Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Semi-Global Robust Stabilization of a Family of Uncertain Nonlinear Systems by Non-Smooth Output Feedback: the Planar Case

Bo Yang * **Wei Lin** **,1 **Yuanzhang Sun** ***

* *Department of Mathematics and Statistics, Texas Tech University, Lubbock, TX 79409, USA*
** *Dept. of Electrical Engineering and Computer Science, Case Western Reserve University, Cleveland, OH 44106, USA*
*** *Dept. of Electrical Eng., Tsinghua University, Beijing, China*

Abstract: We present a preliminary result on robust semi-global stabilization via output feedback for a family of uncertain nonlinear systems which represent a generalized normal form of affine systems in the plane. The main contribution of this paper is to show that with only the knowledge of the bounding system of the uncertain planar system, it is possible to establish the semi-global stabilization result by nonsmooth output feedback, although the uncertain controlled plant is not in a triangular form, non-smoothly stabilizable and non-uniformly observable.

Keywords: Lyapunov Stability, Nonsmooth Output Feedback, Robust Control, Semi-Global Stabilization, Uncertain Nonlinear Systems.

1. INTRODUCTION AND MAIN RESULT

In this paper, we consider a family of single-input-single-output (SISO) uncertain planar systems of the form

$$\begin{aligned} \dot{\eta}_1 &= \eta_2^p + \eta_2^{p-1}\phi_{p-1}(\eta_1, t) + \cdots \\ &\qquad\qquad + \eta_2\phi_1(\eta_1, t) + \phi_0(\eta_1, t) \\ \dot{\eta}_2 &= v \\ y &= \eta_1, \end{aligned} \tag{1}$$

where $\eta = (\eta_1, \eta_2) \in \mathbb{R}^2$, $v \in \mathbb{R}$ and $y \in \mathbb{R}$ are the system state, input and output, respectively. The parameter p is an odd positive integer and the mappings $\phi_i : \mathbb{R} \times \mathbb{R} \to \mathbb{R},\ i = 0, \cdots, p-1$ are C^1 with $\phi_i(0, t) = 0$ for all t, which represent the system uncertainty and need not to be precisely known. However, it is assumed that the system uncertainty satisfies the following condition:

Assumption 1. There exists a known bounding function $\psi : \mathbb{R} \to [0, +\infty)$, which is C^1 with $\psi(0) = 0$ and satisfies for $i = 0, \cdots, p-1$,

$$|\phi_i(\eta_1, t)| \le \psi(\eta_1), \quad \forall(\eta_1, t) \in \mathbb{R} \times \mathbb{R}. \tag{2}$$

It is of interest to note that the form (1) without uncertainty (i.e., $\phi_i(\eta_1, t) \equiv q_i(\eta_1)$) is representative of a class of planar affine systems. In fact, Jakubczyk and Respondek Jakubczyk and Respondek [1990] proved that every smooth affine system in the plane, i.e.,

$$\begin{aligned} \dot{\xi} &= f(\xi) + g(\xi)u \\ y &= h(\xi) \end{aligned}$$

is feedback equivalent to the system (1) without η_2^{p-1} term if $g(0)$ and $\mathrm{ad}_f^p g(0)$ are linearly independent. A more general result was proved in Cheng and Lin [2003] later on, showing that the equation (1) is indeed a special case of the so-called "p-normal form" or Hessenberg form Cheng and Lin [2003]. In other words, system (1) is a normal form of planar affine systems when $\mathrm{rank}[g(0), \mathrm{ad}_f^p g(0)] = 2$.

A distinguished feature of the planar system (1) is that it is in general neither uniformly observable nor smoothly stabilizable when $p > 1$, because on the one hand, the state of (1) can only be represented as a Hölder continuous rather than smooth function of the system input, output, and their derivatives; on the other hand, the linearized system of (1) may have uncontrollable modes associated with eigenvalues on the right-half plane. These points can be seen easily from the simple Example 3 in section 3. In addition, system (1) is not in a triangular form. The lack of uniform observability, smooth stabilizability and a triangular structure makes the robust semi-global stabilization of the uncertain system (1) via output feedback non-trivial. In fact, the existing output feedback stabilization results (e.g., Teel and Praly [1994, 1995], Isidori [2000]) can not be applied to the uncertain planar system (1).

Recently, we have tackled the semi-global stabilization problem by output feedback Yang and Lin [2005*b*], for a class of n-dimensional, non-uniformly observable and nonsmoothly stabilizable systems with a *strict-triangular* form. The work Yang and Lin [2005*b*] has extended the previous output feedback stabilization results such as

[1] This work was supported in part by the U.S. National Science Foundation under Grant ECS-0400413, Air Force Research Laboratory under Grant FA8651-05-C-0110, and the Robert Herbold Faculty Fellow Award.

Theorem 3.5 of Qian and Lin [2004] and Theorem 3.2 of Yang and Lin [2005*a*] in two directions. First, the local output feedback stabilization result in Qian and Lin [2004] (see Theorem 3.5) was extended to the semi-global case without requiring extra growth conditions. Secondly, compared with the global output feedback stabilization results obtained previously in Yang and Lin [2004, 2005*a*], Qian and Lin [2004], the restrictive conditions such as $p_1 = \cdots = p_{n-1}$ and a high-order global Lipschitz-like condition in Yang and Lin [2004], or the growth requirements imposed on the triangular system Qian and Lin [2004] were removed. The trade-off is that semi-global other than global stabilizability was achieved.

Notably, most of the results reviewed so far have an obvious drawback, that is, the design of output feedback controllers still uses a copy of the original system and hence requires the precise information of the controlled plant. Consequently, the output feedback control scheme in Yang and Lin [2005*b*] is not robust with respect to parametric uncertainty. Moreover, it cannot be applied to uncertain nonlinear systems such as (1) which is not in a triangular form. The purpose of this paper is to address the robust issue, and to develop a robust semi-global output feedback control scheme for a family of uncertain systems (1) by using only the knowledge of the upper bound of $\phi_i(\cdot)$, i.e., the function $\psi(\cdot)$. This is one of the main differences between Yang and Lin [2005*b*] and this paper.

The objective of this paper is to show that *robust semi-global* stabilization can be achieved for the whole family of the uncertain systems (1) by a single output feedback controller. Such a robust control problem can be formulated as follows: given a bound $r > 0$, find, if possible, a nonsmooth but C^0 dynamic output compensator, which depends on r and the bounding function ψ, of the form

$$\begin{aligned}\dot{\hat{z}} &= \theta(\hat{z}, y), \qquad \hat{z} \in \mathbb{R}\\ v &= v(\hat{z}, y)\end{aligned} \tag{3}$$

such that the following two properties hold:

- Local Stability: The compensator (3) locally asymptotically stabilizes the whole family of uncertain systems (1) at the origin $(\eta, \hat{z}) = (0, 0, 0)$;
- Semi-Global Attraction: All the trajectories of the closed-loop system starting from the compact set
$$\Gamma_\eta \times \Gamma_{\hat{z}} \triangleq [-r, r]^2 \times [-r, r] \subset \mathbb{R}^3$$
converge to the origin.

The main result of this paper is the following theorem.

Theorem 2. Under Assumption 1, there exists a nonsmooth dynamic output compensator of the form (3), which semi-globally robustly stabilizes the entire family of uncertain planar systems (1).

2. PROOF OF THEOREM 2

In this section, we prove the main result of this paper — Theorem 2. In particular, we shall construct explicitly a nonsmooth robust dynamic output compensator by integrating the tool of adding a power integrator Qian and Lin [2001], the rescaling technique Yang and Lin [2005*a*], and the idea of saturating the estimated states Khalil and Esfandiari [1995].

Proof: First of all, in order to handle the system uncertainty $\phi_i(\eta_1, t)$ and to find a semi-global output feedback controller, we need to develop a robust design technique without using the copy of the system or the precise knowledge of the controlled plant. Inspired by the work Yang and Lin [2005*a*], we introduce a rescaling transformation with a suitable dilation for the original system (1), which turns out to be crucial for dominating the uncertainty of (1). To be precise, let

$$\begin{aligned}x_1 &= \eta_1,\\ x_2 &= \frac{\eta_2}{L},\\ u &= \frac{v}{L^{1+p}},\end{aligned} \tag{4}$$

where $L \geq 1$ is a rescaling factor to be assigned later.

Under the x-coordinate, the uncertain system (1) is rewritten as

$$\begin{aligned}\dot{x}_1 &= L^p x_2^p + L^{p-1} x_2^{p-1} \phi_{p-1}(x_1, t) + \cdots\\ &\qquad\qquad + L x_2 \phi_1(x_1, t) + \phi_0(x_1, t)\\ \dot{x}_2 &= L^p u\\ y &= x_1.\end{aligned} \tag{5}$$

Similar to the work Yang and Lin [2005*b*], one can design recursively a globally stabilizing, nonsmooth state feedback controller of the form

$$\tilde{x}_3^* = -[\xi_2 \beta_2(x_1, x_2^p)]^{1/p}$$

and the Lyapunov function

$$V_c(x) = V_c(x_1, x_2) = \frac{1}{2} x_1^2 + \int_{x_2^*}^{x_2} (s^p - x_2^{*p})^{2-\frac{1}{p}} \mathrm{d}s,$$

which is positive definite and proper Qian and Lin [2001], such that

$$\dot{V}_c(x_1, x_2) \leq L^p[-3(x_1^2 + \xi_2^2) + \xi_2^{2-\frac{1}{p}}(u - \tilde{x}_3^*)], \tag{6}$$

where $x_2^{*p} = -\beta_1(x_1)x_1$ and $\xi_2 = x_2^p - x_2^{*p} = x_2^p + \beta_1(x_1)x_1$ with $\beta_i : R^i \to (0, +\infty)$, $i = 1, 2$ being positive smooth functions independent of L.

Under the coordinate of $x = (x_1, x_2)$, we define the level set $\Omega_x = \{(x_1, x_2) | V_c(x_1, x_2) \leq r_0 + 1\}$, where $r_0 > 0$ is a constant such that

$$\begin{aligned}\Gamma_\eta \subset \Gamma_x &\triangleq \{(x_1, x_2) | |x_1| \leq r, |x_2| \leq r\}\\ &\subset \{(x_1, x_2) | V_c(x_1, x_2) \leq r_0\}.\end{aligned}$$

Moreover, denote

$$M = \max_{x \in \Omega_x} ||x||_\infty$$

as a saturation threshold, which is independent of L (see Figure 1 for details).

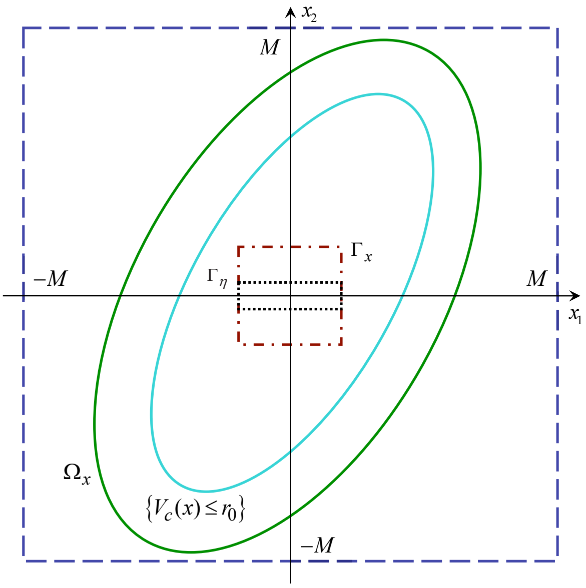

Fig. 1 The level set on x-space and the saturation threshold M.

Clearly, there exists a constant $B_2 > 0$ independent of L, satisfying

$$0 < [\beta_2(x_1, x_2^p)]^{\frac{1}{p}} \leq B_2, \qquad \forall (x_1, x_2) \in \Omega_x. \tag{7}$$

Therefore, on the level set Ω_x, the state feedback controller can be simplified as

$$x_3^* = -B_2 \xi_2^{\frac{1}{p}} \triangleq [u^*(x_1, x_2^p)]^{1/p}.$$

Indeed, because of (7), the derivative of V_c on Ω_x satisfies

$$\dot{V}_c\Big|_{\Omega_x} \leq L^p[-3(x_1^2 + \xi_2^2) + \xi_2^{2-\frac{1}{p}}(u - x_3^*)].$$

Motivated by Yang and Lin [2005*b*], we construct a reduced-order observer to estimate, instead of x_2 itself, the unmeasurable variable $z_2 = x_2^p - Lx_1$.

In view of z_2's dynamics

$$\dot{z}_2 = pL^p x_2^{p-1} u - L\big[L^p x_2^p + L^{p-1} x_2^{p-1} \phi_{p-1}(x_1, t) + \cdots + \phi_0(x_1, t)\big], \tag{8}$$

we design the one-dimensional observer

$$\dot{\hat{z}}_2 = -L^{p+1} \hat{x}_2^p, \quad \text{with} \quad \hat{x}_2^p = \hat{z}_2 + Lx_1. \tag{9}$$

The key difference between the observer (9) and the one in Yang and Lin [2005*b*] is that here we do not use the copy of the system (8). Rather, we build the observer (9) by ignoring the system uncertainty in (8).

By the certainty equivalence principle, we replace the unmeasurable state x_2 in the virtual controller x_3^* by the saturated state estimate $\hat{x}_2$ from the observer (9). In this way, we obtain

$$u = \big[u^*(x_1, [\mathrm{sat}_M(\hat{x}_2)]^p)\big]^{\frac{1}{p}}, \tag{10}$$

or equivalently,

$$v = L^{1+p}\big[u^*(\eta_1, \mathrm{sat}_{M^p}(\hat{z}_2 + L\eta_1))\big]^{\frac{1}{p}}, \tag{11}$$

where $\mathrm{sat}_N(x)$ represents a saturation function with the threshold $N \geq 0$ defined by

$$\mathrm{sat}_N(x) : \mathbb{R} \to \mathbb{R} = \begin{cases} -N & \text{if} \quad x < -N \\ x & \text{if} \quad |x| \leq N \\ N & \text{if} \quad x > N. \end{cases} \tag{12}$$

Clearly, the dynamic output compensator (9)-(11) is implementable.

In what follows, we shall prove that the non-smooth output feedback controller thus constructed, i.e., (9)-(11), semi-globally stabilizes the uncertain system (1).

Let

$$e_2 = z_2 - \hat{z}_2 = x_2^p - \hat{x}_2^p = \frac{\eta_2^p}{L^p} - L\eta_1 - \hat{z}_2 \tag{13}$$

be the estimate error. Then, A straightforward calculation shows that the error dynamics are characterized by

$$\dot{e}_2 = L^p[px_2^{p-1}u - Le_2 - x_2^{p-1}\phi_{p-1}(x_1, t) - \cdots - L^{-(p-1)}\phi_0(x_1, t)]. \tag{14}$$

Now, construct the following Lyapunov function $V(\eta, \hat{z}_2)$ for the whole closed-loop system (1)-(9)-(11):

$$V(\eta, \hat{z}_2) = V_c(x) + V_e(e) = V_c(\eta_1, \frac{\eta_2}{L}) + \frac{\ln(1 + e_2^2)}{\ln(\mu(L))},$$

where $\mu(L) \triangleq 1 + (r^p + Lr + r)^2$ is a polynomial of L.

The corresponding level set can be defined as

$$\Omega = \{(\eta, \hat{z}_2) \in \mathbb{R}^3 \big| V(\eta, \hat{z}_2) \leq r_0 + 1\}.$$

In view of the definition of e_2 in (13), it is easy to verify the level set Ω contains the preset attractive domain $\Gamma_\eta \times \Gamma_{\hat{z}}$ uniformly with respect to $L \geq 1$ since $(\eta_1, \eta_2, \hat{z}_2) \in \Gamma_\eta \times \Gamma_{\hat{z}}$ implies $V(\eta_1, \eta_2, \hat{z}_2) \leq r_0 + 1, \forall L \geq 1$.

Moreover, from the definition of M, it is clear that for all $(\eta_1, \eta_2, \hat{z}_2) \in \Omega$,

$$|x_1| = |\eta_1| \leq M, \quad |x_2| = |\frac{\eta_2}{L}| \leq M.$$

(see the details in Figure 2)

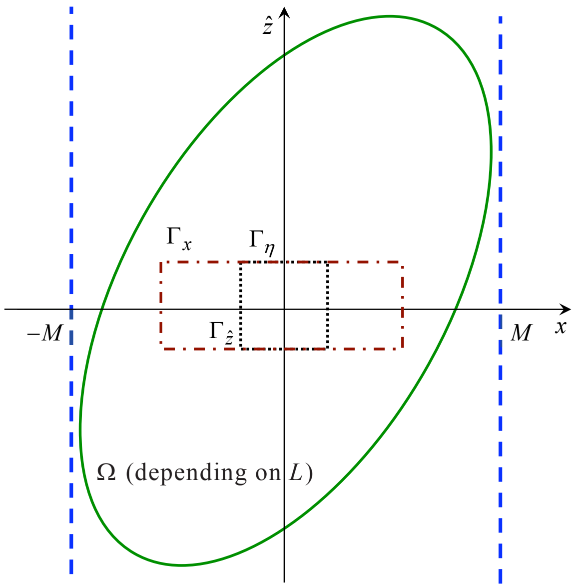

Fig. 2 The level set on $(x, \hat{z})$-space.

Note that the saturation function has the following property:

$$|a - \mathrm{sat}_M(b)| \le 2\min\{|a^p - b^p|^{1/p}, M\},$$
$$\forall a \in [-M, M], \quad \forall b \in \mathbb{R}. \tag{15}$$

Keeping this in mind, the boundedness of x-coordinate on the level set Ω implies there exists a generic constant $C > 0$ independent of L such that

$$\Big|\big[u^*(x_1, [\mathrm{sat}_M(\hat{x}_2)]^p)\big]^{\frac{1}{p}} - \big[u^*(x_1, x_2^p)\big]^{\frac{1}{p}}\Big|\Big|_\Omega$$
$$\le C\min\{|e_2|^{\frac{1}{p}}, 1\}. \tag{16}$$

Hence, in view of (15), (16) and Young's inequality, we have

$$\dot{V}_c\Big|_\Omega \le L^p[-2(x_1^2 + \xi_2^2) + K_1 \min\{e_2^2, 1\}], \tag{17}$$

where $K_1 > 0$ is a constant independent of L.

Similarly, it is easy to see that for all $(i = 0, \cdots, p-1)$ and $L \ge 1$,

$$|L^{-(i-p-1)}\phi_i(x_1, t)|\Big|_\Omega \le \psi(x_1)\Big|_\Omega \le C|x_1|^{1-\frac{i}{p}}. \tag{18}$$

where $C > 0$ is a generic constant independent of L.

Hence, by the Young's inequality, a direct but tedious calculation gives

$$\dot{V}_e\Big|_\Omega \le \frac{L^p}{\ln(\mu(L))(1+e_2^2)}\Big[2p|x_2^{p-1}||e_2||u| - 2Le_2^2$$
$$+2C|e_2||x_2^{p-1}||x_1^{\frac{1}{p}}| + \cdots + 2C|e_2||x_1|\Big]$$

$$\le \frac{L^p\big[-(2L - K_2)e_2^2 + (x_1^2 + \xi_2^2)\big]}{\ln(\mu(L))(1+e_2^2)}$$
$$\le L^p\Big[-\frac{2L - K_2}{\ln(\mu(L))}\frac{e_2^2}{1+e_2^2} + \frac{x_1^2 + \xi_2^2}{\ln(\mu(L))}\Big] \tag{19}$$

with $K_2 > 0$ being a constant independent of L.

Observe that

$$\frac{e_2^2}{1+e_2^2} \ge \frac{1}{2}\min\{e_2^2, 1\}.$$

Putting (17) and (19) together results in

$$\dot{V}\Big|_\Omega \le L^p\Big[-(2 - \frac{1}{\ln(\mu(L))})(x_1^2 + \xi_2^2)$$
$$-\big(\frac{L}{\ln(\mu(L))} - K\big)\min\{e_2^2, 1\}\Big] \tag{20}$$

with $K > 0$ being a constant independent of L.

Since $\mu(L)$ is a fixed polynomial, we have

$$\lim_{L \to +\infty} \frac{L}{\ln(\mu(L))} = +\infty.$$

Therefore, there exists a sufficiently large L such that

$$2 - \frac{K}{\ln(\mu(L))} \ge 1$$
$$\frac{L}{\ln(\mu(L))} - K \ge 1$$

which, in turn, yields

$$\dot{V}\Big|_\Omega \le L^p\Big[-(x_1^2 + \xi_2^2) - \min\{e_2^2, 1\}\Big].$$

This completes the proof of Theorem 2. ∎

3. EXAMPLES AND DISCUSSIONS

The significance of Theorem 2 can be demonstrated by the following two examples.

Example 3. Consider the following non-triangular system with an parameter uncertainty

$$\dot{\eta}_1 = \eta_2^3 + \theta\eta_2\eta_1 + \eta_1$$
$$\dot{\eta}_2 = v$$
$$y = \eta_1, \tag{21}$$

where θ is an unknown parameter bounded by one.

Due to the lack of a triangular structure and the presence of θ, the semi-global design method in Yang and Lin [2005*b*] is no longer valid. However, the uncertain system (21) satisfies Assumption 1. By Theorem 2, the entire family of systems (21) with θ being varied over the interval $[-1, 1]$ is semi-globally stabilizable by a single output feedback controller of the form (9)-(11).

In fact, the Lyapunov function can be chosen as

$$V(\eta, \hat{z}_2) = \frac{1}{2}\eta_1^2 + \int_{x_2^*}^{\frac{\eta_2}{L}} (s^3 - x_2^{*3})^{\frac{5}{3}} + \frac{(\frac{\eta_2^3}{L^3} - L\eta_1 - \hat{z}_2)^2}{\ln(\mu(L))}$$

with $L \geq 1, \mu(L) = 1 + (r^p + Lr + L)^2$ and $x_2^* = [-\frac{\eta_1}{4}(\eta_1^2 + 21)]^{\frac{1}{3}}$. It yields the dynamic compensator in the following form:

$$\dot{\hat{z}}_2 = -L^4[\hat{z}_2 + L\eta_1]$$
$$v = L^4[u^*(\eta_1, \text{sat}_{M^3}(\hat{z}_2 + L\eta_1))]^{\frac{1}{3}},$$

with the function

$$u^*(x_1, x_2^3) = -B_2^3(x_2^3 + x_1\frac{21 + x_1^2}{4})$$

and $B_2, M > 0$ and $L \geq 1$ being appropriate constants.

The simulation shown in Figure 3 demonstrates the transient response of the closed-loop system with $\theta = 1$ and the initial condition $(\eta_1, \eta_2, \hat{z}_2) = (2, 1, 0.5)$ and $B_2 = 7,\ M = 5,\ L = 3$.

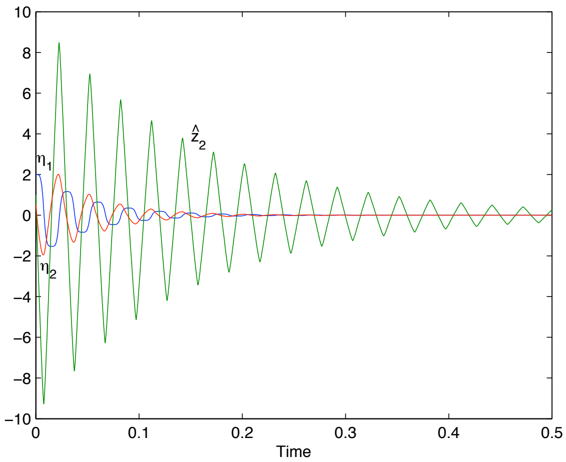

Fig. 3 The transient response of the closed-loop system.

Example 4. Consider the time-varying planar system

$$\begin{aligned}\dot{\eta}_1 &= \eta_2^3 + \ln(1 + \eta_2^2)\eta_1 \sin(\eta_2 t) \\ \dot{\eta}_2 &= v \\ y &= \eta_1.\end{aligned} \quad (22)$$

Although (22) is not precisely in the form (1) (due to the term $\ln(1+\eta_2^2)\eta_1 \sin(\eta_2 t)$), the robust design method used in the proof of Theorem 2 is still valid. Since

$$|\ln(1 + \eta_2^2)\eta_1 \sin(\eta_2 t)| \leq |\eta_2|\ |\eta_1|,$$

it can be concluded that the uncertain system (22), which is non-smoothly stabilizable and non-uniformly observable, is semi-globally stabilizable by the non-smooth dynamic output compensator (9)-(11) as long as the parameter L is large enough.

4. CONCLUSIONS

In this paper, we have presented a preliminary result on how to achieve *robust* semi-global stabilization by non-smooth output feedback for a family of uncertain nonlinear systems. This class of uncertain systems, although they are only two-dimensional, are difficult to be controlled by output feedback for a number of reasons such as the lack of a triangular structure, the presence of the system uncertainty, and the loss of smooth stabilizability and uniform observability. It is worth mentioning that the robust semi-global output feedback stabilization of higher-dimensional nonlinear systems in the "p-normal form" or Hessenberg form Cheng and Lin [2003] is far more complicated and difficult than the planar case. The problem is currently under our investigation.

REFERENCES

Bacciotti, A. (1992). *Local stabilizability of nonlinear control systems.* World Scientific.

Cheng, D. and W. Lin (2003). On p—normal forms of nonlinear systems. *IEEE Trans. Automat. Contr.* **48**, 1242–1248.

Dayawansa, W. (1992). Recent advances in the stabilization problem for low dimensional systems. In: *Proc. IFAC Symp. Nonlinear Control System Design.* Bordeaux. France. pp. 1–8.

Dayawansa, W., C. Martin and G. Knowles (1990). Asymptotic stabilization of a class of smooth two dimensional systems. *SIAM. J. Contr. Optimiz.* **28**, 1321–1349.

Gauthier, J., H. Hammouri and S. Othman (1992). A simple observer for nonlinear systems, applications to bioreactors. *IEEE Trans. Automat. Contr.* **37**, 875–880.

Isidori, A. (2000). A tool for semiglobal stabilization of uncertain non-minimum-phase nonlinear systems via output feedback. *IEEE Trans. Automat. Contr.* **45**, 1817-1827.

Jakubczyk, B. and W. Respondek (1990). Feedback equivalence of planar systems and stability. In: *Robust Control of Linear Systems and Nonlinear Control* (M.A. Kaashoek et al., Ed.). pp. 447–456. Birkhäuser.

Kawski, M. (1989). Stabilization of nonlinear systems in the plane. *Syst. Contr. Lett.,* **12**, 169–175.

Kawski, M. (1995). Geometric homogeneity and applications to stabilization. In: *Proc. of the 3rd IFAC Symp. Nonlinear Control System Design.* Lake Tahoe. CA. pp. 164–169.

Khalil, H. and F. Esfandiari. Semi-global stabilization of a class of nonlinear systems using output feedback. *IEEE Trans. Automat. Contr.* **38**, 1412–1415.

Qian, C. and W. Lin (2001*a*). A continuous feedback approach to global strong stabilization of nonlinear systems. *IEEE Trans. Automat. Contr.* **46**, 1061–1079.

Qian, C. and W. Lin (2004). Recursive observer design and nonsmooth output feedback stabilization of inherently nonlinear systems. In: *Proc. of the 43rd IEEE Conf. Decision Control.* Nassau. The Bahamas. pp. 4927–4932.

Rosier, L. (1992). Homogeneous Lyapunov functions for homogeneous continuous vector field. *Syst. Contr. Lett.* **19**, 467–473.

Teel, A. and L. Praly (1994). Global stabilizability and observability imply semi-global stabilizality by output feedback. *Syst. Contr. Lett.* **22**, 313–325.

Teel, A. and L. Praly (1995). Tools for semiglobal stabilization by partial state and output feedback. *SIAM. J. Contr. Optimiz.* **33**, 1443–1488.

Yang, B. and W. Lin (2004). Homogeneous observers, iterative design and global stabilization of high order nonlinear systems by smooth output feedback. *IEEE Trans. Automat. Contr.* **49**, 1069–1080.

Yang, B. and W. Lin (2005*a*). Robust output feedback stabilization of uncertain nonlinear systems with uncontrollable and unobservable linearization. *IEEE Trans. Automat. Contr.* **50**, 619–630.

Yang, B. and W. Lin (2005*b*). Semi-global output feedback stabilization of non-uniformly observable and nonsmoothly stabilizable systems, In: *Proc. of the 44th IEEE Conf. Decision Control.* Seville. Spain. pp. 4207–4212.