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A Descriptor System Approach to Robust Control for Polytopic Systems with Time Delay and Its Application to Flight Control

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Abstract: This paper investigates the problem of stability analysis for a polytopic system with time-varying delay via parameter-dependent Lyapunov functions. By a relaxation approach with slack matrices and a descriptor model transformation, a new robust delay-dependent stability criterion is expressed as a set of linear matrix inequalities (LMIs) with less computational burden. This criterion combined with fault tolerant techniques can be employed for robust reliable controller synthesis for an aircraft dynamic system with multiple operating points. The resulting flight control system remains stable when actuator faults occur. The simulation results illustrate the effectiveness of the proposed approach.

1. INTRODUCTION

During the past years, the robust stability analysis of linear systems subject to time-invariant uncertainties has attracted considerable attention. For polytopic uncertainty the Edge theorem provides stability conditions. Undoubtedly, the Lyapunov theory is one of the main approaches to deal with such systems. However, the quadratic stability, which uses a single or parameter-independent Lyapunov function for testing the stability, may lead to conservativeness when uncertain parameters are time-invariant. Motivated by this fact, Lyapunov functions depending on uncertain parameters have been proposed to reduce quadratic stability conservatism. Sufficient conditions for the existence of an affine parameter-dependent Lyapunov function have been introduced in (Ramos and Peres, 2002; Vesely, 2003; Cao and Lin, 2004) for polytopic uncertainty. In (Ramos and Peres, 2002), sufficient conditions for robust stability of a polytopic system are proposed based on a set of constraints. The constraints, however, may produce conservativeness. In (Vesely, 2003), by replacing the unity matrix with a positive definite matrix a less conservative result is presented as the modification of that of (Ramos and Peres, 2002). But unfortunately, the modified constraints give rise to conservativeness likewise. Besides, the results of (Gahinet *et al.*, 1996; Ramos and Peres, 2002; Vesely, 2003) are not applicable for controller synthesis. By introducing a slack variable, (Cao and Lin, 2004) recently established a new condition for robust stability of uncertain systems. Beside the reduced conservatism, the conditions do not involve any product of the matrices in the parameter dependent Lyapunov function and system matrices. As such, this stability condition can be adapted for controller synthesis.

Time-delay is a source of performance degradation and instability in many cases. Therefore, the stability problem of time-delay systems is of theoretical and practical importance. Several results on robust control of time-delay systems subject to polytopic uncertainties have been reported in (Souza and Li, 1999; Xia and Jia, 2003; Yu, 2004; Fridman and Shaked, 2003). In (Souza and Li,1999), the authors examine the problem of H_∞ control for uncertain systems with a constant time delay. The obtained results can be easily extended to polytopic systems. Although simulation examples are presented to demonstrate the potentials of the proposed method, the result derived remains conservative. In (Xia and Jia, 2003; Yu, 2004), problems of robust stability and stabilization for polytopic systems with a constant time-delay are considered via parameter-dependent Lyapunov functionals. However, the proposed criterion depends on extra and positive scalar parameters, which increases computational burden and produces conservativeness. In (Fridman and Shaked, 2003), a sufficient condition is proposed for stability of polytopic systems, which ensures a larger upper bound for time-varying delays. But unfortunately, a nonlinear matrix inequality is obtained when this condition is employed for controller synthesis. Consequently, extra scalars that must be positive are introduced to secure a stabilizability condition in terms of LMIs, which causes conservativeness likewise. Recently, a descriptor system approach was proposed for time-delay systems. It reduced significantly the over-design compared with traditional methods (Fridman and Shaked, 2002; Gao and Wang, 2003). This approach was also applicable for polytopic systems.

So, in this paper, the problem of robust stability analysis for polytopic systems with time-varying delays is investigated by parameter-dependent Lyapunov functions. With the introduction of a slack variable, a descriptor system approach is adopted to obtain a new delay-dependent stability criterion in terms of LMIs. This criterion reduces the computational burden involved in solving LMIs. In the derivative of the Lyapunov functional, with the introduction of the augmented vector $\xi(t) \triangleq \begin{bmatrix} x^{\mathrm{T}}(t) & \dot{x}^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$, the term $h^2\dot{x}^{\mathrm{T}}(t)P_2(\lambda)\dot{x}(t)$ is formulated as $\xi^{\mathrm{T}}(t)\begin{bmatrix} 0 & 0 \\ 0 & h^2P_2(\lambda) \end{bmatrix}\xi(t)$, which avoids replacing $\dot{x}(t)$ in $h^2\dot{x}^{\mathrm{T}}(t)P_2(\lambda)\dot{x}(t)$ with the state equation. In consequence, the Lyapunov matrix P_2, which handles time delay, is not involved in any product term with system matrices A and A_d. The result is applied to robust reliable control of an aircraft with multiple operating points. Finally, the performance of the obtained controller is presented based on simulation results.

2. ROBUST STABILITY

Consider the following system with a time-varying delay

$$\begin{aligned} \dot{x}(t) &= A(\lambda)x(t) + A_d(\lambda)x(t-\tau(t)) \\ x(t) &= \phi(t), t \in [-h,0] \end{aligned}, \tag{1}$$

where $x(t) \in R^n$ is the state vector and the initial vector ϕ is a continuously differentiable function from $[-h,0]$ to R^n. We assume that $\tau(t)$ a differentiable function, satisfying

$$0 \le \tau(t) \le h,\ \dot{\tau}(t) \le d < 1. \tag{2}$$

Suppose that system matrices $A(\lambda)$ and $A_d(\lambda)$ belong to a polytopic uncertainty domain Ω_1. In this case, system matrices $(A(\lambda), A_d(\lambda))$ can be written as follows

$$(A(\lambda), A_d(\lambda)) = \sum_{i=1}^{N} \lambda_i (A_i, A_{di}) \in \Omega_1, \tag{3}$$

where $\lambda \triangleq [\lambda_1, \cdots, \lambda_N]^{\mathrm{T}} \in R^N$ denotes a vector of uncertain and time-invariant real parameters satisfying

$$\sum_{i=1}^{N} \lambda_i = 1,\ \lambda_i \ge 0. \tag{4}$$

The following inequalites will be used to prove our results.

Lemma 1. (Kharitonov and Chen, 2003). For any constant matrix $P > 0$ and differentiable vector $x(t)$ we have

$$\begin{aligned} &[\int_{t-\tau(t)}^{t} \dot{x}(s)ds]^{\mathrm{T}} P [\int_{t-\tau(t)}^{t} \dot{x}(s)ds] \\ &\le \tau(t) \cdot \int_{t-\tau(t)}^{t} \dot{x}^{\mathrm{T}}(s)P\dot{x}(s)ds \le h \cdot \int_{t-h}^{t} \dot{x}^{\mathrm{T}}(s)P\dot{x}(s)ds \end{aligned} \tag{5}$$

Lemma 2. (Yao *et al.*, 2004). For any constant matrix R_1 and R_2, a positive definite diagonal matrix U and a time-varying diagonal matrix Σ satisfying $|\Sigma| \le U$, we have $R_1\Sigma R_2 + R_2^{\mathrm{T}}\Sigma^{\mathrm{T}}R_1^{\mathrm{T}} \le \alpha R_1 U R_1^{\mathrm{T}} + \alpha^{-1}R_2^{\mathrm{T}}UR_2$, where scalars $\alpha > 0$, $\Sigma = \mathrm{diag}\{\sigma_1, \sigma_2, \cdots, \sigma_q\}$, $|\Sigma| = \mathrm{diag}\{|\sigma_1|, |\sigma_2|, \cdots, |\sigma_q|\}$.

We represent system (1) in the descriptor form

$$\begin{aligned} \dot{x}(t) &= \eta(t) \\ \eta(t) &= A(\lambda)x(t) + A_d(\lambda)x(t-\tau(t)) \end{aligned}. \tag{6}$$

Now, the following theorem presents a new delay-dependent and rate-dependent robust stability result.

Theorem 1. System (1) with parameter uncertainty (3) and time-varying delay $\tau(t)$ satisfying (2) is robustly asymptotically stable if there exist symmetric positive definite matrices P_{0i}, P_{1i} P_{2i} and matrices P_3, P_4 such that

$$\begin{bmatrix} \Delta_{11} & P_{0i} - P_3^{\mathrm{T}} + A_i^{\mathrm{T}}P_4 & P_3^{\mathrm{T}}A_{di} + P_{2i} \\ * & -P_4^{\mathrm{T}} - P_4 + h^2P_{2i} & P_4^{\mathrm{T}}A_{di} \\ * & * & -\bar{d}P_{1i} - P_{2i} \end{bmatrix} < 0,\ i = 1, \cdots, N, \tag{7}$$

where $\Delta_{11} = P_3^{\mathrm{T}}A_i + A_i^{\mathrm{T}}P_3 + P_{1i} - P_{2i}$, $\bar{d} = 1 - d$.

Proof: Define the following Lyapunov–Krasovskii functional

$$\begin{aligned} V(t,\lambda) &= x^{\mathrm{T}}(t)P_0(\lambda)x(t) + \int_{t-\tau(t)}^{t} x^{\mathrm{T}}(s)P_1(\lambda)x(s)ds \\ &\quad + h \cdot \int_{t-h}^{t} (s-(t-h))\dot{x}^{\mathrm{T}}(s)P_2(\lambda)\dot{x}(s)ds, \end{aligned} \tag{8}$$

where

$$P_0(\lambda) = \sum_{i=1}^{N} \lambda_i P_{0i},\ P_1(\lambda) = \sum_{i=1}^{N} \lambda_i P_{1i},\ P_2(\lambda) = \sum_{i=1}^{N} \lambda_i P_{2i}. \tag{9}$$

Then, the time derivative of $V(t,\lambda)$ is given by

$$\begin{aligned} \dot{V}(t,\lambda) &= x^{\mathrm{T}}(t)P_0(\lambda)\dot{x}(t) + \dot{x}^{\mathrm{T}}(t)P_0(\lambda)x(t) \\ &\quad -(1-\dot{\tau}(t))x^{\mathrm{T}}(t-\tau(t))P_1(\lambda)x(t-\tau(t)) \\ &\quad +x^{\mathrm{T}}(t)P_1(\lambda)x(t) + h^2\dot{x}^{\mathrm{T}}(t)P_2(\lambda)\dot{x}(t) \\ &\quad -h\int_{t-h}^{t} \dot{x}^{\mathrm{T}}(s)P_2(\lambda)\dot{x}(s)ds. \end{aligned} \tag{10}$$

From (10), lemma 1 and Leibniz–Newton formula, we have

$$\begin{aligned} \dot{V}(t,\lambda) &\le x^{\mathrm{T}}(t)P_0(\lambda)\dot{x}(t) + \dot{x}^{\mathrm{T}}(t)P_0(\lambda)x(t) - (1-d)x^{\mathrm{T}}(t-\tau(t)) \\ &\quad \times P_1(\lambda)x(t-\tau(t)) + x^{\mathrm{T}}(t)P_1(\lambda)x(t) + h^2\dot{x}^{\mathrm{T}}(t)P_2(\lambda) \\ &\quad \times \dot{x}(t) - [\int_{t-\tau(t)}^{t} \dot{x}(s)ds]^{\mathrm{T}} P_2(\lambda) [\int_{t-\tau(t)}^{t} \dot{x}(s)ds] \\ &= \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P_0(\lambda) & P_3^{\mathrm{T}} \\ 0 & P_4^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ 0 \end{bmatrix} + \begin{bmatrix} \dot{x}(t) \\ 0 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P_0(\lambda) & 0 \\ P_3 & P_4 \end{bmatrix} \\ &\quad \times \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix} + \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P_1(\lambda) - P_2(\lambda) & P_3^{\mathrm{T}} \\ 0 & P_4^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} x(t) \\ 0 \end{bmatrix} \\ &\quad - x^{\mathrm{T}}(t-\tau(t))(\bar{d}P_1(\lambda) + P_2(\lambda))x(t-\tau(t)) + \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix}^{\mathrm{T}} \\ &\quad \times \begin{bmatrix} 0 & 0 \\ 0 & h^2P_2(\lambda) \end{bmatrix} \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix} + \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P_2(\lambda) \\ 0 \end{bmatrix} x(t-\tau(t)) \\ &\quad + x^{\mathrm{T}}(t-\tau(t)) \begin{bmatrix} P_2(\lambda) & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix}. \end{aligned} \tag{11}$$

Note that one can obtain

$$\begin{bmatrix} x(t) \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix} \tag{12}$$

$$\begin{bmatrix} \eta(t) \\ -\eta(t) + A(\lambda)x(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ A(\lambda) & -1 \end{bmatrix} \begin{bmatrix} x(t) \\ \eta(t) \end{bmatrix} \tag{13}$$

Substituting (6) into (11) and from (12) and (13), we obtain

$$\begin{aligned} \dot{V}(t,\lambda) &\le \xi^{\mathrm{T}}(t) \begin{bmatrix} P_0(\lambda) & P_3^{\mathrm{T}} \\ 0 & P_4^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \eta(t) \\ -\eta(t) + A(\lambda)x(t) \end{bmatrix} \\ &\quad + \xi^{\mathrm{T}}(t) \begin{bmatrix} P_0(\lambda) & P_3^{\mathrm{T}} \\ 0 & P_4^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} 0 \\ A_d(\lambda)x(t-\tau(t)) \end{bmatrix} \end{aligned}$$

$$
\begin{aligned}
&+\begin{bmatrix} \eta(t) \\ -\eta(t)+A(\lambda)x(t) \end{bmatrix}^{\mathrm T} \begin{bmatrix} P_0(\lambda) & 0 \\ P_3 & P_4 \end{bmatrix} \xi(t) + \begin{bmatrix} 0 \\ A_d(\lambda)x(t-\tau(t)) \end{bmatrix}^{\mathrm T} \\
&\times \begin{bmatrix} P_0(\lambda) & 0 \\ P_3 & P_4 \end{bmatrix} \xi(t) + \xi^{\mathrm T}(t) \begin{bmatrix} P_1(\lambda)-P_2(\lambda) & 0 \\ 0 & 0 \end{bmatrix} \xi(t) \\
&- x^{\mathrm T}(t-\tau(t))(\bar d P_1(\lambda)+P_2(\lambda))x(t-\tau(t)) \\
&+ \xi^{\mathrm T}(t) \begin{bmatrix} 0 & 0 \\ 0 & h^2 P_2(\lambda) \end{bmatrix} \xi(t) + \xi^{\mathrm T}(t) \begin{bmatrix} P_2(\lambda) \\ 0 \end{bmatrix} x(t-\tau(t)) \\
&+ x^{\mathrm T}(t-\tau(t)) \begin{bmatrix} P_2(\lambda) & 0 \end{bmatrix} \xi(t) \\
=\ & \xi^{\mathrm T}(t) \begin{bmatrix} P_3^{\mathrm T} A(\lambda) & P_0(\lambda)-P_3^{\mathrm T} \\ P_4^{\mathrm T} A(\lambda) & -P_4^{\mathrm T} \end{bmatrix} \xi(t) + \xi^{\mathrm T}(t) \begin{bmatrix} P_3^{\mathrm T} A_d(\lambda) \\ P_4^{\mathrm T} A_d(\lambda) \end{bmatrix} \\
&\times x(t-\tau(t)) + \xi^{\mathrm T}(t) \begin{bmatrix} A^{\mathrm T}(\lambda)P_3 & A^{\mathrm T}(\lambda)P_4 \\ P_0(\lambda)-P_3 & -P_4 \end{bmatrix} \xi(t) \\
&+ \xi^{\mathrm T}(t) \begin{bmatrix} P_1(\lambda)-P_2(\lambda) & 0 \\ 0 & h^2 P_2(\lambda) \end{bmatrix} \xi(t) \\
&- x^{\mathrm T}(t-\tau(t))(\bar d P_1(\lambda)+P_2(\lambda))x(t-\tau(t)) + \xi^{\mathrm T}(t) \begin{bmatrix} P_2(\lambda) \\ 0 \end{bmatrix} \\
&\times x(t-\tau(t)) + x^{\mathrm T}(t-\tau(t)) \begin{bmatrix} P_2(\lambda) & 0 \end{bmatrix} \xi(t) \\
&+ x^{\mathrm T}(t-\tau(t)) \begin{bmatrix} P_3^{\mathrm T} A_d(\lambda) \\ P_4^{\mathrm T} A_d(\lambda) \end{bmatrix}^{\mathrm T} \xi(t) \\
=\ & \begin{bmatrix} \xi(t) \\ x(t-\tau(t)) \end{bmatrix}^{\mathrm T} \Xi(\lambda) \begin{bmatrix} \xi(t) \\ x(t-\tau(t)) \end{bmatrix},
\end{aligned}
\tag{14}
$$

where

$$
\xi(t) \triangleq \begin{bmatrix} x^{\mathrm T}(t) & \eta^{\mathrm T}(t) \end{bmatrix}^{\mathrm T},
$$

$$
\Xi(\lambda) = \begin{bmatrix} \Gamma(\lambda) & \begin{bmatrix} P_3^{\mathrm T} A_d(\lambda) + P_2(\lambda) \\ P_4^{\mathrm T} A_d(\lambda) \end{bmatrix} \\ * & -\bar d P_1(\lambda) - P_2(\lambda) \end{bmatrix},
$$

$$
\Gamma(\lambda) = \begin{bmatrix} P_3^{\mathrm T} A(\lambda) + A^{\mathrm T}(\lambda) P_3 \\ + P_1(\lambda) - P_2(\lambda) & P_0(\lambda) - P_3^{\mathrm T} + A^{\mathrm T}(\lambda) P_4 \\ * & -P_4^{\mathrm T} - P_4 + h^2 P_2(\lambda) \end{bmatrix}. \tag{15}
$$

According to (7) and (15), we have

$$
\Xi(\lambda) = \sum_{i=1}^{N} \lambda_i \begin{bmatrix} \Gamma_i & \begin{bmatrix} P_3^{\mathrm T} A_{di} + P_{2i} \\ P_4^{\mathrm T} A_{di} \end{bmatrix} \\ * & -\bar d P_{1i} - P_{2i} \end{bmatrix} < 0, \tag{16}
$$

where

$$
\Gamma_i = \begin{bmatrix} P_3^{\mathrm T} A_i + A_i^{\mathrm T} P_3 \\ + P_{1i} - P_{2i} & P_{0i} - P_3^{\mathrm T} + A_i^{\mathrm T} P_4 \\ * & -P_4^{\mathrm T} - P_4 + h^2 P_{2i} \end{bmatrix}. \tag{17}
$$

From (14) and (16), we get $\dot V(t,\lambda) < 0$, which completes the proof according to the Lyapunov theory.

Remark 1. In Theorem 1, with the introduction of the slack variables P_3, P_4 and the corresponding augmented vector $\xi(t) \triangleq \begin{bmatrix} x^{\mathrm T}(t) & \eta^{\mathrm T}(t) \end{bmatrix}^{\mathrm T}$, the robust stability criterion (7) does not involve the product between the Lyapunov matrix P_0 and system dynamic matrices A and A_d. Besides, the augmented vector can be used to formulate $h^2 \dot x^{\mathrm T}(t) P_2(\lambda) \dot x(t)$ as $\xi^{\mathrm T}(t) \begin{bmatrix} 0 & 0 \\ 0 & h^2 P_2(\lambda) \end{bmatrix} \xi(t)$, which avoids replacing $\dot x(t)$ in the term $h^2 \dot x^{\mathrm T}(t) P_2(\lambda) \dot x(t)$ with the state equation and so eliminates the product between the Lyapunov matrix P_2 and system matrices A and A_d. Hence, for (7), P_{2i} are not required to be the same, but the slack variables P_3 and P_4 are. So, it leads to a less conservative condition, as there are no other constraints imposed on P_3 and P_4.

Remark 2. A stability criterion was also given in Theorem 1 of (Fridman and Shaked, 2003). This criterion, however, requires more matrix variables. Consequently, the dimension of (12a) in (Fridman and Shaked, 2003) becomes higher than that of (7) in this paper ($7n$ vs. $3n$). The computational burden involved in solving these inequalities is increased accordingly, which can be verified through the next example.

The example below demonstrates the stability criterion (7).

Example 1: Consider system (1) with the following matrices borrowed from (Xia and Jia, 2002)

$$
A_1 = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.09 \end{bmatrix},\ A_2 = \begin{bmatrix} -2 & -1 \\ 0 & -2 \end{bmatrix},\ A_2 = \begin{bmatrix} -1.9 & 0 \\ 0 & -1 \end{bmatrix},
$$

$$
A_{d1} = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix},\ A_{d2} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ A_{d3} = \begin{bmatrix} -0.9 & 0 \\ -1 & -1.1 \end{bmatrix}.
$$

Several previous stability conditions have been applied to this system. For constant delay, the upper bound h is found to be 0.0853 by (Souza and Xi Li, 1999), 0.4149 by (Xia and Jia, 2002), 4.2423 by (Fridman and Shaked, 2003). According to Theorem 1, it is found that the system is robustly stable for $h = 4.2423$, which means that Theorem 1 yields better results than those obtained in (Souza and Xi Li, 1999; Xia and Jia, 2002) and the same with that of (Fridman and Shaked, 2003). To provide relatively complete information, we calculate the upper bound h for different time-varying cases, listed in Table 1, where the acronyms have the following meaning

SOU stability criterion (Souza and Xi Li, 1999)
XJA stability criterion (Xia and Jia, 2002)
XJII stability criterion (Xia and Jia, 2003)
FSK stability criterion (Fridman and Shaked, 2003)
TEM Theorem 1 proposed in this paper.

Table 1. Delay bounds by different approaches

d	0	0.1	0.5	0.9	Any d
SOU	0.0853	-	-	-	-
XJA	0.4149	-	-	-	-
XJII	0.6142	-	-	-	-
FSK	4.2423	3.3555	1.8088	0.9670	0.7963
TEM	4.2423	3.3555	1.8088	0.9670	0.7963

Besides, the numbers of iterations by (Fridman and Shaked, 2003) and Theorem 1 are presented in Table 2 to compare computational burden. It can be seen from these tables that our method achieves exactly the same upper bound of delay with less computational effort.

Table 2. Number of iterations by different approaches

d	0	0.05	0.1	0.5	0.9
FSK	46	48	46	35	25
TEM	29	29	28	24	17

3. APPLICATION TO FLIGHT CONTROL

In this section, we apply the results of section 2 to a robust reliable flight control problem. The equations of longitudinal motion of the aircraft are described by

$$\begin{bmatrix}\dot\alpha(t)\\ \dot q(t)\end{bmatrix}=\begin{bmatrix}Z'_\alpha(\rho) & Z'_q(\rho)\\ M'_\alpha(\rho) & M'_q(\rho)\end{bmatrix}\begin{bmatrix}\alpha(t-\tau(t))\\ q(t-\tau(t))\end{bmatrix}+\begin{bmatrix}Z_{\delta E}(\rho) & Z_{\delta PTV}(\rho)\\ M_{\delta E}(\rho) & M_{\delta PTV}(\rho)\end{bmatrix}\times\begin{bmatrix}\delta_E(t)\\ \delta_{PTV}(t)\end{bmatrix}+\begin{bmatrix}Z_\alpha(\rho) & Z_q(\rho)\\ M_\alpha(\rho) & M_q(\rho)\end{bmatrix}\begin{bmatrix}\alpha(t)\\ q(t)\end{bmatrix} \tag{18}$$

where $\alpha(t)$ and $q(t)$ represent angle-of-attack (AOA) and pitch rate, respectively; δ_E and δ_{PTV} represent symmetric elevator position and pitch thrust velocity nozzle position, respectively; $\rho=(M,h)$ denotes Mach and altitude; $\tau(t)$ represents a time-varying flight delay. Jet aircrafts typically have multiple operating flight conditions that correspond to the convex combination of given operating points.

Denoting $x(t)=[\alpha(t) \quad q(t)]^{\mathrm T}$, $u(t)=[\delta_E(t) \quad \delta_{PTV}(t)]^{\mathrm T}$, we can rewrite the aircraft system as

$$\dot x(t)=A(\lambda)x(t)+A_d(\lambda)x(t-\tau(t))+B(\lambda)u(t), \tag{19}$$

where $u(t)\in R^m$ is control input, system matrices satisfies

$$(A(\lambda),A_d(\lambda),B(\lambda))=\sum_{i=1}^{3}\lambda_i(A_i,A_{di},B_i)\in\Omega_2 . \tag{20}$$

Let $u^f(t)$ denotes the signal from the actuator that has failed. Then the following actuator fault model is adopted

$$u^f(t)=Fu(t), \tag{21}$$

where the fault matrix F satisfies

$$F\in\psi\triangleq\{F=\mathrm{diag}[f_1,f_2,\cdots,f_m],\ 0\le f_{lj}\le f_j\le f_{uj}, f_{uj}\ge 1, j=1,\cdots,m\}. \tag{22}$$

Define

$$F_0=\mathrm{diag}[f_{01},f_{02},\cdots,f_{0m}],\ W=\mathrm{diag}[w_1,w_2,\cdots,w_m],\ L=\mathrm{diag}[l_1,l_2,\cdots,l_m],\ |L|=\mathrm{diag}[|l_1|,|l_2|,\cdots,|l_m|], \tag{23}$$

where $f_{0j}=\frac{1}{2}(f_{lj}+f_{uj})$, $w_j=\frac{f_{uj}-f_{lj}}{f_{uj}+f_{lj}}$, $l_j=\frac{f_j-f_{0j}}{f_{0j}}$.

From (23), we have

$$F=F_0(I+L), |L|\le W\le I . \tag{24}$$

Obviously when $f_j=0$, the fault model (21) corresponds to the case of the j th actuator fault. When $f_j=1$, it corresponds to the case of no fault in the j th actuator. When $0\le f_{lj}<f_j<f_{uj}, f_{uj}\ge 1$, and $f_j\ne 1$, it corresponds to the case of partial fault in the j th actuator.

The design problem is to find a state feedback controller such that the closed-loop system is robustly stable for all admissible fault matrices F.

Design a state feedback controller

$$u(t)=Kx(t) . \tag{25}$$

Then the closed-loop system is given by

$$\dot x(t)=(A(\lambda)+B(\lambda)FK)x(t)+A_d(\lambda)x(t-\tau(t)) . \tag{26}$$

The following theorem presents a sufficient condition for the existence of the robust reliable controller.

Theorem 2. The system (26) is robustly stable if for all admissible fault matrix F, there exist symmetric positive definite matrices X_{0ri}, X_{1ri} X_{2ri}, matrices X_{3r}, Y_r such that

$$\Pi=\begin{bmatrix}\bar\Delta_{11} & X_{0ri}-X_{3r}+X_{3r}^{\mathrm T}A_i^{\mathrm T}+Y_r^{\mathrm T}FB_i^{\mathrm T} & A_{di}X_{3r}+X_{2ri}\\ * & -X_{3r}-X_{3r}^{\mathrm T}+h^2X_{2ri} & A_{di}X_{3r}\\ * & * & -\bar dX_{1ri}-X_{2ri}\end{bmatrix}<0,\quad i=1,\cdots,N, \tag{27}$$

where $\bar\Delta_{11}=A_iX_{3r}+X_{3r}^{\mathrm T}A_i^{\mathrm T}+B_iFY_r+Y_r^{\mathrm T}FB_i^{\mathrm T}+X_{1ri}-X_{2ri}$. The corresponding state feedback gain is given by $K_r=Y_rX_{3r}^{-1}$.

Proof: By defining, $P_{0i}=P_3^{\mathrm T}X_{0ri}P_3>0$, $P_{1i}=P_3^{\mathrm T}X_{1ri}P_3>0$, $P_{2i}=P_3^{\mathrm T}X_{2ri}P_3>0$, $P_3=X_{3r}^{-1}$, $Y_r=K_rX_{3r}$, multiplying (27) by $\mathrm{diag}\{P_3^{\mathrm T},P_3^{\mathrm T},P_3^{\mathrm T}\}$ and $\mathrm{diag}\{P_3,P_3,P_3\}$, on the left and the right, respectively, we obtain

$$\begin{bmatrix}\Pi_{11} & P_{0i}-P_3^{\mathrm T}+A_{cli}^{\mathrm T}P_3 & P_3^{\mathrm T}A_{di}+P_{2i}\\ * & -P_3^{\mathrm T}-P_3+h^2P_{2i} & P_3^{\mathrm T}A_{di}\\ * & * & -\bar dP_{1i}-P_{2i}\end{bmatrix}<0,\ i=1,\cdots,N, \tag{28}$$

where $\Pi_{11}=P_3^{\mathrm T}A_{cli}+A_{cli}^{\mathrm T}P_3+P_{1i}-P_{2i}$, $A_{cli}=A_i+B_iFK_r$. For the system (26), application of Theorem 1 ends the proof.

A sufficient condition for the existence of robust standard controller without considering any fault is obtained by Theorem 2 with $F=I$.

Corollary 1. The system (26) with $F=I$ is robustly stable if there exist symmetric positive definite matrices X_{0si}, X_{1si} X_{2si} and matrices X_{3s}, Y_s such that

$$\begin{bmatrix}\bar\Delta_{11s} & X_{0si}-X_{3s}+X_{3s}^{\mathrm T}A_i^{\mathrm T}+Y_s^{\mathrm T}B_i^{\mathrm T} & A_{di}X_{3s}+X_{2si}\\ * & -X_{3s}-X_{3s}^{\mathrm T}+h^2X_{2si} & A_{di}X_{3s}\\ * & * & -\bar dX_{1si}-X_{2si}\end{bmatrix}<0,\quad i=1,\cdots,N, \tag{29}$$

where $\bar\Delta_{11s}=A_iX_{3s}+X_{3s}^{\mathrm T}A_i^{\mathrm T}+B_iY_s+Y_s^{\mathrm T}B_i^{\mathrm T}+X_{1si}-X_{2si}$, $\bar d=1-d$. The state feedback gain is given by $K_s=Y_sX_{3s}^{-1}$.

Remark 3. A relevant result was also given in (Fridman and Shaked, 2003). With the introduction of extra scalars α and ε, a stabilizability criterion is derived in terms of LMIs. This criterion depends upon α and ε that must be positive. Thus, this treatment, which estimates α and ε in advance to secure feasible solutions, causes conservativeness.

The design method for robust reliable controller is presented as follows.

Theorem 3. The system (26) is robustly stable if there exist scalars $\alpha_1 > 0$, $\alpha_2 > 0$, symmetric positive definite matrices X_{0ri}, X_{1ri} X_{2ri} and matrices X_{3r}, Y_r such that

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} & A_{di}X_{3r} + X_{2ri} & Y_r^{\mathrm{T}} & Y_r^{\mathrm{T}} \\ * & \Lambda_{22} & A_{di}X_{3r} & 0 & 0 \\ * & * & -\bar{d}X_{1ri} - X_{2ri} & 0 & 0 \\ * & * & * & -\alpha_1 W^{-1} & 0 \\ * & * & * & * & -\alpha_2 W^{-1} \end{bmatrix} < 0, \; i = 1, \cdots, N, \quad (30)$$

where $\Lambda_{11} = A_i X_{3r} + X_{3r}^{\mathrm{T}} A_i^{\mathrm{T}} + B_i F_0 Y_r + Y_r^{\mathrm{T}} F_0 B_i^{\mathrm{T}} + X_{1ri} - X_{2ri} + \alpha_1 B_i F_0 \times W(B_i F_0)^{\mathrm{T}}$, $\Lambda_{12} = X_{0ri} - X_{3r} + X_{3r}^{\mathrm{T}} A_i^{\mathrm{T}} + Y_r^{\mathrm{T}} F_0 B_i^{\mathrm{T}}$, $\Lambda_{22} = -X_{3r} - X_{3r}^{\mathrm{T}} + h^2 X_{2ri} + \alpha_2 B_i F_0 W(B_i F_0)^{\mathrm{T}}$. The corresponding state feedback gain is given by $K_r = Y_r X_{3r}^{-1}$.

Proof: From (24) and (27), we obtain

$$\Pi = \begin{bmatrix} \bar{\Lambda}_{11} & \Lambda_{12} & A_{di}X_{3r} + X_{2ri} \\ * & \bar{\Lambda}_{22} & A_{di}X_{3r} \\ * & * & -\bar{d}X_{1ri} - X_{2ri} \end{bmatrix} + \begin{bmatrix} B_i F_0 L Y_r + (B_i F_0 L Y_r)^{\mathrm{T}} & (B_i F_0 L Y_r)^{\mathrm{T}} & 0 \\ B_i F_0 L Y_r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (31)$$

where $\bar{\Lambda}_{11} = A_i X_{3r} + X_{3r}^{\mathrm{T}} A_i^{\mathrm{T}} + B_i F_0 Y_r + Y_r^{\mathrm{T}} F_0 B_i^{\mathrm{T}} + X_{1ri} - X_{2ri}$, $\bar{\Lambda}_{22} = -X_{3r} - X_{3r}^{\mathrm{T}} + h^2 X_{2ri}$.

From lemma 2, we have

$$\begin{bmatrix} B_i F_0 L Y_r + (B_i F_0 L Y_r)^{\mathrm{T}} & (B_i F_0 L Y_r)^{\mathrm{T}} & 0 \\ B_i F_0 L Y_r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\leq \begin{bmatrix} \alpha_1 B_i F_0 W(B_i F_0)^{\mathrm{T}} + \alpha_1^{-1} Y_r^{\mathrm{T}} W Y_r + \alpha_2^{-1} Y_r^{\mathrm{T}} W Y_r & 0 & 0 \\ 0 & \alpha_2 B_i F_0 W(B_i F_0)^{\mathrm{T}} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (32)$$

From (30)-(32) and using Surch complement we have $\Pi < 0$, which completes the proof according to Theorem 2.

4. SIMULATION

In this section, computer simulations are carried out to confirm the validity of the proposed method. For each operating point, system matrices are given in the Appendix. The flight delay satisfies $\tau(t) = 0.103 + 0.1\sin t$, from which it is known that h and d are 0.203 and 0.1, respectively. According to Corollary 1, we get the following gain matrix K_s of standard controller. Setting $F = \mathrm{diag}\{f_1, f_2\}$, $0 \le f_1 \le 1.2$, and $0 \le f_2 \le 1.6$ and by Theorem 3, gain matrix K_r of reliable controller is obtained as follows.

$$K_s = \begin{bmatrix} 2.4864 & 2.3775 \\ -15.6685 & -11.3463 \end{bmatrix}, K_r = \begin{bmatrix} -0.0020 & 0.0016 \\ -0.0005 & 0.0004 \end{bmatrix}.$$

We compare the performance of the controllers above to demonstrate the effectiveness of our method. Fig. 1-2 show normal state responses using the standard controller and reliable controller, respectively. It is obvious that both controllers make the corresponding flight control system robustly stable in the case of no actuator fault. Especially the former indicates that the closed-loop system can be stabilized with good performance by Theorem 1.

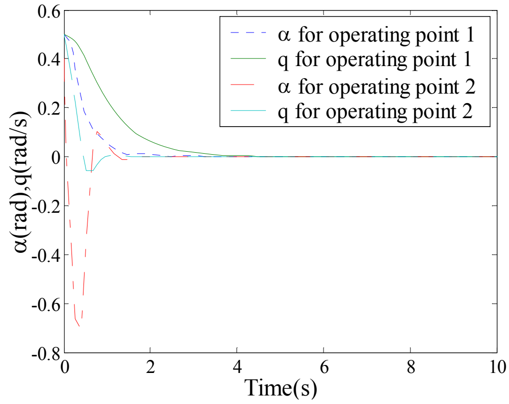


Fig. 1. The state curve of standard control with no fault

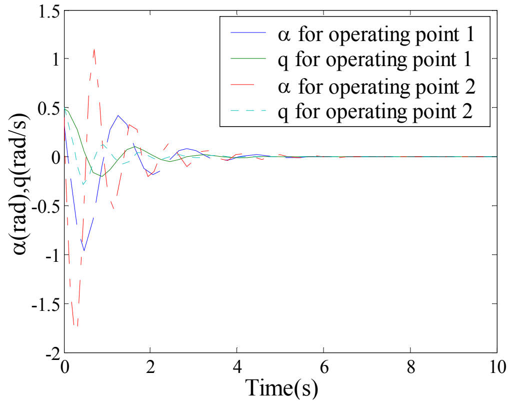
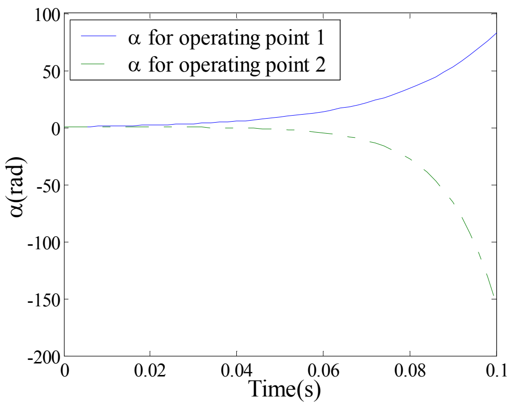


Fig. 2. The state curve of reliable control with no fault

Assume that the first actuator is susceptible to complete fault and the second actuator is normal, that is, $F = \mathrm{diag}\{0,1\}$. In Fig. 3-4, fault state responses using these two controllers are given. It can be seen that the standard controller can not stabilize the closed-loop system when complete fault in the first actuator occurs, while our reliable controller can.



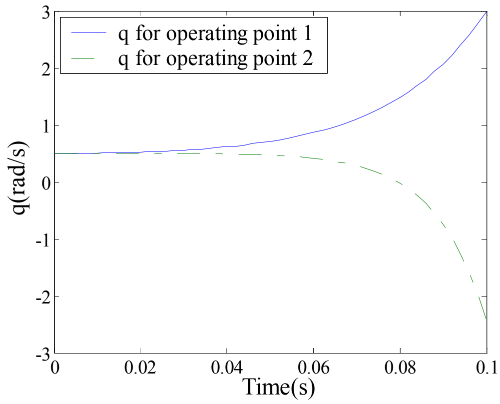


Fig. 3. The state curve of standard control with fault

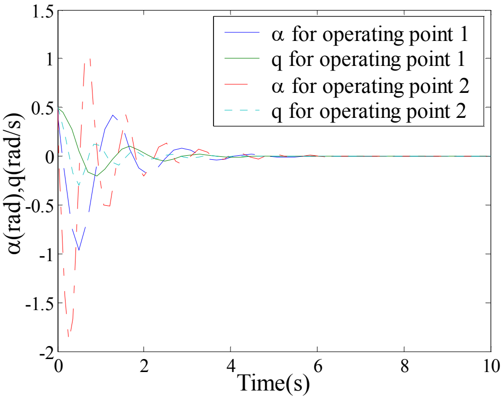


Fig. 4. The state curve of reliable control with fault

5. CONCLUSIONS

This paper proposes a new stability criterion for a time delay system with polytopic uncertainties. The stability criterion is delay-dependent LMIs, which is derived by constructing a parameter-dependent Lyapunov function and employing a descriptor system transformation. A numerical example demonstrates that this criterion achieves exactly the same upper bound of delay with less computational effort. Based on the stability criterion combined with fault tolerant techniques, a robust reliable flight controller is designed for an aircraft. Computer simulations show that our stability criterion is applicable for control synthesis and the stability of the flight control system is guaranteed even in the case of actuator faults.

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Appendix A.

For each operating point, system matrices of the aircraft are

$$A_1=\begin{bmatrix}-1.1750 & 0.9871\\ -8.4580 & 0.8776\end{bmatrix},\ A_{d1}=\begin{bmatrix}-0.3525 & 0.2961\\ -2.5374 & -0.2633\end{bmatrix},$$

$$A_2=\begin{bmatrix}-2.3280 & 0.9831\\ -30.440 & -1.493\end{bmatrix},\ A_{d2}=\begin{bmatrix}-0.6984 & 0.2949\\ -9.1320 & -0.4479\end{bmatrix},$$

$$A_3=\begin{bmatrix}-2.4520 & 0.9856\\ -38.610 & -1.340\end{bmatrix},\ A_{d3}=\begin{bmatrix}-0.7356 & 0.29570\\ -11.583 & -0.4020\end{bmatrix},$$

$$B_1=\begin{bmatrix}-0.194 & -0.0359\\ -19.29 & -3.803\end{bmatrix},\ B_2=\begin{bmatrix}-0.3012 & -0.0587\\ -38.430 & -7.8150\end{bmatrix},$$

$$B_3=\begin{bmatrix}-0.2757 & -0.0523\\ -37.360 & -7.2470\end{bmatrix}.$$