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Switched IMM-EV Algorithms for State Estimation of Some Jump Markov Systems

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Abstract : We consider state estimation for a class of jump Markov linear discrete-time systems. For this, we present an algorithm employing switches among two interacting multiple-model extended-Viterbi (IMM-EV) estimators. The models we adopt for describing the systems can be used in problems such as the tracking of targets capable of abrupt maneuvers and fault detection of systems subject to possible component failures. A maneuver detection scheme, and a method for detecting maneuver termination are integrated into the proposed algorithm. Both methods determine when switches between two IMM-EV algorithms have to be invoked. A numerical example illustrates that the proposed algorithm can be an improvement to several known algorithms. *Copyright © 2008 IFAC*

Keywords: state estimation, jump Markov systems, multiple models, maneuver detection, interacting multiple-model extended-Viterbi (IMM-EV) estimators.

1. INTRODUCTION

This paper is concerned with state estimation for a class of jump Markov linear systems shown in Tugnait (1982), Bar-Shalom and Li (1993). For dealing with the problem, the interacting multiple-model (IMM) algorithm of Blom and Bar-Shalom (1988) has been a popular scheme. Besides, the reweighed IMM (RIMM) algorithm of Johnston and Krishnamurthy (2001), and some of the interacting multiple-model extended-Viterbi (IMM-EV) algorithms of Ho (2003), Ho and Chen (2006) have been proposed. In contrast, we introduce an approach herein using switched IMM-EV algorithms as a viable alternative (in terms of performance improvement) to the aforementioned methods.

The novelty of the proposed approach is that it switches between two IMM-EV algorithms in an adaptive manner. Specifically, the proposed approach utilizes an IMM-EV algorithm for identifying the most likely model path for state estimation if no abrupt system changes take place, and employs an IMM-EV algorithm otherwise which can yield superior performance in state estimation during change occurrences. We use a constant-velocity model which is correct for no change occurrences, and acceleration models for covering the system dynamics. Like the models of Willsky and Jones (1976), the models employed herein can be used to study fault detection and target tracking. We adapt the detection scheme of Bar-Shalom and Birmiwal (1982) and then incorporate it into the proposed approach. Assuming no abrupt changes occur, an appropriate window length is determined so that the fading memory average of innovations from the estimator based on the most probable model identified can approximately serve as a chi-square detector for maneuver detection. When a maneuver is detected, we utilize the IMM-EV which can yield superior performance in state estimation during change occurrences from the maneuver estimated onset time to the estimated time when the maneuver is terminated. During the stage of an assumed change occurrence, a maneuver termination is declared if the constant velocity model is identified as the most likely model for some consecutive time step.

This paper is organized as follows: In Section 2, we briefly describe a class of jump Markov models. In Section 3, we present the approach. In Section 4, we illustrate that the proposed approach can achieve improvements to several schemes in the literature. We conclude this paper in Section 5.

2. MODEL DESCRIPTION

We consider a class of jump Markov linear systems, which can be described by n possible models at each sampling with state and measurement equations given by

$$\mathbf{x}_{k+1} = \mathbf{A}_k^j \mathbf{x}_k + \mathbf{\Gamma}_k^j \boldsymbol{\upsilon}_k^j \quad j = 1,...,n\ ,\ k = 0,1,2,... \tag{1}$$

and

$$\mathbf{z}_k = \mathbf{H}_k^j \mathbf{x}_k + \boldsymbol{\omega}_k^j\ . \tag{2}$$

where $\boldsymbol{\upsilon}_k^j$ and $\boldsymbol{\omega}_k^j$ are independent zero-mean white Gaussian processes; the covariance matrices of process noises are $\mathbf{Q}_k^j = E[(\mathbf{\Gamma}_k^j \boldsymbol{\upsilon}_k^j)(\mathbf{\Gamma}_k^j \boldsymbol{\upsilon}_k^j)^T]$ and the covariance matrices of measurement noises are $\mathbf{R}_k^j = E[\boldsymbol{\omega}_k^j (\boldsymbol{\omega}_k^j)^T]$. The initial state $\mathbf{x}_0$ is assumed to be independent of $\boldsymbol{\upsilon}_k^j$ and $\boldsymbol{\omega}_k^j$. Moreover, we describe the model transition process using a discrete-time and first-order n-state Markov chain with fixed state transition probabilities p_{ij} satisfying $\sum_{j=1}^{n} p_{ij} = 1$, for each $i = 1,...,n$.

The IMM-EV(m) algorithms are summarized in the Appendix A where m denotes the number of most probable model paths at any given time step.

3. THE PROPOSED APPROACH

In this section, we present an approach which switches between the IMM-EV($m = 1$) and the IMM-EV($m = 2$). The reason for this is that as illustrated in earlier works by Ho (2003), Ho and Chen (2006), the IMM-EV($m = 1$) can identify the basic model efficiently and perform better during normal conditions while the IMM-EV($m = 2$) can yield superior performance in state estimation during change occurrences. We adapt the maneuver detection scheme given by Bar-Shalom and Birmiwal (1982) and devise a maneuver termination method to determine switching conditions. For ease of reference, the IMM-EV algorithms are summarized in the Appendix A.

To detect a maneuver, we utilize a fading memory average of the innovations from the estimator based on the model whose model index is determined by eqn. (A6) of of the IMM-EV($m = 1$) as follows:

$$\varepsilon_k = \sum_{i=k-d}^{k} \alpha^{k-i} (\mathbf{v}_i^{j_q})^H (\mathbf{S}_i^{j_q})^{-1} \mathbf{v}_i^{j_q} \tag{3}$$

where $k - d > k_c + 1$ with k_c denoting the most recent past time step at which a switch from the IMM-EV($m = 2$) to the IMM-EV($m = 1$) occurs and with d standing for the window length ; $0 < \alpha < 1$; the superscript j_q indicates the model index obtained from eqn. (A4) of the IMM-EV($m = 1$); $\mathbf{v}_i^{j_q}$ denotes the innovation from the Kalman filter based on the j_q model at the time step i and $\mathbf{S}_i^{j_q}$ stands for the corresponding covariance matrix. Under the Gaussian assumption, we have $(\mathbf{v}_i^{j_q})^H (\mathbf{S}_i^{j_q})^{-1} \mathbf{v}_i^{j_q} \sim \chi^2_{n_z}$ which denotes that $(\mathbf{v}_i^{j_q})^H (\mathbf{S}_i^{j_q})^{-1} \mathbf{v}_i^{j_q}$ has the chi-square distribution with the dimension of the measurement n_z as the degree of freedom.

In the following, we determine the window length d in (3) so that (3) can approximately serve as a chi-square measurement residual detector for maneuver detection.

Proposition 1. For eqn. (3), if the time step k satisfies $k \geq d + 1$ and the window length d satisfies

$$d = INT\left[\frac{\ln(1-\alpha) - \ln n_z}{\ln \alpha} - 1\right] \tag{4}$$

where $INT[a]$ returns the smallest positive integer not less than a, then approximately $\varepsilon_k \sim \chi^2_{n_z/(1-\alpha)}$ by matching the first moment of a chi-square random variable to $E[\varepsilon_k]$.

Proof: It is clear $E[\varepsilon_k] = \frac{(1-\alpha^{d+1})}{(1-\alpha)} n_z$. Suppose (4) holds. Then (4) and $E[\varepsilon_k] = \frac{(1-\alpha^{d+1})}{(1-\alpha)} n_z$ together with $0 < \alpha < 1$ implies $\frac{n_z}{(1-\alpha)} - 1 < E[\varepsilon_k] < \frac{n_z}{(1-\alpha)}$. Using the result given by Bar-Shalom, Y. and K. Birmiwal (1982), we get that matching the first moment of a chi-square random variable to $E[\varepsilon_k]$ can be approximated as matching the first moment of a chi-square random variable to $n_z/(1-\alpha)$. Hence, $\varepsilon_k \sim \chi^2_{n_z/(1-\alpha)}$. ■

Based on Proposition 1, we accept the hypothesis that a maneuver is present if the value of eqn. (4) exceeds a threshold $\chi^2_{n_z/(1-\alpha)}(1-\lambda)$ which corresponds to the value of $100(1-\lambda)$ percent confidence region for $\chi^2_{n_z/(1-\alpha)}$ with a given small tail probability λ.

Suppose a maneuver is declared at time step k. Then the maneuver onset time is estimated as the time step $k-d-1$. Accordingly, we outline the proposed scheme in Table I.

Table I.

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if

$$k > k_c + d + 1 \tag{5}$$

and

$$\varepsilon_k > \chi^2_{n_z/(1-\alpha)}(1-\lambda) \tag{6}$$

then a maneuver is declared at time step k.

if a maneuver is declared at time step k,

then

$$\text{maneuver\_onset} = k - d - 1 \tag{7}$$

A maneuver termination is declared at the time step k if the most likely model index obtained from eqn. (7) of the IMM-EV($m=2$) reverts to the quiescent model index at the time step $k-1$, and the non-maneuvering model index is retained as the most probable model index of the IMM-EV(m=2) at the time step k. The proposed scheme is outlined in Table II.

Table II.

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if eqn. (7) in Step 5 of the IMM-EV($m=2$) satisfies

$$\arg\{\max_{1\le j\le n}[\mu_i(j)]\}\begin{cases}\neq 1 & \text{if } i = k-4\\ = 1 & \text{if } i = k-3,\ldots,k\end{cases} \tag{8}$$

where the model index 1 corresponds to the quiescent model index,

then the termination of a maneuver is declared at the time step k.

An operational cycle of the approach is described as follows. The IMM-EV($m=1$) is employed and at the same time the maneuver detection is performed. When a maneuver is declared at the time step k, a switch from the IMM-EV($m=1$) to the IMM-EV($m=2$) is invoked at the estimated maneuver onset time. And the IMM-EV($m=2$) subsequently updates the state estimates from the estimated maneuver onset time to the time step k. From the time step $k+1$ on, the IMM-EV($m=2$) is employed and the detection of the maneuver termination is performed simultaneously. A switch from the IMM-EV($m=2$) to the IMM-EV($m=1$) takes place when the termination of the maneuver is declared. The operational cycle is repeated. Accordingly, the proposed tracking algorithm is given in Table III.

Table III.

$k=0$. $k=0$. Choice=1. $k_c=1$. Given α with $0<\alpha<1$, a probability λ with $\lambda < 0.1$, and n_z, compute the threshold $\chi^2_{n_z/(1-\alpha)}(1-\lambda)$ and the window length d using (4).

$k = k+1$.

Flag=1.
if Choice=1 & Flag=1, then
{Flag=2;
the IMM-EV($m=1$) is employed for tracking;
the scheme in Table 1 is activated;
if a maneuver is declared at time step k ,
then
{the IMM-EV($m=2$) is initialized by the parameter values from the IMM-EV($m=1$) at the estimated maneuver onset obtained from Table 1, and is employed for modifying the state estimates from the estimated maneuver onset time to time step k ;
Choice=2} }.
if Choice=2 & Flag=1, then
{Flag=2;
the IMM-EV($m=2$) is employed for tracking;
the scheme in Table 2 is activated;
if the termination of target maneuver is declared,
then
Choice=1 and $k_c = k+1$ }.

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4. SIMULATIONS

In this section, we show the performance comparison of the proposed algorithm, the IMM-EV($m=1$), the IMM and the RIMM.

We consider the tracking a maneuvering target example given by Bar-Shalom, Y. and X. R. Li (1993). The example involves tracking a slow 90° turn carried out in 20 sampling periods and a fast 90° turn completed in 5 sampling periods, where the target position is sampled every $T_s = 10\sec$. Before and after the turns, the target moves with constant speed and course. The target state vector is

$$\mathbf{x}_k = \begin{bmatrix} x(k) & \dot{x}(k) & y(k) & \dot{y}(k) \end{bmatrix}^T$$

with the initial conditions $(x(0), y(0)) = (2000, 10000)$ in meters and $(\dot{x}(0), \dot{y}(0)) = (0, -15)$ $m/\sec$. The target's slow turn has accelerations $(\ddot{x}(k), \ddot{y}(k)) = (0.075, 0.075)$ $m/\sec^2$ for $40 \le k \le 60$ and the fast turn has accelerations $(\ddot{x}(k), \ddot{y}(k)) = (0.075, 0.075)$.

The non-maneuvering dynamics is modeled by a constant velocity model with small process noise that accounts for slight changes in velocity. Maneuvers are modeled by a constant velocity model with adequate process noise that accounts for slow and fast variations of accelerations. In eqn. (1), $n=2$,

$$\mathbf{A}_k^1 = A_k^2 = \begin{bmatrix} 1 & T_s & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T_s \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad \mathbf{\Gamma}_k^1 = \mathbf{\Gamma}_k^2 = \begin{bmatrix} T_s/2 \\ 1 \\ T_s/2 \\ 1 \end{bmatrix};$$

in eqn. (2)

$$\mathbf{H}_k^j = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Moreover, $\mathbf{Q}_k^1 = \mathbf{0}_2$; $\mathbf{Q}_k^2 = q * \Gamma_k^2 \Gamma_k^{2^T}$ where $q = 10$ is chosen because $\sqrt{q} \approx 3$ corresponds to the maximum change in velocity per sampling period during the fast maneuver; and

$$\mathbf{R}_k^1 = \mathbf{R}_k^2 = 10^4 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

It is apparent that the constant velocity model should be employed for tracking quiescent motions. Thus in the proposed algorithm, we enforce the eqn. (A4) of the IMM-EV(m=1) to be 1 throughout during target's quiescent motions.

The following parameter values are adopted: $\alpha = 0.8$ for (3), the dimension of the measurement $n_z = 2$, $\lambda = 0.05$ for (6), and the window length $d = 10$ obtained from (4). The threshold in (6) is $\chi_{10}^2(0.95) = 18.3$. With those parameter values, the proposed algorithm employs both the IMM-EV($m=1$) with the and the IMM-EV($m=2$) which is exactly the IMM in this example.

For all algorithms illustrated herein, the model transition probability matrix is $\begin{bmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{bmatrix}$; the initial model probabilities are $\mu_0(j) = 1/2$; the initial state and state error covariance matrices are

$$\hat{\mathbf{x}}_0^j = [2000 \;\; 0 \;\; 10000 \;\; -15]^T$$

$$\hat{\mathbf{P}}_0^j = \begin{bmatrix} 10^4 & 10^4/T_s & 0 & 0 \\ 10^4/T_s & 2\times 10^4/(T_s)^2 & 0 & 0 \\ 0 & 0 & 10^4 & 10^4/T_s \\ 0 & 0 & 10^4/T_s & 2\times 10^4/(T_s)^2 \end{bmatrix}.$$

Simulations are carried out based on 100 Monte Carlo runs. Figs. 1 and 2 show that the proposed algorithm and the IMM yield comparable performance and outperform the IMM-EV($m=1$) substantially during the tracking of maneuvers. The IMM-EV($m=1$) performs slightly better than the proposed algorithm during the 34th-40th time steps because the proposed algorithm modifies the estimates using the IMM in accordance with the estimated maneuver onset time. The performance of the proposed algorithm is superior to that of the IMM before and after maneuvers. It is apparent that the proposed approach and the IMM yield comparable performance when a switch from the IMM-EV($m=1$) to the IMM takes place during the tracking of maneuvers. This is due to the appropriate selection of the window length d for eqn. (3). Moreover, because the detection scheme for maneuver termination is adequate, the proposed algorithm and the IMM perform similarly when a switch from the IMM to the IMM-EV($m=1$) occurs. Accordingly, the proposed algorithm can switch adequately between the IMM-EV($m=1$) and the IMM to achieve desirable performance. The proposed approach can outperform the IMM and the RIMM. We also show that the RIMM may not be an improvement to the IMM. The proposed algorithm and the IMM have similar execution time and perform much faster than the RIMM.

Accordingly, the proposed approach is viable in terms of tracking performance and computational cost.

5. CONCLUSIONS

For state estimation of jump Markov systems described by (1)-(2), we have introduced an algorithm: the combination of the IMM-EV($m=1$) and the IMM-EV($m=2$). The proposed algorithm exploits the combined advantages of two IMM-EV algorithms based on some pre-determined switching rules. To invoke a switch from the IMM-EV($m=1$) to the IMM-EV($m=2$), we have develop a scheme for maneuver detection and onset estimation. Furthermore, we have proposed a method for detecting maneuver termination to invoke a switch from the IMM-EV($m=2$) to the IMM-EV($m=1$). Simulation results have shown that the proposed algorithm is a good alternative to several popular schemes in the literature.

Appendix A. The IMM-EV(m) Algorithms

Given n models and an integer m with $1 \le m \le n$, given probabilities p_{ij} , $i,j=1...n,$ model probabilities $\mu_0(j)$ with $\sum_{j=1}^{n}\mu_0(j)=1$, initial state $\hat{\mathbf{x}}_0^j$ and initial state error covariance $\hat{\mathbf{P}}_0^j$. $k=1,2,...$.

a). For each j, $j=1,2,...,n$, mixing probabilities are

$$\mu_{k-1}(l_{sj} \mid j) = \frac{\max_{s\,1\le i\le n}\{p_{ij}\mu_{k-1}(i)\}}{\sum_{s=1}^{m}\max_{s\,1\le i\le n}\{p_{ij}\mu_{k-1}(i)\}}, \tag{A1}$$

where

$$l_{sj} = \arg_{1\le i\le n}\{\max_s\{p_{ij}\mu_{k-1}(i)\}\} \tag{A2}$$

b). Mixed state estimates and state error covariances are

$$\hat{\mathbf{x}}_{k-1}^{0j} = \sum_{s=1}^{m}\mu_{k-1}(l_{sj} \mid j)\hat{\mathbf{x}}_{k-1}^{l_{sj}},\; j=1...n,$$

$$\mathbf{P}_{k-1}^{0j} = \sum_{s=1}^{m}\mu_{k-1}(l_{sj} \mid j)\{\mathbf{P}_{k-1}^{l_{sj}} + [\hat{\mathbf{x}}_{k-1}^{l_{sj}} - \hat{\mathbf{x}}_{k-1}^{0j}]\times[\hat{\mathbf{x}}_{k-1}^{l_{sj}} - \hat{\mathbf{x}}_{k-1}^{0j}]^T\}. \tag{A3}$$

c). For $j=1...n,$ $\hat{\mathbf{x}}_{k-1}^{0j}$ and $\mathbf{P}_{k-1}^{0j}$ are the inputs to the j^{th} model-based Kalman filtering to yield $\hat{\mathbf{x}}_k^j$, $\hat{\mathbf{P}}_k^j$, the innovation $\mathbf{v}_k^j$ with zero mean and covariance $\mathbf{S}_k^j$, and the model likelihood function $\Lambda_k(j)$.

d). Model probabilities are

$$\mu_k(j) = \frac{\Lambda_k(j)\sum_{s=1}^{m}\max_{s\,1\le i\le n}\{p_{ij}\mu_{k-1}(i)\}}{\sum_{j=1}^{n}\Lambda_k(j)\sum_{s=1}^{m}\max_{s\,1\le i\le n}\{p_{ij}\mu_{k-1}(i)\}} \tag{A4}$$

e). The re-weighted 'm' largest model probabilities of $\mu_k(j)$ are

$$\tilde{\mu}_k(\tilde{l}_s) = \frac{\mu_k(\tilde{l}_s)}{\sum_{s=1}^{m} \mu_k(\tilde{l}_s)}, \; s = 1...m \quad \text{(A5)}$$

where

$$\tilde{l}_s = \arg\{\max_{1 \le j \le n}{}_s \, [\mu_k(j)]\} \quad \text{(A6)}$$

f). The resultant state estimate is

$$\hat{\mathbf{x}}_k = \sum_{s=1}^{m} \tilde{\mu}_k(\tilde{l}_s) \hat{\mathbf{x}}_k^{\tilde{l}_s} \quad \text{(A7)}$$

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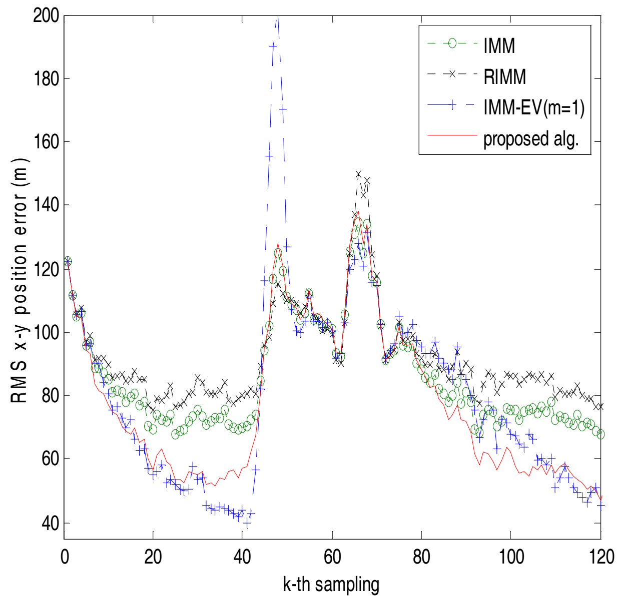

Figure 1. RMS position performance comparison of IMM, RIMM, IMM-EV(m=1) and the proposed algorithm

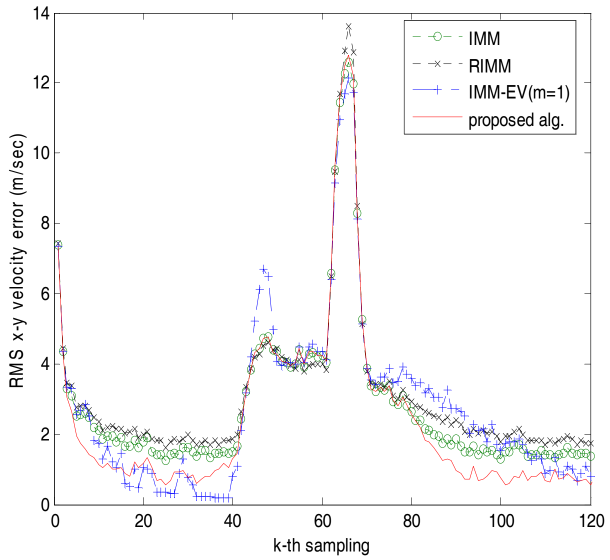

Figure 2. RMS velocity performance comparison of IMM, RIMM, IMM-EV(m=1) and the proposed algorithm