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INTEGRATING THE UTKIN OBSERVER WITH THE UNSCENTED KALMAN FILTER

Martin Ongkosutjahjo and Victor M. Becerra

University of Reading
School of Systems Engineering
Reading RG6 6AY, UK
(Tel: +44-118-3786703; e-mail: v.m.becerra@reading.ac.uk).

Abstract: This paper describes the integration of an Utkin observer with the unscented Kalman filter, investigates the performance of the combined observer, termed the unscented Utkin observer, and compares it with an unscented Kalman filter. Simulation tests are performed using a model of a single link robot arm with a revolute elastic joint rotating in a vertical plane. The results indicate that the unscented Utkin observer outperforms the unscented Kalman Filter.

Keywords: Nonlinear observers, state estimation, Kalman filtering, nonlinear systems.

1. INTRODUCTION

This paper describes the integration of a deterministic observer and a stochastic state estimator. The deterministic observer takes the form of a variable structure system where the driving component of the observer can switch between two values depending on the values of the error between the measured output and the estimated output. This type of state estimator often referred to as sliding mode observer. An Utkin observer is selected as a type of sliding mode observer in this work. The advantage of the Utkin observer is that convergence condition of the estimated state to the true state is an explicit part of the observer design (Edwards and Spurgeon, 1998). Hence, the selection of an appropriate value for the observer gain will guarantee the convergence of any initial estimated state to the true state in some finite time (Edwards and Spurgeon, 1998). Once the estimated state reaches the true state, it will remain on the trajectory of the true state or well within a very small region around the true state. While the convergence of the state is guaranteed by the Utkin observer, an Utkin observer is designed under the assumption that the measurement is not corrupted by noise so it may not perform well under noisy measurement conditions.

The stochastic part of the combined observer is inspired by unscented Kalman filter (UKF). Similar to its predecessor, the extended Kalman filter (EKF), the UKF algorithm involves the propagation of the mean value of the estimated state, the process noise covariance, and the measurement noise covariance. The UKF has two advantages over the EKF. First, no linearization is required which results in a reduction in the complexity of the algorithm (Wan and van der Merwe, 2000). Second, the UKF is accurate up to third order of the Taylor series expansion (Wan and van der Merwe, 2000; Simon, 2006). The UKF is often capable of producing accurate estimates of the state given noisy measurements. However, because the UKF is a stochastic observer, in contrast to the Utkin observer, the convergence condition of the estimated state is not an explicit part of the design. Intuitively a robust observer that is able to achieve convergence and noise rejection is possible by combining the Utkin observer and the UKF, and this is the main motivating idea behind this paper.

2. THE UTKIN OBSERVER

The formulation of the following Utkin observer is largely motivated by Spurgeon and Edwards (1998). Consider a linear system:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, and $p \geq m$. The matrices B and C are assumed to be full rank and the pair (A, C) is assumed to be observable. Now consider a possible transformation $x \mapsto T_c x$ where:

$$T_c = \begin{bmatrix} N_c^T \\ C \end{bmatrix} \tag{3}$$

and the columns of $N_c \in \mathbb{R}^{n\times(n-p)}$ span the null space of C. The linear system in the new coordinates is now given by:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{y}_1(t) \end{bmatrix} = T_c A T_c^{-1} \begin{bmatrix} x_1(t) \\ y_1(t) \end{bmatrix} + T_c Bu(t) \tag{4}$$

With a new output distribution matrix given by:

$$CT_c^{-1} = \begin{bmatrix} 0 & I_p \end{bmatrix} \quad (5)$$

Now let the system in the new coordinates be partitioned as follows:

$$T_c A T_c^{-1} = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \quad (6)$$

$$T_c B = \begin{bmatrix} B_1 \\ \hline B_2 \end{bmatrix} \quad (7)$$

The system can now be written as:

$$\dot{x}_1(t) = A_{11}x_1(t) + A_{12}y(t) + B_1u(t) \quad (8)$$

$$\dot{y}(t) = A_{21}x_1(t) + A_{22}y(t) + B_2u(t) \quad (9)$$

where

$$T_c x = \begin{bmatrix} x_1(t) \\ y_1(t) \end{bmatrix} \begin{matrix} \updownarrow n-p \\ \updownarrow \ \ p \end{matrix} \quad (10)$$

Notice that the output now has been transformed and becomes part of the state equation. Now define an Utkin observer as follows:

$$\dot{\hat{x}}_1(t) = A_{11}\hat{x}_1(t) + A_{12}\hat{y}(t) + B_1u(t) + Lv \quad (11)$$

$$\dot{\hat{y}}_1(t) = A_{21}\hat{x}_1(t) + A_{22}\hat{y}(t) + B_2u(t) - v \quad (12)$$

where $(\hat{x}_1, \hat{y})$ are the estimates of (x_1, y), $L \in \mathbb{R}^{(n-p)\times p}$ is a constant feedback gain matrix, and v is a discontinuous vector defined component-wise by:

$$v_i = K\,\mathrm{sgn}(\hat{y}_{1_i} - y_{1_i}) \quad (13)$$

where $K \in \mathbb{R}_+$. The error dynamics of the system in the new coordinate are given by:

$$\dot{e}_1(t) = A_{11}e_1(t) + A_{12}e_{y_1}(t) + Lv \quad (14)$$

$$\dot{e}_{y_1}(t) = A_{21}e_1(t) + A_{22}e_{y_1}(t) - v \quad (15)$$

Since the pair (A, C) is observable, the pair (A_{11}, A_{21}) must be observable. Matrix L can therefore be chosen so that $\lambda\left(A_{11} + LA_{21}\right) \in \mathbb{C}_-$. Now define a further change of coordinates that is dependent on L (Edwards and Spurgeon, 1998).

$$\tilde{T} = \left[\begin{array}{c|c} I_{n-p} & L \\ \hline 0 & I_p \end{array}\right] \quad (16)$$

The system dynamics on the new coordinates can be written as:

$$\dot{\tilde{x}}_1(t) = \tilde{A}_1\tilde{x}_1(t) + \tilde{A}_{12}\tilde{y}_1(t) + \tilde{B}_1u(t) \quad (17)$$

$$\dot{\tilde{y}}_1(t) = \tilde{A}_{21}\tilde{x}_1(t) + \tilde{A}_{22}\tilde{y}_1(t) + \tilde{B}_2u(t) \quad (18)$$

where

$$\tilde{A} = \left[\begin{array}{c|c} \tilde{A}_{11} & \tilde{A}_{12} \\ \hline \tilde{A}_{21} & \tilde{A}_{22} \end{array}\right] = \left[\begin{array}{c|c} A_{11} + LA_{21} & A_{12} - L\left[A_{11} + LA_{21}\right] + LA_{22} \\ \hline A_{21} & A_{22} - LA_{21} \end{array}\right] \quad (19)$$

$$\tilde{B} = \begin{bmatrix} B_1 + LB_2 \\ \hline B_2 \end{bmatrix} \quad (20)$$

The observer in the new coordinates now has the form:

$$\dot{\hat{\tilde{x}}}_1(t) = \tilde{A}_{11}\hat{\tilde{x}}_1(t) + \tilde{A}_{12}\hat{\tilde{y}}_1(t) + \tilde{B}_1u(t) \quad (21)$$

$$\dot{\hat{\tilde{y}}}_1(t) = \tilde{A}_{21}\hat{\tilde{x}}_1(t) + \tilde{A}_{22}\hat{\tilde{y}}_1(t) + \tilde{B}_2u(t) - v \quad (22)$$

The error dynamics with respect to the new coordinates are:

$$\dot{\tilde{e}}_1(t) = \tilde{A}_{11}\tilde{e}_1(t) + \tilde{A}_{12}\tilde{e}_{y_1}(t) \quad (23)$$

$$\dot{\tilde{e}}_{y_1}(t) = \tilde{A}_{21}\tilde{e}_1(t) + \tilde{A}_{22}\tilde{e}_{y_1}(t) - v \quad (24)$$

Now modify the observer by adding a negative output error feedback term to the Utkin observer (Edwards and Spurgeon, 1998) to give the following observer:

$$\dot{\hat{\tilde{x}}}_1(t) = \tilde{A}_{11}\hat{\tilde{x}}_1(t) + \tilde{A}_{12}\hat{\tilde{y}}_1(t) + \tilde{B}_1u(t) - G_1\tilde{e}_{y_1}(t) \quad (25)$$

$$\dot{\hat{\tilde{y}}}_1(t) = \tilde{A}_{21}\hat{\tilde{x}}_1(t) + \tilde{A}_{22}\hat{\tilde{y}}_1(t) + \tilde{B}_2u(t) - G_2\tilde{e}_{y_1}(t) - v \quad (26)$$

this gives an error dynamics of:

$$\dot{\tilde{e}}_1(t) = \tilde{A}_{11}\tilde{e}_1(t) + \tilde{A}_{12}\tilde{e}_{y_1}(t) - G_1\tilde{e}_{y_1}(t) \quad (27)$$

$$\dot{\tilde{e}}_{y_1}(t) = \tilde{A}_{21}\tilde{e}_1(t) + \tilde{A}_{22}\tilde{e}_{y_1}(t) - G_2\tilde{e}_{y_1}(t) - v \quad (28)$$

By selecting $G_1 = \tilde{A}_{12}$ and $G_2 = \tilde{A}_{22} - A_{22}^S$, where A_{22}^S is any stable design matrix of the appropriate dimensions, the error dynamics are then reduced to:

$$\dot{\tilde{e}}_1(t) = \tilde{A}_{11}\tilde{e}_1(t) \quad (29)$$

$$\dot{\tilde{e}}_{y_1}(t) = \tilde{A}_{21}\tilde{e}_1(t) + A_{22}^S\tilde{e}_{y_1}(t) - v \quad (30)$$

The error dynamics are asymptotically stable for $v \equiv 0$ because the poles of the combined system are given by $\lambda\left(\tilde{A}_{11}\right) \cup \lambda\left(A_{22}^S\right)$ and hence they lie in the open left half of the complex plane. An identical observer has also been reported in (Edwards and Spurgeon, 1994; Edwards and Spurgeon, 1996). Although the Utkin observer assumes a linear model it will be shown below how an Utkin design procedure can be applied to nonlinear systems. The use of linear sliding mode observer for a nonlinear system has also been demonstrated in (Edwards and Spurgeon, 1994).

3. UNSCENTED KALMAN FILTER (UKF)

The UKF estimates the states of nonlinear dynamic systems by calculating the statistics of a set of sample points, referred to as the sigma points, which undergo a nonlinear transformation (Wan and van der Merwe, 2000); Simon, 2006). The method of calculating the statistics of a random variable that undergoes a nonlinear transformation is referred to as the unscented transformation (Wan and van der Merwe, 2000; Simon, 2006).

The UKF algorithm is defined as follows:

$$X_0 = \overline{x} \quad (31)$$

$$X_i = \overline{x} + \left(\sqrt{\left(n_x + \lambda\right)P_x}\right)_i \quad (32)$$

$$i = 1, \ldots, n_x$$

$$X_i = \bar{x} - \left(\sqrt{(n_x + \lambda) P_x}\right)_i \quad (33)$$

$$i = n_x + 1, ..., 2n_x$$

$$W_0^{(mean)} = \frac{\lambda}{n_x + \lambda} \quad (34)$$

$$W_0^{(\text{cov})} = \frac{\lambda}{n_x + \lambda} + \left(1 - \alpha^2 + \beta\right) \quad (35)$$

$$W_i^{(mean)} = W_i^{(\text{cov})} = \frac{1}{2(n_x + \lambda)} \quad (36)$$

$$i = 1, ..., 2n_x$$

where:

$$\lambda = \alpha^2 (n_x + k) - n_x \quad (37)$$

is the scaling factor of the unscented transformation, X_0 is initial state mean, X_i is set of $2n_x + 1$ sigma points, $W_0^{(mean)}$ is the weight of the state mean, $W_0^{(\text{cov})}$ is weight of state mean covariance, $W_i^{(mean)}$ are the weights of the sigma points, $W_i^{(\text{cov})}$ are the weights of the state covariance, n_x is number of states, P_x is state covariance, and α and β are positive scaling parameters used to minimise higher order effect. Furthermore the weights of the sigma points and the covariance are selected so that

$$\sum_{i=0}^{2n_x} W_i^{\text{cov}} = \sum_{i=0}^{2n_x} W_i^{mean} = 1 \quad (38)$$

The UKF algorithm can be described as follows:

1. Initialisation:

$$\bar{x}_0 = E[x_0]$$

$$P_0 = E\left[(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T\right]$$

And the augmented mean and covariance

$$\bar{x}_0^a = E\left[x^a\right] = \left[\bar{x}_0^T \quad 0 \quad 0\right]^T$$

$$P_0^a = E\left[\left(x_0^a - \bar{x}_0^a\right)\left(x_0^a - \bar{x}_0^a\right)^T\right] = \begin{bmatrix} P_0 & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & R \end{bmatrix}$$

2. Sigma points update:

$$X_{t-1}^a = \left[\bar{x}_{t-1}^a \quad \bar{x}_{t-1}^a \pm \sqrt{(n_a + \lambda) P_{t-1}^a}\right]$$

3. Time Update:

a. $X_{t|t-1}^x = f\left(X_{t-1}^x, X_{t-1}^Q\right)$

b. $\bar{x}_{t|t-1} = \sum_{i=0}^{2n_a} W_i^{(mean)} X_{i,t|t-1}^x$

c. $P_{t|t-1} = \sum_{i=0}^{2n_a} W_i^{(\text{cov})} \left[X_{i,t|t-1}^x - \bar{x}_{t|t-1}\right] \times \left[X_{i,t|t-1}^x - \bar{x}_{t|t-1}\right]^T + Q$

d. $Y_{t|t-1} = h\left(X_{t|t-1}^x, X_{t|t-1}^R\right)$

e. $\bar{y}_{t|t-1} = \sum_{i=0}^{2n_a} W_i^{(mean)} Y_{i,t|t-1}$

4. Measurement Update:

a. $P_{\tilde{y}_t \tilde{y}_t} = \sum_{i=0}^{2n_a} W_i^{(\text{cov})} \left[Y_{i,t|t-1} - \bar{y}_{t|t-1}\right] \times \left[Y_{i,t|t-1} - \bar{y}_{t|t-1}\right]^T + R$

b. $P_{x_t y_t} = \sum_{i=0}^{2n_a} W_i^{(\text{cov})} \left[X_{i,t|t-1} - \bar{x}_{t|t-1}\right] \times \left[Y_{i,t|t-1} - \bar{y}_{t|t-1}\right]^T$

c. $K_t = P_{x_t y_t} P_{\tilde{y}_t \tilde{y}_t}^{-1}$

d. $P_t = P_{t|t-1} - K_t P_{\tilde{y}_t \tilde{y}_t} K_t^T$

e. $\bar{x}_t = \bar{x}_{t|t-1} + K_t \left(y_t - \bar{y}_{t|t-1}\right)$

where Q is process noise covariance, R is measurement noise covariance, and Q and R are the tuning parameters of the UKF.

4. AN UNSCENTED UTKIN OBSERVER

The Utkin observer is a deterministic observer which is driven by the system input, the difference between the measurement and the estimated measurement, and a discontinuous vector component

$$v_i = K \operatorname{sgn}(\hat{y}_{1_i} - y_{1_i}) \quad (39)$$

which makes it a robust observer that guarantees a convergence of any initial state to the true state under the assumption that the measurement is not corrupted by any noise.

In contrast to the Utkin observer, the UKF minimises the effect of noise on the estimated states. However, the convergence condition is not explicitly expressed in the UKF algorithm. Convergence in the UKF depends entirely on the selection of the two tuning parameters Q and R. Both the convergence of estimated state and the minimisation in noise effect can be achieved by combining the two observers to form an unscented Utkin observer. For linear systems this is simply done by replacing step 3a of the UKF algorithm:

$$X_{t|t-1}^x = f\left(X_{t-1}^x, X_{t-1}^Q\right) \quad (40)$$

by a stable design of nonlinear Utkin observer.

Note that the linear Utkin observer design procedure can also be applied to nonlinear systems which are linear in the control variable, and where the output equation is linear. This is done by separating the state equation into a linear part and nonlinear part. Let the nonlinear system be described as follows:

$$\dot{x}(t) = Ax(t) + N(t, x) + Bu(t) \quad (41)$$

$$y(t) = Cx(t) \quad (42)$$

Now let A be the linear matrix, N the nonlinearity vector, B the input matrix, and C the output matrix. The system is assumed to be nonlinearly observable (Hermann and Krener, 1977). Applying the transformation given by equation (3) yield:

$$\dot{x}_1(t) = A_{11}x_1(t) + A_{12}y(t) + N_1 + B_1u(t) \quad (43)$$

$$\dot{y}(t) = A_{21}x_1(t) + A_{22}y(t) + N_2 + B_2u(t) \quad (44)$$

Then further transforms the system using the L dependent transformation given by equation (16) to give:

$$\dot{\tilde{x}}_1(t) = \tilde{A}_{11}\tilde{x}_1(t) + \tilde{A}_{12}\tilde{y}_1(t) + \tilde{N}_1 + \tilde{B}_1u(t) \quad (45)$$

$$\dot{\tilde{y}}_1(t) = \tilde{A}_{21}\tilde{x}_1(t) + \tilde{A}_{22}\tilde{y}_1(t) + \tilde{N}_2 + \tilde{B}_2u(t) \quad (46)$$

Thus the modified Utkin observer for a nonlinear case is given by:

$$\dot{\hat{\tilde{x}}}_1(t) = \tilde{A}_{11}\hat{\tilde{x}}_1(t) + \tilde{A}_{12}\hat{\tilde{y}}_1(t) + \hat{\tilde{N}}_1 + \tilde{B}_1u(t) - G_1\tilde{e}_{y_1}(t) \quad (47)$$

$$\dot{\hat{\tilde{y}}}_1(t) = \tilde{A}_{21}\hat{\tilde{x}}_1(t) + \tilde{A}_{22}\hat{\tilde{y}}_1(t) + \hat{\tilde{N}}_2 + \tilde{B}_2u(t) - G_2\tilde{e}_{y_1}(t) + v \quad (48)$$

Now, instead of propagating the sigma points through the model of the plant, the sigma points are now propagated through the Utkin observer and the propagated sigma points will then go through the UKF steps. The unscented Utkin observer can be seen as having $2n_x + 1$ Utkin observers to correct each of the sigma points trajectories before updating them using the UKF algorithm. Although the propagation algorithm (48) and (49) is in a continuous-time form, in practice the Utkin observer requires to be integrated between sampling times to obtain sampled values of the continuous Utkin state estimate, which are required for the UKF part of the combined observer. The combined observer has four design parameters: the process and measurement noise covariances Q and R, and the two design parameters of the Utkin observer L and A_{22}^S.

5. SIMULATION AND RESULTS

The unscented Utkin observer is tested using a simulation of a single link robot arm with an elastic revolute joint rotating in a vertical plane (Koshkouei and Zinober ,2004). The robot arm model is given by:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{F_l}{J_l}x_2 - \frac{glM}{J_l}\sin(x_1) - \frac{k}{J_l}(x_1 - x_3) + \eta_1$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -\frac{F_m}{J_m}x_4 + \frac{k}{J_m}(x_1 - x_3) + \frac{1}{J_m}u + \eta_2$$

where x_1 is the link displacement (rads), x_2 is the link velocity (rads/s), x_3 is the rotor displacement (rads), and x_4 is the rotor velocity (rads/s). J_l is the link inertia, J_m is the motor rotor inertia, k is the elastic constant, M is the mass of the link, l is the centre of the mass, and g is the acceleration due to gravity. F_l and F_m viscous friction coefficients, η_1 and η_2 are random perturbations. For the purpose of simulation the following values are selected: $J_l = 4\,\text{Nm}^2$, $J_m = 4\,\text{Nm}^2$, $l = 0.3\,\text{m}$, $k = 15\,\text{Nm/rad}$, $F_m = 0.005$, $F_l = 0.006$, $M = 0.15\,\text{kg}$, $g = 9.81\,\text{m/s}^2$, $\eta_1 = 0.02\xi$, $\eta_2 = 0.04\xi$, where ξ is Gaussian random noise with variance 0.001. The same system can be written in matrix form where the nonlinear and linear parts are separated as follows:

$$\dot{x}(t) = Ax(t) + N(t,x) + Bu(t) + H\xi$$
$$y(t) = Cx(t) + \upsilon(t)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{J_l} & -\frac{F_l}{J_l} & \frac{k}{J_l} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{J_m} & 0 & -\frac{k}{J_m} & -\frac{F_m}{J_m} \end{bmatrix},$$

$$N = \begin{bmatrix} 0 & -\frac{glM}{J_l}\sin(x_1) & 0 & 0 \end{bmatrix}^T,$$

$$H = \begin{bmatrix} 0 & 0.02 & 0 & 0.04 \end{bmatrix}^T, \; B = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{J_m} \end{bmatrix}^T,$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

where A is a matrix with linear coefficients with respect to the states, N is the vector of nonlinearities, H is the process noise distribution matrix, B is the input matrix, C is the output distribution matrix, and $\upsilon(t)$ represents measurement noise. Now, for the purposes of Utkin observer design, assume that $\xi(t) = \upsilon(t) = 0$, and transform the system into new coordinates using the transformation given by equation (3) to bring the system into the form of equations (43) and (44). Then further transform the system using the transformation matrix of equation (16) to bring the system into the form of equation (45) and (46). The Utkin observer for this particular system is given by:

$$\dot{\hat{\tilde{x}}}_1 = \tilde{A}_{11}\hat{\tilde{x}}_1 + \tilde{A}_{12}\hat{\tilde{y}}_1 + \hat{\tilde{N}}_1 + \tilde{B}_1u - G_1\tilde{e}_{y_1}$$

$$\dot{\hat{\tilde{y}}}_1 = \tilde{A}_{21}\hat{\tilde{x}}_1 + \tilde{A}_{22}\hat{\tilde{y}}_1 + \hat{\tilde{N}}_2 + \tilde{B}_2u - G_2\tilde{e}_{y_1} - v$$

The simulated output is corrupted with a Gaussian noise $\upsilon(t)$ with a variance of 0.001, and the chosen tuning parameters are:

$$Q = \begin{bmatrix} 10^{-3} & 0 & 0 & 0 \\ 0 & 10^{-3} & 0 & 0 \\ 0 & 0 & 10^{-3} & 0 \\ 0 & 0 & 0 & 10^{-3} \end{bmatrix}, R = 10^{-3} ,$$

$L=\begin{bmatrix}-1.5 & -1 & 0\end{bmatrix}^T$, $A_{22}^s=-1$, $K=-0.001$

Note that the transformed form of the model (with states $\tilde{x}_1, \tilde{y}_1$) was used for the purposes of simulation, so that the estimated states obtained by the unscented Utkin observer could be compared with the simulated ones. The same was done with the UKF simulations, to make it possible to compare the results in both cases. Figures 1 to 4 show the simulation results of the unscented Utkin observer. Figures 5 to 8 below show the simulation results of the UKF obtained from the same system simulated under the same conditions and initial states.

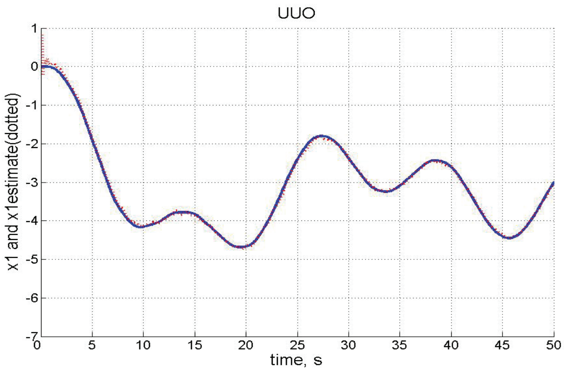

Fig 1. $\tilde{x}_1$ and $\hat{\tilde{x}}_1$ (dotted) obtained through the unscented Utkin observer

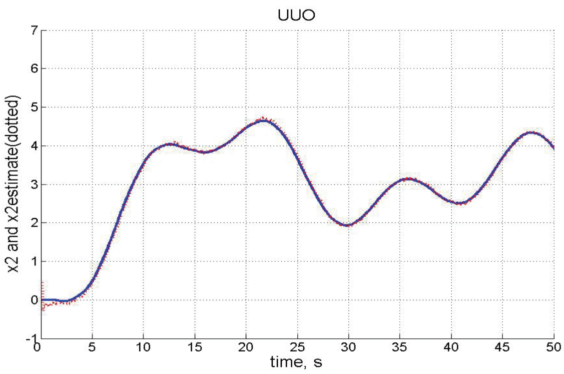

Fig 2. $\tilde{x}_2$ and $\hat{\tilde{x}}_2$ (dotted) obtained through the unscented Utkin observer

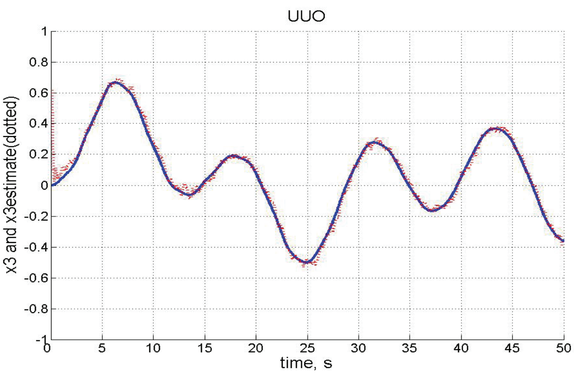

Fig 3. $\tilde{x}_3$ and $\hat{\tilde{x}}_3$ (dotted) obtained through the unscented Utkin observer

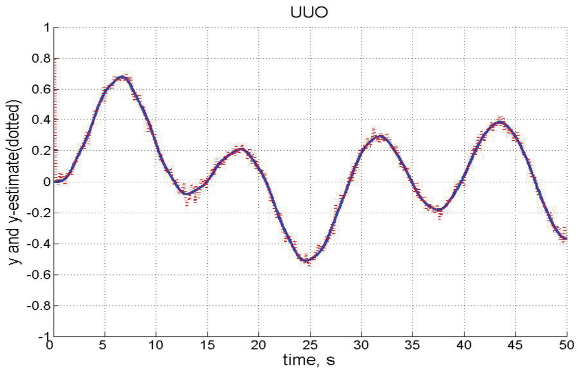

Fig 4. $\tilde{y}_1$ and $\hat{\tilde{y}}_1$ (dotted) obtained through the unscented Utkin observer

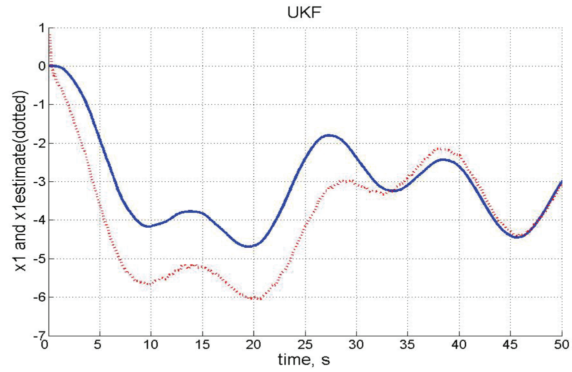

Fig 5. $\tilde{x}_1$ and $\hat{\tilde{x}}_1$ (dotted) obtained through the UKF

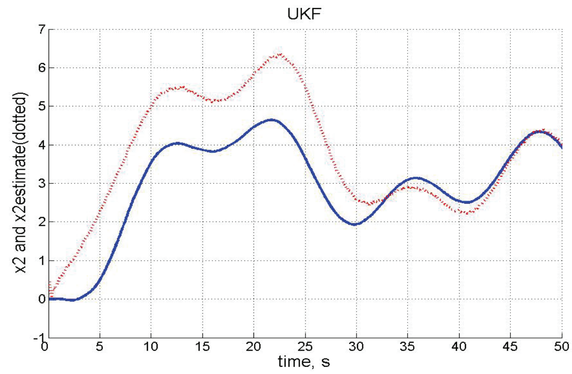

Fig 6. $\tilde{x}_2$ and $\hat{\tilde{x}}_2$ (dotted) obtained through the UKF

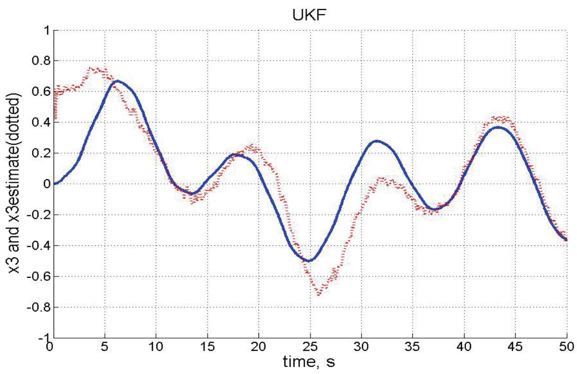

Fig 7. $\tilde{x}_3$ and $\hat{\tilde{x}}_3$ (dotted) obtained through the UKF

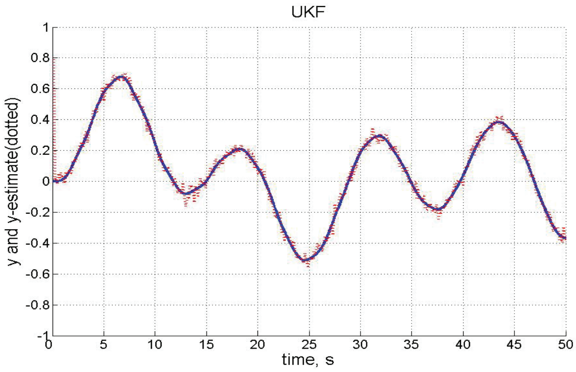

Fig 8. $\tilde{y}_1$ and $\hat{\tilde{y}}_1$ (dotted) obtained through the UKF

Table 1 shows the mean square error of each state from both UKF and unscented Utkin observer obtained from different initial states. The mean square errors of the states are computed from simulations lasting for 100 seconds. The mean square error for each transformed state was computed as follows:

$$MSE_i = \frac{1}{N}\sum_{k=1}^{N}(\hat{\tilde{x}}_{i,k} - \tilde{x}_{i,k})^2 \quad (49)$$

Table 1: Computed Mean Square Errors

Initial states	MSE with Unscented Kalman Filter	MSE with Unscented Utkin Observer
$\tilde{x}_1(0) = 0.8214$	0.62680	0.00250
$\tilde{x}_2(0) = 0.4447$	0.63260	0.00210
$\tilde{x}_3(0) = 0.6154$	0.02060	0.00100
$\tilde{y}_1(0) = 0.7919$	0.00140	0.00140
$\tilde{x}_1(0) = 0.8913$	0.00260	0.00260
$\tilde{x}_2(0) = 0.7621$	0.00230	0.00230
$\tilde{x}_3(0) = 0.4565$	0.00095	0.00093
$\tilde{y}_1(0) = 0.0185$	0.00081	0.00080
$\tilde{x}_1(0) = 0.9501$	0.30700	0.02000
$\tilde{x}_2(0) = 0.2311$	0.30950	0.01950
$\tilde{x}_3(0) = 0.6068$	0.01280	0.00290
$\tilde{y}_1(0) = 0.4860$	0.00100	0.00100
$\tilde{x}_1(0) = 0.9218$	0.03640	0.01280
$\tilde{x}_2(0) = 0.7382$	0.03630	0.01300
$\tilde{x}_3(0) = 0.1763$	0.00190	0.00170
$\tilde{y}_1(0) = 0.4057$	0.00097	0.00096
$\tilde{x}_1(0) = 0.1389$	0.02040	0.01560
$\tilde{x}_2(0) = 0.2028$	0.02050	0.01550
$\tilde{x}_3(0) = 0.1987$	0.00170	0.00160
$\tilde{y}_1(0) = 0.6038$	0.00120	0.00120

Notice that the mean square error is smaller for all states in the case of the Unscented Utkin observer.

6. CONCLUSION

This paper describes the integration of the Utkin observer and the unscented Kalman filter, investigates the performance of the combined observer by simulation using a model of a single link robot arm, and compares it with an unscented Kalman filter. The computed mean square error indicates that the unscented Utkin observer outperforms the unscented Kalman filter. As well as achieving a good convergence and noise rejection, the unscented Utkin observer should also provides robustness due to its discontinuous component. However having established that the unscented Utkin observer outperforms UKF, the unscented Utkin observer is limited to systems whose output has a linear relationship given by equation (42).

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