Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Finite-Dimensional H_∞ Filter Design for Linear Systems with Measurement Delay ⋆

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Abstract: This paper presents the central finite-dimensional H_∞ filter for linear systems with measurement delay, that is suboptimal for a given threshold γ with respect to a modified Bolza-Meyer quadratic criterion including the attenuation control term with the opposite sign. In contrast to the results previously obtained for linear time delay systems, the paper reduces the original H_∞ filtering problem to an H_2 (optimal mean-square) filtering problem, using the technique proposed in [1]. Application of the reduction technique becomes possible, since the optimal filtering equations solving the H_2 (mean-square) filtering problems have been obtained for linear systems with measurement delay. The paper presents the central suboptimal H_∞ filter for linear systems with measurement delay, based on the optimal H_2 filter from [36], where the standard H_∞ filtering conditions of stabilizability, detectability, and noise orthonormality are assumed. Finally, to relax the standard conditions, the paper presents the generalized version of the designed H_∞ filter in the absence of the noise orthonormality. The proposed H_∞ filtering algorithm provides a direct method to calculate the minimum achievable values of the threshold γ, based on the existence properties for a bounded solution of the gain matrix equation. Numerical simulations are conducted to verify performance of the designed central suboptimal filter for linear systems with state delay against the central suboptimal H_∞ filter available for linear systems without delays. The simulation results show a definite advantage in the values of the noise-output transfer function H_∞ norms in favor of the designed filter.

1. INTRODUCTION

Over the past two decades, the considerable attention has been paid to the H_∞ estimation problems for linear and nonlinear systems with and without time delays. The seminal papers in H_∞ control [1] and estimation [2–4] established a background for consistent treatment of filtering/controller problems in the H_∞-framework. The H_∞ filter design implies that the resulting closed-loop filtering system is robustly stable and achieves a prescribed level of attenuation from the disturbance input to the output estimation error in L_2/l_2-norm. A large number of results on this subject has been reported for systems in the general situation, linear or nonlinear (see ([5]–[13]). For the specific area of linear time-delay systems, the H_∞-filtering problem has also been extensively studied (see [14]–[31], [8,11,12]). The sufficient conditions for existence of an H_∞ filter, where the filter gain matrices satisfy Riccati equations, were obtained for linear systems with state delay in [32] and with measurement delay in [33]. However, the criteria of existence and suboptimality of solution for the central H_∞ filtering problems based on the reduction of the original H_∞ problem to the induced H_2 one, similar to those obtained in [1,4] for linear systems without delay, remain yet unknown for linear systems with time delays.

This paper presents the central (see [1] for definition) finite-dimensional H_∞ filter for linear systems with measurement delay, that are suboptimal for a given threshold γ with respect to a modified Bolza-Meyer quadratic criterion including the attenuation control term with the opposite sign. In contrast to the results previously obtained for linear systems with state [32] or measurement delay [33], the paper reduces the original H_∞ filtering problems to H_2 (mean-square) filtering problems, using the technique proposed in [1]. To the best authors' knowledge, this is the first paper which applies the reduction technique of [1] to classes of systems other than conventional LTI plants. Indeed, application of the reduction technique makes sense, since the optimal filtering equations solving the H_2 (mean-square) filtering problems have been obtained for linear systems with state [34,35] or measurement [36] delays. Designing the central suboptimal H_∞ filter for linear systems with measurement delay presents a significant advantage in the filtering theory and practice, since (1) it enables one to address filtering problems for LTV time-delay systems, where the LMI technique is hardly applicable, (2) the obtained H_∞ filter is suboptimal, that is, optimal for any fixed γ with respect to the H_∞ noise attenuation criterion, and (3) the obtained H_∞ filter is finite-dimensional and has the same structure of the estimate and gain matrix equations as the corresponding optimal H_2 filter. Moreover,

⋆ The authors thank The London Royal Society (RS) and the Mexican National Science and Technology Council (CONACyT) for financial support under an RS International Incoming Short Visits 2006/R4 Grant and CONACyT Grants 55584 and 52953.

the proposed H_∞ filtering algorithms provide direct methods to calculate the minimum achievable values of the threshold γ, based on the existence properties for a bounded solution of the gain matrix equation. The corresponding calculations are made for the designed filter.

It should be commented that the proposed design of the central suboptimal H_∞ filters for linear time-delay systems with integral-quadratically bounded disturbances naturally carries over from the design of the optimal H_2 filters for linear time-delay systems with unbounded disturbances (white noises). The entire design approach creates a complete filtering algorithm of handling the linear time-delay systems with unbounded or integral-quadratically bounded disturbances optimally for all thresholds γ uniformly or for any fixed γ separately. A similar algorithm for linear systems without delay was developed in [1].

The paper presents the central suboptimal H_∞ filter for linear systems with measurement delay, based on the optimal H_2 filter from [34], where the standard H_∞ filtering conditions of stabilizability, detectability, and noise orthonormality (see [4]) are assumed. Finally, to relax the standard conditions, the paper presents the generalized version of the designed H_∞ filter in the absence of the noise orthonormality, using the technique of handling non-orthonormal noises carried over from [33].

Numerical simulations are conducted to verify performance of the designed central suboptimal filter for linear systems with measurement delay against the central suboptimal H_∞ filter available for linear systems without delays [4]. The simulation results show a definite advantage in the values of the noise-output transfer function H_∞ norms in favor of the designed filter.

The paper is organized as follows. Section 2 presents the H_∞ filtering problem statement for linear systems with state delay. The central suboptimal H_∞ filter for linear systems with measurement delay is designed in Section 3. An example verifying performance of the H_∞ filter designed in Sections 3 against the central suboptimal H_∞ filter available for linear systems without delays is given in Section 4. The obtained results are generalized to the case of non-orthonormal noises in Section 5. Section 6 presents conclusions to this study.

2. H_∞ FILTERING PROBLEM STATEMENT

Consider the following continuous-time LTV system with measurement delay:

$$\mathscr{M}_1: \ \dot{x}(t) = A(t)x(t) + B(t)\omega(t), \quad x(t_0) = x_0, \tag{1}$$

$$y(t) = C(t)x(t-h) + D(t)\omega(t), \tag{2}$$

$$z(t) = L(t)x(t), \tag{3}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $z(t) \in \mathbb{R}^q$ is the signal to be estimated, $y(t) \in \mathbb{R}^m$ is the measured output, $\omega(t) \in \mathscr{L}_2^p[0,\infty)$ is the disturbance input. $A(\cdot)$, $B(\cdot)$, $C(\cdot)$, $D(\cdot)$, and $L(\cdot)$ are known continuous functions. x_0 is an unknown initial vector. The time delay h is known.

For the system (1)–(4), the following standard conditions ([4]) are assumed:

- the pair (A, B) is stabilizable; $(\mathscr{C}_1)$
- the pair (C, A) is detectable; $(\mathscr{C}_2)$
- $D(t)B^T(t) = 0$ and $D(t)D^T(t) = I_m$. $(\mathscr{C}_3)$

Here, I_m is the identity matrix of dimension $m \times m$. As usual, the first two conditions ensure that the estimation error, provided by the designed H_∞ filter, converge to zero ([37]). The last noise orthonormality condition is technical and corresponds to the condition of independence of the standard Wiener processes (Gaussian white noises) in the stochastic filtering problems ([37]).

Consider a full-order $\mathscr{H}_\infty$ filter in the following form:

$$\mathscr{M}_2: \ \dot{x}_m(t) = A(t)x_m(t) + K_m(t)[y(t) - C(t)x_m(t-h)], \tag{4}$$

$$z_m(t) = L(t)x_m(t), \tag{5}$$

where $x_m(t)$ is the filter state. The gain matrix $K_m(t)$ is to be determined.

Upon transforming the model (1)-(3) to include the states of the filter, the following filtering error system is obtained:

$$\mathscr{M}_3: \ \dot{e}(t) = A(t)e(t) + B(t)\omega(t) - K_m(t)\tilde{y}(t), \tag{6}$$

$$\tilde{y}(t) = C(t)e(t-h) + D(t)\omega(t), \tag{7}$$

$$\tilde{z}(t) = L(t)e(t), \tag{8}$$

where $e(t) = x(t) - x_m(t)$, $\tilde{y}(t) = y(t) - C(t)x_m(t-h)$, and $\tilde{z}(t) = z(t) - z_f(t)$.

Therefore, the problem to be addressed is stated similarly to the H_∞ filtering problem from Section 2: develop a robust $\mathscr{H}_\infty$ filter of the form (4)-(5) for the LTV system with measurement delay ($\mathscr{M}_1$), such that the following two requirements are satisfied:

(1) The resulting filtering error dynamics ($\mathscr{M}_3$) is robustly asymptotically stable in the absence of disturbances, $\omega(t) \equiv 0$;
(2) The filtering error dynamics ($\mathscr{M}_3$) ensures a noise attenuation level γ in an $\mathscr{H}_\infty$ sense. More specifically, for all nonzero $\omega(t) \in \mathscr{L}_2^p[0,\infty)$, the inequality
$$\|\tilde{z}(t)\|_2^2 < \gamma^2 \left\{ \|\omega(t)\|_2^2 + \|x_0\|_{2,R}^2 \right\} \tag{9}$$
holds for $\mathscr{H}_\infty$ filtering problem, where
$\|f(t)\|_2^2 := \int_{t_0}^{\infty} f^T(t)f(t)dt$,
$\|x_0\|_{2,R}^2 = x_0^T R x_0$, R is a positive definite symmetric matrix, and γ is a given real positive scalar.

3. FINITE-DIMENSIONAL H_∞ FILTER DESIGN

The proposed design of the central H_∞ filter (see Theorem 4 in [1]) for LTV systems with measurement delay is also based on the general result (see Theorem 3 in [1]) reducing the H_∞ controller (in, particular, filtering) problem to the corresponding H_2 (i.e., optimal linear-quadratic) controller (or mean-square filtering) problem. Then, the optimal mean-square filter of the Kalman-Bucy type for LTV systems with measurement delay ([36]) is employed to obtain the desired result, which is given by the following theorem.

Theorem 1. The central H_∞ filter for the unobserved state (1) over the observations (2), ensuring the H_∞ noise attenuation condition (9) for the output estimate $z_m(t)$, is given by the equations for the state estimate $x_m(t)$ and the output estimate $z_m(t)$

$$\dot{x}_m(t) = A(t)x_m(t) + P(t)\exp\Big(-\int_{t-h}^{t} A^T(s)ds\Big) \tag{10}$$

$$\times C^T(t)[y(t) - C(t)x_m(t-h)],$$
$$z_m(t) = L(t)x_m(t), \quad (11)$$

with the initial condition $x_m(t_0) = 0$, and the equation for the filter gain matrix $P(t)$

$$dP(t) = (P(t)A^T(t) + A(t)P(t) + B(t)B^T(t) - \quad (12)$$
$$P(t)\exp(-\int_{t-h}^{t} A^T(s)ds)[C^T(t)C(t) -$$
$$\gamma^{-2}L^T(t)L(t)]\exp(-\int_{t-h}^{t} A(s)ds)P(t))dt,$$

with the initial condition $P(t_0) = R^{-1}$.

Proof. Since the filtering error system (6)-(8) is already in the form used in Theorem 3 from [1], then, according to Theorem 3 from [1], the H_∞ filtering problem would be equivalent to the H_2 (i.e., optimal mean-square) filtering problem, where the worst disturbance $w_{worst}(t) = \gamma^{-2}B^T(t)Q(t)e(t)$ is realized, and $Q(t)$ is the solution of the equation for the corresponding H_2 (optimal linear-quadratic) control gain. Therefore, the system, for which the equivalent H_2 (optimal mean-square) filtering problem is stated, takes the form

$$\mathscr{M}_4: \ \dot{e}(t) = A(t)e(t) \quad (13)$$
$$+ \gamma^{-2}B(t)B^T(t)Q(t)e(t) - K_m(t)\tilde{y}(t),$$
$$\tilde{y}(t) = C(t)e(t-h) + \quad (14)$$
$$+ \gamma^{-2}D(t)B^T(t)Q(t)e(t),$$
$$\tilde{z}(t) = L(t)e(t). \quad (15)$$

As follows from Theorem 3 from [1] and Theorem 1 in [36], the H_2 (optimal mean-square) estimate equations for the error states (13) and (15) are given by

$$\mathscr{M}_5: \ \dot{e_m}(t) = A(t)e_m(t-h) - K_m(t)\tilde{y}(t) \quad (16)$$
$$+ P(t)\exp(-\int_{t-h}^{t} A^T(s)ds)$$
$$\times C^T(t)[\tilde{y}(t) - C(t)e_m(t-h)],$$
$$\tilde{z}_m(t) = L(t)e_m(t), \quad (17)$$

where $e_m(t)$ and $\tilde{z}_m(t)$ are the H_2 (optimal mean-square) estimates for $e(t)$ and $\tilde{z}(t)$, respectively, and $A(t)$ is the dynamics matrix in the state equation (1). In the equation (16), $P(t)$ is the solution of the equation for the corresponding H_2 (optimal mean-square) filter gain, where, according to Theorem 3 from [1], the observation matrix $C(t)$ should be changed to $C(t) - \gamma^{-1}L(t)$ ($L(t)$ is the output matrix in (3).

It should be noted that, in contrast to Theorem 3 from [1], no correction matrix $Z_\infty(t) = [I_n - \gamma^{-2}P(t)Q(t)]^{-1}$ appears in the last innovations term in the right-hand side of the equation (16), since there is no need to make the correction related to estimation of the worst disturbance $w_{worst}(t)$ in the error equation (13). Indeed, as stated in ([4]), the desired estimator must be unbiased, that is, $\tilde{z}_m(t) = 0$. Since the output error $\tilde{z}(t)$, satisfying (15), also stands in the criterion (9) and should be minimized as much as possible, the worst disturbance $w_{worst}(t)$ in the error equation (13) should be plainly rejected and, therefore, does not need to be estimated. Thus, the corresponding H_2 (optimal mean-square) filter gain would not include any correction matrix $Z_\infty(t)$. The same situation can be observed in Theorems 1–4 in [4]. However, if not the output error $\tilde{z}(t)$ but the output $z(t)$ itself would stand in the criterion (9), the correction matrix $Z_\infty(t) = [I_n - \gamma^{-2}P(t)Q(t)]^{-1}$ should be included.

Taking into account the unbiasedness (see [4]) of the estimator (16)-(17), it can be readily concluded that the equality $K_m(t) = P(t)\exp(-\int_{t-h}^{t} A^T(s)ds)C^T(t)$ must hold for the gain matrix $K_m(t)$ in (4). Thus, the filtering equations (4)-(5) take the final form (10)-(11), with the initial condition $x_m(t_0) = 0$, which corresponds to the central H_∞ filter (see Theorem 4 in [1]). It is still necessary to indicate the equation for the corresponding H_2 (optimal mean-square) filter gain matrix $P(t)$. In accordance with Theorem 1 from [36], the equation for determining $P(t)$ is given by the equation (12), with the initial condition $P(t_0) = R^{-1}$, which corresponds to the central H_∞ filter (see Theorems 3 and 4 in [4]). Note that the observation matrix $C(t)$ is changed to $C(t) - \gamma^{-1}L(t)$ according to Theorem 3 from [1]. ■

Remark 1. The convergence properties of the obtained estimate (10) are given by the standard convergence theorem (see, for example, [37]): if in the system (1),(2) the pair $(A(t), B(t))$ is uniformly completely controllable and the pair $(C(t)\Phi(t-h,t), A(t))$ is uniformly completely observable, where $\Phi(t,\tau)$ is the state transition matrix for the non-delayed equation (1) (see [39] for definition of matrix Φ), and the inequality $C^T(t)C(t) - \gamma^{-2}L^T(t)L(t) > 0$ holds, then the error of the obtained filter (10)–(12) is uniformly asymptotically stable. As usual, the uniform complete controllability condition is required for assuring non-negativeness of the matrix $P(t)$ (12) and may be omitted, if the matrix $P(t)$ is non-negative definite in view of its intrinsic properties.

Remark 2. The condition $C^T(t)C(t) - \gamma^{-2}L^T(t)L(t) > 0$ assures boundedness of the filter gain matrix $P(t)$ for any finite t, and also as time goes to infinity. Apparently, if $C^T(t)C(t) - \gamma^{-2}L^T(t)L(t) < 0$, then the function $P(t)$ diverges to infinity for a finite time and the designed filter does not work. If the equality $C^T(t)C(t) - \gamma^{-2}L^T(t)L(t) = 0$ holds, then the estimation error is uniformly asymptotically stable, if the state dynamics matrix $A(t)$ itself is asymptotically stable.

Remark 3. According to the comments in Subsection V.G in [1], the obtained central H_∞ filter (10)–(12) presents a natural choice for H_∞ filter design among all admissible H_∞ filters satisfying the inequality (9) for a given threshold γ, since it does not involve any additional actuator loop (i.e., any additional external state variable) in constructing the filter gain matrix. Moreover, the obtained central H_∞ filter (10)–(12) has the suboptimality property, i.e., it minimizes the criterion

$$J = \|\tilde{z}(t)\|_2^2 - \gamma^2 \left\{ \|\omega(t)\|_2^2 + \|x_0\|_{2,R}^2 \right\}$$

for such positive $\gamma > 0$ that the inequality $C^T(t)C(t) - \gamma^{-2}L^T(t)L(t) > 0$ holds.

Remark 4. Following the discussion in Subsection V.G in [1], note that the complementarity condition always holds for the obtained H_∞ filter (10)–(12), since the positive definiteness of the initial condition matrix R implies the positive definiteness of the filter gain matrix gain $P(t)$ as the solution of (13). Therefore, the stability failure is the only reason why the obtained filter can stop working. Thus, the stability margin

$\gamma = \sqrt{\|L^T(t)L(t)\| / \|C^T(t)C(t)\|}$ also defines the minimum possible value of γ, for which the H_∞ condition (9) can still be satisfied.

4. EXAMPLE

This section presents an example of designing the central H_∞ filter for a linear state over linear observations with measurement delay and comparing it to the best H_∞ filter available for a linear system without delay, that is the filter obtained in Theorems 3 and 4 from [4].

Let the unmeasured state $x(t) = [x_1(t), x_2(t)] \in R^2$ (a mechanical oscillator without delay) be given by

$$\dot{x}_1(t) = x_2(t), \quad (19)$$
$$\dot{x}_2(t) = -x_1(t) + w_1(t),$$

with an unknown initial condition $x(0) = x_0$, the delayed scalar observation process satisfy the equation

$$y(t) = x_1(t-5) + w_2(t), \quad (20)$$

and the scalar output be represented as

$$z(t) = x_1(t). \quad (21)$$

Here, $w(t) = [w_1(t), w_2(t)]$ is an L_2^2 disturbance input. It can be readily verified that the noise orthonormality condition (see Section 2) holds for the system (19)–(21).

The filtering problem is to find the H_∞ estimate for the linear state (19) over direct linear observations with measurement delay (20), which satisfies the noise attenuation condition (9) for a given γ, using the designed H_∞ filter (10)–(12). Since the simulation in the interval [0,10] occurs to be insufficient to reveal the convergent properties of the output estimation errors, the filtering horizon is extended and set to $T = 20$.

The filtering equations (10)–(12) take the following particular form for the system (19),(20)

$$\dot{x}_{m_1}(t) = x_{m_2}(t) + (0.2837P_{11}(t) + \quad (22)$$
$$0.9589P_{12}(t))[y(t) - x_{m_1}(t-5)],$$
$$\dot{x}_{m_2}(t) = -x_{m_1}(t) + (0.2837P_{12}(t) +$$
$$0.9589P_{22}(t))[y(t) - x_{m_1}(t-5)],$$

with the initial condition $x_f(0) = 0$, where 0.2837 and 0.9589 are $(1,1)-$ and $(2,1)-$ entries of the exponent of the integral of the reverse-time dynamics matrix for the state (19), $\exp(-\int_{t-5}^{t}[0\ 1 \mid -1\ 0]^T)ds$; and

$$\dot{P}_{11}(t) = 2P_{12}(t) - (1-\gamma^{-2})[0.0805P_{11}^2(t) + \quad (23)$$
$$0.544P_{11}(t)P_{12}(t) + 0.9195P_{12}^2(t)],$$
$$\dot{P}_{12}(t) = -P_{11}(t) + P_{22}(t) -$$
$$(1-\gamma^{-2})[0.0805P_{11}(t)P_{12}(t) + 0.272P_{12}^2(t) +$$
$$0.272P_{11}(t)P_{22}(t) + 0.9195P_{12}(t)P_{22}(t)],$$
$$\dot{P}_{22}(t) = 1 - 2P_{12}(t) - (1-\gamma^{-2})[0.0805P_{12}^2(t) +$$
$$0.544P_{12}(t)P_{22}(t) + 0.9195P_{22}^2(t)],$$

with the initial condition $P(0) = R^{-1}$, where the numerical values are the corresponding entries of the matrix $[\exp(-\int_{t-5}^{t}[0\ 1 \mid -1\ 0]^T)ds][\exp(-\int_{t-5}^{t}[0\ 1 \mid -1\ 0])ds]$.

The estimates obtained upon solving the equations (22),(23) are compared to the conventional H_∞ filter estimates, obtained in Theorems 3 and 4 from [4], which satisfy the following equations:

$$\dot{m}_{K_1}(t) = m_{K_2}(t) + P_{11}(t)[y(t) - m_{K_1}(t-5)], \quad (24)$$
$$\dot{m}_{K_2}(t) = -m_{K_1}(t) + P_{12}(t)[y(t) - m_{K_1}(t-5)],$$

with the initial condition $m_f(0) = 0$;

$$\dot{P}_{11}(t) = 2P_{12}(t) - (1-\gamma^{-2})P_{11}^2(t), \quad (25)$$
$$\dot{P}_{12}(t) = -P_{11}(t) + P_{22}(t) - (1-\gamma^{-2})P_{11}(t)P_{12}(t),$$
$$\dot{P}_{22}(t) = 1 - 2P_{12}(t) - (1-\gamma^{-2})P_{12}^2(t),$$

with the initial condition $P(0) = R^{-1}$.

Numerical simulation results are obtained solving the systems of filtering equations (22),(23) and (24),(25). The obtained estimate values are compared to the real values of the state vector $x(t)$ in (19).

For each of the two filters (22),(23) and (24),(25) and the reference system (19) involved in simulation, the following initial values are assigned: $x_1(0) = 1$, $x_2(0) = 1$, $R = I_2 = diag[1\ 1]$. The L_2 disturbance $w(t) = [w_1(t), w_2(t)]$ is realized as $w_1(t) = 1/(1+t)^2$, $w_2(t) = 2/(2+t)^2$. Since $C(t) = L(t) = [1\ 0]$ in (22),(23) and the minimum achievable value of the threshold γ is equal to $\|L\|/\|C\| = 1$, the value $\gamma = 1.1$ is assigned for the simulations.

The following graphs are obtained: graphs of the output H_∞ estimation error $z(t) - z_m(t)$ corresponding to the estimate $x_m(t)$ satisfying the equations (22),(23) (Fig. 1); graphs of the output H_∞ estimation error $z(t) - z_m(t)$ corresponding to the conventional estimate $m_K(t)$ satisfying the equations (24),(25) (Fig. 2). The graphs of the output estimation errors are shown in the entire simulation interval from $t_0 = 0$ to $T = 20$. Figures 1 and 2 also demonstrate the dynamics of the noise-output H_∞ norms corresponding to the shown output H_∞ estimation errors in each case.

The following values of the noise-output H_∞ norm $\|T_{zw}\|^2 = \|z(t) - z_f(t)\|_2^2/(\|\omega(t)\|_2^2 + \|x_0\|_{2,R}^2)$ are obtained at the final simulation time $T = 20$: $\|T_{zw}\| = 0.8138$ for the H_∞ estimation error $z(t) - z_m(t)$ corresponding to the estimate $x_m(t)$ satisfying the equations (22),(23) and $\|T_{zw}\| = 23.75865$ for H_∞ estimation error $z(t) - z_m(t)$ corresponding to the conventional estimate $m_K(t)$ satisfying the equations (24),(25).

It can be concluded that the central suboptimal multi-equational H_∞ filter (22),(23) provides reliably convergent behavior of the output estimation error, yielding a convincingly lesser value of the corresponding H_∞ norm, in comparison to the assigned threshold value $\gamma = 1.1$. The latter serves as an ultimate bound of the noise-output H_∞ norm as time tends to infinity. In contrast, the conventional central H_∞ filter (24),(25) provides divergent behavior of the output estimation error, yielding a much greater value of the corresponding H_∞ norm, which largely exceeds the assigned threshold. Thus, the simulation results show definite advantages of the designed central suboptimal H_∞ filter for linear systems with measurement delay, in comparison to the previously known conventional H_∞ filter.

5. GENERALIZATIONS

As shown in [33], the noise orthonormality condition ($\mathscr{C}_3$), third standard condition from Section 2 (see also [1,4]), can be omitted. This leads to appearance of additional terms in all H_∞ filtering equations. The corresponding generalizations of the obtained H_∞ filters are given in the following propositions.

Corollary 1. In the absence of the noise orthonormality condition ($\mathscr{C}_3$), the central H_∞ filter for the unobserved state (1) over the observations (2), ensuring the H_∞ noise attenuation condition (9) for the output estimate $z_m(t)$, is given by the following equations for the state estimate $x_m(t)$ and the output estimate $z_m(t)$

$$\dot{x}_m(t) = A(t)x_m(t) + [P(t)\exp(-\int_{t-h}^{t} A^T(s)ds)C^T(t) + \quad (26)$$
$$B(t)D^T(t)]T^{-1}(t)[y(t) - C(t)x_m(t-h)],$$
$$z_m(t) = L(t)x_m(t), \quad (27)$$

with the initial condition $x_m(t_0) = 0$, and the equation for the filter gain matrix $P(t)$

$$dP(t) = (P(t)A^T(t) + A(t)P(t) + B(t)B^T(t) + \quad (28)$$
$$\gamma^{-2}P(t)\exp(-\int_{t-h}^{t} A^T(s)ds)L^T(t)\times$$
$$L(t)\exp(-\int_{t-h}^{t} A(s)ds)P(t) -$$
$$[P(t)\exp(-\int_{t-h}^{t} A^T(s)ds)C^T(t) + B(t)D^T(t)]T^{-1}(t)\times$$
$$[C(t)\exp(-\int_{t-h}^{t} A(s)ds)P(t) + D(t)B^T(t)])dt,$$

with the initial condition $P(t_0) = R^{-1}$, where the matrix $T(t)$ is defined as

$$T(t) = D(t)D^T(t) + C(t)\exp(-\int_{t-h}^{t} A(s)ds)\times$$
$$[\int_{t-h}^{t} \exp(\int_{\tau}^{t} A(s)ds)B(\tau)B^T(\tau)\exp(\int_{\tau}^{t} A^T(s)ds)d\tau]\times$$
$$\exp(-\int_{t-h}^{t} A^T(s)ds)C^T(t).$$

Proof. The proof is straightforwardly delivered using the technique of handling the H_∞ filtering problems for systems with non-orthonormal noises, which can be found in [33].

Remark 5. Since the H_∞ filter designed in Corollary 1 is based on the corresponding H_2 mean-square filter, which is optimal with respect to mean square criteria, Remarks 2–4 remain valid.

6. CONCLUSIONS

This paper designs the central finite-dimensional H_∞ filter for linear systems with measurement delay, that is suboptimal for a given threshold γ with respect to a modified Bolza-Meyer quadratic criterion including the attenuation control term with the opposite sign. The central suboptimal H_∞ filter for linear systems with measurement delay is obtained. Finally, the generalized version of the filter is designed in the absence of the standard noise orthonormality condition.

In the example based on a model of a mechanical oscillator, the numerical simulations are run to verify performance of the designed central suboptimal filter for linear systems with measurement delay against the central suboptimal H_∞ filter available for linear systems without delays. The simulation results show a definite advantage in the values of the noise-output transfer function H_∞ norms in favor of the designed filter. In particular, the estimation errors given by the obtained filter converge to zero, whereas the estimation error of the conventional filter diverges. This significant improvement in the estimate behavior is obtained due to the more careful selection of the filter gain matrix in the designed filter. Although this conclusion follows from the developed theory, the numerical simulation serves as a convincing illustration.

The proposed design of the central suboptimal H_∞ filter for linear time-delay systems with integral-quadratically bounded disturbances naturally carries over from the design of the optimal H_2 filter for linear time-delay systems with unbounded disturbances (white noises). The entire design approach creates a complete filtering algorithm of handling the linear time-delay systems with unbounded or integral-quadratically bounded disturbances optimally for all thresholds γ uniformly or for any fixed γ separately. The presented approach would be applied in the future to obtain the central suboptimal H_∞ filters for nonlinear polynomial and nonlinear polynomial time-delay systems.

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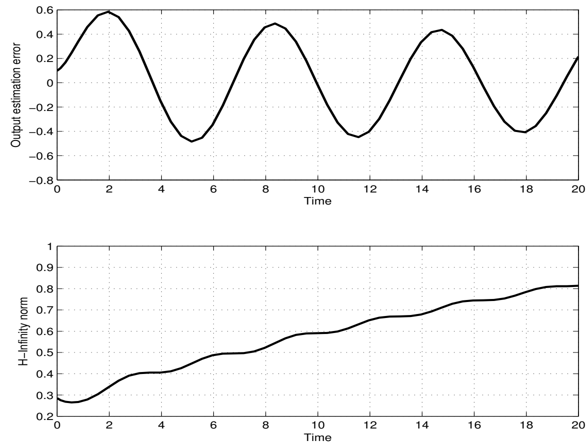

Fig. 1. **Above.** Graph of the output H_∞ estimation error $z(t) - z_m(t)$ corresponding to the estimate $x_m(t)$ satisfying the equations (22),(23), in the simulation interval [0,20].**Below.** Graph of the noise-output H_∞ norm corresponding to the shown output H_∞ estimation error, in the simulation interval [0,20].

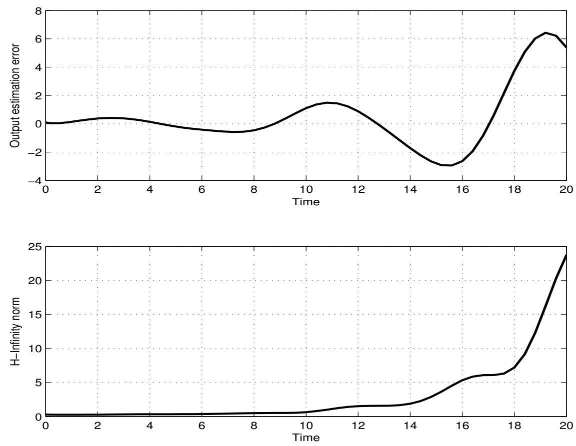

Fig. 2. **Above.** Graph of the output H_∞ estimation error $z(t) - z_m(t)$ corresponding to the estimate $m_K(t)$ satisfying the equations (24),(25), in the simulation interval [0,20].**Below.** Graph of the noise-output H_∞ norm corresponding to the shown output H_∞ estimation error, in the simulation interval [0,20].