Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Fault Detection, Identification and Accommodation for an Electro-hydraulic System: An Adaptive Robust Approach ⋆

Shreekant Gayaka † and **Bin Yao** †,‡

† *Ray W. Herrick Laboratories, School of Mechanical Engineering, Purdue University, West Lafayette, IN 47907, USA (e-mail: sgayaka@purdue.edu)*
‡ *State Key Laboratory of Fluid Power Transmission and Control, Zhejiang University, Hangzhou, China (e-mail: byao@purdue.edu)*

Abstract: In the present work, we use an adaptive robust approach for fault detection and accommodation in electro-hydraulic systems. It is well known fact that any realistic model of a hydraulic system suffers from significant extent of uncertain nonlinearities and parametric uncertainties. An adaptive robust scheme is robust to such uncertainties and tracks the change in parameters reliably. Consequently, such a scheme becomes a natural choice for designing robust fault detection algorithms for electro-hydraulic systems. In this paper, we present the main results obtained by using adaptive robust state reconstruction and adaptive robust observers for fault detection in electro-hydraulic systems. Furthermore, the useful information about faults contained in the residual is used for designing an active fault-tolerant controller. We give an outline of the stability analysis for the faulty closed loop system, which shows that all states remain bounded and desired performance is restored to acceptable limits after the occurrence of fault. Simulation results show the effectiveness of the proposed scheme.

Keywords: Fault Detection, fault identification, fault tolerant systems, nonlinear systems, electro-hydraulic systems

1. INTRODUCTION

Electro-hydraulic systems are used in many industrial applications - ranging from earth moving heavy equipments to lawn mowers, as they have excellent size-to-power ratio and are able to generate large forces. Due to their ever increasing use in industry and elsewhere, it is essential that such systems be made more reliable. Hence, fault detection and diagnosis (FDD) of electro-hydraulic systems has become an important area of research.

Fault diagnosis for electro-hydraulic systems is a very challenging task, as it is extremely difficult to model such systems precisely owing to their nonlinear dynamics. Other sources of modeling errors include uncertain nonlinearities, unmodeled dynamics and parametric uncertainties. Examples of such uncertainties would include deadband, hysteresis in the valves, nonlinear pressure/ flow relations and variation in fluid volumes due to actuator movement.

In our approach, we use a nonlinear system model to account for the nonlinearities in the system and categorize the modeling errors in two main groups based on their source - parametric uncertainties and uncertain nonlinearities. Parametric uncertainties include variations due to change in temperature and pressure, and component changes due to wear. Other uncertainties such as external disturbances, leakage and friction cannot be modeled exactly and are lumped together as uncertain nonlinearities. Any reliable fault diagnosis scheme must take into account these factors.

Many techniques have been proposed in the literature for fault diagnosis of mechatronic systems. Nonlinear plant based observers have been used in Frank (1994), Hammouri et al. (2002) and Preston et al. (1996). Parameters estimation based techniques have been used in Tan and Sepehri (2002), Yu (1997). An informative statistical study of commonly occurring faults in hydraulic systems can be found in Munchhof (2006). On the other hand, fault tolerant control system design for electro-hydraulic systems has received relatively less attention so far. But, with the increase in demand for robustness and reliability, it is fast becoming an active area of research. In Tan and Sepehri (2002), the authors present a fault accommodation (FA) technique based on the linear model of the system. Blanke et al. (2003) gives a good overview of the various techniques used in fault tolerant control system design. In the present work, we present a comprehensive scheme for FDD and fault accommodation, taking into account various uncertainties present in the system.Figure 2 represents the architecture of the proposed scheme. The scheme is based on adaptive robust philosophy proposed in Yao and Tomizuka (1997). Using the FDD results presented in Garimella and Yao (2005), we propose an adaptive robust fault accommodation technique for leakage and contamination faults. The proposed scheme can be easily integrated with the backstepping controller design for

⋆ This work was supported in part by the US National Science Foundation Grant (No. CMS-0600516).

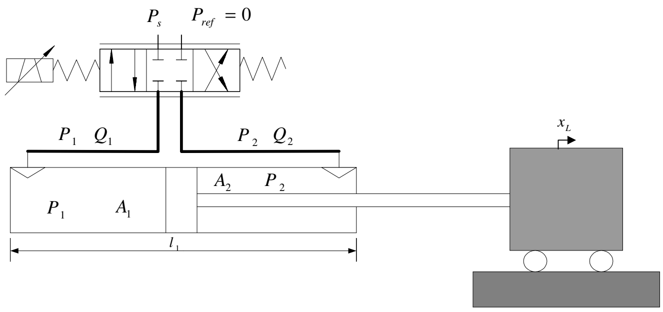

Fig. 1. Schematic of single-rod electro-hydraulic servo system

the nominal system. Furthermore, we prove that the performance can be restored to acceptable limits by using this technique. Although, developed for electro-hydraulic system, the fault accommodation scheme can be easily extended to any system which uses a backstepping based controller.

The paper is organized as follows. Section 2 describes the system model, various assumptions and the objective of present work. In section 3, an overview of the adaptive robust framework for FDD of the electro-hydraulic system is presented. Section 4 presents a novel technique for accommodation of two commonly occurring faults - contamination and leakage fault. In section 5, simulation results validating the proposed scheme is presented and we conclude with section 6.

2. PROBLEM FORMULATION

2.1 System Model

The system model used in this paper is same as used in Yao et al. (2000a). The schematic is depicted in Fig.1. The objective of the control system design is to track a specified trajectory as closely as possible. The dynamics of the inertia load and actuator can be written as,

$$m\ddot{x}_L = P_1A_1 - P_2A_2 - b\dot{x}_L - F_{fc}(\dot{x}_L) + \tilde{f} \tag{1}$$

$$\frac{V_1}{\beta_e}\dot{P}_1 = -A_1\dot{x}_L - C_{tm}(P_1 - P_2) - C_{em1}(P_1 - P_r) + Q_1 \tag{2}$$

$$\frac{V_2}{\beta_e}\dot{P}_2 = A_2\dot{x}_L + C_{tm}(P_1 - P_2) - C_{em2}(P_2 - P_r) - Q_2 \tag{3}$$

where, b is the combined coefficient of the modeled damping and viscous friction forces, F_{fc} is the modeled Coulomb friction, $\tilde{f}$ is the lumped uncertainty, $V_i = V_{h_i} \pm A_i x_L$ is the total control volume of the first and second chamber for $i = 1, 2$ respectively, V_{h_i} is the chamber volume when $x_L = 0$, A_i is the ram area, β_e is the effective bulk modulus, C_{tm} is the coefficient of internal leakage, C_{emi} is the coefficient of external leakage and Q_i is the supplied and return flow rate for $i = 1, 2$ respectively.

Q_1 and Q_2 are related to the spool valve displacement of the servovalve x_v by,

$$Q_1 = k_{q1}x_v\sqrt{\Delta P_1}, \quad \Delta P_1 = \begin{cases} P_s - P_1 & \text{for } x_v \geq 0 \\ P_1 - P_r & \text{for } x_v < 0 \end{cases} \tag{4}$$

$$Q_2 = k_{q2}x_v\sqrt{\Delta P_2}, \quad \Delta P_2 = \begin{cases} P_2 - P_r & \text{for } x_v \geq 0 \\ P_s - P_2 & \text{for } x_v < 0 \end{cases} \tag{5}$$

It is assumed that x_v is related to the actual control input by a static mapping and therefore, x_v can be treated as the control input for controller design. Also, the difference in order of magnitude of various variables like pressure and position can cause problem in gain tuning and parameter estimation. As in Yao et al. (2000b), we will use constants S_{c3} and S_{c4} for scaling pressure and valve opening respectively. Thus, the various scaled variables are, $[x_1 \;\; x_2 \;\; x_3 \;\; x_4]^T = [x_L \;\; \dot{x}_L \;\; \bar{P}_1 \;\; \bar{P}_2]^T$ and the uncertain parameters are $\theta_1 = \frac{S_{c3}A_1}{m}, \theta_2 = d_n, \theta_3 = \frac{\beta_e S_{c4} k_{q1}}{V_{h1}\sqrt{S_{c3}}}, \theta_4 = \frac{C_{tm}\beta_e}{V_{h1}}, \theta_5 = \frac{C_{em1}\beta_e}{V_{h1}}, \theta_6 = \frac{C_{em2}\beta_e}{V_{h1}}$. Now, in state-space representation, the system dynamics can be written as,

$$\dot{x}_1 = x_2 \tag{6}$$

$$\dot{x}_2 = \theta_1(x_2 - \bar{A}x_3) + \theta_2 + \Delta_1 \tag{7}$$

$$\dot{x}_3 = \theta_3(-\bar{A}_1x_2 + g_3u) - \theta_4(x_3 - x_4) - \theta_5(x_3 - Pr) + \Delta_2 \tag{8}$$

$$\dot{x}_4 = \theta_3(\bar{A}_2x_2 - g_4u) + \theta_4(x_3 - x_4) - \theta_6(x_4 - Pr) + \Delta_3 \tag{9}$$

where g_3 and g_4 are the scaled nonlinear function corresponding to Q_1 and Q_2 respectively, and $\bar{A}_i$ represents the scaled ram areas, and $\bar{A} = \frac{A_2}{A_1}$. The reader is referred to Yao et al. (2000b) for details regarding the model and various parameter values.

2.2 Assumptions

Assumption 1: The unknown but constant parameters lie in a known bounded region i.e.,

$$\theta_i \in \Omega_\theta \Rightarrow \{\theta_i : \theta_{i,min} \leq \theta_i \leq \theta_{i,max}\} \tag{10}$$

Assumption 2: The unmodeled dynamics and uncertain nonlinearities are bounded by known constants i.e.,

$$\Delta_i \in \Omega_\Delta \Rightarrow \{\Delta_i : \Delta_{i,min} \leq \Delta_i \leq \Delta_{i,max}\} \tag{11}$$

Fact 1. We will briefly state the main results obtained in Yao et al. (2000a) for the fault-free system (6-9). Let $z = [z_1 \;\; z_2 \;\; z_3]^T = [x_1 - x_{1,des} \;\; x_2 - \alpha_2 \;\; P_L - \alpha_3]$, be the error variables in backstepping design where, $P_L = x_3 - \bar{A}x_4$ and α_i represents the virtual control law. Then, using assumptions (10-11), a discontinuous projection based adaptive robust controller (ARC) can be designed such that the output tracking error z_1 and the transformed states $z = [z_1 \;\; z_2 \;\; z_3]^T$ are bounded. Furthermore, a positive definite (p.d) function of the form $V_N = \sum_{i=1}^{3} \frac{1}{2} w_i z_i^2, w_i > 0$ is bounded above by,

$$V_N(t) \leq e^{-\lambda_V t}V_N(0) + \frac{\varepsilon}{\lambda_V}(1 - e^{-\lambda_V t}) \tag{12}$$

where, λ_V and ε are functions of controller gains and can be adjusted freely. Thus, the output tracking error exponentially converges to a residual ball, whose size can be made arbitrarily small. Moreover, in absence of uncertain nonlinearities, the parameter adaptation guarantees asymptotic output tracking.

2.3 Fault Model

We will now describe the fault model used for the present work. Since, we consider the fault accommodation problem for only leakage and contamination type of faults, we will focus on these two faults. But, other additive state faults and component faults can be dealt with similarly.

Leakage fault: A leakage fault is said to have occurred when, $\theta_4 \geq \theta_{4,max}$ such that it lies outside the convex region Ω_θ defined earlier and can no longer be passively compensated by the nominal controller. This causes significant deviation of the performance variable from its desired value. The fault is assumed to occur at an unknown time.

Remark 1: Under normal operating conditions, we neglect the leakage terms for the purpose of controller design, as its effect is relatively small on the system dynamics and can be compensated by a robust control law. But, when the fault occurs, the effect of the leakage term cannot be neglected anymore and it needs to be considered explicitly for controller design. Consequently, the nominal controller may not be able to guarantee the stability of the closed loop system or that the prespecified goals are met. For the present work, it will be assumed that the sensitivity of the detection algorithm ensures the fault is detected early enough such that system states remain bounded until it is detected. At this instant, the fault accommodation module is activated.

We shall model the leakage fault as,

$$f_l(x,t) = \theta_{4f}(x_3 - x_4) + \Delta_l(x,u,t) \tag{13}$$

where θ_{4f} represents the estimated coefficient of leakage after the occurrence of fault and $\Delta_l(x,u,t)$ is the error in modeling the leakage fault, which is bounded above by the constant δ_l.

Contamination fault: A contamination fault is said to have occurred when, $\theta_3 \geq \theta_{3,max}$ or $\theta_3 \leq \theta_{3,min}$ and results in significant deviation of the performance variable from its desired value.

Remark 2: The parameter θ_3 is a measure of the effective bulk modulus β_e of the system. Air can decrease the value of β_e, and water can increase it. Such a change in the effective bulk modulus ultimately affects the natural frequency of the system, which has serious impact on the performance of the closed loop system. Although, a robust control law can passively tolerate these variations to certain extent, excessive contamination can significantly degrade the performance of the system. It should be understood that the nominal controller is designed under the assumption that θ_3 lies in Ω_θ. Once θ_3 is outside this region, the nominal controller may not be able to guarantee the stability of the system.

After the occurrence of fault, we can rewrite the effective bulk modulus of system as,

$$\theta_{3f} = \theta_{3,min}(1 - \theta_f), \qquad \text{when} \quad \theta_{3f} \leq \theta_{3,min} \tag{14}$$

$$\theta_{3f} = \theta_{3,max}(1 + \theta_f), \qquad \text{when} \quad \theta_{3f} \geq \theta_{3,max} \tag{15}$$

i.e., the system model needs to be changed after the fault is detected for the purpose of controller design. As mentioned previously, such a structural change may destabilize the system. The nominal control law can take care of the parametric uncertainties, as long as it stays within Ω_θ. This is the motivation for splitting θ_{3f}. We only want to compensate for the parametric change which lies outside Ω_θ, without discarding the nominal controller. The fault compensation controller guarantees the system stability in spite of the structural change and ensures that the desired performance is achieved.

2.4 Objective

From equations (6-9), we see that the system model has parametric uncertainties, as well as uncertain nonlinearities. A robust FDD scheme which does not use the information about the structure of the uncertainties must use the worst-case scenario bound, which significantly reduces the fault sensitivity of the algorithm. Also, the nominal controller is designed based on certain predetermined performance criteria, and hence, it is desirable to keep this controller in loop. Now, we will state our two objectives:

(1) to detect incipient and small faults reliably in presence of various uncertainties and,
(2) to accommodate the fault by augmenting the nominal control law and restore the performance to acceptable limits.

3. FAULT DETECTION AND DIAGNOSIS

In this section, we present the main results of an adaptive robust technique for FDD of electro-hydraulic systems. The overall scheme is shown in fig 2.

3.1 State Fault Detection

An adaptive robust state reconstruction (ARSR) scheme is used for detecting additive state faults. The dynamics of the reconstructed states is given by,

$$\dot{\hat{x}}_2 = k_2(x_2 - \hat{x}_2) + \hat{\theta}_1(x_2 - \bar{A}x_3) + \hat{\theta}_2 \tag{16}$$

$$\dot{\hat{x}}_3 = k_3(x_3 - \hat{x}_3) + \hat{\theta}_3(-\bar{A}_1 x_2 + g_3 u) - \hat{\theta}_4(x_3 - x_4) - \hat{\theta}_5(x_3 - Pr) \tag{17}$$

$$\dot{\hat{x}}_4 = k_4(x_4 - \hat{x}_4) + \hat{\theta}_3(\bar{A}_2 x_2 - g_4 u) + \hat{\theta}_4(x_3 - x_4) - \hat{\theta}_6(x_4 - Pr) \tag{18}$$

where batch RLS algorithm is used to obtain the parameter estimate $\hat{\theta}_i$. The parameter estimation scheme will be described in the next section. We use state-estimation error $\tilde{x}_i = \hat{x}_i - x_i$ as residual for detecting state-faults. The threshold for the residual evaluation is chosen such that it upper bounds the uncertain nonlinearities, and incorporates the estimated parameters to reduce the bound on parametric uncertainties. This guarantees that no false alarms are set-off and the algorithm is robust to various uncertainties.

3.2 Component Fault Detection

Component faults are detected by comparing the estimated parameters with nominal parameter values obtained by off-line system identification. In practice, it is difficult to always satisfy the PE condition. Hence, we use batch RLS, with explicit monitoring of PE condition to ensure that parameters are updated only when the regressor is rich enough i.e., if $R(kT) = \int_{(k-1)T}^{kT} \Omega(\tau)\Omega^T(\tau)d\tau$, then the adaptation law is given by,

$$\hat{\theta}_{(\bullet)}(kT) = \begin{cases} R(kT)^{-1} \displaystyle\int_{(k-1)T}^{kT} \Omega(\tau)z(\tau)d\tau, & \\ \qquad\qquad \text{if} \quad R(kT) > \alpha(kT)I & \\ \hat{\theta}_{(\bullet)}((k-1)T), \quad \text{otherwise} & \end{cases} \tag{19}$$

Such an intelligent scheme significantly improves the reliability of parametric or component fault detection, and avoids false alarm. Once the parameter estimate crosses the preset threshold value, an alarm indicating the fault is generated.

3.3 Sensor Fault Detection

For sensor fault detection, we use adaptive robust observers for reconstructing the states. The estimation error is used as residual. The details of the observer design can be found in Garimella and Yao (2002). Due to space limitations, we will only state the main points in the design of nonlinear adaptive robust observer,

(1) Most of the observer design techniques assume an *a priori* known perfect model of the system and cannot handle parametric uncertainties and uncertain nonlinearities simultaneously. On the other hand, an adaptive robust observer design technique is robust to all the aforementioned modeling uncertainties. This makes it an ideal choice for estimating states of an electro-hydraulic system.
(2) The use of batch RLS reduces the extent of parametric variation and consequently, improves the sensitivity of the detection scheme.
(3) Discontinuous projection mapping ensures that the parameter and state estimates remain bounded.

It should be noted that in addition to fault detection, isolation and identification is also achieved under single-fault hypothesis, using the residual signals generated. Without fault isolation and identification, it would not be possible to develop any fault accommodation scheme.

4. FAULT ACCOMMODATION

In this section, we present some results on fault accommodation for leakage and contamination faults. These faults can be linearly parameterized in terms of unknown parameters with known structure. It should be noted that there is no restriction on the size of the fault, as it can grow unboundedly if the states associated with the known structure become unbounded.

The nominal control law is usually designed based on certain predetermined performance criteria. Hence, after the occurrence of the fault, it is desirable to keep the nominal controller in loop and augment it with a *fault compensation* controller which can counteract the effect of fault.

The basic idea of an adaptive robust fault accommodation scheme (ARFA) is to compensate for the fault by estimating the parameters in fault function, and compensate

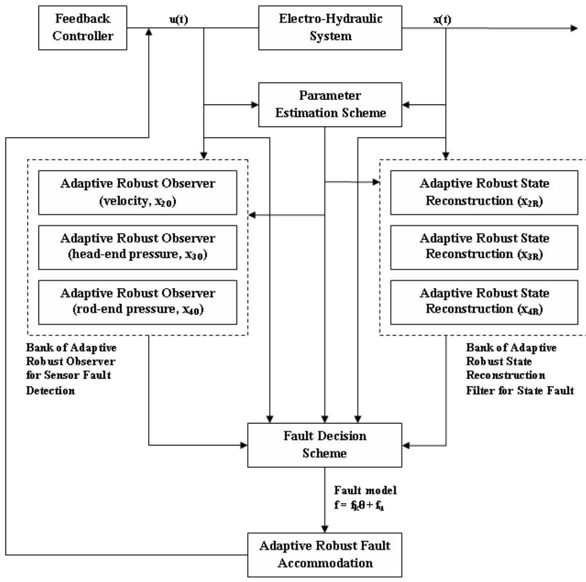

Fig. 2. An adaptive robust fault detection and accommodation scheme for electro-hydraulic systems

for the fault estimation mismatch by using a robust term in the fault compensation controller. For such purposes, it will be assumed that the extent of parametric uncertainties in the fault function is known. In practice, the extent will be decided by various constraints like the maximum allowable size of the fault which can be safely tolerated without shutting down the machine and statistical information available about the fault.

Theorem 2. For the system (6-9), if a leakage fault occurs, it can be accommodated by adding the following fault compensating control law u_R to the nominal control law u_N,

$$u_R = u_{Ra} + u_{Rs} \tag{20}$$

$$u_{Ra} = \frac{\hat{\theta}_{4f}}{\hat{\theta}_3} \frac{(1+\bar{A})}{(g_3+\bar{A}g_4)}(x_3 - x_4) \tag{21}$$

$$\dot{\hat{\theta}}_{4f} = -\gamma_{4f} w_3 z_3 (1+\bar{A})(x_3 - x_4) \tag{22}$$

where smooth projection, as defined in Yao and Tomizuka (1997) is used. Also, the robust part of the control law - u_{Rs}, is such that,

$$\begin{aligned} (a)\ & w_3 z_3[\theta_3(g_3+\bar{A}g_4)u_{Rs} - \tilde{\theta}_3(g_3+\bar{A}g_4)u_{Ra} \\ & +\tilde{\theta}_{4f}(1+\bar{A})(x_3-x_4) - (1+\bar{A})\Delta_l] \le \varepsilon_{4f} \end{aligned} \tag{23}$$

$$(b)\ z_3 u_{Rs} \le 0 \tag{24}$$

Theorem 3. For the system (6-9), if a contamination fault occurs, it can be accommodated by adding the following fault compensation control law u_R to the nominal control law u_N,

$$u_R = u_{Ra} + u_{Rs} \tag{25}$$

$$u_{Ra} = \frac{\hat{\theta}_f}{1-\hat{\theta}_f}\left(u_N - \bar{A}_1 x_2 \frac{(1+\bar{A}^2)}{g_3+\bar{A}g_4}\right) \tag{26}$$

$$\dot{\hat{\theta}}_f = -\gamma_f w_3 z_3 \{(g_3+\bar{A}g_4)(u_{Ra}+u_{Rs})$$

$$-\bar{A}_1(1+\bar{A}^2)x_2\} \tag{27}$$

where smooth projection is used and u_{Rs} is such that,

$$(a)\ w_3 z_3\big[(1-\theta_f)(g_3+\bar{A}g_4)u_{Rs} - \tilde{\theta}_f \bar{A}_1(1+\bar{A}^2)x_2$$
$$+\tilde{\theta}_f(g_3+\bar{A}g_4)(u_{Ra}+u_{Rs})\big] \le \varepsilon_{3f} \tag{28}$$
$$(b)\ z_3 u_{Rs} \le 0 \tag{29}$$

Remark 3: In the above stated theorems, we have used the direct ARC approach for fault accommodation, and hence, once the fault is detected, the update law for the fault function parameters are derived using a Lyapunov function based approach. This estimation scheme runs in parallel with the batch RLS for parameter estimation of the overall system. In indirect ARC approach, this redundant estimation scheme could be removed. But, for the present work, we shall stick to this approach.

5. RESULTS

In order to demonstrate the effectiveness of the proposed scheme, simulations using the swing-arm model of an electro-hydraulic robot arm are presented here. The model parameters used can be obtained from Yao et al. (2000b).

Leakage fault: The leakage fault is inserted at t=8 seconds. As seen from fig.3, the fault is detected at t=10 seconds, and the fault accommodation module is activated at the same instant. Fig.4 shows the position tracking error for swing arm. As expected, the error increases significantly after the occurrence of fault. But, once the FA module becomes active, the fault compensating controller reduces this error, as seen from fig.4.

Remark 4: It should come as no surprise that the parameters do not converge to their true value, as there is significant amount of unmodeled dynamics in the system. But, it should be noted that a projection type mapping ensures the parameter estimates do not drift away even in presence of modeling errors and disturbances.

Contamination fault: The contamination fault is introduced at t=8 seconds. The fault is detected at t=11 seconds, as seen in fig.5, and the alarm generated activates the FA module. Once again, there is noticeable degradation in performance after the occurrence of fault which is restored to acceptable limits after the FA module starts functioning, as seen from fig.6.

6. CONCLUSION

In this paper, we presented a nonlinear model-based adaptive robust approach for fault detection, diagnosis and accommodation in electro-hydraulic systems. The main contribution of the present work is the development of a systematic framework to deal with various uncertainties encountered in a typical electro-hydraulic system. Adaptation is used to reduce the large extent of parametric uncertainties which makes the detection algorithm more sensitive without compromising its robustness against modeling errors. The FDD scheme developed here can detect state, sensor and component faults.

A novel fault accommodation scheme for leakage and contamination faults is also presented in this paper. The

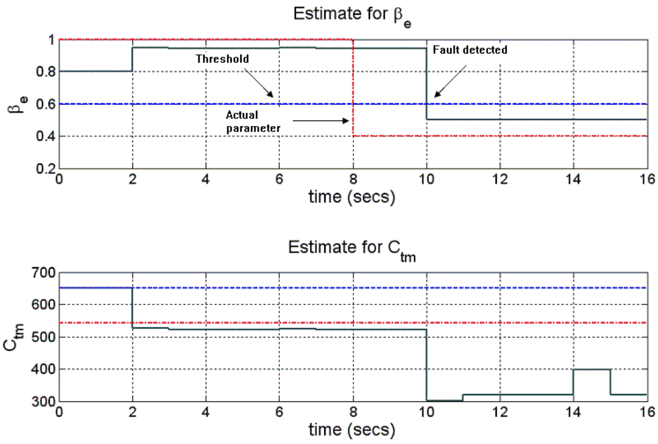

Fig. 3. Detection of contamination fault

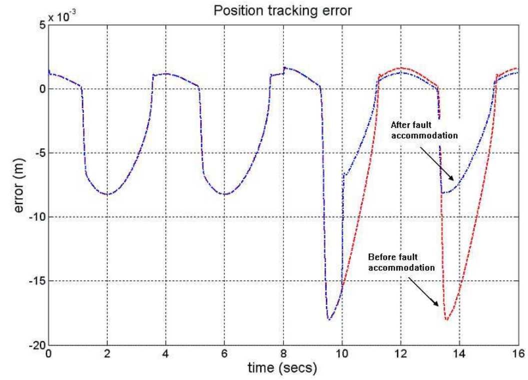

Fig. 4. Fault accommodation for contamination fault

accommodation technique uses the structural information of the faults to augment the nominal controller with a fault compensation controller. As seen from the simulation results, the accommodation scheme successfully compensates the fault.

REFERENCES

M. Blanke, M. Kinnaert, J. Lunze, and M. Staroswiecki. *Diagnosis and Fault-Tolerant Control.* Springer, 2003.

P. M. Frank. Online fault detection in uncertain nonlinear systems using diagnostic observers: a survey. *International Journal of System Science*, 25(12):2129–2154, 1994.

P. Garimella and Bin Yao. Nonlinear adaptive robust observer for velocity estimation of hydraulic cylinders using pressure measurement only. In *the ASME International Mechanical Engineers Congress and Exposition (IMECE), IMECE02-DSC-32077*, pages 1–10, 2002. The revised full version has been submitted to *the IEEE/ASME Transaction on Mechatronics.*

P. Garimella and Bin Yao. Model based fault detection of an electro-hydraulic cylinder. In *American Control Conference*, pages 484–489, 2005.

H. Hammouri, P. Kabore, S. Othman, and J. Biston. Failure diagnosis and nonlinear observer. application to a hydraulic process. *Journal of the Franklin Institute*, 339:455–478, 2002.

M. Munchhof. *Model-Based Fault Detection for a Hydraulic Servo Axis.* PhD thesis, Technischen Univesitat Darmstadt, Darmstadt, Germany, 2006.

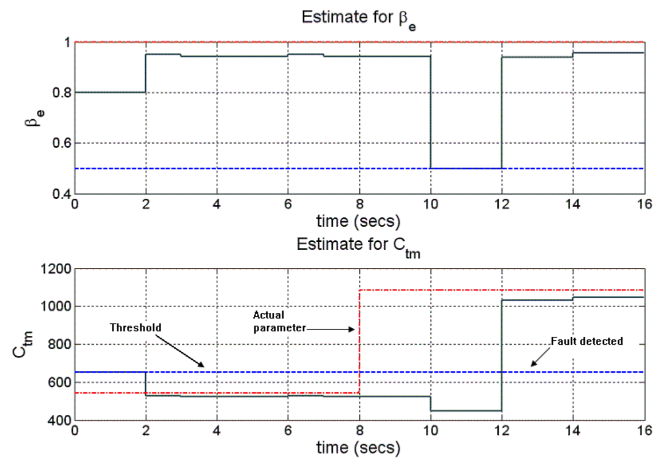

Fig. 5. Detection of leakage fault

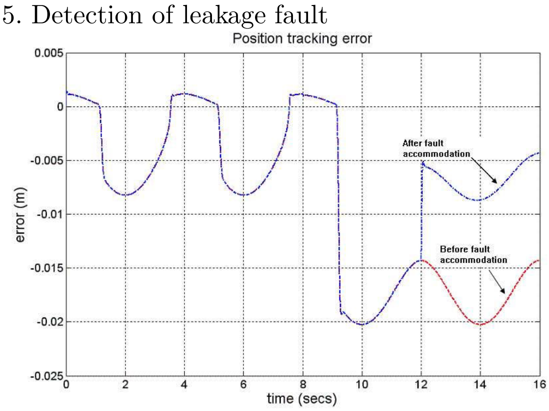

Fig. 6. Fault accommodation for leakage fault

G. J. Preston, D. N. Shields, and S. Daley. Application of a robust nonlinear fault detection observer to a hydraulic system. In *Proceedings of UKACC International Conference Control, U.K.*, pages 1484–1489, 1996.

H. Z. Tan and N. Sepehri. Parametric fault diagnosis for electrohydraulic cylinder drive units. *IEEE Transactions on Industrial Electronics*, 49(1):96–106, February 2002.

Bin Yao and M. Tomizuka. Adaptive robust control of SISO nonlinear systems in a semi-strict feedback form. *Automatica*, 33(5):893–900, 1997. (Part of the paper appeared in Proc. of 1995 American Control Conference, pp2500-2505, Seattle).

Bin Yao, F. Bu, J. Reedy, and G.T.C. Chiu. Adaptive robust control of single-rod hydraulic actuators: theory and experiments. *IEEE/ASME Trans. on Mechatronics*, 5(1):79–91, 2000a.

Bin Yao, Fanping Bu, and George T. C. Chiu. Nonlinear adaptive robust control of electro-hydraulic systems driven by double-rod actuators. *International Journal of Control*, 74(8):761–775, 2000b.

D. Yu. Fault diagnosis for a hydraulic drive using a parameter estimation method. *Control Engineering Practice*, 5(9):1283–1291, 1997.

Appendix A. PROOF OF THEOREM 1

After the occurrence of fault, the dynamics of $z_3 = P_L - \alpha_2$ is given by,

$$\dot z_3 = -\bar A_1\theta_3(1+\bar A^2)x_2 + \theta_3(g_3+\bar A g_4)u - \Delta_l(1+\bar A) - \theta_{4f}(1+\bar A)(x_3-x_4) + \Delta_2 - \bar A\Delta_3 - \dot\alpha_2 \qquad (A.1)$$

$$\Rightarrow \dot z_3 = \dot z_{3n} + \theta_3(g_3+\bar A g_4)u_R - \theta_{4f}(1+\bar A)(x_3-x_4) - \Delta_l(1+\bar A) \qquad (A.2)$$

$$\Rightarrow \dot z_3 = \dot z_{3n} + \hat\theta_3(g_3+\bar A g_4)u_{Ra} - \hat\theta_{4f}(1+\bar A)(x_3-x_4) + \theta_3(g_3+\bar A g_4)u_{Rs} + \tilde\theta_{4f}(1+\bar A)(x_3-x_4) - \tilde\theta_3(g_3+\bar A g_4)u_{Ra} - \Delta_l(1+\bar A) \qquad (A.3)$$

where $\dot z_{3n}$ is the derivative of the z_3 error variable when the system is healthy. The derivative of the p.d function V_N after the occurrence of fault is given by,

$$\dot V_N = \sum_{i=1}^{3}\frac{\partial V_N}{\partial z_i}(F_{z_i}(z,\theta) + \theta^T G_{z_i}(z)u_N + \Delta_{z_i}) + w_3 z_3\left(\hat\theta_3(g_3+\bar A g_4)u_{Ra} - \hat\theta_{4f}(1+\bar A)(x_3-x_4)\right) + w_3 z_3[\theta_3(g_3+\bar A g_4)u_{Rs} - \tilde\theta_3(g_3+\bar A g_4)u_{Ra} + \tilde\theta_{4f}(1+\bar A)(x_3-x_4) - (1+\bar A)\Delta_l] \qquad (A.4)$$

where, F_z, G_z and Δ_z represent the system model in the transformed coordinates, and has the same form for healthy as well as the faulty system. Now, choosing u_R as given by (20-22), we obtain the following result,

$$\dot V_N \le -\lambda_N V_N + \varepsilon' \qquad (A.5)$$

where, $\varepsilon' = \varepsilon + \varepsilon_{4f}$. Now, from fact 1 and equation (12), we can conclude that the performance of the faulty closed loop system has been restored to acceptable limits. Additionally, in presence of parametric uncertainties only, the adaptation law for faulty parameters guarantee asymptotic output tracking.

Appendix B. PROOF OF THEOREM 2

As mentioned previously, after the occurrence of contamination fault, we will θ_{3f} as $\theta_{3f} = \theta_{3,min}(1-\theta_f)$. With this in mind, the dynamics of z_3 can be rewritten as,

$$\dot z_3 = -\bar A_1\theta_{3,min}(1-\theta_f)(1+\bar A^2)x_2 + \theta_{3,min}(1-\theta_f)(g_3+\bar A g_4)u + \Delta_2 - \bar A\Delta_3 - \dot\alpha_2 \qquad (B.1)$$

$$\Rightarrow \dot z_3 = \dot z_{3n} - \theta_f\theta_{3,min}[-\bar A_1 x_2(1+\bar A^2) + (g_3+\bar A g_4)u_N] + (1-\theta_f)\theta_{3,min}(g_3+\bar A g_4)u_R \qquad (B.2)$$

$$\Rightarrow \dot z_3 = \dot z_{3n} + (1-\theta_f)\theta_{3,min}(g_3+\bar A g_4)u_{Ra} - \theta_f\theta_{3,min}[-\bar A_1(1+\bar A)x_2 + (g_3+\bar A g_4)u_N] + \theta_{3,min}(\tilde\theta_f\{(g_3+\bar A g_4)(u_{Ra}+u_N) - \bar A_1(1+\bar A^2)x_2\} + (1-\theta_f)(g_3+\bar A g_4)u_{Rs}) \qquad (B.3)$$

Now, choosing u_R as given by equations (25-29), we get

$$\dot V_N \le -\lambda_N V_N + \varepsilon'' \qquad (B.4)$$

where, $\varepsilon'' = \varepsilon + \varepsilon_{3f}$, which implies that the performance has been restored to acceptable limits and as in the previous case, in presence of parametric uncertainties only, the adaptation law for faulty parameters guarantee asymptotic output tracking.