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Disturbance Attenuation and Fault Detection via Zero-pole Assignment: a Dynamic Observer Approach ⋆

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Abstract: In this paper, a zero-pole assignment approach is proposed for disturbance attenuation in dynamic fault detection observer design. The properties of dynamic observers are analysed and it is shown that the poles of a dynamic observer can be shifted and the additional zeros can be assigned arbitrarily. Then, a novel pole-zero assignment approach is proposed and its application to a continuous system is presented. In the simulation, the disturbances are low-frequency signals (< 1 Hz), which is more difficult to be attenuated compared to high-frequency disturbances. The dynamic observer shows the capabilities to attenuate such low-frequency disturbances. The zeros assignment in observer design would be the main contribution of this paper.

Keywords: Fault Detection, Robust Observer Design, Dynamic Observer

1. INTRODUCTION

The primary objective of this paper is to present a constructive robust dynamic observer design procedure integrating zero-pole assignment techniques, for fault detection in a multivariable system corrupted by band-limited disturbances. The design of Robust Fault Detection Observer (RFDO) has received much attention during the last two decades. (e.g., Frank and Ding [1997], Chen and Patton [1999], Wang et al. [1993], Gao et al. [2007]), and a variety of approaches have been proposed, such as state observers or filters, full disturbance decoupling observers, UIOs (Unknown Input Observers), H_2, H_∞, H_- based robust observers, etc..

Even though the observer based fault detection theories become rich, the basic structures are confined in traditional *static observers*, because of their simplification. The poles of static observers can be assigned to desired positions arbitrarily through eigenstructure assignment. However, its zeros are invariant. From the view of system performance, the pole positions are insufficient for achieving an optimal observer and the system behaviour is also greatly affected by the zeros. Zero invariance puts a limitation on the performance of disturbance attenuation in fault detection.

Therefore, some researchers proposed *dynamic observers* for improving the performance (Park et al. [2002]). Compared to static observers where only one real constant coefficient gain matrix needs to be optimised, in dynamic observers there are four matrices K_1, K_2, K_3, K_4 to be optimised in dynamic observers. Dynamic observers provide more freedom raising both advantages and challenges. Some preliminary works have been done on dynamic observers, but the attention is mainly on the poles assignment. Some researchers (Duan et al. [2003], Chang [2006]) proposed PIO (Proportional Integral Observer) and treat the PIO as a static observer with an additional integral term to deal with the steady state error. Park et al. [2002] proposed a dynamic observer design method as a dual of control design for the state estimation.

It is felt that taking into account the locations of zeros is helpful: it takes advantage of the additional freedom to attenuate disturbances and reduces the computation costs in optimisaion by diminishing the search space. The disturbance can be attenuated further if the zeros are close to the disturbance frequency. Furthermore, by specifying the zero positions, the search space of free parameters is diminished. Thus the 'curse of dimension' is relieved and a global optimal solution is more possible.

Although the multivariable system zeros were first proposed by Rosenbrock over 30 years ago (Rosenbrock [1973]), the system zeros study received relatively less attention than the poles research. For more information on system zeros, please see Schrader and Sain [1989], Smagina [2002], Al-Assadi [2007]. There has not been a known result of utilizing the zero assignment technique to design a fault detection observer.

Different from all the reported works on dynamic observer design, this paper aims to establish a zeros assignment approach in dynamic observer design and get systematic study on its zeros. The properties of dynamic observer zeros, and the possibility of zeros assignment are studied

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in section 2 and 3. An application is illustrated in section 5. It has been shown that the zeros assignment in dynamic observer design is possible and a better disturbance attenuation can be achieved. This would be the main contribution of this paper.

2. PROBLEM FORMULATION

Consider a completely controllable and observable multivariable system whose states/outputs are corrupted by disturbances/faults (Chen and Patton [1999]):

$$\begin{cases} \dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u(t) + \mathbf{B}_f f(t) + \mathbf{B}_d d(t) \\ y(t) = \mathbf{C}x(t) + \mathbf{D}u(t) + \mathbf{D}_f f(t) + \mathbf{D}_d d(t) \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^p$ the input, $y \in \mathbb{R}^r$ the output. $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\mathbf{B} \in \mathbb{R}^{n\times p}$, $\mathbf{C} \in \mathbb{R}^{r\times n}$, $\mathbf{D} \in \mathbb{R}^{r\times p}$ and $f(t)$, $d(t)$ are the general fault vector and disturbance vector, respectively.

For such a system (1), a mth-order dynamic observer is defined as

$$\begin{cases} \dot{z}(t) = \mathbf{K}_1 z(t) + \mathbf{K}_2 r(t) \\ v(t) = \mathbf{K}_3 z(t) + \mathbf{K}_4 r(t) \end{cases} \tag{2}$$

and

$$\begin{cases} \dot{\hat{x}}(t) = \mathbf{A}\hat{x}(t) + \mathbf{B}u(t) + v(t) \\ \hat{y}(t) = \mathbf{C}\hat{x}(t) + \mathbf{D}u(t) \end{cases} \tag{3}$$

where $z \in \mathbb{R}^m$ is the *dynamic feedback state vector*, $v \in \mathbb{R}^n$ the output of dynamic feedback and

$$r(t) = y(t) - \hat{y}(t) \tag{4}$$

is the residual. $r(t)$ is designed for indicating failures. It also works as a correction term to reduce the effects due to the disturbance $d(t)$.

Comparing to the static observer, the dynamic observer have the similar forward model (3). The main difference is the feedback path: the constant gain matrix K in static observers is replaced with a dynamic system (2). The transfer function matrix (TFM) of dynamic feedback gain (2) is given by

$$H(s) = \mathbf{K}_3(sI - \mathbf{K}_1)^{-1}\mathbf{K}_2 + \mathbf{K}_4 \tag{5}$$

It worthy noting that the TFM of the feedback gain in the static observer is a real coefficient constant K without frequency complex variable s. Therefore, the feedback path of the static observer does not change the frequency characteristics of the correction term $r(t)$.

The overall dynamics of the dynamic observer (2), (3) can be rewritten in an augment form:

$$\begin{cases} \begin{pmatrix} \dot{\hat{x}} \\ \dot{z} \end{pmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{K_4C} & \mathbf{K}_3 \\ -\mathbf{K}_2\mathbf{C} & \mathbf{K}_1 \end{bmatrix} \begin{pmatrix} \hat{x} \\ z \end{pmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} u \\ \qquad + \begin{bmatrix} \mathbf{K}_4 \\ \mathbf{K}_2 \end{bmatrix} (y - \mathbf{D}u) \\ \hat{y} = [\mathbf{C} \quad \mathbf{0}] \begin{pmatrix} \hat{x} \\ z \end{pmatrix} + \mathbf{D}u \end{cases} \tag{6}$$

Defining $e(t) = x(t) - \hat{x}(t)$ and subtracting (3) from the disturbance/fault corrupted system (1) give

$$\begin{cases} \begin{pmatrix} \dot{e}(t) \\ \dot{z}(t) \end{pmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{K_4C} & -\mathbf{K}_3 \\ \mathbf{K}_2\mathbf{C} & \mathbf{K}_1 \end{bmatrix} \begin{pmatrix} e(t) \\ z(t) \end{pmatrix} \\ \qquad + \begin{bmatrix} \mathbf{B}_d - \mathbf{K}_4\mathbf{D}_d \\ \mathbf{K}_2\mathbf{D}_d \end{bmatrix} d(t) + \begin{bmatrix} \mathbf{B}_f - \mathbf{K}_4\mathbf{D}_f \\ \mathbf{K}_2\mathbf{D}_f \end{bmatrix} f(t) \\ r(t) = [\mathbf{C} \quad \mathbf{0}] \begin{pmatrix} e(t) \\ z(t) \end{pmatrix} + \mathbf{D}_d d(t) + \mathbf{D}_f f(t) \end{cases} \tag{7}$$

It can be seen that $r(t)$ in (7) is not zero even if the observer (7) is stable and the state estimation error $e(t)$ is zero. Both $d(t)$ and $f(t)$ contribute to the non-zero residual. The negative effects of $d(t)$ in $r(t)$ degrade the performance of fault detection. Thus it is essential to attenuate the disturbance effects in $r(t)$ and enhance the sensitivity $r(t)$ to $f(t)$.

For simple notification, equation (7) is rewritten as

$$\begin{cases} \dot{\widetilde{x}}(t) = \widetilde{A}\widetilde{x}(t) + \widetilde{B}_d d(t) + \widetilde{B}_f f(t) \\ r(t) = \widetilde{C}\widetilde{x}(t) + \widetilde{D}_d d(t) + \widetilde{D}_f f(t) \end{cases} \tag{8}$$

where

$$\widetilde{x} = [e^T(t)\ z^T(t)]^T, \qquad \widetilde{A} = \begin{bmatrix} \mathbf{A} - \mathbf{K}_4\mathbf{C} & -\mathbf{K}_3 \\ \mathbf{K}_2\mathbf{C} & \mathbf{K}_1 \end{bmatrix},$$

$$\widetilde{B}_d = \begin{bmatrix} \mathbf{B}_d - \mathbf{K}_4\mathbf{D}_d \\ \mathbf{K}_2\mathbf{D}_d \end{bmatrix}, \quad \widetilde{B}_f = \begin{bmatrix} \mathbf{B}_f - \mathbf{K}_4\mathbf{D}_f \\ \mathbf{K}_2\mathbf{D}_f \end{bmatrix},$$

$$\widetilde{C} = [\,\mathbf{C}\ 0\,], \qquad \widetilde{D}_d = \mathbf{D}_d, \quad \widetilde{D}_f = \mathbf{D}_f$$

For a better disturbance attenuation, we now study the zero properties of the dynamic observer first, and propose an approach utilising the zeros assignment methodology.

3. TFMS OF DYNAMIC OBSERVER

It can be seen from (8) that the process dynamics are canceled, and the residual $r(t)$ is affected by the fault as well as the disturbance. s-transforming (8) gives the TFM relating $d(t)$, $f(t)$ to $r(t)$:

$$r(s) = G_f(s)f(s) + G_d(s)d(s) \tag{9}$$

where

$$\begin{cases} G_f(s) = \widetilde{C}(sI - \widetilde{A})^{-1}\widetilde{B}_f + \widetilde{D}_f \\ G_d(s) = \widetilde{C}(sI - \widetilde{A})^{-1}\widetilde{B}_d + \widetilde{D}_d \end{cases} \tag{10}$$

Alternatively, the observer behaviour can be described by the (Rosenbrock) system matrix $P_{dob}(s)$:

$$\left[\begin{array}{cc|cc} sI - A + K_4C & K_3 & B_d - K_4D_d & B_f - K_4D_f \\ -K_2C & sI - K_1 & K_2D_d & K_2D_f \\ \hline -C & 0 & D_d & D_f \end{array}\right]$$

3.1 Poles of Dynamic Observer

From the simplified expression (8) of the dynamic observer, it can be seen that the stability of the dynamic observer is determined by the matrix $\widetilde{A}$. According to linear system theory, the poles of the dynamic observer (7) are the eigenvalues of $\widetilde{A}$. The dynamic observer is stable if and only if all the eigenvalues are on the left half s-plane.

3.2 Zeros of Dynamic Observer

The following theorem gives the relationship between the zeros of the dynamic observer and the zeros of the plant system.

Theorem 1. The disturbance/fault (transmission) zeros of the dynamic observer (7) are the disturbance/fault transmission zeros of the plant system (1) together with the eigenvalues of dynamic gain (2).

Proof: According to the definition of zeros, the disturbance/fault transmission zeros $Z_1$of system (1) are those zeros of TFMs relating the disturbance $d(t)$, fault $f(t)$ to the system output $y(t)$. That is

$$Z_1 = \{s| \text{ rank } P_{ydf}(s) < n + \min(r, d)\} \tag{11}$$

where

$$P_1(s) = \text{rank} \left[\begin{array}{c|cc} sI - A & B_d & B_f \\ \hline -C & D_d & D_f \end{array}\right] \tag{12}$$

with normal rank n+$\min(r, d)$. Note that disturbance(fault) zeros in (11) are different from the system input/output zeros relating u to y.

For the dynamic observer (7), the disturbance/fault transmission zeros are the set of complex numbers s such that $P_{dob}(s)$ losses rank locally.

$$Z_2 = \{s|\text{rank } P_{dob}(s) < n + m + \min(r, d)\} \tag{13}$$

where the size of $P_{dob}(s)$ is $(n+m+r) \times (n+m+d)$ and its normal rank is $min(n + m + r, n + m + d)$.

The rank of $P_{dob}(s)$ (11) is calculated by

$$\begin{aligned}
&\text{rank}P_{dob}(s) \\
&\quad(\text{row}_3 \times K_4 + \text{row}_1 \to \text{row}_1) \\
&= \text{rank} \begin{bmatrix} sI - A & K_3 & B_d & B_f \\ -K_2C & sI - K_1 & K_2D_d & K_2D_f \\ -C & 0 & D_d & D_f \end{bmatrix} \\
&\quad(\text{row}_2 - \text{row}_3 \times K_2 \to \text{row}_2) \\
&= \text{rank} \begin{bmatrix} sI - A & K_3 & B_d & B_f \\ 0 & sI - K_1 & 0 & 0 \\ -C & 0 & D_d & D_f \end{bmatrix} \\
&= \text{rank} \left[\begin{array}{ccc|c} sI - A & B_d & B_f & K_3 \\ -C & D_d & D_f & 0 \\ \hline 0 & 0 & 0 & sI - K_1 \end{array}\right]
\end{aligned} \tag{14}$$

It follows that Z_2 are those values of s for which

$$\text{rank} \begin{bmatrix} sI - A & B_d & B_f \\ -C & D_d & D_f \end{bmatrix} < n + \min(r, d) \tag{15}$$

or

$$\text{rank } [sI - K_1] < m \tag{16}$$

Recalling the definition (11), one can see that (15) coincides with Z_1. Equation (16) shows that the eigenvalues of K_1 compose another subset of the dynamic observer zeros. The result of the theorem follows. **Q.E.D.**

Remark 1: Theorem 1 implies that a mth-order dynamic gain (2) introduces m additional transmission zeros in the observer, located at the poles of the dynamic feedback gain. Theorem 1 can be understood as a generalisation of the well-known SISO dynamic feedback control result that closed-loop zeros are zeros in the forward-path and poles in the feedback-path.

Remark 2: Theorem 1 verifies that the transmission zeros are invariant in static observers and shows the possibility of zero assignment in dynamic observer by introducing additional zeros. Due to the zero invariance, one can not shift the positions of zeros in a static filter. In dynamic filters, however, the extra disturbance zeros introduced by K_1 can be arbitrarily assigned. This is the main implication of Theorem 1.

Remark 3: Compare to the static observer whose number of free parameters is $n \times r$, the dynamic observer has 4 matrices with $(m + n) \times (m + r)$ parameters. For most current dynamic observers, the design concept are nearly the same as static observers, only the poles are considered and the free parameters are determined roughly by optimisation algorithms. The additional zeros introduced by K_1 are ignored. In our algorithm, the freedom of assigning additional zeros are utilised. This technique, on one side, is able to assign zeros close to the disturbance frequency for further attenuation, and, on the other side, to set more constrains on the free parameters with more possibility of avoiding local optimal solution.

4. DESIGN PROCEDURE

The zero assignment solution to dynamic robust fault detection observer (DRFDO) can now be stated as follows:

Given a system (1) corrupted by the band-limited disturbance $d(t)$, if the main frequency contents of residuals $r(t)$ can be estimated at w_r, then, (a) *assigning the eigenvalues of K_1 to $\pm jw_r$;* (b) *assigning the eigenvalues of $\widetilde{A}$ on the left half s-plane and* c *minimising the following performance index*

$$J = \frac{\left|\left|G_d(s))\right|\right|_{s=jw_r}}{\rho + \left|\left|G_f(s)\right|\right|_{s=0}} \tag{17}$$

give the stable and optimal gain matrix K_1, K_2, K_3, K_4 such that $d(t)$ is attenuated and the sensitiveness to $f(t)$ is enhanced at the greatest extent. Here, ρ is a small real number to guarantee the denominator will not be zero.

The detailed design procedure is:

(1) Estimate the disturbance frequency w_r via the spectrum analysis of the residual. The residual can be generated by any stable static observer;
(2) Determine the order m of the dynamic gain (2) according to the estimated disturbance frequencies and assign the eigenvalues of K_1 close to the disturbance frequencies. Note that, the eigenvalues of K_1 must be self-conjugate such that K_1, K_2, K_3, K_4 are real matrices;
(3) Select desired regions of dynamic observer poles.
(4) Select initial values for K_2, K_3, K_4 and find their optimal values through optimisation algorithms.
(5) The fault can be detected during plant operation by using scheme as follows

$$\|r(t)\| > \tau \Rightarrow \text{a fault has happened}$$

where τ is the pre-specified threshold.

Remark 4: The key step in this zero assignment method is to assign the eigenvalues of K_1 to $\pm jw_r$. And the optimisation step aims at (1) stablising the dynamic observer by assigning pole to desired places; (2) optimising the robustness to disturbances and the sensitivity to faults.

5. APPLICATION AND RESULTS

To illustrate the proposed dynamic observer design approach, this section considers robust fault detection of a 2 inputs 2 outputs system:

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -0.943 & 0.1601 \\ 3.9439 & -3.234 \end{bmatrix} x(t) + \begin{bmatrix} 86.794 & 40.312 \\ 154.691 & 81.275 \end{bmatrix} u(t) \\ y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t) \end{cases} \tag{18}$$

Clearly, the open-loop poles of system (18) are $\{\lambda_1 = -0.6940, \lambda_2 = -3.4833\}$. Both of their real parts are negative which assures the plant system is stable.

The disturbance model is assumed as

$$B_d = B_f = B, \quad D_d = D_f = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \tag{19}$$

where the fault matrix $B_f = B$ for actuator faults. Note that, $B_d = B$ too. This simulation is more practicable and closer to industrial applications. Because, in the industrial environment, it is common that the disturbance enters the system by corrupting the input signal. Another advantage is that the estimation of disturbance matrix B_d is avoided.

It is also worthy noting that, in this configuration ($B_d = B_f = B$), the widely used H_∞/H_∞ static observer design may fail without using weighting functions, because $\|G_f(s)\|_\infty$ and $\|G_d(s)\|_\infty$ are identical and the performance index $\|G_f(s)\|_\infty/\|G_d(s)\|_\infty$ is always 1.

The disturbance injected to the system is

$$\mathbf{d}(t) = \begin{pmatrix} d_1(t) \\ d_2(t) \end{pmatrix} = \begin{pmatrix} 0.5\sin(5t) \\ 0.4\cos(5t) \end{pmatrix} \tag{20}$$

In the simulation, the inputs $u(t)$ are unit step signals, and both the amplitudes of $d_1(t), d_2(t)$ are about 50% of the the input signals. Moreover, attenuation of such low frequency disturbances is more difficult than that of high frequency disturbances.

A dynamic observer is now designed for this system.

Step 1. In order to estimate the disturbance frequency, a static observer is first constructed via *K0=place(A',C',[-1, -1])'*. A 4096-point FFT is employed to calculate the spectrum of $r(t)$. Fig.1 provides an illustration of the spectrum. From the zoomed in plot on the left of Fig.1,

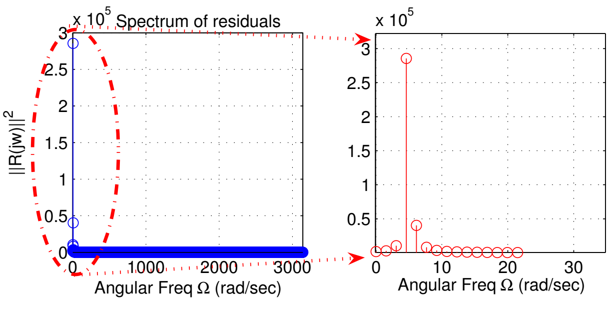

Fig. 1. Spectrum of Residuals (4096 point FFT)

it can be seen clearly that $r(t)$ has one main frequency components corresponding to the disturbance frequency $\omega = 5.0$. This estimated frequency agrees with the true disturbance frequency. Hence, set $w_r = 5$.

Step 2. Since the number of the estimated disturbance frequencies is 1, the number of the dynamic gain (2) is $m = 2$ and the desired zero positions are $\pm 5j$. Let

$$K_1 = \begin{bmatrix} 0 & -5 \\ +5 & 0 \end{bmatrix} \tag{21}$$

to assign the zeros to $\pm 5j$. Then the dynamic observer structure is $K_1 \in \mathbb{R}^{2\times 2}$, $K_2 \in \mathbb{R}^{2\times 2}$, $K_3 \in \mathbb{R}^{2\times 2}$, $K_4 \in \mathbb{R}^{2\times 2}$.

step 3. Set the desired regions of poles are: (a) their real parts are less than -1, and (b) their imaginary parts are close to real axis as much as possible.

step 4. Set the initial values of K_2, K_3 randomly. In this simulation, in order to reduce the observer complexity, the matrix K_4 is not used and set as zero. By using *fmincon* command, the optimal K_2, K_3 are found as

$$K_2 = \begin{bmatrix} 0.1430 & -2.6552 \\ -4.3399 & 2.8362 \end{bmatrix}, \quad K_3 = \begin{bmatrix} -1.1410 & 1.0494 \\ 5.4621 & -0.5900 \end{bmatrix}, K_4 = 0_{2\times 2} \tag{22}$$

which give the resulting dynamic observer

$$\widetilde{A} = \left[\begin{array}{cc|cc} -0.942 & 0.160 & 1.141 & -1.049 \\ 3.944 & -3.235 & -5.462 & 0.590 \\ \hline 0.143 & -2.655 & 0 & -5.000 \\ -4.340 & 2.836 & 5.000 & 0 \end{array}\right]$$

$$\widetilde{B}_d = \left[\begin{array}{cc} 86.794 & 40.312 \\ 154.691 & 81.275 \\ \hline 0 & 0 \\ 0 & 0 \end{array}\right], \widetilde{B}_f = \left[\begin{array}{cc} 86.794 & 40.312 \\ 154.690 & 81.274 \\ \hline 0 & 0 \\ 0 & 0 \end{array}\right] \tag{23}$$

$$\widetilde{C} = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right), \ \widetilde{D}_d = 0, \ \widetilde{D}_f = 0$$

And the four poles of the dynamic observer are:

$$-0.9166 \pm 0.0678\mathbf{j}, \text{ and } -1.1721 \pm 0.0907\mathbf{j} \tag{24}$$

As discussed before, since the disturbance distribution matrix $B_d = B_f = B$, the TFMs relating residual $r(t)$ to disturbance $d(t)$, fault $f(t)$ are the same, as shown in (25).

It is easy to verify that the normal rank of (25) is 2 and it reduces to 1 when $s = \pm 5i$. These results are consistent with Theorem 1.

The bode plot of $G_f(s)$ and $G_d(s)$ are shown in Fig. 2. It can be seen that there are dips around the frequency 5 rad/sec, and they contribute the attenuation of disturbances of 5 rad/sec.

For fair comparison with the conventional static observer, a static observer K_{place} is designed by using *place* function. The resulting static observer gain matrix is given by

$$K_{place} = \begin{pmatrix} -0.0260 & 0.0923 \\ 4.0117 & -2.3182 \end{pmatrix} \tag{26}$$

Because there are only two poles in the static observer and the zeros are of interest, the effects of different poles between dynamic and static observers should be reduced as much as possible. As the dominant zeros in the dynamic observer (23) are $[-0.9166 \pm 0.0678\mathbf{i}]$, they are selected as the desired poles for static observer K_{place}.

5.1 Residuals without fault

In this simulation, no fault happens. The norms of $r(t)_{dyn}$ and $r(t)_{place}$, which are the residuals of our dynamic observer and the static observer K_{place}, respectively, are

$$G_f(s) = G_d(s) = \frac{\begin{bmatrix} 86.7941(s+4.606)(s^2-1.086s+3.12) & 40.3116(s+4.859)(s^2-1.302s+3.041) \\ 154.6907(s-0.5563)(s^2+3.712s+20.47) & 81.2747(s-0.4981)(s^2+3.397s+20.32) \end{bmatrix}}{(s^2+1.833s+0.8447)(s^2+2.344s+1.382)} \quad (25)$$

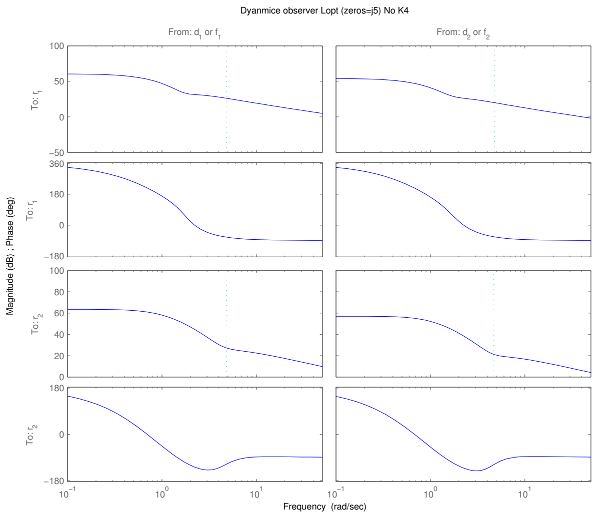

Fig. 2. Bode plots of the fault TFM $G_f(z)$ and the disturbance TFM $G_d(s)$. Note that, $G_d(s)$ and $G_f(s)$ are same, because $B_d = B_f = B$.

shown in Fig. 3. The disturbance attenuation in dynamic observer is more apparent than that of K_{place}. In the time domain, $\|r(t)\|_{dyn}$ has a large overshoot at the beginning due to the transient process of the dynamic observer. In the steady sate, the maximum magnitude of $\|r(t)\|_{dyn}$ is below than 14, however that of $\|r(t)\|_{place}$ is over 17.5.

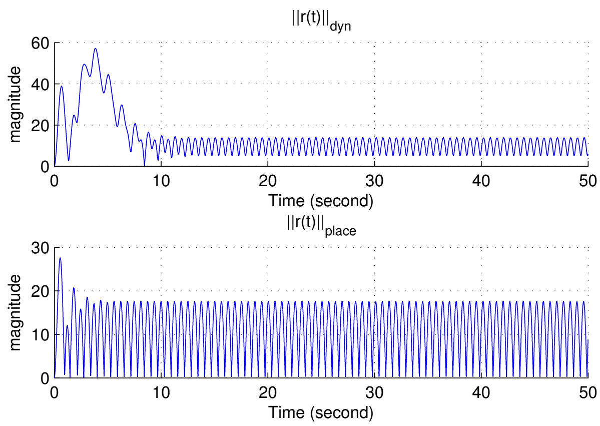

Fig. 3. Fault free residuals corresponding to dynamic observer and static observer K_{place}, respectively.

5.2 Residuals of abrupt faults

Abrupt Actuator Faults In order to simulate the happenings of two successive abrupt actuator faults at two input channels respectively, the fault function can be represented as $f_a(t) = [f_1(t)\ f_2(t)]^T$ and

$$f_1(t) = \begin{cases} 0 & (t < 20) \\ 0.05 & (t \geqslant 20) \end{cases} \quad f_2(t) = \begin{cases} 0 & (t < 30) \\ 0.05 & (t \geqslant 30) \end{cases}$$

Fig. 4 shows $\|r(t)\|_{dyn}$ of the dynamic observer and

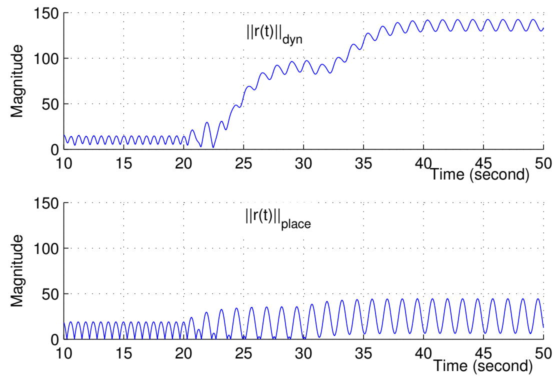

Fig. 4. Residuals of the dynamic observer and K_{place} in the case where two abrupt actuator faults occurs at 20s and 30s, respectively

$\|r(t)\|_{place}$ of K_{place}. From Fig. 4, two step increases in $\|r(t)\|_{dyn}$ can be seen clearly only 3 seconds after each fault occurs.

The static observer, however, fails to detect such abrupt faults. Although the peak values of $\|r(t)\|_{place}$ show step increases, there is no clear interval between the normal residuals and faulty residuals. Therefore, missed alarms may exist.

5.3 Residuals of incipient faults

Single Incipient Actuator Fault In this simulation, the gradual fault injected to the input signal is $f_a(t) = [f_1(t)\ f_2(t)]^T$ and

$$f_1(t) = \begin{cases} 0 & (t < 20) \\ 0.0025(t-20) & (k \geqslant 20) \end{cases} ; \quad f_2(t) = 0 \quad (27)$$

and the corresponding plant outputs and residuals are depicted in Fig. 5. Due to the large output values and the small fault size, the changes in the outputs can not be noticed. However, this fault can be seen clearly from the residuals. $\|r(t)\|_{dyn}$. responses the incipient fault (27) with a straight line which increases at about 4.8 unit per second. Whereas $\|r(t)\|_{place}$ is with significant disturbances and its increase rate is only 0.8 unit per second. This comparison verifies that the dynamic observer is able to detect a smaller gradual fault earlier and more significant.

Single Incipient Sensor Fault This simulation shows the detection of a single incipient sensor fault. The fault function added to the two outputs are:

$$f_1(t) = \begin{cases} 0 & (t < 20) \\ 0.1(t-20) & (k \geqslant 20) \end{cases} ; \quad f_2(t) = 0 \quad (28)$$

respectively. The plant outputs and residuals are shown in Fig.6. Similar to the incipient actuator case, the dynamic

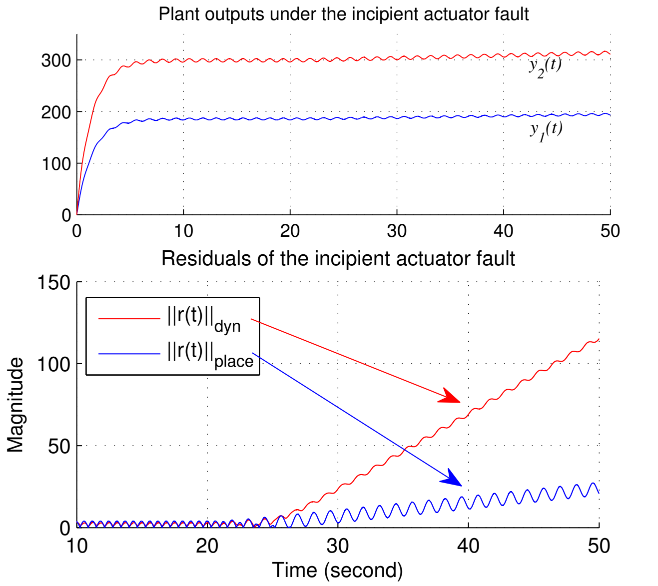

Fig. 5. Plant outputs and residuals under an incipient actuator fault at the first input channel. The actuator fault occurs at 20 second with slope rate 0.0025

observer shows a great improvement on detecting such a incipient sensor fault.

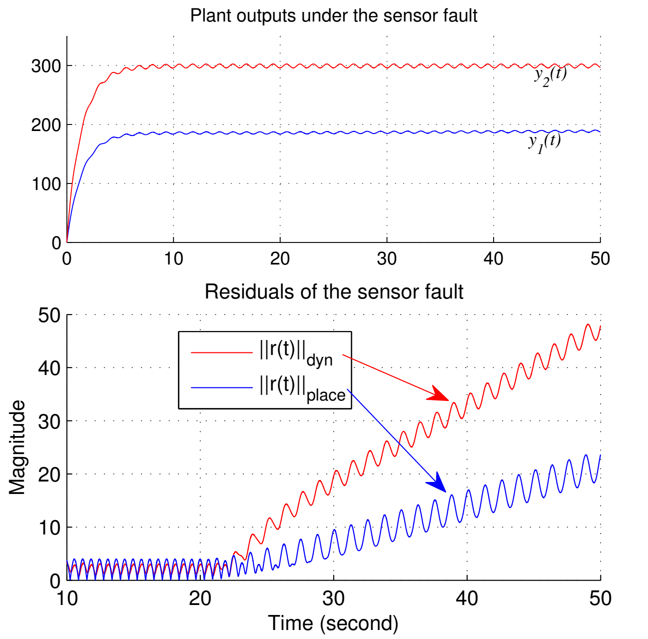

Fig. 6. Plant outputs and residuals in the case of the single incipient actuator fault occurring at 20 sec in the first output channel

It should be note that, in our design procedure, because $D_f = 0$, the output sensor fault are not considered during the optimisation. But the results of the successful detection of incipient sensor fault shows the advantages of zeros assignment. Since the zeros are assigned closer to the disturbance frequency, the disturbances can be attenuated even the observer is not optimised for such a novel fault.

6. CONCLUSION

In this paper, a systematic study of dynamic observers' zeros is presented. The structure and properties of the proposed dynamic observer are analysed and its capacities for fault detection are illustrated by simulations.

The proposed dynamic observer differs from the classical static observer and a dynamic feedback gain is introduced. The additional degree of design freedom in the dynamic observer is used to assign the additional zeros for attenuating disturbances further. In the application, the attenuation of low frequency disturbances is given higher priority, and the zero assignment approach shows its ability to attenuate such disturbances and detect both the actuator and sensor faults. Hence, we are able, on one side, to formulate and get a better observer in the sense of robustness and disturbance rejection, and, on the other side, to obtain new insight into the observer construction for fault detection.

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