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# Adaptive Formation Control using Artificial Potentials for Euler-Lagrange Agents ⋆

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**Abstract:** In this paper, we present a formation control strategy for a group of agents modeled as Euler-Lagrange systems. The formation is achieved by means of a desired kinematic model generated by artificial potentials. The system uncertainties are compensated by binary adaptive control which combines the good transient properties and robustness of Sliding Mode Control with the desirable steady-state properties of parameter adaptive systems. Furthermore, an important advantage with respect to sliding mode control is that the proposed controller generates a continuous signal so that control chattering is avoided. A simplified version of the controller is also proposed, which does not require the knowledge of the velocities of the neighboring vehicles.

Keywords: formation control; binary adaptive control; artificial potentials; multiagent systems.

## 1. INTRODUCTION

Formation control is an important field in multiagent coordinated control. Basically, a group of autonomous agents are required to accomplish desired tasks cooperatively, maintaining a specific pattern. Multiagent systems have several advantages over a single agent, such as, greater flexibility, robustness, efficiency and redundancy (Chen and Wang [2005]).

The formation control system should be able to perform several functions, such as formation keeping, inter-agent collision avoidance, navigation and obstacle avoidance. To this end, different strategies have been proposed in the literature, e.g., leader-follower (Desai et al. [1988], Das et al. [2002] e Shao et al. [2007]), behavior-based (Balch and Arkin [1998], Lawton et al. [2003]), virtual structure (Tan [1996]) and artificial potentials (Leonard and Fiorelli [2001]).

In control application, and in particular in formation control, it is important to achieve robustness and stability in the presence of unknown disturbances and parameter uncertainties. To address these issues, Gazi [2005] considered a control strategy based on artificial potentials and sliding mode control (SMC). Other papers proposed the utilization of adaptive control for group formation (Hadaegh et al. [1998]), (Wong et al. [2001]), (Semsar and Khorasani [2006]).

Adaptive control systems with conventional update law may exhibit undesirable transient behavior. Moreover, the basic adaptive systems is nonrobust with respect to unmodeled dynamics or external disturbances (Rohrs et al. [1990]) and modifications of the basic adaptation law may be required (Ioannou and Sun [1996]). On the other hand, SMC can generate undesirable high frequency switching of the control signal known as control chattering.

In this paper, we try to circumvent the above mentioned drawbacks while preserving the advantages od adaptive control and SMC. This is accomplished by utilizing an adaptive control scheme based on the *binary model reference adaptive control* (B-MRAC) proposed by Hsu and Costa [1990]. Basically, the B-MRAC consists of a high gain gradient adaptive law with parameter projection to maintain the adaptive parameter vector within some closed finite ball in the parameter space. Stemming from the binary control theory introduced in (Emelyanov [1987]), the B-MRAC combines the good transients of Variable Structure (VS) adaptive control systems (Hsu et al. [1994]) with the desirable steady-state properties of parameters adaptive controllers. Essentially, this is achieved by exploiting the good properties of a well known adaptation scheme when the adaptation gain is sufficiently high. As the gain is increased, the controller tends to behave as a sliding mode controller.

*Notation and Terminology*: $\lambda_M(\cdot)(\lambda_m(\cdot))$ denotes the largest (smallest) eigenvalue of a matrix. ISS (IOpS) means Input-to-State-Stable (Input-to-Output-practically-Stable) (Jiang et al. [1994]). The Euclidean norm of a vector $v$ and the corresponding induced norm of a matrix $A$ are denoted by $|v|$ and $|A|$, respectively. For any measurable function function $u : \mathbb{R}_+ \to \mathbb{R}^m$, $\|u\|$ denotes ess $\sup\{|u(t)|, t \geq 0\}$.

The paper is organized as follows. In Section II, we present the dynamic model of the agents and a desired kinematic model derived from a given potential function. In Section III, we develop stable binary adaptive control schemes in two possible scenarios. First, the velocities of the neighbors are assumed available for each agent. Then, this is relaxed

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to the partially decentralized case when only the individual velocities are available. In Section IV, simulations are presented to illustrate the performance of the proposed strategies. Finally, conclusions are presented in Section V.

## 2. MATHEMATICAL MODEL AND PROBLEM STATEMENT

Consider a multi-agent system consisting of N vehicles fully actuated and modeled by the following dynamics

$$H^i(y^i)\ddot{y}^i + C^i(\dot{y}^i, y^i)\dot{y}^i = \tau^i, \quad i = 1, ..., N. \tag{1}$$

where $H^i \in \mathbb{R}^{n\times n}$ is the inertia matrix, $C^i \in \mathbb{R}^{n\times n}$ represent the centripetal and Coriolis forces and $\tau^i \in \mathbb{R}^n$ is a vector of independent control forces. The model corresponds to a class of Euler-Lagrange system which has the following properties: 1) for all $i$ the inertia matrix $H^i$ is positive definite and satisfies $h^i_m\|x\|^2 \le x^T H^i(y_i)x \le h^i_M\|x\|^2$, with positive constant $h^i_m$ and $h^i_M$; 2) the inertia matrix $H^i$ is differentiable; 3)the matrix $C^i$ is chosen based on the Christoffel symbols so that $\dot{H}^i - 2C^i$ is skew-symmetric and thus,

$$v^T\left(\dot{H}^i - 2C^i\right)v = 0 \quad \forall v \in \mathbb{R}^n \tag{2}$$

We consider the particular flocking problem which consists in designing control laws so that all N agents stop at a desired configuration defined by given relative positions of the agents, i.e., $\dot{y}^i(t) \to 0$ and $\left|y^i(t) - y^j(t)\right| \to d^{ij}$ for given constants $d^{ij} \ge 0$, $i, j = 1, ..., N$.

In what follows, we assume that the model is uncertain, i.e., the parameters of model (1) are known only nominally.

### *2.1 Artificial potential function approach*

In this section we derive a kinematic model to be followed by the agents based on an artificial potential function (APF) which is specified by the designer to generate interaction rules among the group members, and also the environment, through attraction/repulsion forces. To this end, the APF will be composed of two parts, one concerning the interagent interactions and the other taking care of the interactions with the environment. The former includes functions of the distance between a pair of agents and allows generating a desired geometric pattern with a prescribed inter-vehicle distance between group components. The latter can be used to define the navigation function of the formation, e.g., virtual leaders tracking (Leonard and Fiorelli [2001]), obstacle avoidance (Gazi and Pasino [2004]) and target pursuing (Yao et al. [2006]).

Following the idea of potential function, the motion of each agent is required to obey the following first order kinematic model

$$\dot{y}^i = -\nabla_{y^i} J(y) = -g^i(y) \tag{3}$$

where $y^i \in \mathbb{R}^n$ is the position of the i-th agent, $y^T = [y^{1T}, \ldots, y^{NT}]$ and $J : \mathbb{R}^{n\times N} \times \mathbb{R}^n \to \mathbb{R}$ is an artificial potential function (APF), assumed twice differentiable.

### *2.2 The sliding function*

Let us define the sliding function [1] $s^i$ as

$$s^i = \dot{y}^i + g^i(y) \tag{4}$$

The control objective is to make $s^i(t) \to 0$ as $t \to \infty$, so that each agent obeys the desirable kinematic model (3) asymptotically. This could be achieved using discontinuous control functions to reach the sliding manifold $s^i = 0$ in finite time. However, in order to avoid high frequency control switching (chattering phenomena), we will design continuous control laws in what follows.

## 3. BINARY ADAPTIVE CONTROL OF MULTI-AGENT SYSTEMS

The next step consists in the design of the control signals $\tau^i$ such that the sliding functions tend to zero in spite of the system uncertainties.

The derivative of (4) is given by

$$\dot{s}^i = \ddot{y}^i + \dot{g}^i(y, \dot{y}) \tag{5}$$

Premultiplying (5) by $H^i$ and considering (1), one gets

$$H^i\dot{s}^i + C^i s^i = \tau^i + H^i\dot{g}^i + C^i g^i \tag{6}$$

Assume that one can write the linear parametrization $Y^i\theta^{*i} = -(H^i\dot{g}^i + C^i g^i)$, where $Y^i$ is a regressor matrix composed of known functions of $y$ and $\dot{y}$ and $\theta^{i*} \in \mathbb{R}^m$ parameter vector ($m$ is number of matching parameters) and substituting in (6) we obtain is a vector of constant parameters. Such vector is assumed uncertain in the sense that it is known only nominally. For simplicity, we will assume that the nominal value is zero [2]. Now, equation (6) can be written as

$$H^i\dot{s}^i + C^i s^i = \tau^i - Y^i\theta^{i*} \tag{7}$$

Then, the following control law is proposed

$$\tau^i = Y^i\theta^i - K^i_D s^i \tag{8}$$

and introducing the parameter mismatch $\tilde{\theta}^i = \theta^i - \theta^{i*}$, one rewrite (7) as

$$H^i\dot{s}^i + C^i s^i = Y^i\tilde{\theta}^i - K^i_D s^i \tag{9}$$

which is a well known form in the adaptive control theory of robot manipulators (Slotine and Li [1991]).

### *3.1 Centralized binary adaptive control*

In this section we assume that the position and velocity of all neighbors of each agent are available for control. This scenario corresponds to a centralized control system. The following adaptation law based on binary model reference adaptive control (B-MRAC), (Hsu and Costa [1987] - see Appendix A) is proposed

$$\dot{\theta}^i = -\sigma\theta^i - \Gamma^i Y^{i^T} s^i \tag{10}$$

[1] which corresponds to the switching function in SMC.
[2] If this was not the case, it would suffice to add to $\tau^i$ a nominal control of the form $Y^i\theta^i_{nom}$

The $\sigma$-factor, also called projection factor (PF), is defined as:

$$\sigma = \begin{cases} 0 & ; if \left|\theta^i\right| < M_\theta^i \ or \ \sigma_{eq} < 0 \\ \sigma_{eq} & ; if \left|\theta^i\right| \geq M_\theta^i \ and \ \sigma_{eq} \geq 0 \end{cases} \tag{11}$$

where $\sigma_{eq} = -\theta^{i^T}\Gamma^i Y^{i^T} s/|\theta^i|^2$and $M_\theta^i(> |\theta^*|)$ is a constant. Let $B_\theta^i = \{\theta^i : |\theta^i| \leq M_\theta^i\}$. Assuming that $|\theta^i(0)| \in B_\theta^i$, the projection factor acts as follows. If at any time $\theta^i(t)$ is on the sphere $|\theta^i| = M_\theta^i$ and the term $-\Gamma^i Y^{i^T} s^i$ points outwards such sphere, the update vector is projected onto the tangent plane of the sphere; alternatively, if it points inwards, the $\sigma$-factor is equal zero and $\theta^i(t)$ moves to the interior of the sphere. Then, it is easy to prove that the closed ball $B_\theta^i$ is invariant (Hsu and Costa [1987]), i.e., $\forall t \geq 0, \quad |\theta^i(t)| \in B_\theta^i$ .

For stability analysis, consider system (9), (10), (11), and assume $\theta^i(0) \in B_\theta^i$ with constant $M_\theta^i \leq \left|\theta^{i^*}\right|$. We can show that $s^i \to 0$ as $t \to \infty$. To this end, consider the following candidate Lyapunov function

$$V^i = \frac{1}{2} s^{i^T} H^i s^i + \tilde{\theta}^{i^T} \Gamma^{i^{-1}} \tilde{\theta}^i \tag{12}$$

For simplicity of analysis and without loss of generality, we define $\Gamma^i = \gamma I$. Using the skew-symmetry property, (2) holds and the time-derivative of (12) is thus given by

$$\dot{V} = -s^{i^T} K_D^i s^i - \frac{\sigma}{\gamma}(\tilde{\theta}^i + \theta^{i^*})^T \tilde{\theta}^i \tag{13}$$

The last term of (13) is nonpositive (see Hsu and Costa [1987] or Ioannou and Sun [1996]) and therefore

$$\dot{V}^i \leq -s^{i^T} K_D^i s^i \leq 0 \tag{14}$$

One can show using Barbalat's lemma that $s^i \to 0$ as $t \to \infty$. Therefore, each agent obeys the desired dynamics asymptotically.

### *3.2 Decentralized binary adaptive control*

We now consider a partially decentralized control system scenario where the velocities of the neighbors are not available for each agent.

We first define $w^i = H^i \dot{g}^i$, where $w^i \in \mathbb{R}^n$, and assume that $\left|w^i\right| \leq \bar{w}$, where $\bar{w}$ is a positive constant. The stability analysis of the closed loop system will show that for each $\bar{w}$, there exists a ball of initial conditions for which the bound $\bar{w}$ holds. Moreover, the radius of the ball can be made arbitrarily large for increasing $\bar{w}$. As mentioned, we consider the term $H^i \dot{g}^i$ as a disturbance that is to be rejected by the control action. In this way, we define $Y^i \theta^{i^*} = -\left[C^i g^i\right]$ and from (6), we obtain

$$H^i \dot{s}^i + C^i s^i = \tau^i - Y^i \theta^{i^*} + w^i \tag{15}$$

The control law for this case is proposed as

$$\tau^i = Y^i \theta^i - \bar{w} sat(\gamma_w s^i) - K_D^i s^i \tag{16}$$

where for $v \in \mathbb{R}^n, \quad sat(v) = [sat(v_1), ..., sat(v_n)]^T$ and $\gamma_w$ is a positive constant. Substituting this control law in (15), we have

$$H^i \dot{s}^i + C^i s^i = Y^i \tilde{\theta}^i - K_D^i s^i + w^i - \bar{w} sat(\gamma_w s^i) \tag{17}$$

*Stability analysis* The model of the overall multiagent system can be regarded as being composed of the agent subsystems (17) interconnected by the dynamic subsystems corresponding to

$$w^i = H^i \dot{g}^i(y, \dot{y}) \tag{18}$$

Figure 1 represents this in terms of a block diagram. For

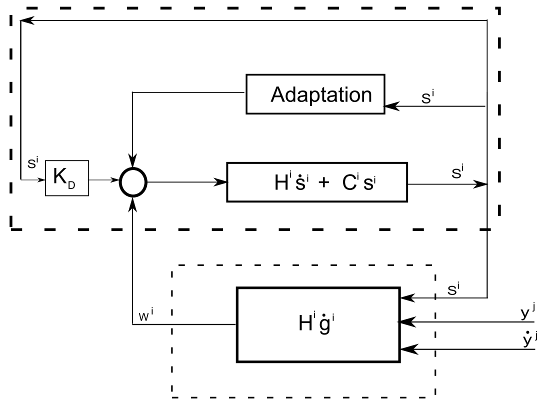


Fig. 1. Block diagram for the $i$-th agent

the upper block, the Lyapunov function is chosen as

$$V^i = \frac{1}{2} s^{i^T} H^i s^i + \frac{1}{2} \tilde{\theta}^{i^T} \Gamma^{i^{-1}} \tilde{\theta}^i \tag{19}$$

For simplicity of analysis and without loss of generality, we define $\Gamma^i = \gamma I$. Then, we can write the inequality

$$V^i \leq h_M^i \left|s^i\right|^2 + \frac{1}{2\gamma} \left|(\theta^i - \theta^{i^*})\right|^2 \tag{20}$$

Since $\left|\theta^i(t)\right| \leq M_\theta^i$, we obtain

$$V^i \leq h_M^i \left|s^i\right|^2 + \frac{2M_\theta^{i^2}}{\gamma} \tag{21}$$

The time-derivative of (19) is given by

$$\dot{V}^i = -s^{i^T} K_D^i s^i - \frac{\sigma}{\gamma} \theta^{i^T} \tilde{\theta}^i + s^{i^T} w^i(t) - s^{i^T} \bar{w} sat(\gamma_w s^i) \tag{22}$$

Since the second term is nonpositive, we can write

$$\dot{V}^i \leq -\lambda_m(K_D^i) \left|s^i\right|^2 + \left|s_r^i\right| \left|w^i\right| - \gamma_w \left|s_r^i\right|^2 \tag{23}$$

where $s_r^i$ is a reduced vector composed of non saturated elements. The last two terms have a maximum value at $\frac{w^{i^2}}{4\gamma_w}$. Thus, we have

$$\dot{V}^i \leq -\lambda_m(K_D^i) \left|s^i\right|^2 + \frac{w^{i^2}}{4\gamma_w} \tag{24}$$

Manipulating (21) and substituting in (24)

$$\dot{V}^i \leq -\lambda_1 V^i + \lambda_1 \frac{2M_\theta^{i^2}}{\gamma} + \frac{w^{i^2}}{4\gamma_w} \tag{25}$$

where $\lambda_1 = \lambda_m(K_D^i)/h_M^i$. By using a comparison lemma, one can show that

$$V_i \leq c_1 e^{-\lambda_1 t} V(0) + \int e^{-\lambda_1(t-\xi)} u d\xi \tag{26}$$

where $u = \lambda_1 \frac{2M_\theta^{i^2}}{\gamma} + \frac{w^{i^2}}{4\gamma_w}$

Using the inequality (21) and upper bounding the integral term of (26) in terms of $||w^i||$ we get, after some algebraic manipulations,

$$\left|s^i(t)\right|^2 \leq c_1 e^{-\lambda_1 t}\left|s^i(0)\right|^2 + k_1 \frac{M_\theta^{i^2}}{\gamma} + k_2 \frac{\|w^i\|^2}{\gamma_w} \tag{27}$$

$\forall t \geq 0$, where $c_1, k_1, k_2$ are positive constants. This results in the following bound

$$|s(t)| \leq \beta_1(|s(0)|, t) + \gamma_w^{-1/2} k_3 \|w\| + \gamma^{-1} d_1. \tag{28}$$

where $s^T(t) = [s^{1T}(t), \ldots, s^{NT}(t)]$, $w^T = [w^{1T}, \ldots, w^{NT}]$, $k_3$ and $d_1$ are positive constants and $\beta_1$ is a class $KL$ function (Jiang et al. [1994]). Now, concatenating the equations (18) for $i = 1, ..., N$, we have

$$w = H\dot{g} = H\frac{\partial g(y)}{\partial y}\dot{y} \tag{29}$$

$$\dot{y} = -g(y) + s. \tag{30}$$

Equations (29) and (30) can be interpreted as a nonlinear dynamic system with input $s$ and output $w$. Assuming that this subsystem is ISS, one has

$$|w(t)| \leq \beta_2(|y(0)|, t) + \eta_2 \|s(t)\| \tag{31}$$

where $\beta_2$ is class $KL$ and $\eta_2$ is a positive constant. From a small gain theorem for ISS systems (Jiang et al. [1994]), equations (28) and (31) impliy that system (17)(29-30) is (locally) IOpS with null inputs, i.e., practically asymptotically stable, with residual set of size $O(1/\sqrt{\gamma})$. As a consequence $\left|y^i - y^j\right| - d^{ij} \to O(1/\sqrt{\gamma})$.

Noting that the bounds in (28) and (31) are independent of $\bar{w}$, we can further conclude that the system is semi-globally stable with respect to the parameter $\bar{w}$.

This result holds under the ISS assumption for system (29-30). It is possible to show that for the class of quadratic potential function this assumption is valid. Moreover, based on the results of simulations, we conjecture that this assumption could be shown to hold for more general class of potential functions.

## 4. SIMULATIONS

### 4.1 Illustration example

We consider the control of a group of three vehicles modeled as point masses moving on a plane. The objective is to achieve an ultimate triangular pattern. The dynamics of each agent is described by

$$M^i \ddot{y}^i + D^i \dot{y}^i = \tau^i \tag{32}$$

where $y^i \in \mathbb{R}^2$ is the position of the vehicles, $M^i$ and $D^i$ represent scalar mass and damping constants, respectively. Substituting (4) and (5) in (32) we obtain

$$M^i \dot{s}^i + D^i s^i = \tau^i + M^i \dot{g}^i + D^i g^i \tag{33}$$

For the centralized control (Sec. 3.1), the control law is given by (8), where $Y^i = -[\dot{g}^i \quad g^i]$ and $\theta^{i*} = [M^i \quad D^i]^T$. In the partially decentralized case (Sec. 3.2), the control is given by (16), where $Y^i = -[g^i]$, $\theta^i = D^i$ and $w^i(t) = M^i \dot{g}^i$.

We have used the potential function given in Gazi [2005]

$$J(y) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left[\frac{a^{ij}}{2}\left|y^i - y^j\right|^2 + \frac{b^{ij} c^{ij}}{2} exp\left(-\frac{\left|y^i - y^j\right|^2}{c^{ij}}\right)\right]$$

where $a^{ij}$ is an attraction constant and $b^{ij}$ is a repulsion constant. The parameter $c^{ij}$ is defined by the following

$$c^{ij} = \frac{d^{ij^2}}{log\left(\frac{b^{ij}}{a^{ij}}\right)}. \tag{34}$$

The constants $d^{ij}$ give the desired inter-vehicular distances. The agent parameters are identical and given by , $\forall i$, $M^i = 50$ and $D^i = 5$. The initial velocities were set equal to zero. For the APF parameters, we have chosen $\forall i, j$, $a^{ij} = 0.5$, $b^{ij} = 50$, $d^{ij} = 10$ and thus $c^{ij} = 14.4765$. For the centralized control case, the performance of three controllers were compared: a controller with fixed nominal parameters ($\tau^i = Y^i \theta^i_{nom} - K^i_D s^i$), the adaptive controller with the conventional update law, and the binary adaptive controller. The parameters for the first controller were chosen as $\theta^i_{nom} = [10 \quad 1]^T$, $K^i_D = 1$. For the second and third controllers, $\theta^i(0) = [10 \quad 1]^T$ (initialized at nominal values), $K^i_D = 1$ and $\Gamma^i = 20 I_2$. In particular, for the binary adaptive controller, $M_\theta = 1.1\left|\theta^{i*}\right|$. Figure 2

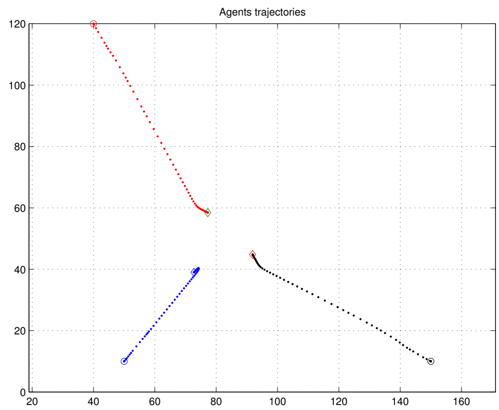


Fig. 2. Trajectories of the agents forming a triangle

shows the trajectories of the three agents moving towards their prescribed interagent distances and the desired formation pattern. The initial positions are represented by circles. From Figure 3, we can see that the formation error ($E_f = \sum(|y^i - y^j| - d^{ij})$ and the magnitude of the sliding functions, represented by $\sum|s^i|$, tend to zero faster in the adaptive cases. In Figure 4, we can note that the binary

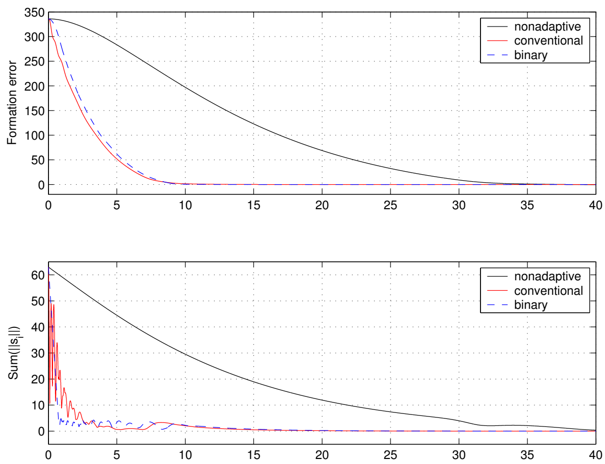


Fig. 3. Formation error and $\sum\left|s^i\right|$

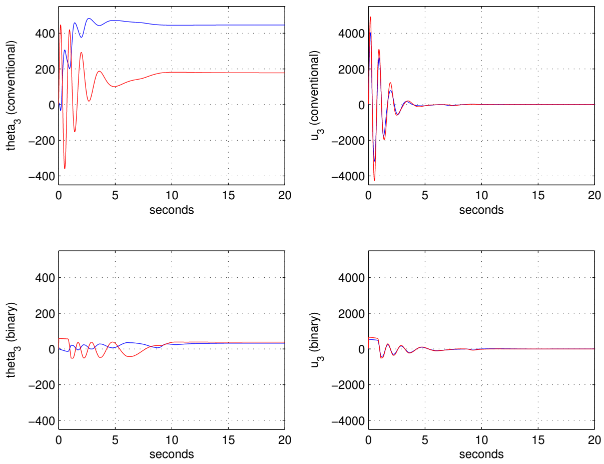


Fig. 4. $\theta^3(t)$ and $u^3(t)$

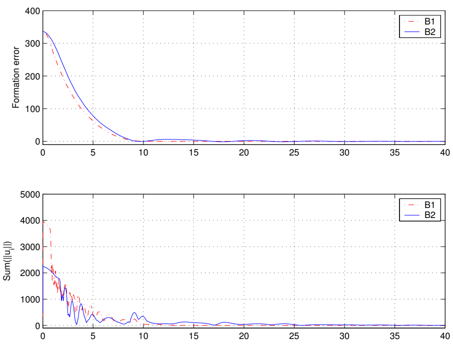


Fig. 5. Formation error and $\sum |u^i|$

adaptation leads to smoother adaptive parameters (only shown for $i = 3$) compared to the conventional adaptation. Since the adaptive parameters are kept within prescribed limits by ($|\theta^i(t)| \leq M_\theta^i$), significantly less control action is required for binary adaptive control. In the second set of simulations, two binary adaptive controllers were compared: controller B1 (centralized) and controller B2 (decentralized). For controller B1, the system parameters and initial conditions were set as in the previous case. For controller B2, the parameters were set to: $K_D^i = 1$, $\gamma^i = \gamma_w^i = 20$, $M_\theta = 1.1\left|\theta^{i*}\right|$, $\bar{w} = 10$. In this case $\theta^i$ (scalar) was initialized at a nominal value $\theta^i(0) = 1$. According to Figure 5, the error formation response for both controllers B1 and B2 are close and the control actions are of similar magnitude.

### 4.2 Obstacle avoidance and other formation shapes

The obstacle avoidance problem is addressed here by considering the obstacles as fixed vehicles. The potential function for the obstacles consists only of the repulsion part. For different shapes of formation, it is sufficient to set different values for $d^{ij}$. As an example, for a regular hexagonal pattern (Figure 6), $\mathbf{D}_{ij} = \{d^{ij}\}$ can be defined as

$$\mathbf{D}_{ij} = \begin{bmatrix} 0 & 1 & 1 & \sqrt{3} & 2 & \sqrt{3} \\ 1 & 0 & \sqrt{3} & 1 & \sqrt{3} & 2 \\ 1 & \sqrt{3} & 0 & 2 & \sqrt{3} & 1 \\ \sqrt{3} & 1 & 2 & 0 & 1 & \sqrt{3} \\ 2 & \sqrt{3} & \sqrt{3} & 1 & 0 & 1 \\ \sqrt{3} & 2 & 1 & \sqrt{3} & 1 & 0 \end{bmatrix} * d \qquad (35)$$

Figure 7 shows the vehicles forming a regular hexagon with obstacle avoidance. A triangular formation for six agents is shown in Figure 8.

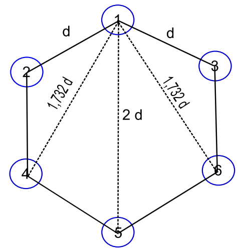


Fig. 6. Regular hexagonal pattern

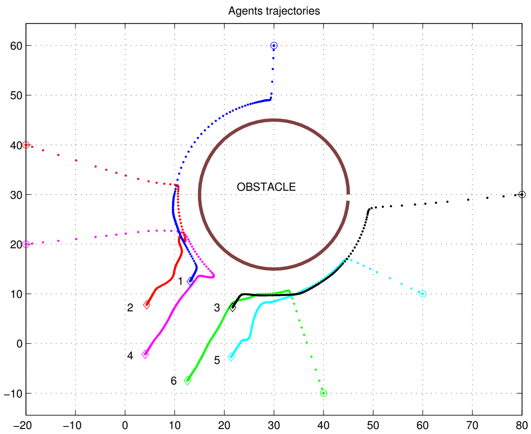


Fig. 7. Agents forming a regular hexagon

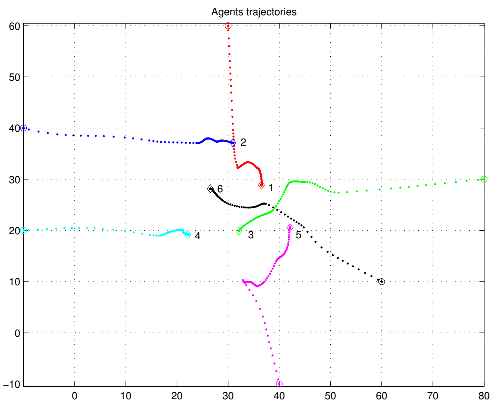


Fig. 8. Trajectories of the six agents forming a triangle

It should be noticed however that potential functions may have local minima problems and, depending on the initial conditions, the desired pattern may not be reached. This is an issue for further research.

## 5. CONCLUSIONS

We have proposed a framework to design formation control for a group of Euler-Lagrange agents. The system uncertainties are dealt with using binary adaptive control, which can guarantee robustness and chattering avoidance since it delivers continuous control signals. Artificial potential functions (APF) were used to generate a desired a convergent geometric pattern with a prescribed interagent distance. A simplified version of the controller, which does not require the knowledge of the velocities of the neighbors for each agent, was shown to be possible with semiglobal stability property. Simulations confirm satisfactory results. Future work include the stability analysis for general potential functions and extension to nonholonomic agents. Other issues include responses to typical real-world situations such as range measurement dropouts, communication delays etc.

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## Appendix A. BINARY MODEL REFERENCE ADAPTIVE CONTROL

The well known error equation of MRAC is of the form

$$\dot{e} = Ae - \bar{b}(u + {\theta^*}^T\omega) \tag{A.1}$$
$$e_o = y - y_m = h^T e$$
$$u = \theta^T \omega$$

where $e \in \mathbb{R}^{3n-2}$ is the state error vector, $n$ is the order of the plant, $u$ is the input, $\theta \in \mathbb{R}^{2n}$ is the adjustable parameter vector, $\theta^*$ is the model matching parameter vector, $\omega \in \mathbb{R}^{2n}$ is the regressor vector, $y$ is the plant output, $y_m$ is the model output, $e_o$ is the output or tracking error, $\bar{b} = (\theta^*_{2n})^{-1}b$, $(A, b, h)$ is an appropriate nonminimal realization of the reference model transfer function assumed SPR.

The gradient adaptation law with a $\sigma$-modification (Ioannou and Kokotovic [1984]) is given by

$$\dot{\theta} = -\sigma\theta - \gamma e_o \omega, \quad \sigma > 0, \tag{A.2}$$

In VS adaptive control, according to Hsu et al. [1994] the input $u$ can be

$$u = -f(\omega)sgn(e_o), \quad f(\omega) > \left|{\theta^*}^T\omega\right| \tag{A.3}$$

For instance, $u = -M_\theta\,|\omega|\,sgn(e_o), \quad M_\theta > |\theta^*|$

A binary version of (A.3) is given by Emelyanov [1987] as follows

$$u = M_\theta\,|\omega|\,\mu(t), \tag{A.4}$$

$$\dot{\mu}(t) = \begin{cases} -\alpha sgn(e_o) & ; for\, |\mu(t)| \le 1, \\ -\beta\mu(t) & ; for\, |\mu(t)| > 1, \quad |\mu(t_o)| \le 1, \; t > t_o \end{cases} \tag{A.5}$$

where $\alpha$ and $\beta$ are positive constants and $t_o$ is the initial time. It can be shown that all such solutions satisfy $|\mu(t)| \le 1, \forall t > t_o$ and, moreover, when $\alpha \to \infty$, (A.5) becomes the bang-bang law $\mu = -sgn(e_o)$. Thus, the binary controller (A.4) and (A.5) tends to the VS law (A.3) as $\alpha \to \infty$, in some sense.

It was proved in Hsu and Costa [1990] that a B-MRAC with predictable and uniform transient behavior can be derived from the M-RAC by using a projection factor (PF) and by (essentially) increasing the speed of adaptation, while keeping the adjustable parameter vector $\theta$ inside some finite ball of appropriate radius. The PF is given by (11) where $\sigma_{eq} = -\gamma e_o \theta^T \omega / |\theta|^2$.

The B-MRAC has excellent adaptation properties for large enough $\gamma$. This results from the fact if $M_\theta > |\theta^*|$, then $|e(t)|^2$ tends exponentially fast to some residual value of order $O(1/\gamma)$. The foregoing properties were proved in Hsu and Costa [1990].