Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Modelling and H_∞ low order control of web handling systems with a pendulum dancer

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Abstract: The plant considered in this paper is an unwinding - winding system with pendulum dancer mechanism for elastic webs. First, a non linear mathematical model of the web process line is presented. A state space model is deduced which helps in the synthesis of the H_∞ controllers around the set points given by the reference signals. Industrials systems typically use decentralized PI controllers. In this paper, performances of low order H_∞ controllers for such systems are analyzed.

Index Terms—Pendulum dancer mechanism, industrial winder control, H_∞ robust control, fixed order control.

1. INTRODUCTION

The systems transporting metal foils, paper, plastic films or fabric are very common in the industry. The main goal is to increase the web transport velocity as much as possible while controlling the tension of the web. The main concern is the coupling existing between web velocity and tension. There are many sources of velocity disturbance: roller non-circularity, web sliding… Due to the elasticity of the web, these disturbances are transmitted to the web tension. Several studies concerning web handling control [1][2][3] generally use PID, fuzzy or neural approaches. Multivariable control strategies recently have been proposed for industrial metal transport systems [4][5], and the decoupling of web tension and velocity with H_∞ robust control was presented in [6][7].

An important number of web process lines use dancer mechanism for attenuating web tension disturbances. There has been recently a growing interest in studying dancer behavior, in comparing active and passive dancers [8], and in improving active dancers due to their ability to attenuate disturbances [9][10]. All those studies deals with linear dancers, nevertheless pendulum dancers are also very common in industry. Their mechanical description and behavior is more complex as they are rotating around a revolute joint, thus changing the direction of web velocity and tension with time.

Here we studied a winding plant test bench close to industrial situations, and having a pendulum dancer (see figure 1). A non-linear model of this plant and its description is presented. An industrial control structure with PI controllers is used to track the web tension and velocity references and to decouple from each other. Nevertheless, the obtained performances are sensitive to parameters changes. Therefore, a robust H_∞ controller will be synthesized. For industrial implementation purpose, this controller has to be low order.

2. PLANT MODEL

The model of a web transport system is built using the model of a web tension between two consecutive rolls and the dynamical model of each roll.

A. Web tension calculation

Modeling of web transport systems is based on three laws:

- Hooke's law which introduces web elasticity.
- Coulomb's law describes web tension variations due to friction and the contact between web and roll.
- The Mass Conservation law which describes coupling between web velocity and web tension.

These laws allow the calculation of web tension between two rolls.

1) *Hooke's law*: the tension T of an elastic web is function of the strain ε:

$$T = ES\varepsilon = E\,S\,\frac{L - L_0}{L_0} \tag{1}$$

where ε is the web strain, E the Young modulus, S the web section, L the web length under stress and L_0 the web length without stress.

2) *Coulomb's law*: the study of a web tension on a roll can be considered as a problem of friction between solids [11]. On the roll, the web tension is constant on a sticking zone.

3) *The Mass* C*onservation law*: we consider an element of the web of length $l = l_0\ (1+\varepsilon)$ of density ρ, under unidirectional stresses. The cross-section can be considered constant. According to the law of mass conservation, the mass of the web remains constant between the state without stress and the state under stress:

$$dm = \rho\, S\, l = \rho_0\, S\, l_0 \Rightarrow \frac{\rho}{\rho_0} = \frac{1}{1+\varepsilon} \tag{2}$$

4) *Tension-velocity relationship*: the continuity equation, applied to the web [11], gives:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho V)}{\partial x} = 0 \tag{3}$$

By integrating on the variable x from 0 to L, taking into account the equation (2), we obtain [12]:

$$\frac{d}{dt}\left(\frac{L_2}{1+\varepsilon_2}\right) = \frac{V_2}{1+\varepsilon_1} - \frac{V_3}{1+\varepsilon_2} \tag{4}$$

This relation can be simplified by deriving the left term [12], using the approximation $\varepsilon \ll 1$:

$$L_2 \frac{dT_2}{dt} = ES(V_3 - V_2) + T_1 V_2 - T_2(2V_2 - V_3) \tag{5}$$

B. Web velocity calculation

The velocity of roll k is obtained from the torque balance:

$$\frac{d(J_k \frac{V_k}{R_k})}{dt} = R_k(T_k - T_{k-1}) + K_k U_k + C_{fk} \tag{6}$$

where $K_k U_k$ is the motor torque (if the roll is driven) and C_{fk} is the friction torque which is the sum of the static friction C_{fks} and of the dynamic friction torque C_{fkd}. The dynamic friction is modeled as modeled as $C_{fkd} = f_k\omega_k$, with f_k an experimental constant. Note that unwinder and winder inertia J_k and Radius R_k are time dependent and vary substantially during process processing.

2.2 Pendulum dancer modelling

A dancer is a free roll (roll R_4) at the end of an arm (Fig 1); the other end of the arm is connected to the machine frame using a revolute joint O, therefore the dancer moves along an arc of circle. Its utility is double: it imposes the web tension (in steady state operation) and filters the variations of tension mechanically. The web tension is fixed by an actuator which is an air jack in most industrial situations and also in this experiment. The dancer movement, actuated by a jack (see figure 1) and by the web tensions, is measured by the angle α.

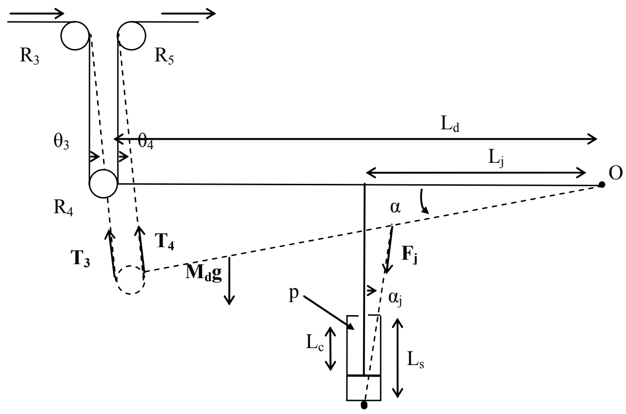

Fig. 1. Pendulum dancer in the web handling process; dancer and jack parametrization, with air pressure in the upper side of the jack.

The dancer is defined as the union of the arm and the free roll R_4. The dancer mechanism is characterized by the following parameters (fig. 1): L_d is the dancer arm length, J_d is the rotational inertia of the dancer (arm and free roll), L_j is the distance between the jack and the revolute joint, M_d is the mass of the dancer, L_{gd} the distance between the dancer center of mass and the revolute joint.

The dancer dynamic behavior is described by the fundamental principle of dynamics applied to the dancer (fig. 2)

$$-T_3(L_d + R_4)\cos(\alpha - \theta_3) - T_4(L_d - R_4)\cos(\alpha - \theta_4) + M_d g L_{gd}\cos\alpha$$
$$+ F_j L_j \cos(\alpha + \alpha_j) - C_{ffO} - C_{ffB} - C_{ff4} = J_d \frac{d^2\alpha}{dt^2} \tag{7}$$

Where C_{ffO}, C_{ffB} and C_{ff4} are the friction torque on the rotational joints. The jack is parameterized (fig. 1) by a fixed length L_m (mount and rod), a stroke L_s, a length of the jack chamber L_c under air pressure p (upper side of the jack in order to produce a resultant force downward) and S the surface under pressure of the jack piston. The minimum length of the jack is L_s+L_m, the maximum length is $2L_s+L_m$, and the length in operation is $2L_s+L_m-L_c$.The jack is actuated with adjustable pressure p. This pressure is fixed in operation to a nominal value related to the desired web tension. In operation, the nominal pressure is calculated in order to get a dancer in an horizontal position ($\alpha=0$) when the web tensions T_3 and T_4 are equals to the desired nominal tension T_n. Using dancer equilibrium equation (eq. 7.) with $T_3=T_4 = T_n$ and $\alpha=0$, the nominal jack force F_{jn} should be

$$F_{jn} = \frac{T_n(L_d + R_4) + T_n(L_d - R_4) - M_d g L_{gd}}{L_j} \tag{8}$$

And therefore the air pressure inside the jack is fixed to

$$p = \frac{F_{jn}}{S} = \frac{2T_n L_d - M_d g L_{gd}}{L_j S} \tag{9}$$

The jack has a stiffness and viscous dynamic behavior represented respectively by K and N. The jack force F_j actually applied to the dancer corresponds to:

$$\left\| \vec{F_j} \right\| = K\ x + N\dot{x} + pS \tag{10}$$

where x corresponds to the jack piston linear displacement in the cylinder.

$$x = 2L_s - L_c + L_m - \left[2L_s - L_c + L_m - 2L_j \sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\right] a\cos\alpha_j \quad (11)$$

The jack stiffness is calculated using the classical relation [22]:

$$K = \frac{BS^2}{V} \quad (12)$$

where B is the bulk modulus, and the V the volume of air inside the jack.

The bulk modulus for gases B is given by

$B = \gamma p$, where γ is called the adiabatic index ($\gamma = 1.4$ for dry air at operating conditions) [23] and p the pressure. By replacing in the equation, it can be seen that the stiffness K is related to the jack nominal force:

$$K = \frac{BS}{L_C} = \frac{\gamma p S}{L_C} = \frac{\gamma F_{jn}}{L_C} \quad (13)$$

The jack resultant force on the dancer can thus be written

$$\left\| \vec{F}_j \right\| = \frac{\gamma F_{jn} x}{L_c} + N\dot{x} + F_{jn} \quad (14)$$

The movement of the dancer induces a change in the length of the web span upstream and downstream the dancer. When α changes, the length L_3 changes from its initial length L_{30} to

$$L_3 = \sqrt{\left[L_{30} + (L_d + R_4)\sin\alpha\right]^2 + \left[2(L_d + R_4)\sin^2\frac{\alpha}{2}\right]^2} \quad (15)$$

as well as L_4 changes to

$$L_4 = \sqrt{\left[L_{40} + (L_d - R_4)\sin\alpha\right]^2 + \left[2(L_d - R_4)\sin^2\frac{\alpha}{2}\right]^2} \quad (16)$$

The computation of web tension T_3 upstream and T_4 downstream the dancer must take into account the change in the position of the dancer (roll R_4). The hypothesis classically used [19] is that the web speed is equal to the roll velocity when it is in contact with the roll; according to this hypothesis, there is no sliding between the web and the dancer roll.

The dancer roll tangential velocity is not the same when the web comes in contact with the roll on the left side, and when the web leaves the roll on the right side. The roll linear velocity in a Galilean reference frame on the left side is called V_{4L}, and the velocity on the right side is called V_{4R}.

The rotation of the dancer roll R_4 creates a difference in tangential velocity:

$V_{4L} = V_4 + (L_d + R_4)\dot{\alpha}$ and

$$V_{4R} = V_4 - (L_d - R_4)\dot{\alpha} \quad (17)$$

And the tension velocity relations (eq. 5) used to calculate the web tension upstream and downstream the dancer become:

$\frac{d}{dt}\left(\frac{L_3}{1+\varepsilon_3}\right) = \frac{V_3}{1+\varepsilon_2} - \frac{V_{4L}}{1+\varepsilon_3}$ and

$$\frac{d}{dt}\left(\frac{L_4}{1+\varepsilon_4}\right) = \frac{V_{4R}}{1+\varepsilon_3} - \frac{V_5}{1+\varepsilon_4} \quad (18)$$

2.3 Modelling of the two motors setup with pendulum dancer

The model studied here stands for a working bench representing an industrial installation using a pendulum dancer (fig. 2). It consists of two main rolls which are each actuated by a brushless motor: the unwinder at the beginning of the process, and the winder at the end. Therefore, the radius of the unwinder R_u and the radius of the winder R_w are changing with time. The web is then passing through several free rolls (of radius R), which are fixed to the frame of the test bench, except the roll of the dancer (roll 4). The web velocity on the roll i is referred as V_i, where i is the number of the roll (i from 1 to 7). T_i stands for the web tension of the web span between roll i and $i+1$, which length is L_i. Web tensions are experimentally measured with load cells on roll 2 and 6.

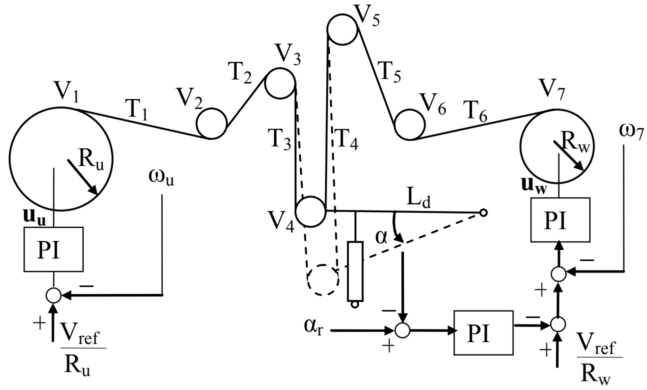

Fig. 2. Industrial controllers using PI control.

A new model of a web transport system is built from the equations of web tension behavior between two consecutive rolls and the equations describing the velocity of each roll, by taking into account the material properties and the friction phenomena (static and viscous frictions).

The obtained non-linear model is programmed in Matlab/Simulink environment.

A linear model can be deduced by linearization of the non-linear model around a setting point. There are several ways to choose the setting point, and several parameters have to be fixed. The linearization is made for a nominal web tension T_0, a nominal web velocity V_0 and unwinder and winder radii R_{u0} and R_{w0}. The linearised relation obtained is

$$L_2 \frac{dT_2}{dt} \cong (ES + T_0)(V_3 - V_2) + V_0(T_1 - T_2) \quad (18)$$

The behavior of the dancer is described by the fundamental principle of dynamics applied to the dancer, and linearised around the setting point (α=0, thus $\theta_3=\theta_4=\alpha_j=0$), and the viscous behavior N of the jack is neglected as it cannot be easily measured and fixed in industrial jacks. The jack displacement x

$$x = 2L_s - L_c + L_m - \left[2L_s - L_c + L_m - 2L_j \sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\right] a\cos\alpha_j \quad (19)$$

becomes $x = -L_j \alpha$

The jack behavior is linearised around a working point of $L_j = L_d/2$ and a nominal tension T_0

Therefore the nominal jack force F_{jn} fixed by the air pressure and changing when T_n is changing

$F_{jn} = \dfrac{2T_n L_d - M_d g L_{gd}}{L_j}$ becomes

$$F_{jn0} = \frac{2T_0 L_d - M_d g L_{gd}}{L_j} \qquad (20)$$

The resultant jack force Fj

$$\left\| \vec{F_j} \right\| = F_{jn}\left(1 - \frac{\gamma L_j \alpha}{L_c}\right) \qquad (21)$$

must be linearised because it is the product of a command (jack force reference) and of a state variable α. Therefore, F_{jn} is linearised in F_{jn0} which is the value of F_{jn} for the tension set point T_0; the jack force becomes

$$\left\| \vec{F_j} \right\| = F_{jn} - \frac{F_{jn0} \gamma L_j \alpha}{L_c} \qquad (22)$$

F_{jn0} is computed only once at the beginning of each simulation and doesn't change, while F_{jn} may change during the simulation as T_n is changing during the simulation. The linearised dancer equation is:

$$-T_3(L_d + R_4) - T_4(L_d - R_4) + M_d g L_{gd} + L_j F_{jn} - \frac{\gamma L_j^2 F_{jn0} \alpha}{L_c} = J_d \frac{d\dot{\alpha}}{dt^2} \qquad (23)$$

The tension velocity relationship (see eq. 5) upstream and downstream the dancer, has to be linearised while taking into account the variation of the length between two rolls:

$$L_3 \frac{dT_3}{dt} = E_0(V_4 - V_3) + V_0(T_2 - T_3) + (L_d + R_4)\dot{\alpha} E_0 \qquad (24)$$

And

$$L_4 \frac{dT_4}{dt} = E_0(V_5 - V_4) + V_0(T_3 - T_4) + (L_d - R_4)\dot{\alpha} E_0 \qquad (25)$$

The friction torque C_{fk} applied to roll k is the sum of the static friction C_{sk} and the viscous friction $C_{vk} = f_k \omega_k$. The static friction can be neglected as high velocity.

The state space representation can be expressed as:

$$\frac{dX}{dt} = A(t)X + B(t)U \qquad (26)$$

$$Y = C\,X$$

Practically, the controller parameters have been determined according to the classical procedure employed in industrial systems. The PI controllers for velocity control of the unwinder and of the winder have been designed independently, considering that the winder and the unwinder are not connected to the rest of the web handling line. The last PI controller is designed to regulate the tension on the entire web handling system. The controller parameters are tuned one the PI regulating velocity have been fixed. The behavior of the system is simulated with tensions references varying from 5 to 15 N and web velocities references from 1.66 m/s to 3.2 m/s (100 to 200 m/min).

A positive web velocity is observed during the first seconds corresponding to the first tension step. The other perturbation at 80s corresponds of the web tension to velocity coupling.

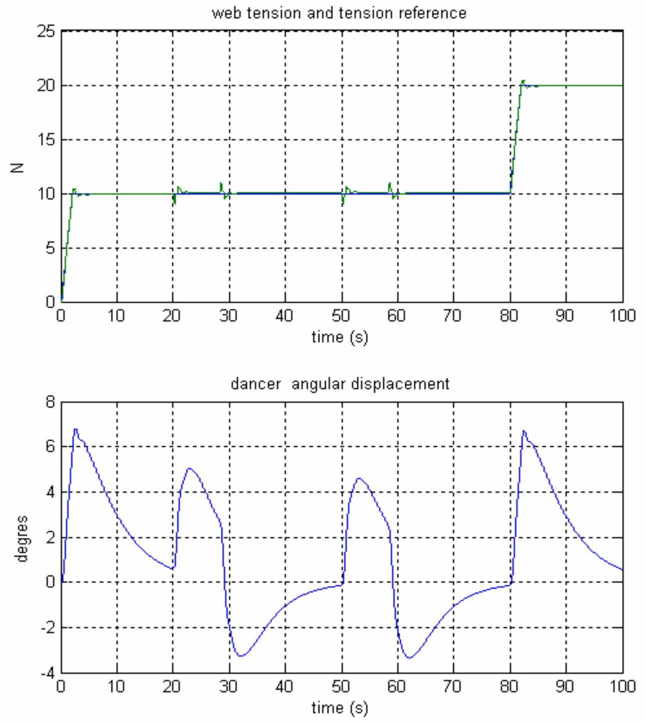

Fig 3. : web tension and dancer angular displacement

These controllers have been synthesized for a jack stiffness value of K=0.1. Anyway, K is subjected to changes with time, maintenance operations, changes in mechanical design…The robustness of the PI controller to K variations is illustrated in the following pictures.

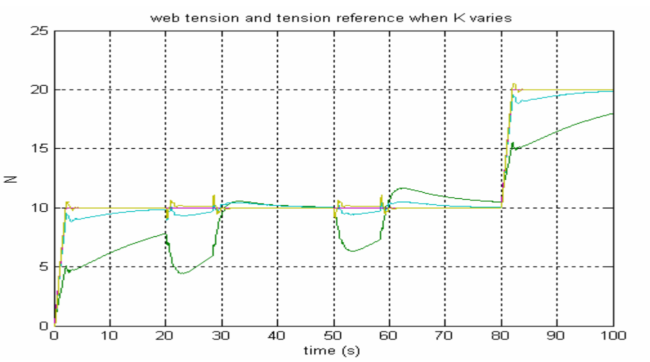

Fig 4. : Web tension with varying value of K (0.01 to 10)

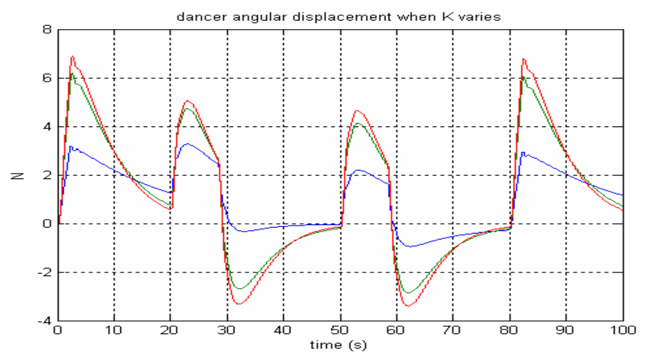

Fig 5. : Angular displacement with varying value of K (0.01 to 10)

It can be observed that the velocity response is rather robust to stiffness variations, while tension response is more sensitive to variations of K.

5. REDUCED ORDER CENTRALIZED CONTROL DESIGN

Robust H_∞ control is a powerful tool to synthesize multivariable controllers with interesting properties of robustness and disturbances rejection. The synthesis should be done using a linear model corresponding to the starting phase, i.e. an empty roller at the winder. The starting phase is very important: if a problem occurs in this phase, most likely, the roller will be badly wounded.

A major drawback of standard H_∞ design algorithms is the high order of the computed controllers. Indeed, the order of the controller is typically equal to the order of the plant plus the order of the frequency weighting functions. With current model reduction techniques, the controller order cannot always be reduced a posteriori while preserving stability and a satisfying performance. In any case, it is an additional computational burden. It is therefore highly relevant, especially for industrial applications, to develop design algorithms producing fixed-order controllers from the outset. After more than four decades of intensive research efforts, it turns out that, deceptively, efficient software for designing fixed-order controllers is still not readily available. The underlying mathematical problem seems to be difficult since fixed-order controller design can be formulated as a typically nonsmooth (nondifferentiable) affine problem in the nonconvex cone of stable matrices (or, equivalently, stable polynomials). However, recent progress in nonlinear variational analysis, tailored at solving H_∞ fixed-order control problems [14] paved the way for the development of nonsmooth optimization algorithms based on quasi-Newton (BFGS), bundling and gradient sampling. A MATLAB software called HIFOO (H-Infinity Fixed-Order Optimization) has been released in late 2005 [15], and uses local optimization techniques. This software, used in this work, has also been used for the reduced order controller calculation in web handling systems with load cells [17].

We synthesized H_∞ controllers with output weighting and model matching (S/KS/T weighting scheme with model matching). The weighting functions W_p, W_u, and W_t appear in the closed loop transfer matrix:

$$T_{zr} := \begin{bmatrix} W_p(M_0 - T_{yr}) \\ W_u K\ S \\ W_t T \end{bmatrix} \tag{27}$$

where S is the sensitivity function :

$$S = (I + GK)^{-1} \tag{28}$$

and T is the complementary sensitivity function :

$$T = I - S \tag{29}$$

The weighting function W_p has a high gain at low frequency in order to reject low frequency disturbances. The form of W_p is as following [23] :

$$W_p(s) = \frac{\dfrac{s}{M} + \omega_B}{s + \omega_B \varepsilon_0} \tag{30}$$

where M is the maximum peak magnitude of S,

$$\| S \|_\infty \leq M \tag{31}$$

ω_B is the required frequency bandwidth ε_0 is the steady-state error allowed.

The weighting function W_u is used to avoid large control signals and the weighting function W_t increases the roll-off at high frequencies.

The controller K is calculated using the "γ-iteration" [22] :

$$\| \gamma . T_{zr} \|_\infty = \sup_\omega \sigma_{\max}(\gamma . T_{zr}(j\omega)) < 1 \tag{32}$$

For calculating the H_∞ controller for the second subsystem (dancer and winder), the adopted control strategy is represented in Fig. 6. The controller C_2 is used to control the dancer angle, and a velocity loop is also used, like the industrial strategy given in Fig. 3. This controller has to be calculated in order to minimize the H_∞ norm of the transfer function between the inputs and the weighted outputs, by using the HIFOO software. The synthesis problem consists now to compute a low order H_∞ output feedback controller for the augmented subsystem G_2 (including the dancer and winder modelling, weighting filters, integrators). If the controller C_2 is chosen as a zero-order controller, we obtain an H_∞ based PI controller.

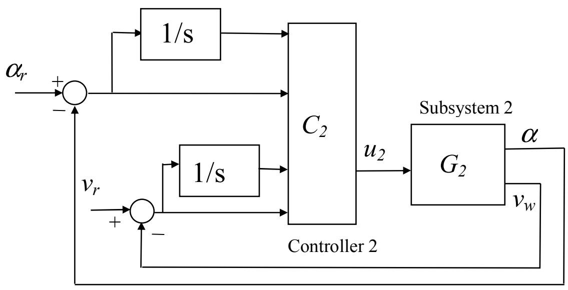

Fig. 6 : control strategy of subsystem 2

The first subsystem, including only the unwinder, is velocity controlled and therefore imposes the web velocity of the plant.

It has been observed that the coupling between web velocity to web tension is significantly reduced by using a controller with a order higher than two. The figure 7 gives the web tension results for different values of K, by using an H_∞ controller of order 3. An improved robustness to K variations has been achieved with this control strategy.

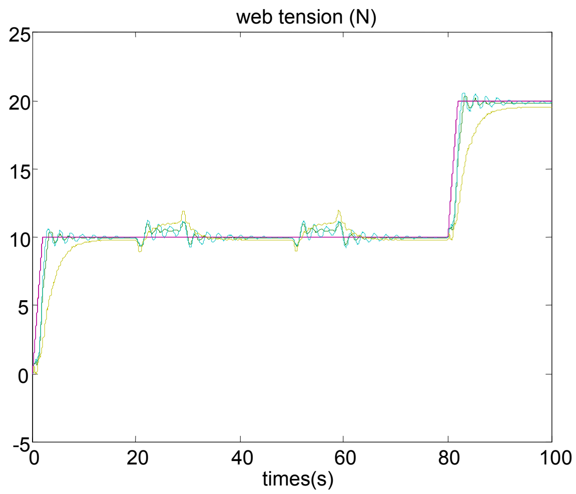

Fig. 7 . Simulation results for different values of K

6. CONCLUSION

The non-linear model of a system including a winder and a pendulum dancer mechanism gives the web tension and velocity and the dancer displacements. During velocity changes, dancer displacements must be well controlled. Unfortunately, web elasticity strongly couples web velocity and tension.

A new control strategy for such system, a fixed order H_∞ controller, has been synthesized. This control strategy is compared to an industrial approach using PI controllers. The robust H_∞ controller reduces coupling between web velocity and web tension and offers improved performances.

ACKNOWLEDGEMENT

This work was supported in part by the French Ministry of Research through the project "Winding and high velocity handling of flexible webs" (ERT Enroulement, contract number 01 B 0395).

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