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# A Novel RSMI based on Regression and Natural Power Method

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**Abstract:** In this paper, we propose a new recursive subspace model identification (RSMI) based on regression and natural power method (NP) which is an array signal processing algorithm with excellent convergence properties. We call this new algorithm as 'R-NP'. The basic idea of the algorithm is to utilize an unstructured least squares linear regression approach at the updating observation vector step and the close relationship between RSMI with NP. This algorithm has simpler procedures than other RSMI algorithms. A numerical example illustrates that R-NP method is efficient and have a better performance in terms of transient behavior with respect to EIVPAST. In this paper, we consider the case where the order of system to be identified is a priori known.

Keywords: System identification; Subspace method; Recursive algorithm.

## 1. INTRODUCTION

Subspace model identification (SMI) has developed for more than a decade. A number of papers about SMI have been published. Several representative algorithms have been proposed, e.g. CVA (Larimore [1990]), MOESP (Verhaegen & Dewilde [1992]), N4SID (Van Overschee & De Moor [1994]). All these SMIs fall into the unifying theorem proposed by Van Overschee & De Moor [1995]. In the literature, these algorithms can be interpreted as singular value decomposition just with different weighted matrices.

Most of the past SMIs are considered for linear time invariant (LTI) systems in off-line field. But most practice systems are time variant, even nonlinear. It's natural to develop recursive SMI (RSMI) for identifying these systems. RSMI has been an active research area for last decade( Gustafsson [1997], Oku & Kimura [2002], Lovera [2003], Mercère & Lovera [2007]). As we know, RSMI methods are derived from the off-line versions of SMI. Most of available SMI techniques are based on how to obtain the extended observability matrix which is derived from the singular value decomposition (SVD) of a certain matrix consists of given input and output data. In order to estimate recursively the observability matrix, it is necessary to determine, at each time step, an accurate update of the observation vector or matrix first. In Gustafsson [1997], the author estimated the Toeplitz matrix of future input term by utilizing the structure of the Toeplitz matrix and eliminate the future input term. Other methods (Gustafsson et al. [1998] Lovera [2003] Mercère et al. [2005]) were based on RQ decomposition of MOESP to avoid estimating the Toeplitz matrix. These methods made use of a sequence of Givens rotations to eliminate the future input term and obtained an observation matrix with the same information of observation vector(Mercère et al. [2004]). Oku (Oku & Kimura [2002]) provided the updating compressed I/O data concept to updating the same information of observation vector. But these methods seems a little fussy and complicated. In this paper, we introduce a new method based on unstructured least squares linear regression approach to eliminate the future input term directly. This thought derive from Jansson [2003].

The computation of the SVD has been the bottleneck in RSMI. Gustafsson et al. [1998] presented recursive algorithms to directly update an estimate of the extended observability matrix when the order of a system to be identified is a priori know. Updating a vector was to be fed into the IV-PAST algorithm given by (Gustafsson [1998], Gustafsson [1997]), Oku & Kimura [2002] had presented another recursive algorithm using gradient type subspace tracking. Mercère et al. [2005] proposed an algorithm through another array signal processing called propagator method (PM). The projection approximation subspace tracking (PAST) was originally introduced into the array signal processing by Yang [1995],Yang [1996]. The basic idea of PAST is that a projection like unconstrained criterion is approximated using a clever projection approximation, which leads to a recursive least squares (RLS)-like algorithm for tracking the signal subspace. However, because of the approximation, an estimate converges to a matrix whose column vectors span a slightly different subspace from the one obtained by the original minimization

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problem. Gradient type subspace tracking is a straightforward resolution for minimizing the unconstrained criterion. PAST, and gradient type subspace tracking can all be viewed as some heuristic variations of a classic method for subspace computation, known as the power method Golub et al. [1996]. In this paper, we adopt a natural version of the power method (NP) to search the global minimizer. The natural power method is shown to have the fastest convergence rate among the power-based methods (Hua et al. [1999]).

In this paper, we combine this unstructured least squares linear regression approach and NP method to form a new RSMI method called R-NP. A numerical example illustrates that R-NP has a good convergence rate and accurate system parameters estimation. Its updating procedure is simpler and more intuitionistic than other methods, and it seems to have a better performance in the view of transient behavior than the EIVPAST method.

This paper is organized as follows: in section 2, the problem and notation are introduced, in section 3, the basic estimation method is described. Section 4 is dedicated to the main results that illuminate the new algorithm based on regression and NP method. Finally, section 5 gives a numerical example to illustrate that our algorithm and compare these two results. And in section 6, the conclusion is presented. A detail of R-NP algorithm is provided in appendix.

## 2. PROBLEM AND NOTATION

Consider an $n-th$ order linear time-invariant system in the innovation state space form which is equivalent with process form:

$$x_{t+1} = Ax_t + Bu_t + Ke_t \tag{1a}$$

$$y_t = Cx_t + Du_t + e_t \tag{1b}$$

Where $y_t \in R^l$, $u_t \in R^m$, $x_t \in R^n$ and $e_t \in R^l$ are the system output, input, state and innovation, respectively. $A$, $B$, $C$ and $D$ are system matrices with appropriate dimensions. $K$ is the Kalman filter gain.

The problem is to estimate recursively a state-space realization from the updates of the disturbed I/O data $u(t)$ and $y(t)$. To establish the statistical consistency of the SIM under open-loop condition, we have the following assumptions :
A1: $(A,C)$ is observable.
A2: $(A,[B\ K])$ is controllabe.
A3: The eigenvalues of $A-KC$ are strictly inside the unit circle.
A4: The input $u$ and innovation $e$ are jointly stationary. And

$$\overline{E}[e(k)e(l)^T] = R_e\delta_{kl}, \tag{2}$$

$$\overline{E}[e(k)u(l)^T] = 0, \forall k,l \tag{3}$$

where $\overline{E}$ is defined as in Ljung [1999] :

$$\overline{E}\{\cdot\} = \lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}E\{\cdot\}. \tag{4}$$

A5: The input signal is persistency-of-excitation (PE) of a sufficient order.
A6: The order $n$ of system (1) is priori known.

For the propose of identifying, we introduce some notations by convention:

$$y_f(t) = \begin{bmatrix} y_t^T\ y_{t+1}^T \cdots y_{t+f-1}^T \end{bmatrix}^T$$

where $f > n$ is a user defined integer which is called future horizon. From the same way, we can formulate the $u_f(t)$, $w_f(t)$, and $e_f(t)$. We also denote that the extended observability matrix of the system:

$$\Gamma_f = \begin{bmatrix} C^T\ CA^T\ \cdots\ (CA^{f-1})^T \end{bmatrix}^T$$

Then we can obtain an extended input-output equation:

$$y_f(t) = \Gamma_f x(t) + H_f u_f(t) + G_f e_f(t) \tag{5}$$

where $H_f$ and $G_f$ are two lower triangular Toeplitz matrices.

$$H_f = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{f-2}B & CA^{f-3}B & \cdots & D \end{bmatrix}$$

$$G_f = \begin{bmatrix} I & 0 & \cdots & 0 \\ CK & I & \cdots & 0 \\ CAK & CK & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{f-2}K & CA^{f-3}K & \cdots & I \end{bmatrix}$$

We know the procedure of subspace identification approach is: first, the (Kalman filter) states or observability matrix are estimated directly from input-output data, then the system matrices can be obtained (Van Overschee & De Moor [1996]). As long as we know the estimation $\hat{\Gamma}_f$ of the observability matrix, we can estimate $\hat{A}$, $\hat{C}$, $\hat{B}$, $\hat{D}$, $\hat{K}$ up to within a similarity transformation, and $\hat{R}$. The main subject of RSMI is to work out an approach to compute the estimation of a basis for the column subspace of matrix $\Gamma_f$ with a reduced computational cost.

For most RSMIs, the first step is to obtain the new vector or matrix called observation vector or matrix which contain the new information of system (Mercère et al. [2004]). It seems natural to consider $z_f(t) = y_f(t) - H_f u_f(t)$ as the observation vector.

## 3. ESTIMATION METHOD

As in the introduction, we have shown that there are some methods to obtain the observation vector or matrix. In Gustafsson [1997], it pointed out that the first block of $H_f$ at t time can be approximately given by $[\hat{D}^T(t-1)(\hat{\Gamma}_f(t-1)_{(1:p(f-1),:)}\hat{B}(t-1)^T]^T$ which $\hat{D}^T(t-1)$ and $\hat{B}(t-1)$ is the previous step estimated $D$, $B$. Here, we adopt the notation used in the MATLAB. For example, $X_{(1:l,:)}$ denotes the first $l$ rows of a matrix $X$. Similarly, we can form the entire $H_f$ at t time from the structure of $H_f$. From the procedure, it's easy to show that asymptotic properties is not good enough, since the next step updating using the last estimated system parameters. Other methods are based on RQ decomposition of MOESP to avoid estimating the Toeplitz matrix. Through the Givens rotations of R matrix of RQ decomposition to eliminate the future input term (more details see, e.g.Gustafsson et al. [1998], Lovera [2003], Mercère et al. [2005], and the references therein).

The algorithms proposed in Oku [2000] are the ways of compressing given I/O data recursively into a matrix with a fixed size, not the ones of updating an estimate of the extended observability matrix directly. These methods are not intuitionistic enough and a little complicated. And most algorithm are only based on MOESP. In this section, we propose a simple and effective way to achieve the update step.

From the above equation(5), it is easy to show that we can subtract $H_f u_f(t)$ from $y_f(t)$ if the matrix $H_f$ was a priori known. And then compute SVD of the term $z_f(t)$ to estimate $\Gamma_f(t)$. Let us rewrite (1) as follows:

$$x_{t+1} = A_k x_t + B_k u_t + K y_t \tag{6a}$$

$$y_t = C x_t + D u_t + e_t \tag{6b}$$

where $A_k = A - KC$, $B_k = B - KD$. From (6), it is also clear that:

$$x(t) = \sum_{i=0}^{p-1} A_k^i \left[ K y(t-i-1) \;\; B_k u(t-i-1) \right] + A_k^p x_{t-p} \tag{7}$$

And since assuming $A_k$ is stable according A3, if choosing $p$ large enough, we can consider that $A_k^p \approx 0$. Then we can estimate the state by a linear combination of past inputs and outputs as follows:

$$\hat{x}(t) = \mathcal{L} p(t) \tag{8}$$

where $\mathcal{L}$ is unknown expression of $A_k$, $B_k$ and $K$. $p(t)$ is a vector which consist of delayed inputs and outputs $p$ steps back:

$$p(t) = \left[ y^T(t-1) \cdots y^T(t-p) \;\; u^T(t-1) \cdots u^T(t-p) \right]^T$$

Substitute $\hat{x}(t)$ for $x(t)$ in (5), we can obtain:

$$y_f(t) \approx \Gamma_f \mathcal{L} p(t) + H_f u_f(t) + G_f e_f(t) \tag{9}$$

This above equation indicates that $y_f(t)$ is regressed on $p(t)$ and $u_f(t)$. So we can estimate unstructured form $\widehat{\Gamma_f \mathcal{L}}$ and $\hat{H}_f$ of $\Gamma_f \mathcal{L}$ and $H_f$. Because $H_f$ is a lower triangular matrix. We could use a constrained linear square regression to estimate $H_f$. This guarantees the form of $H_f$. We denote $z_f(t) = y_f(t) - \hat{H}_f u_f(t)$ then as follows:

$$z_f(t) \approx \Gamma_f x(t) + G_f e_f(t) \tag{10}$$

The first step of RSMI can be easily solved by (9). For improving the statistical properties of estimated $H_f$, we must utilize the past samples as many as possible. At the time $t+i$, new data samples $y_f(t+i+f-1)$ and $u_f(t+i+f-1)$ are acquired, then we can reconstruct (9) as follows:

$$\begin{aligned} \left[ y_f(t) \;\cdots\; y_f(t+i) \right] &\approx \mathcal{L} \left[ p(t) \;\cdots\; p(t+i) \right] + \\ H_f \left[ u_f(t) \;\cdots\; u_f(t+i) \right] &+ G_f \left[ e_f(t) \;\cdots\; e_f(t+i) \right] \end{aligned} \tag{11}$$

Then we can get the new observation vector:

$$z_f(t+i) = y_f(t+i) - \hat{H}_f u_f(t+i) \tag{12}$$

and the new input-output equation become:

$$z_f(t+i) \approx \Gamma_f x(t+i) + G_f e_f(t+i) \tag{13}$$

From (11)-(13), we can obtain the new observation vector $z_f(t+i)$, then we have to compute the SVD decomposition from $z_f(t+i)$ to estimate the column subspace spanned by $\Gamma_f$.

**[Remark 1]** This method can provide an accurate estimation of observation vector. But it seems having a lumbersome computation on least squares linear regression. This is a trade-off between accuracy and computation burden. For a LTI system, we don't have to compute least squares linear regression every step, just as the past horizon $p$ in (7) is enough, we could obtain an accurate estimation of $H_f$.

## 4. RECURSIVE SMI BASED ON NATURAL POWER METHOD

Most important calculation is to compute the SVD of recursive observation vector $z_f(t)$ in RSMI. But if we did it at each step time, it would cost lots of computation, obviously it's not suitable for online implementation. Thus, we have to develop new techniques to avoid the use of such burdensome computation. Here, we introduce a new algorithm to update SVD based on subspace tracking algorithm. The observation model generally assumed in antenna array signal processing has the following form :

$$r_t = A(t) s_t + n_t \tag{14}$$

In the above equation, the $n \times 1$vector $r_t$ denotes the observation, $A(t)$ is a deterministic $n \times p$ matrix, $s_t$ is a random $p \times 1$ vector which denotes the source vector and $n_t$ stands for noise. In the array signal processing field, several adaptive algorithms were suggested to estimate the signal subspace recursively.

### *4.1 Review of IVPAST*

Projection Approximation Subspace Tracking (PAST) algorithm Yang [1995] was proposed by Yang to deal with array signal processing problem.In this method,Yang introduced an unconstrained criterion to estimate the range of $A(t)$ as follows :

$$V(W) = E\|r - WW^T r\|^2 \tag{15}$$

where the matrix argument $W \in \mathscr{R}_{n \times p}$ and $n > p$. $\|\cdot\|$ is the Euclidean vector norm and $E[\cdot]$ is the expectation operator.

Yang Yang [1995] have proved the global minimum of $V(W)$ is attained if and only if $W = QT$ where $Q$ contains the $n$ dominating eigenvectors of $R_r = E[rr^T]$. Here $T$ is an arbitrary unitary matrix. Furthermore, all other stationary points are saddle points. From the minimization of (15), it provides an expression particular basis of $A(t)$. The expectation operator in (15) is replaced with exponentially weighted sum to obtain a recursive update.

$$V(W) = \sum_{k=1}^{t} \lambda^{t-k} \parallel r(k) - W(t) W^T(t) r(k) \parallel^2 \tag{16}$$

where $\lambda$ is a forgetting factor($0 < \lambda < 1$). And replace $R_r$ with $R_r(t) = \sum_{k=1}^{t} \lambda^{t-k} r(k) r^T(k)$. The key idea of PAST is to replace $W^T(k) r(k)$ with

$$h(k) = W^T(k-1) r(k) \tag{17}$$

This is so-called projection approximation. Substitute (17) for $W^T(k) r(k)$ in (16),

$$\bar{V}(W(t)) = \sum_{k=1}^{t} \lambda^{t-k} \parallel r(k) - W(t) h(k) \parallel^2 \tag{18}$$

then $V(W)$ can be minimized by

$$W(t) = R_{rh}(k) R_h^{-1}(t) \tag{19}$$

In Yang [1995] an efficient recursive RLS-like algorithm have been given. An IV generalization of PAST has been proposed in Gustafsson [1998]. IVPAST use an instrumental variable vector $\xi \in \mathscr{R}^{\gamma \times 1}$ to deal with the situation in which the measurements of $r(t)$ are affected by noise with arbitrary and unknown covariance matrix. This algorithm is similar to PAST , except replace $r$ with $R_{r\xi}$ in (15), where $R_{r\xi} = \sum_{k=1}^{t} \lambda^{t-k} r(k)\xi^T(k)$.

**[Remark 2]**Theoretically, the columns $V(W)$ minimizing the criterion $V(W)$ are orthonormal. But the minimization of $\bar{V}(W)$ leads to a matrix having columns that are not exactly orthonormal. This can be interpreted as a slow change of basis. And it might arise a problem in the estimation of the state space system parameters (Mercère et al. [2004]).

### *4.2 Natural power method*

In this paper, we bring a new subspace tracking algorithm into RSMI. The data under consideration is a sequence of $n \times 1$ random vector $\{r(t)\}$. The correlation matrix of the sequence is denoted by $R(t) = E[r(t)r(t)^T]$, which is always assumed positive definite. The principal subspace spanned by the sequence, of dimension $p < n$, is defined to be the span of the $p$ principal eigenvector of the correlation matrix can be obtained by the classic power method as follows Hua et al. [1999]:

$$w(t+1) = Rw(t) \tag{20}$$

where the $n \times 1$ weight vector $w(t)$ is the estimate of the first principal eigenvector at the $t$th iteration.

A natural choice of the scaling can be obtained as

$$w(t+1) = Rw(t)(w^T(t)R^2w(t))^{-1/2} \tag{21}$$

where the added scaling term guarantees that $w(i+1)$ is normalized to prevent $w(i)$ from becoming too large or too small. And the correlation matrix $R$ can be replaced recursively as:

$$R(t+1) = \alpha R(t) + r(t+1)r(t+1)^T \tag{22}$$

where $\alpha$ is a forgetting factor which range will be $0.99 \leq \alpha \leq 1$. For computing a principal subspace of dimension $p$,the generalized matrix version of (21) can be summarize :

$$W(t+1) = R(t+1)W(t)(W^T(t)R^2(t+1)W(t))^{-1/2} \tag{23}$$

From the above equations, one can gets a simple NP algorithm for subspace tracking as follow:

Choose the initial value $W(0)$ which is positive definite. Assume the parameter $\alpha$ satisfies $0.99 \leq \alpha \leq 1$. Suppose the $R(t)$,$r(t+1)$,$W(t)$ have already known, the $(t+1)$th estimate of $W(t+1)$ can be updated recursively as:

$$R(t+1) = \alpha R(t) + r(t+1)r^T(t+1) \tag{24a}$$

$$Z(t+1) = W^T(t)R^2(t+1)W(t) \tag{24b}$$

$$W(t+1) = R(t+1)W(t)Z(t+1)^{-1/2} \tag{24c}$$

If the covariance matrix R remains constant, and the $p$th and $(p+1)$th eigenvalues of the covariance matrix are distinct, and the initial weight matrix $W(0)$ meets a mild condition.Then the natural power algorithm 1 globally and exponentially converges to the principal subspace. An $O$(np) implementation of algorithm 1 and a detailed proof of convergence property have been proposed in Hua et al. [1999].

**[Remark 3]** We can obviously see that if we choose an initial weight matrix $W(0)$ which satisfies positive definite($W^T(0)R^2W(0) > 0$), then from the iteration (21), we can obtain an always orthogonal basis because of $W(t)$ remains orthonormal. This is an advantage over PAST. But we must see that IVPAST can deal with more complicated noise. From Hua et al. [1999], the restrictions on the noise seems loose. And a more detailed analysis should be considered. This is further work to be done.

### *4.3 Recursive SMI based on regression and NP*

RSMI is dealt with the recursive estimation system parameters from the updates of the input-output data $u(t)$ and $y(t)$. So Recursive SMI has two main procedures:
a) update the observation vector $z_f(t)$.
b) compute SVD of $z_f(t)$ to estimate $\Gamma_f$.
From the above analysis, we can obviously establish a analogy between (13) and (14). From this analogy, the algorithm described in section 4.2 can be applied to model(13). So we can easily estimate the column space spanned by the extended observability matrix and the system parameters recursively as following steps:

**Algorithm**: Suppose the new input-output sample data $u_f(t+i)$ and $y_f(t+i)$ are acquired. The $(t+i)$th estimate of the system parameters can be obtain recursively as following steps:
Step1: From (11)-(12), estimate $\hat{H}_f$ to attain $z_f(t+1)$ .
Step2: Substitute $z_f(t+i)$ for $r(t+i)$ in (23), and with the known $\Gamma_f(t+i-1)$ to compute $\Gamma_f(t+i)$ which has the estimated basis of $\hat{\Gamma}_f$.
Step3: Estimate the corresponding system parameters. $A$, $B$, $C$, $D$, $K$ and $R$ from $\hat{\Gamma}_f$ and state space model (1).
A detail of this algorithm is proposed:

$$\begin{aligned}
\Gamma_f(t+i) &= R(t+i)\Gamma_f(t+i-1)Z(t+i-1)^{-1/2} \\
Z(t+i-1) &= \Gamma_f^T(t+i-1)\Gamma_f(t+i-1) \\
R(t+i) &= \alpha R(t+i) + z_f(t+i)z_f^T(t+i) \\
z_f(t+i) &= y_f(t+i) - \hat{H}_f(t+i)u_f(t+i) \\
\hat{H}_f(t+i) &= \widehat{\Gamma_f \mathcal{L} H_f}(t+i)_{(:,f(l+m)+1:f(2m+l))} \\
\widehat{\Gamma_f \mathcal{L} H_f}(t+i) &= [y_f(t) \; y_f(t+1) \; \cdots \; y_f(t+i)] / \\
[p(t) \; p(t+1) \; \cdots \; p(t+i) \; & u_f(t) \; u_f(t+1) \; \cdots \; u_f(t+i)]
\end{aligned}$$

## 5. NUMERICAL SIMULATION

In this Section, the performance of the NP are illustrated with an MIMO system simulation study. In particular, the identification of a time invariant system of the R-NP method is compared with the EIVPAST algorithm and discussed.

Consider the 4th order linear time invariant system described by the equations:

$$x(t+1) = \begin{bmatrix} 0.603 & 0.603 & 0 & 0 \\ -0.603 & 0.603 & 0 & 0 \\ 0 & 0 & -0.603 & -0.603 \\ 0 & 0 & 0.603 & -0.603 \end{bmatrix} x(t) + \begin{bmatrix} 1.1650 & -0.6965 \\ 0.6268 & 1.6961 \\ 0.0751 & 0.0591 \\ 0.3516 & 1.7971 \end{bmatrix} u(t) + v(t)$$

$$y(t) = \begin{bmatrix} 0.2641 & -1.4462 & 1.2460 & 0.5774 \\ 0.8717 & -0.7012 & -0.6390 & -0.3600 \end{bmatrix} x(t) + \begin{bmatrix} -0.1356 & -1.2704 \\ -1.3493 & 0.9846 \end{bmatrix} u(t) + w(t)$$

The input $u(t)$ is the sum of a zero mean white noise sequence ($variance = 1$) filtered with a second order Butterworth filter (cutoff 0.5 times the Nyquist frequency, sampling Time $T = 1$) and $v(t)$ and $w(t)$ are zero mean white noise sequence of variance 0.1.

The above system has been used in order to carry out a comparison between the algorithms proposed in this paper (R-NP) and the EIVPAST algorithm Gustafsson [1998]. For the sake of completeness, we choose the future horizon as 8. The forgetting factor is fixed as 0.99. And the sample number is 1060. Both algorithms are the same parameters setting.

Fig.1 illustrates the comparison between the R-NP method and the EIVPAST method in estimating the eigvalues of system parameter A. Fig.2 shows the comparison in the view of the principal angles between the extended observability matrix and its estimate. As can be seen from these figures, R-NP and EIVPAST can yield consistent estimates. And the subspace angles between the estimate and the true the extended observability matrix also can converge to 0 which means the estimate subspace converge to the true ones. But we also can see all these converge trajectories, it seems the R-NP method show a better performance than the EIVPAST.

## 6. CONCLUSION

In this paper, a new recursive subspace model identification algorithm called R-NP have been proposed. We simply the procedure of estimating the Toeplitz matrix, and introduce a new array signal processing into RSMI. Furthermore, the performance of the R-NP and EIVPAST method have been compared by two simulation examples. The results show that the R-NP method can estimate consistent system parameters and have a fast global convergence property. We also compare the R-NP method with the EIVPAST method. From the results, we can see that the R-NP method have a better performance. Future work will aim at further exploring the issues related with the convergence of the class of RSMI algorithms and extend the algorithm R-NP to the RSMI for closed-loop data.

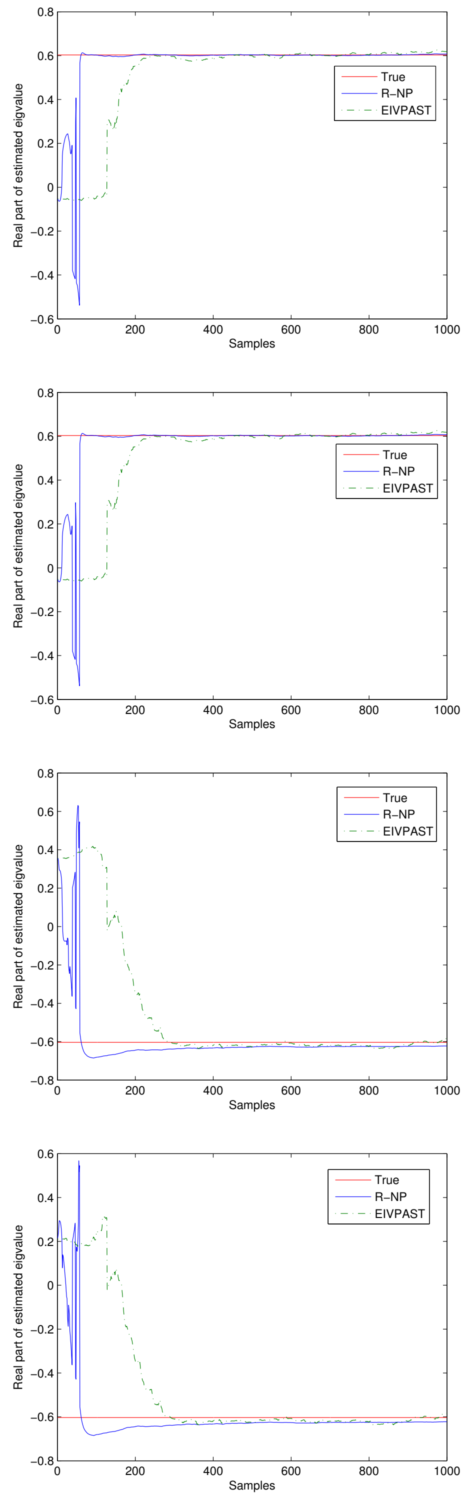


Fig. 1. The estimated eigvalues of A by R-NP and EIVPAST method. MIMO case

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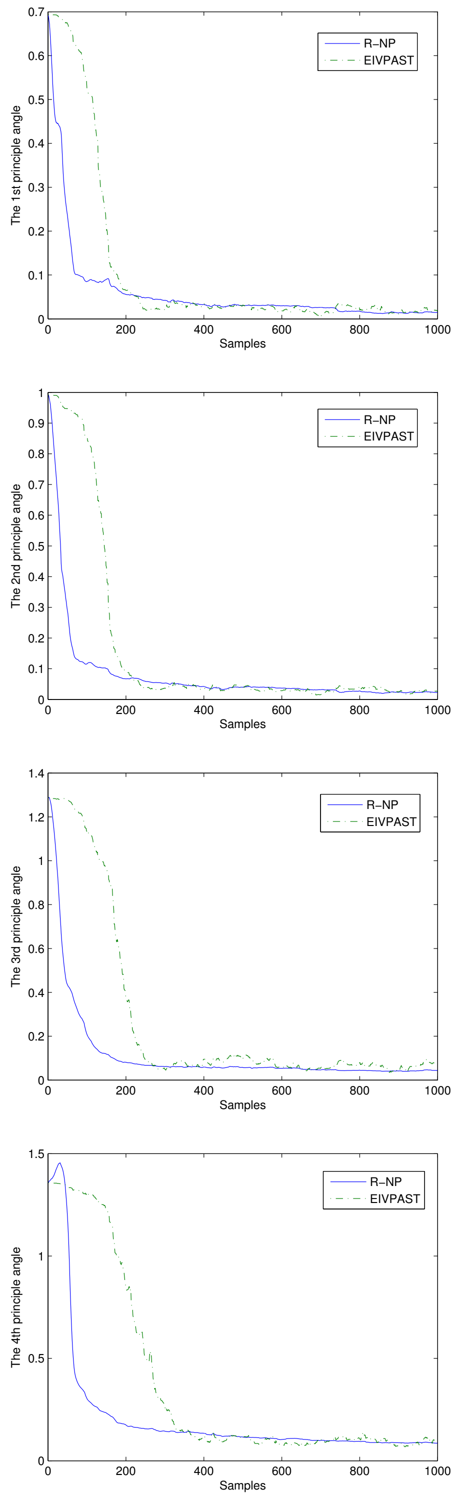


Fig. 2. The principal angles calculated by R-NP and EIVPAST method. MIMO case

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