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Energy Based Mode Tracking of Hybrid Systems

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Abstract: Hybrid systems consist of continuous and discrete dynamics. These systems can be described with a set of modes, where in each mode the system is governed by a continuous dynamics, and different modes correspond to different continuous models. For hybrid systems, model-based fault detection and isolation (FDI) is a challenging task, since system's prevailing dynamical model and the system's current mode (discrete state) are mutual dependent and intertwined. This paper introduces a new energy based approach for mode tracking of physical systems with hybrid dynamics. A Hybrid Bond Graph systematic analysis is utilized to characterize each system's mode with compact energy relations. To track system's modes, we monitor these energy relations in real time. The result is an energy based mode tracker that is more efficient in terms of computational resources than existing techniques.

1. INTRODUCTION

A well-known approach to health monitoring of dynamical systems is model based (Frank, 1990). This approach utilizes dynamical model of a system as a reference to its normal behaviour. The difference between system behaviour (measured with sensors) and its expected behaviour (computed with model) is expressed by variables, known as residues. As long as these residues are within bounds with predetermined thresholds, we say that the monitored system behaves as its model and is at its normal operational state. Successful implementation of model based techniques requires accurate and updated system models. Failing this can lead to deceptive results such as false alarms. This problem can be escalated when the system under consideration is hybrid. Hybrid systems consist of discrete and continuous dynamics. Each discrete state of the system is named a mode, characterized by a continuous model (see Fig. 1), and different modes correspond to different continuous models. Monitoring the system behaviour requires measurement or estimation of continuous states variables as well as tracking the system discrete dynamics (mode evolution). Discrete events force the system to move from one mode to another. These changes are referred as mode changes. Some of the discrete events are known (e.g. initiated by a supervisory controller) and some are unknown but measurable. The main obstacles of applying model based FDI to hybrid systems are unobservable events (such as unknown discrete inputs) and unknown discrete dynamics. In this case, unpredicted and unmeasured mode changes can happen at any time and in any order. In model based FDI of hybrid systems, the residues will show normal behaviour if the monitored system is in normal operation and its current mode is known. However if unobservable mode change has occurred, the prevailing continuous model in the health monitoring process is no longer valid for the monitored system. As a result, the FDI residues will exhibit abnormal behaviour which in turn can be interpreted as system's component-faults (i.e. these faults are not represented as modes), or as a change of mode. In health monitoring of hybrid systems, it is essential to distinguish between these two scenarios and to identify the new system's mode, in a case of mode change.

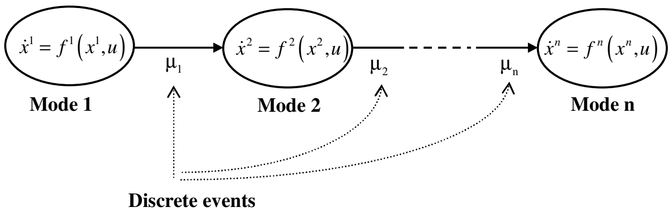


Fig. 1. Mode and model-change of a hybrid system

A common approach to model based FDI of hybrid systems is to develop a monitoring system with two modules. One module is in charge of the continuous monitoring of each mode (e.g. using continuous observer), and the other part is in charge of providing (if possible) the current system's. In this paper we address the second module, and propose an efficient method for mode tracking of hybrid systems.

In (Narasimhan and Biswas, 2007), a hybrid observer is proposed as part of a model based FDI paradigm for hybrid systems. This approach is based on a combination of Kalman filter (for continuous tracking of the plant states within any mode) and a mode change detector. The discrete event types that considered in (Narasimhan and Biswas, 2007) are either known controlled mode changes (i.e. caused by a supervisory controller) or autonomous mode changes (i.e. changes that are triggered by the plant states). Once the mode-change conditions are detected, a Finite State Machine (FSM) is utilized to determine the new mode. The state-space model of the new mode is dynamically computed and applied to the Kalman filter. The difficulty in this approach arises from the indirect way in which a new mode is identified. New mode identification is based on detection of the mode-change causing events rather than a direct identification of the monitored system continuous dynamics in its new mode. As a result, this method can't work with unobservable events (such as unknown discrete inputs), requires support of discrete model, e.g., FSM, as well as the discrete initial condition.

A different approach is taken in (Balluchi *et al.*, 2002). The authors suggested a dynamical observer design for hybrid systems. This observer consists of two parts: a location observer (which identifies the current location of the hybrid system) and a continuous observer. Based on the known or partially known discrete dynamics of the hybrid system, a current location tree is used to narrow in a subset of possible locations of the system. Under the assumption of linear continuous model in each mode, a bank of N Luenberger observers (one for each suspected system dynamics) is used to generate N residuals. These residuals are then used to identify the continuous dynamics to which the system subjects. Extending this approach, a robust hybrid observer for a class of uncertain hybrid nonlinear systems with unknown mode transition function is proposed in (Wang *et al.*, 2007). This observer consists of a mode observer (MO) for discrete mode estimation, and a continuous observer (CO) for continues state estimation. Based on the robust hybrid observer, the authors of (Wang *et al.* 2007) proposed a robust fault diagnosis scheme, for faults modelled as discrete modes with unknown transition function. In (Wang *et al.*, 2007) the MO consists of a bank of mode isolators, each is a UIEKO (i.e. Unknown Input Extended Kalman Observer) and has similar structure but different parameters.

The common principle of the last two methods is a simultaneous observers running of all suspected modes, and a search for the one that fits best to the observation of the monitored system. The disadvantage of these methods is their high demand for computational resources. The number of states in these hybrid observers is equal to the product of the number of monitored system's states and the number of suspected modes. Therefore these methods may not be applicable to complex systems with large number of states and modes. Our motivation is to develop a method that can overcome this problem and can be integrated into our bond graph based FDI paradigm for hybrid systems (this paper describes only the mode tracker, however, the use of the bond graph as the preferred modelling technique is motivated also by the FDI paradigm). Our solution for the mode tracking problem is based on characterization of each system's mode with a compact energy relation (contrary to a detailed dynamical model). Consequently, the method is more efficient in terms of computational resources than existing techniques.

Another advantage of the proposed method comes from the use of hybrid bond graph and its clear presentation of the configuration of a hybrid system. This property lays a foundation for a systematic approach for power relations development in hybrid systems.

The paper is organized as follows: Section 2 introduces the concept of power-nets for mode characterization and suggests a new method for power-nets derivation from the hybrid bond graph model of the monitored system. Section 3 presents an example of two-tank system, followed by numerical results. Section 4 concludes the paper.

2. POWER NETS AND SYSTEM'S MODES

In a physical system, we can distinguish three different quantities of energy: $E_{in} = \int P_{in} dt$ the inserted-energy to the system, H the stored-energy in the system and $E_{diss} = \int P_{diss} dt$ the dissipated-energy from the system; the term P represents power. From the principle of energy conservation, it is clear that:

$$\int \dot{H} dt = \int \left(P_{in} - P_{diss} \right) dt \qquad (1)$$

Energy balance (such as (1)) was suggested by (Fantuzzi and Secchi, 2004) for fault detection in *continuous* systems. As this balance depends on the initial conditions of the system, which always have some degree of uncertainty, it is not suitable for the development of mode tracking strategies. Thus, we use time derivative of (1)

$$\dot{H}(x) = P_{in}(u,x) - P_{diss}(x) \qquad (2)$$

where $x = x(t)$ indicates the system states and $u = u(t)$ is the system continuous input. We refer relationships of the form (2) as Power-Net (PN). We wish to characterize each mode of the hybrid system with a unique PN. If a system has n modes, then our goal is a set of n-unique PNs, referred as

$$\dot{H}^i(x) = P_{in}^i(u,x) - P_{diss}^i(x) \qquad (3)$$

where the superscripts $i = 1, \cdots, n$ index the modes. We evaluate all PNs of possible modes in parallel. If a PN of a certain mode matches the monitored system's observations, then we deduce that the monitored system is in this mode.

In practice and most likely, we measure neither the energy of the system's components nor the power, but we measure the state variables of the monitored system. In this work we assume that we measure the complete set of the state variables. We also assume the knowledge of the monitored system continuous input. With these measurements, we can evaluate (3) for $i = 1, \cdots, n$.

In order to avoid numerical differentiation of $H(x)$ which is contaminated with measurement noises, we formulate (3) in the following state space form:

$$\left. \begin{aligned} \dot{H}^i(x) &= \underbrace{P_{in}^i(u,x) - P_{diss}^i(x)}_{U^i} \\ Y^i &= H^i(x) \end{aligned} \right\} \Rightarrow \begin{aligned} \dot{H}^i &= U^i \\ Y^i &= H^i \end{aligned} \qquad (4)$$

where H^i is a measurable state (i.e mode tracker state) and U^i is a known input. To check if relation (4) holds in the monitored system we use the Luenberger observer (5) to estimate H^i.

$$\dot{\hat{H}}^i = U^i + L\left(H^i - \hat{H}^i\right) = P_{in}^i - P_{diss}^i + L\left(H^i - \hat{H}^i\right) \quad (5)$$

Each observer i implements a dynamical model that is compatible with only one of the system's modes (i.e. mode i) and the observers performances are an indication to the system's current mode. If the current mode of the monitored system is characterized by PNi then:

$$\dot{e}^i = \left(\dot{H}^i - \dot{\hat{H}}^i\right) = -L\left(H^i - \hat{H}^i\right) = -Le^i. \quad (6)$$

Choosing $L > 0$ will make the observer error of mode i converge asymptotically to zero, i.e. $\lim_{t\to\infty} e^i(t) = 0$. In practice, as a result of measurement noises and modelling uncertainties, we check convergence of the observer error to a threshold.

When each mode of the system can be characterized by a unique single PN, then the number of mode tracker's states that should be running simultaneously is equal to the number of suspected modes (and not to the number of suspected modes times the number of system's states as in (Balluchi *et al.*, 2002) and (Wang *et al.* 2007)).

To characterize system's modes by PNs, a model that presents clearly the various energy relations of the monitored system is required. Before presenting our solution, we continue with rather intuitive physical analysis of the problem. The system's components can be classified into three groups regarding their role in the power balance (PN): 1) the group of energy sources (contributing to the term P_{in}), 2) the group of energy storages (contributing to $\dot{H}$) and 3) the group of energy dissipaters (contributing to P_{diss}). For any mode, we use this classification to build a set of PNs. Some of these PNs might include only a subset of the components in the system due to discrete event nature of the hybrid system under consideration.

The following example demonstrates the concept of mode characterization in terms of PNs. Consider an electrical circuit, as shown in Fig. 2, with two modes, determined by a switch s. The switch has two states: zero and one. The system has four components: two storage elements L and C, one source element E and one dissipation element R.

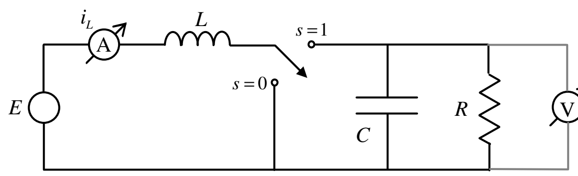


Fig. 2. An electric circuit

When the switch is at the $s = 1$ state or, equivalently, we say the system is in its mode 1, all four elements are coupled in one circuit. For this mode we can write the following PN (i.e. PN1).

$$\underbrace{P_L + P_C}_{\dot{H}^1} = \underbrace{P_E}_{P_{in}^1} - \underbrace{P_R}_{P_{diss}^1} \quad \Rightarrow \quad \text{PN1} \quad (7)$$

When the switch is at the $s = 0$ state and we call that the system is in its mode 0, the circuit is switched to two decoupled and smaller circuits, each one includes two components, E and L in one, and C and R in another. We can write two PNs that exist only in this particular mode (PN0.1 and PN0.2).

$$\underbrace{P_L}_{\dot{H}^{0.1}} = \underbrace{P_E}_{P_{in}^{0.1}} \quad \Rightarrow \quad \text{PN0.1} \quad , \quad \underbrace{P_C}_{\dot{H}^{0.2}} = -\underbrace{P_R}_{P_{diss}^{0.2}} \quad \Rightarrow \quad \text{PN0.2} \quad (8)$$

It is easy to see that if more than one PN exist in a certain mode then any linear combination of these PNs produces a new PN which is also valid for the same mode. In the example $\text{PN0.1} + \text{PN0.2} = \text{PN1}$. We understand that PN1 is valid for both modes and therefore, if stand by itself, is insufficient for mode tracking. Our goal is to characterize each system's mode with a unique PN that comprises as much information as possible of power relations in that mode. In the last example, this goal can be achieved if we use different linear combination of PN0.1 and PN0.2 such as subtraction, which leads to PN0. PN0 is valid only for the mode where $s = 0$.

$$\underbrace{P_L - P_C}_{\dot{H}^0} = \underbrace{P_E}_{P_{in}^0} + \underbrace{P_R}_{P_{diss}^0} \quad \Rightarrow \quad \text{PN0} \quad (9)$$

Clearly, using PN0 and PN1 enables us to trace system's modes and its discrete dynamics. The number of observer states that we need to run simultaneously is two (namely, H^0 and H^1). This number is equal to the number of system's modes. As all of the system's modes can be distinguished by PNs, we say that this system is PN-mode identifiable.

Definition 1: A hybrid system is said to be PN-mode identifiable if each one of its modes can be characterised with a unique set of PNs.

To facilitate the PN approach, we need a model that describes the physical structure of the system, as well as the various physical components and the system's modes. To fill this need we use the Hybrid Bond Graph (HBG) model of the system (i.e. bond graph enhanced with controlled-junctions) (Mosterman and Biswas, 1995). Bond graph (Karnopp *et al.*, 2006) is an energy based model, that presents clearly how the energy is distributed in the system, and components are connected to each other representing power flow. When dealing with hybrid systems, hybrid bond graph has a great advantage in presenting a global picture, i.e., presenting the system over all its various modes in a single picture.

Contrary to the physical analysis that was presented, where we needed to analyze each mode separately, the HBG-based method that we propose is based on a so called global analysis (global in the sense that we do not need to analyze each mode separately). From this analysis we produce PNs that are a function of the hybrid bond graph controlled-junctions' state, we call these PNs, Global PNs (GPNs). From the GPNs one can easily drive sets of PNs to characterize each one of the system's modes. We discuss the suitability of bond graph to the PN approach, and exemplify the GPN method on the electrical circuit example.

The hybrid bond graph model of the electrical circuit is given in Fig 3. Controlled junctions are numbered with small index, next to the junction type, in order to distinguish them from standard '0' and '1' type junctions. An interpretation to the hybrid bond graph model in Fig 3 is given as follows: The resistor and capacitor parallel-connection (on the left hand side of the circuit) is modelled by a 0-junction, and a 1-junction is used to model the series-connection of the source and the inductance. When the switch is at the $s=0$ state, there is no power exchange between the two sides of the circuit. From physical point of view, when the switch is $s=0$, it enforces zero voltage (*effort*) to the left (to the series connection) and zero current (*flow*) to the right (to the parallel connection). As a single controlled-junction can enforce only one type of power variable (i.e. *effort* or *flow*) to zero, the physical switch modelling requires two successive controlled-junctions from different types. In addition, a unified control-signal (ON/OFF) is used to control the two junctions simultaneously (where OFF corresponds to $s=0$). When the two controlled-junctions are ON, the median branch (including bond-4, bond-5 and the two controlled-junctions) is equivalent to a single bond.

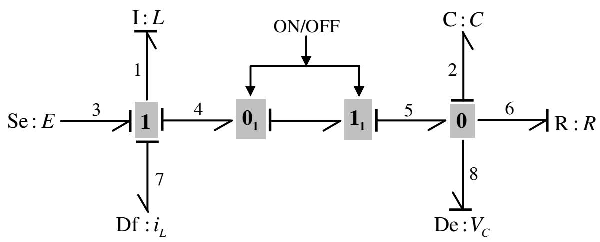
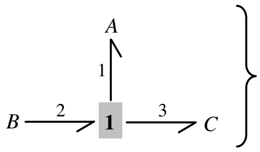


Fig. 3. Bond graph model of electric circuit

Bond graph presents the power flow in the system. The junctions in bond graph terminology represent interior connection of subsystems and are based on the energy conservation principle, i.e. the power that enters the junction is equal to the power that leaves the junction (Fig. 4). The junction type ('0' or '1') and the causality assignment do not have any influence on this relation (however, they do play a role when driving relations between power and measurements). We adopt this principle and continue with the electric circuit example to formulate the method.



$$\Rightarrow \quad \begin{aligned} p_2 &= p_1 + p_3 \\ p_B &= p_A + p_C \end{aligned}$$

Fig. 4. Power relation in bond graph junction

The circuit has two sensors i_L and V_C, and each sensor is utilized for GPN derivation. To derive a GPN, we start from the sensor and spread through the bond graph power paths, until only components' power and controlled-junctions appear in the GPN (The sensor power is assumed zero). The result is given in (10), where the discrete variables (0_1) and (1_1) represent the controlled-junctions discrete state (when the controlled-junction is ON the variable is equal to 1 and when the junction is OFF the variable is equal to 0)

$$\begin{aligned} &(i_L): \quad P_7 = P_3 - P_1 - P_4 = P_3 - P_1 - (0_1)(1_1)P_5 \\ &\qquad\qquad = P_3 - P_1 - (0_1)(1_1)(P_2 + P_6 + P_8) \\ &\Rightarrow \quad (\text{GPN1}) \quad 0 = P_E - P_L - (0_1)(1_1)(P_C + P_R) \\ &(V_C): \quad P_8 = P_5 - P_2 - P_6 = (0_1)(1_1)P_4 - P_2 - P_6 \\ &\qquad\qquad = (0_1)(1_1)(P_3 - P_1 - P_7) - P_2 - P_6 \\ &\Rightarrow \quad (\text{GPN2}) \quad 0 = (0_1)(1_1)(P_E - P_L) - P_C - P_R \end{aligned} \tag{10}$$

By applying the controlled-junctions' discrete state of each mode to the GPNs, we can achieve the set of PNs that characterizes each system's mode. Note that, the hybrid bond graph systematic approach result given in (11) is identical to (7) and (8) (which were achieved by an intuitive analysis).

$$\begin{aligned} &\text{For mode 1 } \left((0_1) = (1_1) = 1\right) \\ &\quad \text{GPN1} \quad \Rightarrow \quad 0 = P_E - P_L - P_C - P_R \quad (\Rightarrow \text{PN1}) \\ &\quad \text{GPN2} \quad \Rightarrow \quad 0 = P_E - P_L - P_C - P_R \quad (\Rightarrow \text{PN1}) \\ &\text{For mode 0 } \left((0_1) = (1_1) = 0\right) \\ &\quad \text{GPN1} \quad \Rightarrow \quad 0 = P_E - P_L \quad (\Rightarrow \text{PN0.1}) \\ &\quad \text{GPN2} \quad \Rightarrow \quad 0 = -P_C - P_R \quad (\Rightarrow \text{PN0.2}) \end{aligned} \tag{11}$$

The last step in the mode tracker design is to find relations between power and measurements. We use the hybrid bond graph to obtain (12). These relations can be substituted into the two GPNs, for practical implementation of the method.

$$\begin{aligned} &P_E = P_3 = e_3 f_3 = E i_L \quad , \quad P_R = P_6 = e_6 f_6 = V_C i_R = \frac{V_C^2}{R} \\ &P_L = P_1 = e_1 f_1 = V_L i_L = L \frac{di_L}{dt} i_L = \frac{d}{dt}\left(L i_L^2 / 2\right) \\ &P_C = P_2 = e_2 f_2 = V_C i_C = V_C C \frac{dV_C}{dt} = \frac{d}{dt}\left(C V_C^2 / 2\right) \end{aligned} \tag{12}$$

In general, power-measurements relations can be controlled-junctions dependent and substituted into the GPNs as a function of controlled-junctions' state.

The following procedure summarizes the proposed method for mode tracking of hybrid systems:

1. Develop the hybrid bond graph model of the system.
2. For each sensor develop a GPN. Start from the sensor, and spread through the bond graph power paths, until only components' power and controlled-junctions appear in the GPN.
3. For each GPN, substitute the different controlled-junctions' states to develop a set of PNs which are correspond to the system's modes.
4. For each mode, use linear combination of PNs to achieve two goals: a) find out if PNs characterize more than one

mode. b) Characterize each system's mode with a single and unique PN. The result of this stage is the PN-combined-set

5. Use the hybrid bond graph to develop power-measurements relations and substitute these relations into the GPNs (and consequently into the PN-combined-set).

6. For any PN of the PN-combined-set, formulate in a state space form (see (4)), and implement a Luenberger observer as described in (5).

7. Run all Luenberger observers of suspected modes simultaneously. If only one observer (which is attached to a certain mode) converges to a predefined threshold, then we deduce the current mode of the system.

3. TWO-TANK SYSTEM EXAMPLE

In this section, we exemplify the PN mode tracking approach, on a two-tank system (Fig. 5).

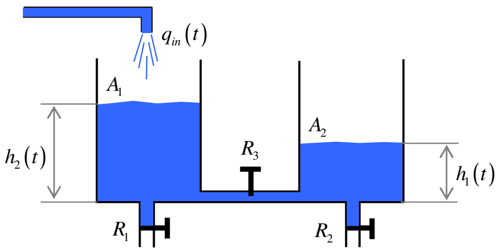


Fig. 5. Two-tank system example

The system consists of two tanks, flow source and three valves. Each valve can be open or closed, the valves discrete dynamics is unknown (i.e. any valve can get opened or closed at any time by unknown input); in addition, valves' state can not be measured. $\{A_1, A_2\}$ are the two tanks cross section areas, and $q_{in}(t)$ is the inflow to the first tank. The system equipped with two level sensors, namely $h_1(t)$ and $h_2(t)$. When a valve is open its continuous dynamics is nonlinear according to:

$$f(t) = (1/R_i) \cdot \mathrm{sign}(\Delta h(t)) \cdot \sqrt{|\Delta h(t)|} \quad (13)$$

Where $f(t)$ is the volumetric flow of the liquid through the valve, Δh is the liquid level difference across the valve (that is proportional to the pressure difference), R_i is the valve discharge coefficient (assumed constant), and $\mathrm{sign}(\cdot)$ is a function used to adjust the direction of flow.

The hybrid bond graph model of the two-tank system is given in Fig. 6. In this graph the *flow* variable is the liquid volumetric flow (i.e. $[\mathrm{m}^3/\mathrm{s}]$), and the *effort* variable is the liquid height (i.e. $[\mathrm{m}]$). As the product of these two variables is not a physical power, this bond graph is named a pseudo bond graph. However, this fact does not interfere with our results and the methods can be implemented as shown below.

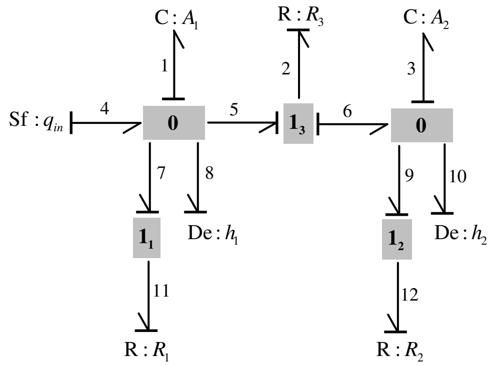


Fig. 6. The HBG of the two-tank system example

Based on the two level sensors, we derive the two GPNs in (14), where H_{Ai} represents the energy stored in tank A_i (i.e. $\dot{H}_{Ai} = P_{Ai}$).

$$\begin{aligned}
&(\mathrm{GPN1}) \quad \dot{H}_{A1} + (1_3)\dot{H}_{A2} = P_{qin} - (1_1)P_{R1} \\
&\qquad\qquad - (1_3)(1_2)P_{R2} - (1_3)P_{R3} \\
&(\mathrm{GPN2}) \quad (1_3)\dot{H}_{A1} + \dot{H}_{A2} = (1_3)P_{qin} - (1_3)(1_1)P_{R1} \\
&\qquad\qquad - (1_2)P_{R2} - (1_3)P_{R3}
\end{aligned} \quad (14)$$

Due to space limitation, a table that shows the complete sets of PNs for each mode (that is derived from (14)) is not presented. However, that table reveals the fact that the system is PN-mode identifiable and in order to produce a unique PN for each mode both GPNs are required. In complex systems with large number of modes and states it is important to reduce the number of states in the mode tracker, as much as possible (probably with some reduction in robustness). Consequently, we wish to characterize each system's mode with a unique PN. To achieve this, we produce new PNs, from linear combination of existing PNs. In our example, subtraction of GPN2 from GPN1 for the first four modes, and the existing PNs of the last four modes, form a new set of eight PNs (i.e. PN0-PN7) that characterize uniquely each mode of the hybrid system (In this example, summation of GPN1 and GPN2 for the first four modes can be considered as well). This result is given in Table 1.

To illustrate the effectiveness of the PN mode tracker, we present simulation results of the two-tank system example. The hybrid bond graph model of the system (i.e. the behaviour model) was constructed using MATLAB-SIMULINK bond graph block-set, developed by the authors. Due to space limitations the block diagrams are not presented in the paper. The system's physical parameters are the following: the tanks cross-section area is $A_1 = A_2 = 1\mathrm{m}^2$, the initial conditions are $h_1(0) = 1.4\mathrm{m}$, $h_2(0) = 0.2\mathrm{m}$, the valves coefficients are $R_1 = R_2 = 100\,\mathrm{s}/\mathrm{m}^2$, $R_3 = 50\,\mathrm{s}/\mathrm{m}^2$, the observers' gain is $L = 30$ and $q_{in} = 0$.

The simulation presents the following scenario: In the beginning only valve R_1 is open, and the system is in mode one. At $t = 10\sec$ valve R_3 is opened and the system is in mode five for 15 seconds (i.e. at $t = 25\sec$ the system is turning-back to mode one). At $t = 40\,\text{sec}$ valve R_2 is opened and the system's mode is changing to mode three. At $t = 55\,\text{sec}$ valve R_1 is closed and the system is turning to mode two. The relevant observers' errors are presented in Fig. 7. Each observer error represents a PN of a particular mode. It can be seen that each observer's error has significant smaller absolute value in the time periods when the system is in the corresponding mode (i.e. the system is in the mode where the PN is valid). Comparison of these observers' errors to a suitable threshold enables tracking of the system's modes.

Table 1.
The PN-combined-set of the two-tank system example

Mode No.	(1_3)	(1_2)	(1_1)	PN0 - PN7 (PN-combined-set)
0	0	0	0	$\dot{H}_{A1} - \dot{H}_{A2} = P_{qin}$
1	0	0	1	$\dot{H}_{A1} - \dot{H}_{A2} = P_{qin} - P_{R1}$
2	0	1	0	$\dot{H}_{A1} - \dot{H}_{A2} = P_{qin} + P_{R2}$
3	0	1	1	$\dot{H}_{A1} - \dot{H}_{A2} = P_{qin} - P_{R1} + P_{R2}$
4	1	0	0	$\dot{H}_{A1} + \dot{H}_{A2} = P_{qin} - P_{R3}$
5	1	0	1	$\dot{H}_{A1} + \dot{H}_{A2} = P_{qin} - P_{R3} - P_{R1}$
6	1	1	0	$\dot{H}_{A1} + \dot{H}_{A2} = P_{qin} - P_{R3} - P_{R2}$
7	1	1	1	$\dot{H}_{A1} + \dot{H}_{A2} = P_{qin} - P_{R3} - P_{R2} - P_{R1}$

4. CONCLUSIONS AND FURTHER WORK

The method proposed in this paper, for mode tracking of hybrid systems, is efficient and simple. The goal of the method is to assist model based fault detection and isolation in hybrid systems. The method is based on a unique representation of each system's mode, with compact power relations, named power-nets (PNs). The hybrid bond graph model of the system exploits the advantages of energy based modelling approach to generate global power-nets (GPNs). The concept of global power relations is found comprehensive to produce the particular PNs of the system's modes.

The advantages of our method with respect to existing methods are: more efficiency in computational resources, no requirement for discrete dynamical model, and a systematic analysis that is based on the hybrid bond graph model of the system.

Our future work will be dedicated to more complex systems i.e., systems with large number of states and modes, and to the cases where not all of the system's state variables are measurable.

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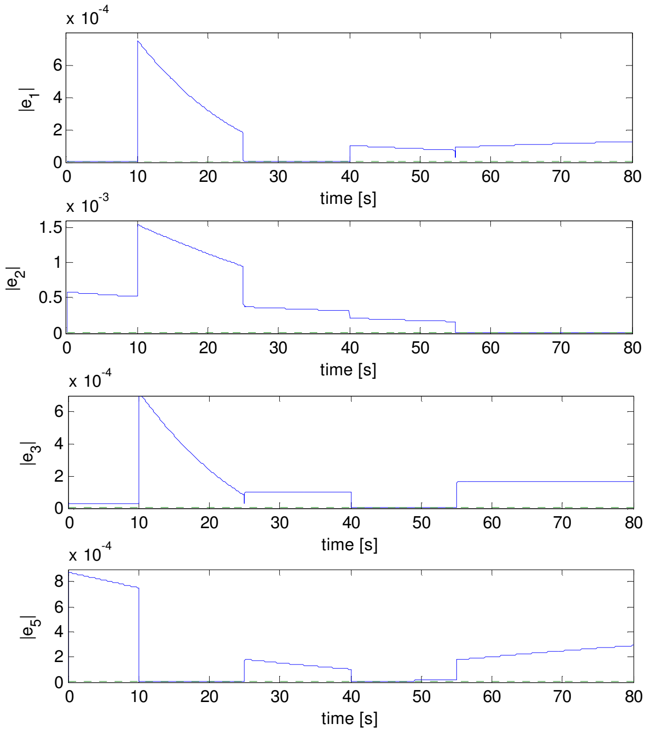

Fig. 7. Absolute value of observers' errors

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