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Nonlinear H_∞ Control of a Bilateral Nonlinear Teleoperation System

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Abstract: In this paper, we present a nonlinear H_∞ control technique for the bilateral teleoperation of a nonlinear master-slave system. The proposed controller guarantees robust stability in the presence of uncertainty in operator and environment impedances. The guidelines to include nonlinear intervening tool between master and slave robot are suggested. The proposed technique enables adjusting weighting of position and force tracking error functions for both amplitude and frequency domain. To solve the corresponding partial differential equation, called HJI, an approximation method based on Taylor series expansion of the solution is used. A numerical simulation demonstrates that the linear controller tends to instability in contact tasks while the third order approximated nonlinear controller yields desired performance.

1. INTRODUCTION

Bilateral teleoperation systems have been a benchmark for a variety of control systems, from Lyapunov-based designs (Lee et. al., 2006), predictive controls (Sirouspour et. al., 2006) and H_∞ techniques (Leung et. al., 1995) to passivity control (Lee et. al., 2006) and adaptive control approaches (Hashtrudi et. al., 1996). It is challenging, since it seeks for robust stability to the uncertain human and environment impedance and time-delayed feedback signals. Moreover, unlike many control problems which are formulated within a regulation problem, it asks for impedance matching. The impedance of the environment should be transmitted to the human operator with minimum distortion. Ideally, a teleoperation system should be transparent. More recent applications of teleoperation systems enable user to touch the virtual environment. Thus, with the incorporation of digital computers, quantizing feedback and control signals and the sampling rate should also be considered in the performance and stability of the overall system.

Historically, early approaches to the control of teleoperation systems discuss different control aspects of a linear 1-DOF robotic systems (Lawrence, 1993 and Cavusoglu et. al., 2002). However, recent works apply the design process directly to nonlinear n-DOF robotic systems (Lee et. al., 2006). Although this complicates the design extremely, the final results could be directly applied to the nonlinear system without the need of an intermediate linearization step. The control of robotic manipulator using nonlinear H_∞ control has been reported in (Yazdanpanah et. al., 1998 and Park et. al., 2000); however, there are not important works about the application of this approach to a teleoperation system. This work presents a nonlinear control framework for a teleoperation system. The approach used in this paper is nonlinear H_∞ control. Using advantages of this approach in controlling a teleoperation system, nonlinear objectives such as amplitude dependant error functions and nonlinear intervening tool are handled.

2. PRELIMINARIES

2.1 Modelling

Consider a teleoperator system consisting of two master and slave robots which are governed by two *n*-DOF nonlinear mechanical systems as follows:

$$\begin{aligned} M_1(q_1)\ddot{q}_1(t) + C_1(q_1,\dot{q}_1)\dot{q}_1(t) + g_1(q_1) &= T_1(t) + F_1(t) \\ M_2(q_2)\ddot{q}_2(t) + C_2(q_2,\dot{q}_2)\dot{q}_2(t) + g_2(q_2) &= T_2(t) + F_2(t) \end{aligned} \quad (1)$$

where q_i, F_i, $T_i \in \Re^n$ are the position coordinates, human and environment forces, and the control signals respectively, $M_i \in \Re^{n\times n}$ are symmetric and positive definite inertia matrices, and $C_i \in \Re^{n\times n}$ are Coriolis matrices, with i=1,2.

The main objective in controller design of a teleoperation system is to transfer the mechanical impedance of the remote task to the operator in a way that s/he feels as if s/he is performing the task directly. This should be done with minimum distortion in the frequency and amplitude of interest.

2.1 Performance criteria

Trade-off between performance and stability plays an important role in control system design of a teleoperation system; since, it asks the multivariable performance objective (i.e. impedance match) while maintaining stability in the presence of the uncertainty of operator's hand impedance and the one's of the environment under manipulation as a part of the closed loop system. A state of the art controller design framework has to incorporate this trade-off in the design process in order to be applicable.

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The performance of the controller design, i.e. the impedance match as described above, requires the equality between two force signals, F_1 and F_2, and two position signals, q_1 and q_2. Position tracking can be achieved through minimizing the following frequency and amplitude dependant error function:

$$\begin{aligned} z_1(t) &= h_p\left(e_1(t)\right)\times a_p\left(e_1(t)\right) \\ e_1 &= q_1 - \alpha_p q_2 \end{aligned} \tag{2}$$

where α_p is the scaling factor for the position tracking, h_p is the output of a linear low pass filter used to weight position tracking error over frequencies, and static map a_p determines the amplitude of interest for tracking. Incorporation of the amplitude in z_1 is due to the fact that frequency bandwidth of the signals produced by the operator is mainly amplitude-dependant. Human operator could produce high frequency movements only in low amplitudes while movements with high amplitude are restricted to low frequencies. One possible choice of a_p can be the following Gaussian function:

$$a_p\left(x\right) = e^{-k\,(x^T x)} \tag{3}$$

A more extensive criterion for force tracking adds a virtual tool intervening between the operator and the environment. This could be achieved through minimization of the following error function:

$$z_2(t) = F_1 - \left(z_t(q_1) + \alpha_f F_2\right) \tag{4}$$

where α_f is the scaling factor for force tracking and z_t is the output of a desirable nonlinear dynamic of the intervening tool.

2.3 *Consideration of Uncertainties*

The force applied by the operator to the master robot, F_1, can be modeled by the following equation:

$$\begin{aligned} F_1 &= F_1^* - F_{1h} = F_1^* - z_h q_1 \\ z_h &= M_h \frac{d^2}{dt^2} + B_h \frac{d}{dt} + K_h \end{aligned} \tag{5}$$

where F_1^* is a L_2-*bounded* exogenous force generated by the operator's hand and z_h is a mass-damper-spring model of the hand's muscular system. M_h, B_h, K_h are unknown positive definite matrices with H-infinity norms belonging to a known bound. It is reported in several works that a linear model could capture the dynamics of the human's arm (Cavusoglu et. al., 2002, Hashtrudi et. al., 1996). Moreover, the assumption of L_2-*boundedness* of F_1^* generalizes the design to a large class of applications.

In a similar way, the environment can be modeled as follows:

$$\begin{aligned} F_1 &= F_2^* - F_{1e} = F_1^* - z_e q_1 \\ z_h &= .M_e \frac{d^2}{dt^2} + B_e \frac{d^2}{dt^2} + K_e \end{aligned} \tag{6}$$

3. NONLINEAR H$_\infty$ CONTROL

We present a brief review of nonlinear H$_\infty$ control problem in this section. The application of this approach to the control of teleoperation systems is then presented in the next section.

Consider the nonlinear system

$$\begin{aligned} \dot{x} &= f\,(x, w, u) \\ z &= h(x, w, u) \end{aligned} \tag{7}$$

where $x \in \Re^n$ is the state vector, $w \in \Re^{m1}$ is the exogenous input containing disturbance signals to be rejected and references to be tracked, $u \in \Re^{m2}$ is the control vector, and $z \in \Re^p$ is the controlled output. We want to design a controller to stabilize the system and attenuate the effect of the exogenous input w on the controlled output z. Consider the following Hamiltonian function

$$H(x, w, u, p) = p^T f(x, w, u) - \gamma^2 \|w\|^2 + \|z\|^2 \tag{8}$$

where $p^T = V_x$. If there exist a nonnegative function $V(x)$ vanishing at $x=0$ such that Hamilton-Jacobi-Isaacs (HJI) inequality

$$\mathrm{H(x,w,u,Vx)} \le 0 \tag{9}$$

holds, the resulting closed loop system with the control

$$u = u_*(x, V_x^T(x)) \tag{10}$$

is stable and dissipative with respect to the supply rate

$$s(w, z) = \gamma^2 \|w\|^2 - \|z\|^2 \tag{11}$$

Hence, the closed loop system has an L_2 gain less than or equal to γ. Moreover, it is locally asymptotically stable if the system is zero state detectable (Van der Schaft, 2004).

In general, it is not possible to find the explicit solution of the HJI partial differential inequality (8). One practical way is to consider the Taylor expansion of the solution up to a desirable order and, then, find the solution by means of a polynomial approximation method.

We briefly explain the procedure of finding the approximated solution of the HJI inequality (8). Consider the series expansion for $v_*(x)$ and $V(x)$ up to order d

$$\begin{aligned} v_*(x) &= v_x^{(1)}(x) + v_x^{(2)}(x) + \ldots + v_x^{(d)}(x) = \sum_{i=1}^{d} v_x^{(i)}(x) \\ V(x) &= V^{(2)}(x) + V^{(3)}(x) + \ldots + V^{(d+1)}(x) = \sum_{i=2}^{d+1} V^{(i)}(x) \end{aligned} \tag{12}$$

where $v=[w^T\ u^T]^T \in \Re^{m1+m2}$ in which $v_x^{(d)}$ is all the polynomials of the form

$$c_k x_1^{j_1} x_2^{j_2} \ldots x_n^{j_n} \qquad \text{with } j_1 + j_2 + \ldots + j_n = d \tag{13}$$

The function $V^{(2)}$,and consequently $v_x^{(1)}$, is determined by solving the corresponding algebraic Riccati equation (ARE) of the linearized model of the system (6). Consider the linearization of the system around the origin

$$\begin{aligned} \dot{x} &= Ax + Bv \\ z &= Cx + Dv \end{aligned} \tag{14}$$

with

$$\begin{aligned} A &= \frac{\partial}{\partial x} f(x,v)\Big|_{(0,0)} & B &= \frac{\partial}{\partial v} f(x,v)\Big|_{(0,0)} \\ B &= \frac{\partial}{\partial x} h(x,v)\Big|_{(0,0)} & D &= \frac{\partial}{\partial v} h(x,v)\Big|_{(0,0)} \end{aligned} \tag{15}$$

and the corresponding ARE as follows

$$F^T K + KF - KBR^{-1}B^T K + Q = 0 \tag{16}$$

with

$$\begin{aligned} F &= A - BR^{-1}D^T C, \\ R &= DTD - \begin{bmatrix} \gamma^2 I_{m1} & 0 \\ 0 & 0 \end{bmatrix}, \\ Q &= C^T C - C^T DR^{-1}D^T C \end{aligned} \tag{17}$$

Then, the solution to the linear H$_\infty$ problem may be computed as

$$\begin{aligned} V^{(2)}(x) &= x^T Kx \\ v_*^{(1)} &= -R^{-1}(B^T K + D^T C)x = -\bar{K}x \end{aligned} \tag{18}$$

3.1 Approximated controller

By using Eq.8 and the fact that v_* defines a saddle point for the Hamiltonian function, we get the following equations (Christen, et. al., 1997)

$$\begin{aligned} & V_x^{(m)}(x)(A - B\bar{K})x = \left[-V_x Bv * -V_x \tilde{f}(x, v_*) - g_1(x, v_*) - v_*^T R v_* \right]^{(m)} \\ & m = 3,4,.. \\ & v_*^{(k)} = \frac{-1}{2} R^{-1} \left(\left(\frac{\partial}{\partial v} \tilde{g} \right)^T \Big|_{v=v_*} + B^T V_x^T + \left(\frac{\partial}{\partial v} \tilde{f} \right)^T \Big|_{v=v_*} V_x^T \right)^{(k)} \\ & k = 2,3,... \end{aligned} \tag{19}$$

where

$$\tilde{f} = f - Ax, \quad \tilde{g} = h - Cx - Dv \tag{20}$$

The write-hand side of Eq.19 is determined by the first (m-1) terms of *V(x)* and the first (m-2) terms of $v_*(x)$, while $v_*^{(k)}$ depends on the first (k-1) terms of $v_*(x)$ and (k+1) terms of *V(x)*. Hence, the consecutive terms of $v_*(x)$ and *V(x)* can be computed in the following sequence up to a desirable order

$$V^{(2)}, v_*^{(1)}, V^{(3)}, v_*^{(2)}, ... \tag{21}$$

4. APPLICATION TO A TELEOPERATION SYSTEM

In the preceding section, we discussed a state space approach to the design of the nonlinear H$_\infty$ control system. Therefore, we proceed with deriving the state space model of the robots in this section. Performance objectives are included by introducing some exosystems and augmenting their states to the states of the robots.

Using (5) and (6), Equation (1) reads

$$\begin{aligned} M_1(q_1)\ddot{q}_1(t) + C_1(q_1,\dot{q}_1)\dot{q}_1(t) &= T_1(t) + F_1^* - F_{1h} \\ M_2(q_2)\ddot{q}_2(t) + C_2(q_2,\dot{q}_2)\dot{q}_2(t) &= T_2(t) - F_2^* - F_{2e} \end{aligned} \tag{22}$$

By choosing $q_1, \dot{q}_1, q_2,$ and $\dot{q}_2$ as the states of the system we obtain the state space equations in the form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = g(x,u,w) = \begin{bmatrix} x_2 \\ -M_1^{-1}(x_1)\left(C_1(x_1,x_2)x_2 + u_1 + w_1 - w_3\right) \\ x_4 \\ -M_2^{-1}(x_3)\left(C_1(x_3,x_4)x_4 + u_2 - w_2 - w_4\right) \end{bmatrix} \tag{23}$$

where

$$\begin{aligned} & \begin{bmatrix} x_1^T & x_2^T & x_3^T & x_4^T \end{bmatrix}^T = \begin{bmatrix} q_1^T & \dot{q}_1^T & q_2^T & \dot{q}_2^T \end{bmatrix}^T \\ & w = \begin{bmatrix} F_1^{*T} & F_2^{*T} & F_{1h}^T & F_{2e}^T \end{bmatrix}^T, \begin{bmatrix} u_1^T & u_2^T \end{bmatrix}^T = \begin{bmatrix} T_1^T & T_2^T \end{bmatrix}^T \end{aligned} \tag{24}$$

Force signals applied by human and environment are considered disturbances to the system. Force sensors may be used at each side of a teleoperation system to measure the force signals. These signals may also be incorporated as inputs to the control system. We determine the bandwidth of the measurements by two low-pass filters. Consider the following two low-pass filters

$$\begin{aligned} \tilde{F}_1 &= F_{lp1}\left(w_1 - w_3\right) \\ \tilde{F}_2 &= F_{lp2}\left(-w_2 - w_4\right) \end{aligned} \tag{25}$$

The state space model of Eq.25 may be written in the form of

$$\begin{aligned} \begin{bmatrix} \dot{x}_5 \\ \dot{\tilde{x}}_5 \end{bmatrix} &= f_{lp1}(x_5, w_1, w_3) \\ \begin{bmatrix} \dot{x}_6 \\ \dot{\tilde{x}}_6 \end{bmatrix} &= f_{lp2}(x_6, w_2, w_4) \end{aligned} \tag{26}$$

where $\dot{\tilde{x}}_5, \dot{\tilde{x}}_6$ are the outputs of the filter.

Now, consider the position tracking error function, (2). Say x_7 to be the states of the low-pass filter h_p. We can write the equation of the exosystem as

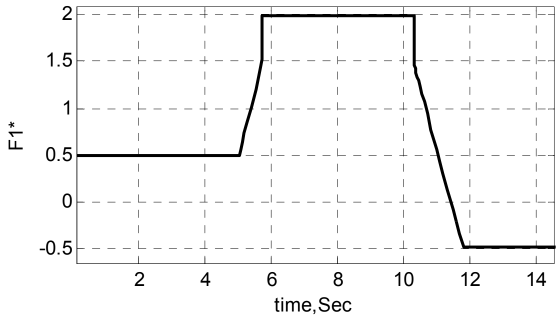

Fig.1 Exogenous force signal produced by the operator

$$\dot{x}_7 = f_p(x_7, x_1, x_3), \quad z_1(t) = h_1(x_7) \tag{27}$$

Also, by choosing x_8 as the states of the intervening tool, z_t, one may write the exosystem equations as

$$\dot{x}_8 = f_t(x_8, x), \quad z_2(t) = h_2(x_8, x) \tag{28}$$

The closed-loop system should also be robustly stable to the uncertain human and environment impedances. Considering (5), for the human operator, the following conditions should hold

$$\frac{\|q_1\|_2}{\|F_h\|_2} \le \frac{1}{\overline{K}_\infty}, \quad \frac{\|\dot{q}_1\|_2}{\|F_h\|_2} \le \frac{1}{\overline{B}_\infty}, \quad \frac{\|\ddot{q}_1\|_2}{\|F_h\|_2} \le \frac{1}{\overline{M}_\infty} \tag{29}$$

where $\|X\|_\infty \le \overline{X}_\infty$. To add this conditions to the control objectives, fictitious outputs will be added as follows,

$$z_3^T = \left[\overline{M}_{h_\infty}\ddot{q}_1^T \quad \overline{B}_{h_\infty}\dot{q}_1^T \quad \overline{K}_{h_\infty}q_1^T\right] \tag{30}$$

This definition of outputs adds the acceleration to the measurement which is undesirable. Hence, one may use the approximation of the acceleration signals in the desirable frequency domain, i.e.

$$\ddot{q}_1 \cong \frac{s}{as+1}\dot{q}_1 \tag{31}$$

where s is the Laplace variable and a is the cut-off frequency of the low pass filter. This approximation will add additional states. Similar formulation is also used for the environment impedance,

$$z_4^T = \left[\overline{M}_{e_\infty}\ddot{q}_2^T \quad \overline{B}_{e_\infty}\dot{q}_2^T \quad \overline{K}_{e_\infty}q_2^T\right] \tag{32}$$

Finally, the overall nonlinear system reads

$$\dot{x} = f(x, w, u), \quad z = h(x, w) \tag{33}$$

where

Table.1 Parameters used in the simulation

$\overline{M}_{h_\infty} = \overline{M}_{e_\infty}$	0.2	$\overline{B}_{h_\infty} = \overline{B}_{e_\infty}$	2
$\overline{K}_{h_\infty} = \overline{K}_{e_\infty}$	20		

$$f(x,u,w) = [g^T \quad f_{lp1}^T \quad f_{lp2}^T \quad f_p^T \quad f_t^T]^T, \quad z = [z_1^T \quad z_2^T \quad z_3^T \quad z_4^T]^T \tag{34}$$

5. ILLUSTRATIVE EXAMPLE

In this example, we consider a pair of two 1-DOF direct-drive robots. The dynamics of the master and slave robots are governed by the following equations

$$\begin{aligned} &0.1\ddot{q}_1(t) + 0.1\dot{q}_1(t) = T_1(t) + F_1^* - F_{1h} \\ &0.1\ddot{q}_2(t) + 0.1\dot{q}_2(t) - 0.1sin(q_2) = T_2(t) - F_2^* - F_{2e} \end{aligned} \tag{35}$$

as it can be seen, the master robot is linear mass-damper system, while the slave robot incorporates a nonlinear term due to the gravitational force. Two low pass force measurement filters, see (25), are defined as

$$F_{lp1} = F_{lp2} = \frac{1}{(0.01s+1)} \tag{36}$$

the exosystem defined for the position tracking objective is defined as,

$$\dot{x}_7 = -10x_7 + 10(x_1 - x_3), \quad z_1(t) = 5x_7 \tag{37}$$

which is the realization of a low pass filter with cutoff frequency equal to 0.1. This specifies the desirable bandwidth of the position tracking. Also, force tracking is achieved through minimizing of the following output

$$\begin{aligned} &\dot{x}_8 = -10x_8 + 10(F_1 - F_2), \\ &z_2(t) = 0.5(x_8 - 0.1\sin x_1) \\ &F_1 = w_1 - w_3,\ F_2 = w_2 - w_4 \end{aligned} \tag{38}$$

which adds a virtual gravitational effect of a mass at the master side same as the slave side. The values of upper bounds of the uncertainties are summarized in table.1.

Having discussed the problem, we now present the numerical solution of the problem. As mentioned earlier, the first step is to find the solution to the ARE and then constructing the higher order terms will be computed consecutively. The first order linear time-invariant controller should be as good as the nonlinear while initial conditions are near the origin and disturbance signals are low amplitude. The attenuation gain, γ, is set to one.

To demonstrate the performance of the controller we should consider a scenario. The human operator is modeled by a PD position tracking controller using spring and damping gains as 10 and 1 respectively. At the beginning, he pushes the master robot to the position 0.8. Then, he tries to push the master robot to the position 1.5. While he is moving the robot to this target, he realizes the existence of a hard wall while receiving step-like force feedback at the position 1. The hard wall is simulated by a spring with 30 N/m stiffness. Fig.1 shows the exogenous force exerted by the operator used in this simulation. The results of the first order controller is depicted in Fig.2. In the first five second, F_1^* is low amplitude, no contact in occurred (*Fe*=0). As expected, position and force tracking is obtained. When F_1^* become large oscillations are seen in response and error signals become larger. For this system, stability is no violated in the presence of linear controller. Fig.3 shows the results of the third order controller. In comparison to the first order controller, It shows that less oscillations is occurred in contact and less error in the output corresponding with intervening tool is occurred.

6. CONCLUSIONS

In this paper, a nonlinear H_∞ control technique for control of a nonlinear master-slave teleoperation system was presented. The proposed technique includes the most important objectives of the control of a teleoperation system like: motion and force scaling, intervening tool between master and slave robots, and stability to the uncertainty of operator and environment impedances. Corresponding HJI equation is solved by Taylor series approximation of the solution. Simulation results on a 1-DOF master-slave system shows that the first order approximated linear controller can not achieve desired performance and oscillation in the force response occurs during the contact. It was observed that the third order controller for the example in this paper is sufficient to achieve good performance. Experimental results with a multi-DOF robotic system have to be done in the future to demonstrate the effectiveness of the proposed method.

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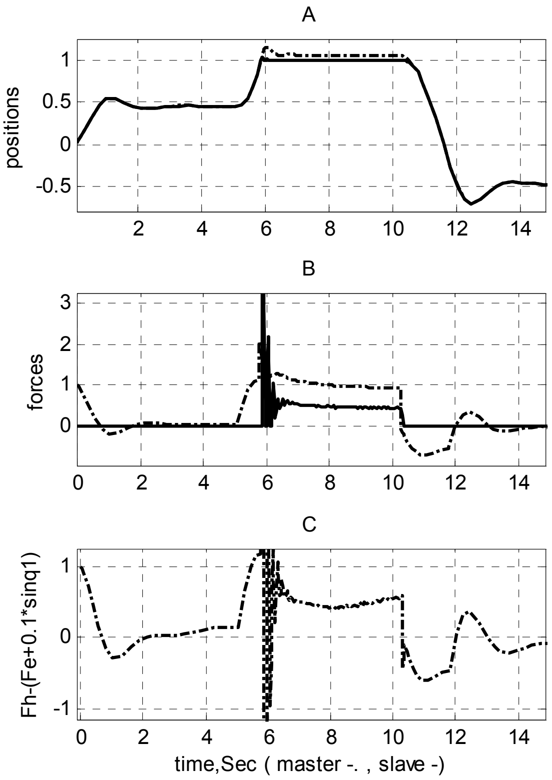

Fig.2 Results of the first order controller, A. master and slave positions, B. master and slave forces, C. the output corresponding with the intervening impedance, (z_2).

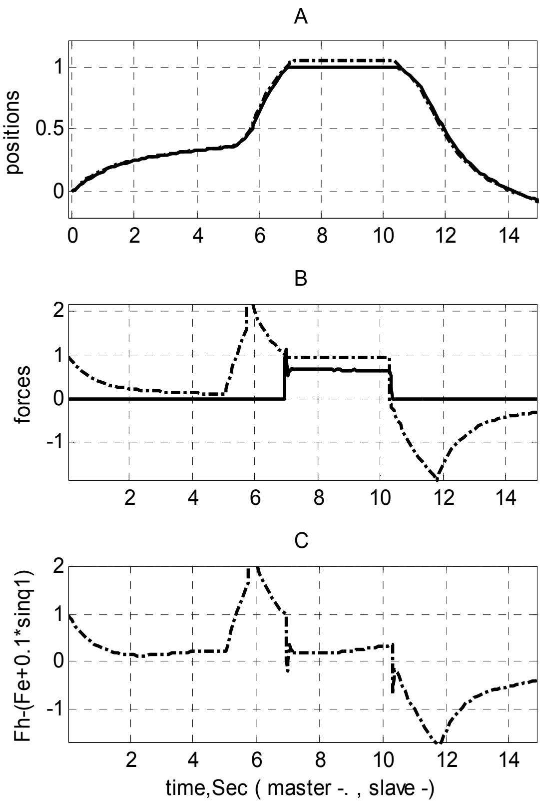

Fig.3 Results of the third order controller, A. master and slave positions, B. master and slave forces, C. the output corresponding with the intervening impedance, (z_2).