(t))h_j(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{x}_r(t)) \\ &\times \left(\hat{\boldsymbol{A}}_i - \hat{\boldsymbol{B}}_i\hat{\boldsymbol{K}}_j\right) \begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{x}}_r(t) \\ \boldsymbol{e}(t) \end{bmatrix} \end{aligned} \quad (35)$$

The feedback gain $\hat{\boldsymbol{K}}_i$ is determined by solving Theorem 3. Note that (36), (37) and (38) are represented in terms of LMIs. Hence we can effectively determine the feedback gains by computer software like MATLAB.

Theorem 3. If there exist positive definite matrix $\boldsymbol{X}$ and $\hat{\boldsymbol{M}}_i$ satisfying (36), (37) and (38), then the fuzzy system (33) can be stabilized by the dynamic fuzzy controller (34).

$$\boldsymbol{X} > \boldsymbol{0}, \quad (36)$$

$$\hat{\boldsymbol{A}}_i\boldsymbol{X} + \boldsymbol{X}\hat{\boldsymbol{A}}_i^T - \hat{\boldsymbol{B}}_i\hat{\boldsymbol{M}}_i - \hat{\boldsymbol{M}}_i^T\hat{\boldsymbol{B}}_i^T < \boldsymbol{0},\ \forall i, \quad (37)$$

$$\begin{aligned} &\hat{\boldsymbol{A}}_i\boldsymbol{X} + \boldsymbol{X}\hat{\boldsymbol{A}}_i^T + \hat{\boldsymbol{A}}_j\boldsymbol{X} + \boldsymbol{X}\hat{\boldsymbol{A}}_j^T \\ &\quad -\hat{\boldsymbol{B}}_i\hat{\boldsymbol{M}}_j - \hat{\boldsymbol{M}}_j^T\hat{\boldsymbol{B}}_i^T - \hat{\boldsymbol{B}}_j\hat{\boldsymbol{M}}_i - \hat{\boldsymbol{M}}_i^T\hat{\boldsymbol{B}}_j^T < \boldsymbol{0}, \quad (38) \\ &\quad \forall i,\ i < j, \end{aligned}$$

where $\hat{\boldsymbol{K}}_i = \hat{\boldsymbol{M}}_i\boldsymbol{X}^{-1}$.

By using the designed dynamic fuzzy controller, we can make the output $\boldsymbol{y}(t)$ of the nonlinear system (24) converge to the output $\boldsymbol{y}_r(t)$ of the reference system (26).

Remark 2. In general, differentiating dynamics of a nonlinear system with respect to time makes the differential equation complicated. The complexity makes the fuzzy model construction and controller design difficult. In [8, 9, 10], we have proposed the switching fuzzy model which can be automatically constructed by solving optimization conditions, and derived controller design conditions in terms of LMIs. By utilizing switching fuzzy control approach, we can automatically and effectively design the switching fuzzy controller by computer software for such a complicated system.

The dynamic controller may cause slow convergence and large error. By using the following theorem, we can guarantee the maximum value of error vector $\boldsymbol{e}(t)$.

Theorem 4. [6] Assume that initial condition $\tilde{\boldsymbol{x}}(0) = [\dot{\boldsymbol{x}}(0)\ \dot{\boldsymbol{x}}_r(0)\ \boldsymbol{e}(0)]$ is known. The constraint $\|e_\ell(t)\| \le \mu_\ell$ is enforced at all times if the LMIs

$$\begin{bmatrix} 1 & \tilde{\boldsymbol{x}}^T(0) \\ \tilde{\boldsymbol{x}}(0) & \boldsymbol{X} \end{bmatrix} \ge \boldsymbol{0}, \quad \begin{bmatrix} \boldsymbol{X} & \boldsymbol{X}\tilde{\boldsymbol{C}}_\ell^T \\ \tilde{\boldsymbol{C}}_\ell \boldsymbol{X} & \mu_\ell^2 \boldsymbol{I} \end{bmatrix} \ge \boldsymbol{0},$$

hold, where $\tilde{\boldsymbol{C}}_\ell$ is the vector to determine which error is constrained, that is, $e_\ell(t) = \tilde{\boldsymbol{C}}_\ell \tilde{\boldsymbol{x}}(t)$, where,

$$\tilde{\boldsymbol{C}}_\ell = [\overbrace{\boldsymbol{0}}^{n}\ \overbrace{\boldsymbol{0}}^{n}\ \overbrace{\tilde{\boldsymbol{c}}_\ell}^{q}]$$

$\tilde{\boldsymbol{c}}_\ell \in \boldsymbol{R}^{1\times q}$ is a vector whose ℓth element is 1 and all the other elements are 0.

4. DESIGN EXAMPLE

To illustrate the utility of this model following control approach, we show a simulation example.

Consider the following nonlinear system.

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ -\sin x_1(t) + (x_2(t)+6)\,u(t) \end{bmatrix} \tag{39}$$

$$y(t) = [1\ 0] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{40}$$

where we assume that $|x_1(t)| \le 10$, $|x_2(t)| \le 5$, $|u(t)| \le 5$. In many cases, these assumptions are determined from the physical specification of the system. For the above nonlinear system, we consider the model following control problem to make the output $y(t)$ converge to the output of the following reference system.

$$\begin{bmatrix} \dot{x}_{r1}(t) \\ \dot{x}_{r2}(t) \end{bmatrix} = \begin{bmatrix} -0.01x_{r1}(t) + x_{r2}(t) \\ -x_{r1}(t) - 0.01x_{r2}(t) \end{bmatrix} + \begin{bmatrix} -3 \\ -1 \end{bmatrix} \tag{41}$$

$$y_r(t) = [1\ 0] \begin{bmatrix} x_{r1}(t) \\ x_{r2}(t) \end{bmatrix} \tag{42}$$

We define the error system $e(t)$ as follows:

$$e(t) = y(t) - y_r(t) \tag{43}$$

Firstly, by differentiating the nonlinear system (39), the reference system (41) and the error system (43) with respect to time t, we can obtain the following augmented system.

$$\begin{bmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \\ \\ \ddot{x}_{r1}(t) \\ \ddot{x}_{r2}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} \dot{x}_2(t) \\ -\dot{x}_1(t)\cos x_1(t) \\ \quad +\dot{x}_2(t)u(t) + (x_2(t)+6)\dot{u}(t) \\ -0.01\dot{x}_{r1}(t) + \dot{x}_{r2}(t) \\ -\dot{x}_{r1}(t) - 0.01\dot{x}_{r1}(t) \\ \dot{x}_1(t) - \dot{x}_{r1}(t) \end{bmatrix} \tag{44}$$

Then, by applying sector nonlinearity concept to the nonlinear terms $\cos x_1(t)$, $u(t)$ and $x_2(t)$ in the augmented system, the augmented T-S fuzzy model is constructed as follows:

$$\begin{bmatrix} \ddot{\boldsymbol{x}}(t) \\ \ddot{\boldsymbol{x}}_r(t) \\ \dot{\boldsymbol{e}}(t) \end{bmatrix} = \sum_{i=1}^{8} h_i(\boldsymbol{x}(t), \boldsymbol{u}(t)) \times \left(\hat{\boldsymbol{A}}_i \begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{x}}_r(t) \\ \boldsymbol{e}(t) \end{bmatrix} + \hat{\boldsymbol{B}}_i \dot{\boldsymbol{u}}(t) \right) \tag{45}$$

where

$$\hat{\boldsymbol{A}}_1 = \hat{\boldsymbol{A}}_5 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 5 & 0 & 0 & 0 \\ 0 & 0 & -0.01 & 1 & 0 \\ 0 & 0 & -1 & -0.01 & 0 \\ 1 & 0 & -1 & 0 & 0 \end{bmatrix},$$

$$\hat{\boldsymbol{A}}_2 = \hat{\boldsymbol{A}}_6 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 5 & 0 & 0 & 0 \\ 0 & 0 & -0.01 & 1 & 0 \\ 0 & 0 & -1 & -0.01 & 0 \\ 1 & 0 & -1 & 0 & 0 \end{bmatrix}$$

$$\hat{\boldsymbol{A}}_3 = \hat{\boldsymbol{A}}_7 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & -5 & 0 & 0 & 0 \\ 0 & 0 & -0.01 & 1 & 0 \\ 0 & 0 & -1 & -0.01 & 0 \\ 1 & 0 & -1 & 0 & 0 \end{bmatrix},$$

$$\hat{\boldsymbol{A}}_4 = \hat{\boldsymbol{A}}_8 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & -5 & 0 & 0 & 0 \\ 0 & 0 & -0.01 & 1 & 0 \\ 0 & 0 & -1 & -0.01 & 0 \\ 1 & 0 & -1 & 0 & 0 \end{bmatrix}$$

$$\hat{\boldsymbol{B}}_1 = \hat{\boldsymbol{B}}_2 = \hat{\boldsymbol{B}}_3 = \hat{\boldsymbol{B}}_4 = \begin{bmatrix} 0 \\ 11 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$\hat{\boldsymbol{B}}_5 = \hat{\boldsymbol{B}}_6 = \hat{\boldsymbol{B}}_7 = \hat{\boldsymbol{B}}_8 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$h_1(\boldsymbol{x}(t), u(t)) = \hat{h}_{11}(x_1(t)) \times \hat{h}_{21}(u(t)) \times \hat{h}_{31}(x_2(t))$$
$$h_2(\boldsymbol{x}(t), u(t)) = \hat{h}_{12}(x_1(t)) \times \hat{h}_{21}(u(t)) \times \hat{h}_{31}(x_2(t))$$
$$h_3(\boldsymbol{x}(t), u(t)) = \hat{h}_{11}(x_1(t)) \times \hat{h}_{22}(u(t)) \times \hat{h}_{31}(x_2(t))$$
$$h_4(\boldsymbol{x}(t), u(t)) = \hat{h}_{12}(x_1(t)) \times \hat{h}_{22}(u(t)) \times \hat{h}_{31}(x_2(t))$$
$$h_5(\boldsymbol{x}(t), u(t)) = \hat{h}_{11}(x_1(t)) \times \hat{h}_{21}(u(t)) \times \hat{h}_{32}(x_2(t))$$
$$h_6(\boldsymbol{x}(t), u(t)) = \hat{h}_{12}(x_1(t)) \times \hat{h}_{21}(u(t)) \times \hat{h}_{32}(x_2(t))$$
$$h_7(\boldsymbol{x}(t), u(t)) = \hat{h}_{11}(x_1(t)) \times \hat{h}_{22}(u(t)) \times \hat{h}_{32}(x_2(t))$$
$$h_8(\boldsymbol{x}(t), u(t)) = \hat{h}_{12}(x_1(t)) \times \hat{h}_{22}(u(t)) \times \hat{h}_{32}(x_2(t))$$

$$\hat{h}_{11}(x_1(t)) = \frac{\cos x_1(t) + 1}{2}, \ \hat{h}_{12}(x_1(t)) = \frac{1 - \cos x_1(t)}{2}$$
$$\hat{h}_{21}(u(t)) = \frac{u(t) + 5}{10}, \ \hat{h}_{22}(u(t)) = \frac{5 - u(t)}{10}$$
$$\hat{h}_{31}(x_2(t)) = \frac{x_2(t) + 5}{10}, \ \hat{h}_{32}(x_2(t)) = \frac{5 - x_2(t)}{10}$$

For the augmented fuzzy model (45), by simultaneously solving Theorems 3 and 4 with the initial condition $\tilde{\boldsymbol{x}}(0) = [-5 \ 4 \ -2 \ -1 \ 0]$, $\tilde{\boldsymbol{C}}_5 = [0\ 0\ 0\ 0\ 1;]$ and $\mu_5 = \sqrt{10}$, we can obtain the following feedback gains.

$$\boldsymbol{P} = \boldsymbol{X}^{-1} = \begin{bmatrix} 0.0367 & 0.0032 & -0.0383 & -0.0045 & 0.1088 \\ 0.0032 & 0.0003 & -0.0034 & -0.0004 & 0.0093 \\ -0.0383 & -0.0034 & 0.0419 & 0.0047 & -0.1152 \\ -0.0045 & -0.0004 & 0.0047 & 0.0025 & -0.0134 \\ 0.1088 & 0.0093 & -0.1152 & -0.0134 & 0.4281 \end{bmatrix}$$

$$\boldsymbol{K}_1 = [0.1049 \ 0.0103 \ -0.1089 \ -0.0132 \ 0.2825] \times 10^3$$
$$\boldsymbol{K}_2 = [0.1057 \ 0.0104 \ -0.1095 \ -0.0133 \ 0.2839] \times 10^3$$
$$\boldsymbol{K}_3 = [0.1089 \ 0.0098 \ -0.113 \ -0.0137 \ 0.2935] \times 10^3$$
$$\boldsymbol{K}_4 = [0.109 \ 0.0097 \ -0.1129 \ -0.0137 \ 0.2932] \times 10^3$$
$$\boldsymbol{K}_5 = [0.8892 \ 0.0872 \ -0.9233 \ -0.1136 \ 2.4459] \times 10^3$$
$$\boldsymbol{K}_6 = [0.9496 \ 0.0932 \ -0.9854 \ -0.1216 \ 2.6187] \times 10^3$$
$$\boldsymbol{K}_7 = [0.6815 \ 0.0646 \ -0.7077 \ -0.0874 \ 1.8830] \times 10^3$$
$$\boldsymbol{K}_8 = [0.7720 \ 0.0734 \ -0.8013 \ -0.0992 \ 2.1383] \times 10^3$$

Figures 1 and 2 show the control result and the control input, where initial states are $\boldsymbol{x}(0) = [-5 \ 4]$ and $\boldsymbol{x}_r(0) = [-2 \ -1]$. By using the designed controller, the output of the nonlinear system (39) converges to the output of the reference system (41).

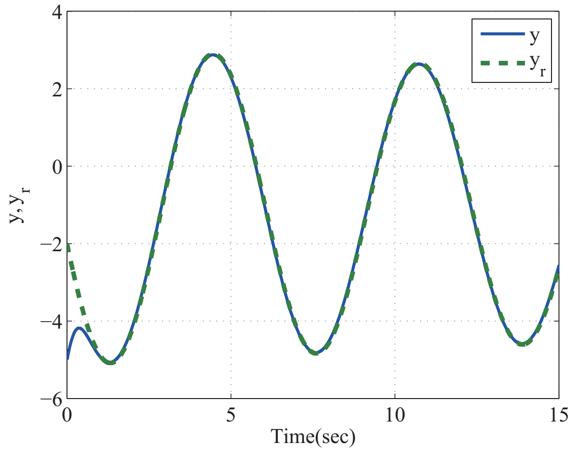

Fig. 1. Control result.

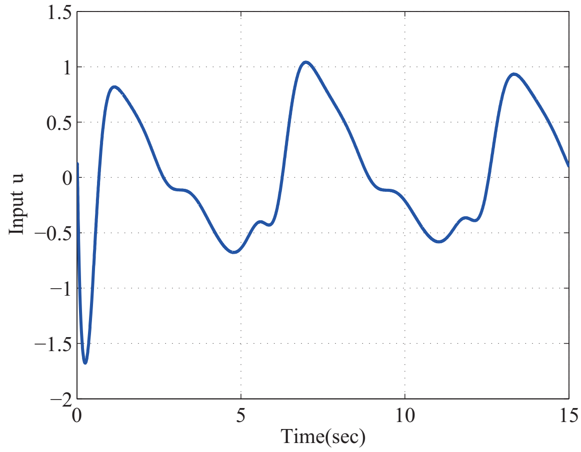

Fig. 2. Control input.

5. CONCLUSIONS

This paper has presented model following control for a class of nonlinear systems using the fuzzy model-based control approach. We have shown the construction method of augmented fuzzy control system for continuous-time nonlinear systems by differentiating the original nonlinear system and the reference system. Moreover, we have introduced the dynamic fuzzy controller which can make the output of the nonlinear system converge to the output of the reference system, and derived the controller design conditions in terms of LMIs.

Our future work is to apply this approach to real complicated systems.

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