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Parametric Identification of Nonlinear Hysteretic systems

Y. Rochdi *. F. Giri * . F.Z Chaoui *. J. Rodellar^. F. Ikhouane^

**Greyc, Caen, France*
(e-mail: youssefrochdi@ yahoo.fr, fouadgiri@yahoo.fr).
^Departament de Matemàtica Aplicada III, Universitat Politècnica de Catalunya, Barcelona,Spain

Abstract: The hysteretic behaviour is an essential feature of many physical systems. Such a feature is conveniently accounted for in hysteretic systems modelling through the well known nonlinear Bouc-Wen equations. But these involve several unknown parameters and internal signals that are not all accessible to measurements. These difficulties make the identification of hysteretic systems a challenging problem. To cope with these issues, previous works are generally based on simplifying assumptions that amount to supposing, among others, that the Bouc-Wen equations describe an isolated physical element in which 'hysteretis' is the only dynamic feature. The point is that, even such a case, the control input should be an external driving force and no the displacement. In this paper, the hysteretic equations are let to be what they really are in most practical situations: just a part of the system dynamics. A multi-stage parametric identification scheme is designed and shown to recover consistently the system unknown parameters. The proposed solution is suitable for systems not tolerating large displacements (e.g. like buildings) as well as for situations where force sensors are not available.

1. INTRODUCTION

Hysteresis is a memory feature that characterises a wide variety of nonlinear systems. To describe the behaviour of hysteretic processes several mathematical models have been proposed (Macki, *et al*, 1993). In this paper the focus is made on the Bouc-Wen model (Wen, 1976) which is suitable for smooth hysteresis. It has received a great deal of interest and, in particular, it has been used to model piezoelectric actuators, base isolation devices and magnetorheological dampers (Nagarajaiah and Xiaohong, 2000), (Ni *et al,* 1998, Savaresia *et al*, 2005). In this model, the restoring force $\Phi_{BW}(t)$ is related to the displacement $x(t)$ as follows:

$$\Phi_{BW}(t) = \alpha k x(t) + (1-\alpha) D k z(t) \qquad (1)$$

$$\dot{z}(t) = D^{-1}\left(A\dot{x}(t) - \beta|\dot{x}(t)||z(t)|^{n-1} z(t) - \gamma \dot{x}(t)|z(t)|^{n} \right) \qquad (2)$$

where the parameters $0 < \alpha < 1$, $k > 0$, $D > 0$, $A > 0$, $-\beta < \gamma \leq \beta$, $n \geq 1$ determine the shape and size of the hysteresis loop.

Identification of the model (2) amounts to determining the parameters α, β, γ, A, D, k, n. This is not a trivial task as the state $z(t)$ is not accessible to measurements. In particular, the parameter n comes nonlinearly in the model and is not necessarily an integer. Many identification methods have been proposed to get estimates of the Bouc-Wen model parameters. A number of early methods have been reported in (Wen, 1976) and more recent ones have been presented in (Smith et *al.,* 1999; Ni et *al.*, 1998; Ikhouane and Rodellar, 2005b). However, the proposed methods are generally based on simplifying assumptions on the system or on the signals. For instance, in (Smith et *al.,* 1999) and (Ni et *al.*, 1998) it was supposed that some system parameters (e.g. mass, friction coefficient) are known and the second derivative of *x(t)* is measurable. In (Smith *et al.,* 1999), it was assumed that *n* is an integer and bounded by a known integer. In (Ikhouane and Rodellar, 2005b) it is supposed that the restoring force $\Phi_{BW}(t)$ is accessible to measurements and the displacement *x(t)* is the actual control input of the system.

In this paper, the hysteretic element (1)-(2) is considered as a part of a more complete mechanical structure. Accordingly, the displacement *x(t)* undergoes a Newton motion equation that is driven by an external force. Furthermore, the displacement measurement is affected by an additive noise. The global structure is controlled by the driving force (input signal) and observed by the (noisy) measured displacement (output signal). The more realistic system thus defined is characterized by equations (1)-(2) together with the Newton and output equations. The Newton equation introduces additional unknown parameters (inertia and friction coefficients). An identification scheme is designed to get estimates of all unknown parameters without resorting to those unrealistic assumptions such as supposing accessible to measurement the restoring force and the displacement derivatives or supposing the parameter *n* to be an integer. The proposed method operates in three main stages. In each stage a part of the unknown parameters and unmeasured signals are estimated and the estimates obtained in one stage are based upon in the next stage. The proposed identification method relies on simple experiments that only necessitate sine wave exciting signals. Finally, note that consistency properties are formally established for all involved estimators, while most earlier works were limited to simulations (Wen, 1976; Smith et *al.,* 1999; Ni et *al.*, 1998).

2. IDENTIFICATION PROBLEM STATEMENT

2.1 Class of identified systems

We are interested in (mechanical/structural) systems that involve the hysteresis feature. This is for instance the case of base isolators installed to supply passive or active (through actuators) protection of huge buildings against earthquakes. The motion of the base is described applying Newton's law:

$$m\ddot{x}(t) + f\dot{x}(t) + \Phi_{BW}(t) = u(t) \quad (5a)$$

$$x_d(t) = x(t) + \xi(t) \quad (5b)$$

where $u(t)$ represents the excitation force; $x(t)$ denoted the real displacement and $x_d(t)$ is measured value; $\xi(t)$ is a zero-mean ergodic stochastic process accounting for external disturbances. The constants m and f are respectively the inertia and the friction coefficient. $\Phi_{BW}(t)$ denotes the nonlinear restoring force that is assumed to undergo the normalized Bouc-Wen model:

$$\Phi_{BW}(t) = k_x x(t) + k_w \omega(t) \quad (6)$$

$$\dot{\omega}(t) = \rho(\dot{x}(t) - \sigma|\dot{x}(t)||\omega(t)|^{n-1}\omega(t) - (1-\sigma)\dot{x}(t)|\omega(t)|^n) \quad (7)$$

where $k_x > 0,\ k_w > 0, \rho > 0, \sigma \geq \frac{1}{2}$ and $n \geq 1$.

The normalized model (6)-(7) involves only five unknown parameters, while the initial model (1)-(2) involved seven parameters. Equation (6) shows that the restoring force $\Phi_{BW}(t)$ is the superposition of an elastic component $k_x x(t)$ and a hysteretic component $k_w\omega(t)$. The signal $\omega(t)$, which is an internal state, is not supposed to be available. Consequently, the restoring force is in turn not available. In fact, $u(t)$ and $x_d(t)$ are the measurable signals. In particular, $x(t)$ and its derivatives $\dot{x}(t)$ and $\ddot{x}(t)$ are not supposed to be measurable.

Remark 2.1.
1) The fact that none of $\Phi_{BW}(t)$, $x(t)$, $\dot{x}(t)$ and $\ddot{x}(t)$ is supposed to be measurable does constitute a substantial progress with respect to the existing literature (e.g. (Smith *et al*., 1999); (Ni *et al*., 1998); (Ikhouane and Rodellar, 2005b)).
2) Note also that, unlike (Ikhouane and Rodellar, 2005b), the internal signal $x(t)$ is not considered to be the actual control input. In fact, taking $x(t)$ as the control input would be possible if the hysteretic model (1)-(2) could exist as an autonomous physical element. The fact is that, in most situations, equations (1)-(2) come in as a part of a more general model; they just point out the fact that hysteresis is a feature of the considered system. As a matter of fact, the system is driven by an external force $u(t)$. Consequently, there is no way to enforce $x(t)$ to fit the non-smooth triangular signal resorted to in (Ikhouane and Rodellar, 2005a-b).
3) An other interesting feature of the present study, compared to the previously mentioned works, is that a measurement noise is accounted for in the displacement, making the present study more realistic.

2.2 Identification objective

Our purpose is to design an identification scheme that provides asymptotically accurate estimates of the unknown parameters $m,\ f,\ k_x, k_w,\ \rho,\ \sigma$ and n.

3. IDENTIFICATION SCHEME DESIGN

3.1 Hysteretic model re-parameterization

The proposed identification scheme is based on a new re-parameterization of the system. To this end, we rewrite equations (6) and (7) as follows:

$$\Phi_{BW}(t) = k_x x(t) + v(t) \quad (8)$$

$$\dot{v}(t) = a\dot{x}(t) - b|\dot{x}(t)||v(t)|^{n-1}v(t) + c\dot{x}(t)|v(t)|^n \quad (9)$$

where:

$$v(t) = k_w\omega(t) \quad (10a)$$

$$a = \rho k_w \quad ; \quad b = \frac{\rho\sigma}{k_w^{n-1}}; \quad c = \frac{\rho(\sigma-1)}{k_w^{n-1}} \quad (10b)$$

In this re-parameterized model, the unknown parameters are n, m, f, k_x, a, b, and c. The parameters k_w, ρ, γ can be obtained from the parameters a, b, c and n using the following equations:

$$k_w = \left(\frac{a}{b-c}\right)^{1/n}; \rho = \frac{a}{k_w}; \sigma = \frac{b}{a}(k_w)^n \quad (11)$$

3.2 Estimation of the true displacement x(t)

First, recall that (see e.g. (Smith *et al*., 1999)) when the input $u(t)$ is periodic, the undisturbed output $x(t)$ is in turn periodic, in steady-state, with the same period. In fact, in steady-state, one observes in practice a hysteretic limit cycle.
The proposed identification scheme necessitates three experiments all of them involving a periodic input signal. More specifically, the input signals considered are simple sine waves. Consequently, in all experiments, the undisturbed output $x(t)$ turns out to be, in the steady-state stage, a periodic signal with the same period as the input $u(t)$. This property, together with those of the external disturbance $\xi(t)$, makes it possible to consistently estimate $x(t)$, using the measured signal $x_d(t)$. In effect, letting T denotes the period of the input signal, the displacement $x(t)$ can estimated averaging the measured output $x_d(t)$ as follows:

$$\bar{x}(t,N) = \frac{1}{N}\sum_{i=1}^{N} x_d(t+iT) \quad (12)$$

Proposition 3.1. The estimator (12) is consistent i.e
$\bar{x}(t,N) \rightarrow x(t)$ (w.p. 1) as $N \rightarrow \infty$ □

Proof. The proof is omitted due to the limitation of the length of the paper. ■

3.3. Outline of the proposed identification scheme

Before presenting in detail the different components of the identification scheme, it is convenient to first sketch a general view. Roughly speaking, the proposed identification scheme includes three main steps. First, the parameters m and f are estimated ($\hat{m}$ and $\hat{f}$), and $\dot{x}(t)$ and $\ddot{x}(t)$ are estimated from $\bar{x}(t)$ ($\dot{\bar{x}}(t)$ and $\ddot{\bar{x}}(t)$). Then, an obvious estimator of $\Phi_{BW}(t)$ would simply be:

$$\hat{\Phi}_{BW}(t) = u(t) - \hat{m}\ddot{\bar{x}}(t) - \hat{f}\dot{\bar{x}}(t) \qquad (13)$$

In the second step, the parameter k_x is estimated and the estimate $\hat{k}_x$ is used in the following estimator of the internal variable $v(t)$:

$$\hat{v}(t) = \left(\hat{\Phi}_{BW}(t) - \hat{k}_x \bar{x}(t) \right) \qquad (14)$$

In the third step, the rest of the parameters (i.e. a, b, c, n) are estimated based on (9), using $\hat{v}(t)$. The estimates thus obtained are used to get estimates for the parameters k_w, ρ, γ making use of (11).

3.4 *Step 1:* Estimation *of m, f and* $\Phi_{BW}(t)$

As already mentioned, in this paper we only consider sine input signals i.e.

$$u(t) = U_m \cos(\omega t) \qquad (15)$$

It has also been mentioned earlier that the resulting (steady-state) displacement $x(t)$ is periodic with period $T = 2\pi / \omega$. Then, invoking the Fourier series theory, the displacement can be decomposed as follows:

$$x(t) = X_1 \cos(\omega t - \varphi) + x_{har}(t) \qquad (16a)$$

with:

$$x_{har}(t) = \sum_h X_h \cos(h\omega t - \varphi_h) \;\; (h \geq 2) \qquad (16b)$$

Furthermore, it can be shown (see e.g. (Ikhouane and Rodellar, 2005a)), that for small displacements the relationship between $\Phi_{BW}(t)$ and $x(t)$ becomes linear in steady state, i.e.:

$$\Phi_{BW}(t) = (k_x + a)x(t) \qquad (17)$$

This means that for a periodic displacement with small magnitude the hysteretic feature has no substantial effect on the displacement. In view of (17), equation (5) becomes:

$$m\ddot{x}(t) + f\dot{x}(t) + Kx(t) \cong u(t) \qquad (18a)$$

with:

$$K = k_x + a \qquad (18b)$$

The smaller is $\max_t(x(t))$, the more accurate the linear equation (18a). This in turn implies that $x_{har}(t) \to 0$ as $\max_t(x(t)) \to 0$.

The second-order linear equation (18a) is particularly suitable to get estimates of the unknown parameters m, f, K. But, it is important to recall that here $x(t)$ is the output of the system that is driven by $u(t)$, and furthermore $x(t)$ is not directly measurable. So, the question is: *how to be sure that such equation actually holds ?*

To answer the above question, we make use of the fact that $\bar{x}(t, N)$ is a consistent estimate of the (unavailable) signal $x(t)$ and the fact that $x_{har}(t) \to 0$ as $\max_t(x(t)) \to 0$. The idea is to drive the system into a 'small-signals' operation regime, by tuning U_m (and possibly ω) and observing the size of $x_{har}(t)$. This is precisely formulated in the following research procedure:

Small Signals Operation (SSO)

1) Develop the T-periodic signal $\bar{x}(t)$ in Fourier series.

2) Compute the distortion ratio: $D_R = \dfrac{\sum_h (\bar{X}_h)^2}{\bar{X}_1^2}$

3) If $D_R > \varepsilon$, then tune U_m or ω to decreasing x(t) and go to step 1. Else, note the values of U_m , ω and $\bar{X}_1$ and end the procedure.

In Step 3, the parameter ε is a real threshold whose choice is let to the designer. The outcome of the SSO procedure is a quadruplet $\left(U_m, \omega, \varphi(\varepsilon), \bar{X}_1(\varepsilon)\right)$ corresponding to conditions where equations (17)-(18a) hold with an error that depend on ε. The smaller is ε, the more accurate are (17)-(18a). However, a too small value of ε would necessitates a too long time for the SSO procedure to end up research. Therefore, ε should be chosen bearing in mind the above two requirements. Our simulations has shown that that the choice ε=0.05 a satisfactory compromise.

The above procedure can be run on several times leading to different quadruplets. Let $\left(U_{m1}, \omega_1, \varphi_1(\varepsilon), \bar{X}_{11}(\varepsilon)\right)$ and $\left(U_{m2}, \omega_2, \varphi_2(\varepsilon), \bar{X}_{12}(\varepsilon)\right)$ denote two such quadruplets.

On the other hand, from (18a) one gets the following expressions:

$$\left|\frac{\bar{X}_1}{U_m}\right| = \frac{1}{\sqrt{(f\omega)^2 + (K - m\omega^2)^2}} \qquad (19a)$$

$$\tan\varphi = \frac{f\omega}{K - m\omega^2} \qquad (19b)$$

In effect, these characterize the harmonic behavior of any linear second-order system. Substituting, successively $\left(U_{m1}, \omega_1, \varphi_1(\varepsilon), \bar{X}_{11}(\varepsilon)\right)$ and $\left(U_{m2}, \omega_2, \varphi_2(\varepsilon), \bar{X}_{12}(\varepsilon)\right)$ in the above expressions, one gets four equations involving the unknown parameters m, f and K. Then estimates $\hat{m}(\varepsilon), \hat{f}(\varepsilon), \hat{K}(\varepsilon)$ can then be obtained by simply solving the obtained equations. The quality of the estimates depends on the value of ε. The smaller is ε, the more accurate the estimates. The result thus established is simply formulated as follows:

$$\hat{m}(\varepsilon) = m + O(\varepsilon)\,; \qquad \hat{f}(\varepsilon) = f + O(\varepsilon) \quad (20a)$$

$$\hat{K}(\varepsilon) = K + O(\varepsilon) \quad (20b)$$

Remark 3.1. In (Ikhouane and Rodellar, 2005b), no method has been given to check whether (18a) holds or not. In the light of these observations, the procedure SSO presented above turns out to be a significant progress.
Moreover, since the signal x(t) and its derivatives $\dot{x}(t)$ and $\ddot{x}(t)$ have all been supposed to be unavailable, the estimation of the parameters m, f, K based on the parametrization (18a) and the least-squares is not possible.

3.5 Step 2: estimation of k_x, a and v(t)

In this section, the amplitude of the output signal *x(t)* is no longer supposed to be small. In such a case, the curve $(u(t), \bar{x}(t))$ leads to a hysteretic limit cycle.

Estimation of k_x and a. To this end, we perform two experiments that consist in exciting the system, successively, with the two following signals:

$$u_1(t) = U_m \cos(2\pi t / T) \quad (21a)$$

$$u_2(t) = U_m \cos(2\pi t / T) + U \quad (21b)$$

where U_m and U are any real constants. Let $x_1(t)$ and $x_2(t)$ denote the resulting (undisturbed) displacements. These are *T*-periodic because the input signals $u_1(t)$ and $u_2(t)$ are so. Then, consistent estimates $\bar{x}_1(t,N)$ and $\bar{x}_2(t,N)$ of $x_1(t)$ and $x_2(t)$ are obtained using (12). Let us introduce the notations where $(i = 1,2)$:

$$\bar{x}_{i\,max}(N) = \max_t \bar{x}_i(t,N), \quad \bar{x}_{i\,min}(N) = \min_t \bar{x}_i(t,N)$$

Through the multiple experiments, we always observe that :

$$\bar{x}_{2max}(N) - \bar{x}_{2min}(N) = \bar{x}_{1max}(N) - \bar{x}_{1min}(N) + \delta(N) \quad (22a)$$

$$\bar{x}_2(t) - \bar{x}_{2min} = \bar{x}_1(t) - \bar{x}_{1min} + \delta(N) \quad (22b)$$

$$\delta(N) \to 0 \text{ as } N \to \infty \quad (22c)$$

Using these observations, consistent estimators are constructed and presented in the following proposition.

Proposition 3.2. Let the system described by (5a-b) and (8)-(9) be successively excited with the two input signals defined by (17a-b). Let $x_1(t)$, $x_2(t)$ denote the resulting displacements and $\bar{x}_1(t)$, $\bar{x}_2(t)$ be their estimates obtained applying (12). Consider the following estimators for the parameters k_x and a:

$$\hat{k}_x(N) = \frac{U}{\bar{x}_{2min}(N) - \bar{x}_{1min}(N)} \quad (23)$$

$$\hat{a}(N) = \hat{K}(N) - \hat{k}_x(N) \quad (24)$$

Then, one has:
$\hat{k}_x(N) \to k_x$ and $\hat{a}(N) \to a$ w.p.1 as $N \to \infty$ □

Proof. The proof is omitted due to the limitation of the length of the paper. ■

Estimator for v(t).
If the derivatives $\dot{x}(t)$ and $\ddot{x}(t)$ were measurable, then a consistent estimator of *v(t)* could readily be obtained from (5a), (6) and (10a). Specifically, the obtained estimator would be:

$$\hat{v}(t,N,\varepsilon) = u(t) - \hat{m}(\varepsilon)\ddot{x}(t) - \hat{f}(\varepsilon)\dot{x}(t) - \hat{k}_x(N)x(t) \quad (25)$$

However, accurate sensors of the displacement derivatives are generally not available in practice. Furthermore, it is generally not possible to design accurate estimators for arbitrary signals derivatives. Nevertheless, we will show that this is presently possible, thanks to the (steady-state) *T*-periodicity of the involved signals. Indeed, as *x(t)* is periodic (in steady-state) with period $T = 2\pi / \omega$, it can be developed in Fourier series:

$$x(t) = x_0 + \sum_{h=1}^{\infty} X_h \cos(h\omega t - \varphi_h) \quad (26)$$

The derivatives $\dot{x}(t)$ and $\ddot{x}(t)$ are in turn given the following Fourier expansions:

$$\dot{x}(t) = -\sum_{h=1}^{\infty} h\omega X_h \sin(h\omega t - \varphi_h) \quad (27a)$$

$$\ddot{x}(t) = -\sum_{h=1}^{\infty} (h\omega)^2 X_h \cos(h\omega t - \varphi_h) \quad (27b)$$

The above expressions show that if the Fourier coefficients X_h of *x(t)* were available, then it would be possible to obtain the derivatives $\dot{x}(t)$ and $\ddot{x}(t)$. The point is that the measurements $x_d(t)$ of *x(t)* are noisy. Nevertheless, we do have a consistent estimator of *x(t)*, namely $\bar{x}(t,N)$. Then, instead of (26) we consider the Fourier series of $\bar{x}(t)$:

$$\bar{x}(t,N) = \bar{x}_0(N) + \sum_{h=1}^{\infty} \bar{X}_h(N) \cos(h\omega t - \varphi_h(N)) \quad (28)$$

Then, deriving twice (28) and truncating the obtained developments, one gets the following estimators for $\dot{x}(t)$ and $\ddot{x}(t)$:

$$\dot{\bar{x}}(t,N,M) = -\sum_{h=1}^{M} h\omega \bar{X}_h(N) \sin(h\omega t - \varphi_h(N)) \quad (29a)$$

$$\ddot{\bar{x}}(t,N,M) = -\sum_{h=1}^{M} (h\omega)^2 \bar{X}_h(N) \cos(h\omega t - \varphi_h(N)) \quad (29b)$$

where *M* is any positive integer. Given the above estimates of $\dot{x}(t)$ and $\ddot{x}(t)$, equation (25) suggests the following estimator of *v(t)*:

$$\hat{v}(t,N,M,\varepsilon) = u(t) - \hat{m}(\varepsilon)\ddot{\bar{x}}(t,N,M) - \hat{f}(\varepsilon)\dot{\bar{x}}(t,N,M) - \hat{k}_x(N)\bar{x}(t,N) \quad (30)$$

Proposition 3.3. The estimators (29a-b) and (30) have the following properties:
1) (25a-b) are consistent, i.e.
a) $\dot{\bar{x}}(t,N,M) \to \dot{x}(t)$ (w.p. 1) as N, M $\to \infty$
b) $\ddot{\bar{x}}(t,N,M) \to \ddot{x}(t)$ (w.p. 1) as N, M $\to \infty$
2) (30) is consistent up to an error that depends on ε, the threshold introduced in the procedure SSO. More precisely, we have:
$\hat{v}(t,N,M) - v(t) \to O(\varepsilon)$ (w.p. 1) as N, M $\to \infty$
where $O(\varepsilon) \to 0$ if $\varepsilon \to 0$ □

Proof. The proof is omitted due to the limitation of the length of the paper. ■

Remark 3.2. The results of Proposition 3.3 suggest that the integers N and M should be sufficiently large. The larger are these integers the batter the quality of the estimates. As long as M is concerned, equations (23a-b) show that a convenient choice is one such that: $h^2 X_h << X_1$ for all $h > M$.

3.6 Step 3: Estimation of the remaining parameters

Though the signal *v(t)* is accurately estimated, the estimation of the remaining parameters in (9) (namely *b*, *c* and *n*) is not that easy. One key idea to overcome these difficulties is to notice that equation (9) considerably simplifies in each quadrant of the plane $(v(t), \dot{x}(t))$. More specifically, it follows from (9) that:

if $\dot{x}(t) > 0, v(t) > 0$, $\dot{v}(t) = \left[a - (b-c)v(t)^n\right]\dot{x}(t)$ (31a)
if $\dot{x}(t) > 0, v(t) < 0$, $\dot{v}(t) = \left[a + (b+c)|v(t)|^n\right]\dot{x}(t)$ (31b)
if $\dot{x}(t) < 0, v(t) < 0$, $\dot{v}(t) = \left[a + (-b+c)|v(t)|^n\right]\dot{x}(t)$ (31c)
if $\dot{x}(t) < 0, v(t) > 0$, $\dot{v}(t) = \left[a + (b+c)v(t)^n\right]\dot{x}(t)$ (31d)

These equations are completed with (10a-b). It follows from (31a) that:

$$\frac{\dot{v}(t)}{\dot{x}(t)} - a = (c-b)v(t)^n \qquad (32)$$

On the other hand, one gets from (10) that:

$$c - b = -\frac{\rho}{k_w^{n-1}} = -\frac{a}{k_w^n} < 0$$

Then, taking logarithms of both sides of (32), yields:

$$log\left(-\vartheta(t) + a\right) = log(b-c) + n\,log\left(v(t)\right) \qquad (33a)$$

$$\vartheta(t) \overset{def}{=} \frac{\dot{v}(t)}{\dot{x}(t)} \qquad (33b)$$

Equation (33a) is quite interesting because: i) the unknown parameters *n* and $log(b-c)$ come in linearly, ii) accurate estimates are available for the involved signals, namely $log(-\vartheta + a)$ and $log(v)$. Therefore, the unknown parameters can be recovered applying the least-squares estimator to the equation:

$$log\left(-\hat{\vartheta}(t,N,M,L,\varepsilon) + \hat{a}(N)\right) = log(b-c) + n\,log\left(\hat{v}(t,N,M,\varepsilon)\right) \qquad (34a)$$

where:

$$\hat{\vartheta}(t,N,M,L,\varepsilon) = \frac{\hat{\dot{v}}(t,N,M,L,\varepsilon)}{\dot{\bar{x}}(t,N,M)} \qquad (34b)$$

where $\hat{\dot{v}}(t,N,M,L,\varepsilon)$ denotes a consistent estimator (up to an error $O(\varepsilon)$) of the derivative $\dot{v}(t)$. $\hat{\dot{v}}(t,N,M,L,\varepsilon)$ is constructed making use of the fact that *v(t)* is T-periodic (in steady-state) and $\hat{v}(t,N,M,\varepsilon)$ is a consistent estimator (up to $O(\varepsilon)$). This construction is based on the Fourier series expansion of $\hat{v}(t,N,M,\varepsilon)$ up on order *L*, just as this was done to get the estimators $\dot{\bar{x}}(t,N,M)$ and $\ddot{\bar{x}}(t,N,M)$ (see (29a-b)) making use of the T-periodicity of *x(t)* and the fact that $\bar{x}(t,N)$ is a consistent estimator.
The least-squares estimator should be run on every time $\dot{\bar{x}}(t,N,M) > 0$ and $\hat{v}(t,N,M,\varepsilon) > 0$. Doing so, one gets consistent estimates of $(b-c)$ and *n*.
Similarly, considering the case where $\dot{x}(t) > 0$ and $v(t) < 0$, it follows from (31b) and (10a-b) that:

$$\frac{\dot{v}(t)}{\dot{x}(t)} - a = (c+b)|v(t)|^n \qquad (35a)$$

$$c + b = \frac{\rho(2\sigma - 1)}{k_w^{n-1}} \geq 0 \quad (\text{because } \sigma \geq 1/2) \qquad (35b)$$

These imply:

$$log\left(\vartheta(t) - a\right) = log(b+c) + n\,log(|v(t)|) \qquad (36)$$

There too, the unknown parameters, $log(b+c)$ and *n*, come in linearly and consistent estimators are available for the involved signals i.e. $\vartheta(t)$ and *v(t)*. Then, the unknown parameters can be recovered applying the least-squares estimator to the equation:

$$log\left(\hat{\vartheta}(t,N,M,L,\varepsilon) - \hat{a}(N)\right) = log(b+c) + n\,log\left(\hat{v}(t,N,M,\varepsilon)\right) \qquad (37)$$

The present least-squares estimator is run on every time $\dot{\bar{x}}(t,N,M) > 0$ and $\hat{v}(t,N,M,\varepsilon) < 0$. The consistent estimate thus obtained for $(b+c)$ is combined with that obtained previously for $(b-c)$ to get consistent estimates for *b* and *c*.

4. EVALUATION OF THE IDENTIFICATION SCHEME

To illustrate the efficiency of the proposed identification scheme, we consider a system described by (5a-b)-(6)-(7) where the parameters take the following values:

$\rho = 6; n = 2.7; m = 3; f = 7; k_x = k_w = 2; \sigma = 0.7$

So the parameters to identified, according the new re-parameterisation (8), are from (10), (18b) and (31c):

$a = 12;\ b = 1.292;\ c = -0.554;\quad K = 14$

The disturbance is a sequence of uniformly distributed random numbers over the interval $[-0.04\quad 0.04]$.

First step: we proceed with two experiments using the followings input signals:

$u_1(t) = 0.4\sin(\pi t/20)$ and $u_2(t) = 0.5\sin(\pi t/10)$

The obtained estimates of the parameters are regrouped in table 1.

Table1 Estimation of parameters *m, f, K*

Estimates for	m=3	f=7	K=14
N=50	3.534	7.201	14.075
N=200	3.070	7.184	14.009

***Second step*:** we proceed with two experiments that using the followings input signals:

$u_1(t) = 2\sin(\pi t/10)$ and $u_2(t) = 2\sin(\pi t/10) + 1$

The corresponding undisturbed outputs $\bar{x}_1(t,N)$, $\bar{x}_2(t,N)$ are constructed with *N=200.* Using (23) one has: $\hat{k}_x = \dfrac{U}{\bar{x}_2 - \bar{x}_1} = \dfrac{1}{0.5} = 2$ and $\hat{a} = \hat{K} - \hat{k}_x = 14.009\text{-}2 = 12.009$

***Third step*:** In this step the system is excited by the following input signal:

$u_3(t) = 2\sin(\pi t/10)$

Then in steady state, for $N=800,\ M=8,\ L=8$ and $\varepsilon = 0.05$, one computes successively $\bar{x}(t,N)$, $\dot{\bar{x}}(t,N,M)$, $\ddot{\bar{x}}(t,N,M)$, $\hat{v}(t,N,M,\varepsilon)$ $\hat{\dot{v}}(t,N,M,\varepsilon,L)$ and $\hat{\vartheta}(t,N,M,L,\varepsilon)$ using respectively (12), (25a-b), and (30), (34b). By running the least-squares estimator to the equation (34), one gets :

$$\hat{n} = 2.652 \text{ and } \hat{b} - \hat{c} = 1.877 \qquad (38)$$

In the same way, by running the least-squares estimator to the equation (37), one gets:

$$\hat{n} = 2.860 \text{ and } \hat{b} + \hat{c} = 0.662 \qquad (39)$$

Combining (38) and (39), one gets:

$$\hat{n} = (2.86 + 2.652)/2 = 2.75,\ \hat{b} = 1.269,\ \hat{c} = 0.607 \quad (40)$$

From (11) one gets : $\hat{k}_w = 1.963$; $\hat{\rho} = 6.112$; $\hat{\sigma} = 0.675$.

The obtained estimates are close to their true values, despite the presence of disturbances. In the figure 1 we have plotted the limit cycle $(x_d(t), \Phi_{BW}(t))$ (Black) obtained from the true model, and the estimated limit cycle $(x_d(t), \Phi_{BW}(t))$ (White) obtained from the estimated model. It is important to notice the level of disturbances.

5. CONCLUSIONS

We have considered the problem of identifying systems whose dynamic behaviour involves a hysteretic feature. Such a feature has been accounted for using the well known Bouc-Wen equations. This model (5)-(7) involves internal signals that are not necessarily accessible to measurements (e.g. $\Phi_{BW}(t)$, $v(t)$, $\dot{x}(t), \ddot{x}(t)$). Moreover, the displacement $x(t)$ stands as an internal signal and, consequently, cannot be considered as the system control input. The identification scheme we have designed operates in three main steps. The involved estimators are generally shown to be consistent. Compared to the existing works, our solution does not suppose that the internal signals ($\Phi_{BW}(t)$, $v(t)$, $\dot{x}(t), \ddot{x}(t)$) are available, the parameter n is known or is an integer. Moreover, the system actual control input is let be what it really is in practice i.e. the driving force $u(t)$. Finally, the proposed solution requires simple experiments as these only involve sine input signals.

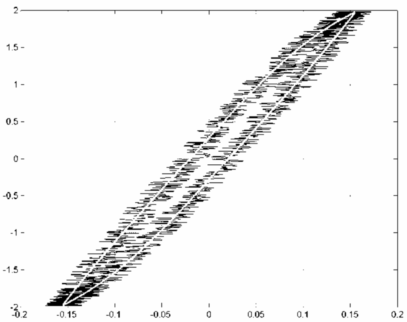

Fig. 1. Limit cycle $(x_d(t), \Phi_{BW}(t))$ for the true model (back) and for the estimated model (white)

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