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Optimal Fractional Order Proportional Integral Controller for Varying Time-Delay Systems

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Abstract: In many industrial processes, the first order plus time delay (FOPDT) is still being widely used. FOPDT systems are also called "KLT systems (gain, delay, and time constant)." Considering uncertainties in the time delay, this paper attempts to answer this research question: "*Will a fractional order controller help and do better?*" In this paper, we first focus on fractional order proportional and integral controller (FOPI) for varying time-delay systems. Based on our previously proposed FOPI controller tuning rules using fractional M_s constrained integral gain optimization (F-MIGO), we tried to simultaneously maximize the jitter margin and ITAE performance (minimize ITAE performance index) for a set of hundred KLT systems having different time-constants and time-delay values. We observed that the optimization results in enlarged jitter margin of all systems at expense of a slight decrease in ITAE performance of delay dominated systems. Further, the F-MIGO optimization based tuning rules were summarized by approximation of optimized gain parameters and fractional orders α of the FOPI controller. Simulation results are presented to verify the proposed new tuning rules for best jitter margin and ITAE performance.

Keywords: Fractional calculus; fractional order controller, varying time-delay system, FMIGO algorithm, multi-objective Optimization, jitter margin, ITAE performance index.

1. INTRODUCTION

Time-delays are responsible for poor performance, controller complexity and even instability of system in many chemical, biological, mechanical and transportation processes. Extensive simulation results on how the jitter in the loop can degrade system performance and lead to instability of system can be found in thesis works of, e.g., [Marti, 2002, Cervin, 2000]. Ensuring the stability of systems with varying time-delays has always been an interesting area of research for control engineers [Wu et al., 2003, Phat and Niamsup, 2006, Kao and Rantzer, 2007]. This paper introduces a new jitter-robust controller design by optimizing the gain parameters of Fractional Order Proportional Integral (FOPI) controller based on Fractional M_s constrained Integral Gain Optimization (F-MIGO) algorithm [Koštial et al., 2007, Bhaskaran et al., 2007a, Eriksson and Johansson, 2007a,b]. This controller design is helpful in finding the maximum value of jitter (variance in time-delay) at which the system remains stable. The Integral of Time weighted Absolute Error (ITAE) performance of the proposed controller is better than the best integer order PID controller.

The reason we focused on PI/D (proportional integral derivative) controllers is that they are the most popular controllers used in industry due to their simplicity, performance robustness and availability of many effective yet simple tuning methods based on minimum plant model knowledge [Zeigler and Nichols, 1942]. A survey has shown that 90% of control loops are of PI or PID structure [Koivo and Tanttu, 1991, Yamamoto and Hasimoto, 1991]. As for the reason of considering fractional order controllers, we remark that, dynamic systems characterized using fractional order differential equations are based on fractional calculus, or calculus of non-integer order. The past decade has seen an increase in research efforts related to fractional calculus [Debnath, 2004, Magin, 2004] and its applications to control theory [Vinagre and Chen, 2002, Xue and Chen, 2002, Chen, 2006, Xue et al., 2006]. Hence, our objective is to apply the Fractional-Order Control (FOC) to enhance the (integer order) dynamic system control performance [Vinagre and Chen, 2002, Xue et al., 2006]. Pioneering works in applying fractional calculus in dynamic systems and controls and the recent developments can be found in [Manabe, 1960, 1961, Oustaloup, 1981, Axtell and Bise, 1990, Vinagre and Chen, 2002, Editor), 2002, Ortigueira and Editors), 2003].

In this paper, a test batch of hundred FOPDT systems is considered and FOPI tuning rules are used to compute the proportional gain, integral gain and non-integer order α of the integrator for each system. The gain and α values so obtained for each system are then used to compute the ITAE performance and jitter margin. Finally multi-objective optimization algorithm is applied to optimize these gains and α values for each system. Jitter margin and ITAE value are calculated at these optimum values as in [Eriksson and Johansson, 2007a,b].

The main contribution of this paper lies in answering this research question: "*Will a fractional order controller help and do better?*" when there are uncertainties in the time delay due to e.g., jitter in the loop.

This paper is organized as follows: Section 2 provides an introduction to FOPDT model, FOPI controller & FMIGO tuning rules and briefly defines the jitter margin, ITAE performance index and the multi-objective optimization method. Section 3 focuses on optimal tuning of FOPI controller followed by Sec. 4 which aims at approximation of optimized gain parameters K_p, K_i and α to get new set of optimal FOPI tuning rules that ensures the best jitter margin and ITAE performance. Finally, Sec. 5 concludes this paper with remarks on future research work.

2. BASIC CONCEPT AND TERMINOLOGIES

This work is based on design of optimum FOPI controller to control a class of systems which can be approximated by FOPDT model, also called KLT model.

2.1 FOPDT Model

A FOPDT system can be represented mathematically as in (1):

$$G(s) = K\frac{e^{-Ls}}{Ts+1}, \tag{1}$$

where K is static gain or steady-state gain of the system, L is the time-delay and T is the time-constant of the system. These three model parameters can be obtained by drawing the S-shaped open-loop step response or reaction curve of the system as shown in Fig. 1. In the open-loop step response curve, K is the ratio of the final open-loop output step response value to the initial input value of open-loop step response of the system; L is the time at which the tangent to the maximum slope intersects the time axis and T is the time at which the tangent to the maximum slope of the system intersects the final response of the system. Another important characteristic of FOPDT system is its relative time-delay, τ, represented by (2).

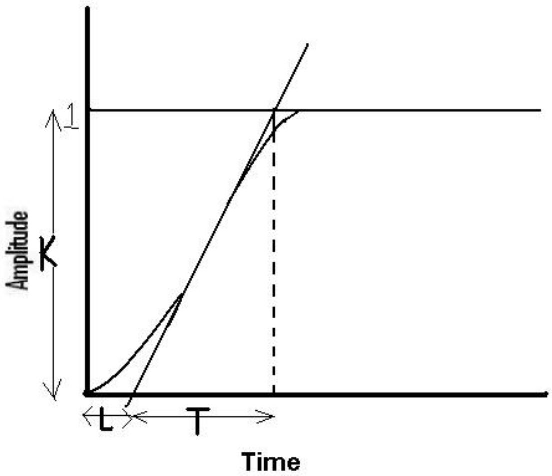

Fig. 1. Determining system parameters of FOPDT model from step response

$$\tau = \frac{L}{L+T} \tag{2}$$

Systems with $\tau > 0.6$ are called delay-dominated and $\tau < 0.1$ are called lag-dominated. Making generalizations, any system plant with $T > L$ is lag-dominated plant and with $T < L$ is delay-dominated plant [Eriksson and Johansson, 2007b].

2.2 FOPI Controller and F-MIGO Tuning Rules

As in [Koštial et al., 2007], in time-domain, if $u(t)$ is the control input, $r(t)$ is the set-point signal and $y(t)$ is the output, the fractional PI^α controller is represented by (3) as:

$$u(t) = K_p(r(t) - y(t)) + K_i D_t^{-\alpha}(r(t) - y(t)), \tag{3}$$

where $D_t^\alpha x$ is the fractional differointegral operator. We adopt the following definition for the fractional derivative of order α of function $f(t)$ [Oldham and Spanier, 1974],

$$\frac{\mathrm{d}^\alpha}{\mathrm{d}t^\alpha} f(t) = \begin{cases} f^{(n)}(t) & \text{if } \alpha = n \in N, \\ \dfrac{t^{n-\alpha-1}}{\Gamma(n-\alpha)} * f^{(n)}(t) & \text{if } n-1 < \alpha < n, \end{cases} \tag{4}$$

where the $*$ denotes the time convolution between two functions.

In frequency-domain, the FOPI controller $C(s)$ is simply written as

$$C(s) = K_p + \frac{K_i}{s^\alpha} \tag{5}$$

where K_p and K_i are the proportional and integral gain parameters of the fractional controller and α is the non-integer order of the integrator. Note that the delay in the system is after the plant $G(s)$. How to tune the gains K_p, K_i and the non-integer order α has been studied in [Bhaskaran et al., 2007b] and experimentally validated in [Bhaskaran et al., 2007a]. The tuning rules developed in [Bhaskaran et al., 2007b] are summarized as:

$$K_p = \frac{0.2978}{K(\tau + 0.000307),} \tag{6}$$

$$K_i = \frac{K_p(\tau^2 - 3.402\tau + 2.405)}{0.8578T},$$

$$\alpha = \begin{cases} 0.7, & \text{if } \tau < 0.1 \\ 0.9, & \text{if } 0.1 \le \tau < 0.4 \\ 1.0, & \text{if } 0.4 \le \tau < 0.6 \\ 1.1, & \text{if } \tau \ge 0.6 \end{cases}.$$

based on fractional M_s constrained integral gain optimization method (F-MIGO) and the detailed information can be found in [Bhaskaran et al., 2007b].

2.3 Multi-objective Optimization Problem

A multi-objective optimization method is used which simultaneously minimizes n objective functions $O(x)$ which are functions of decision variables x bounded by some nonlinear equality and inequality constraints. This is represented mathematically as:

$$\min_x O(x) \tag{7}$$

subject to the following equality and inequality constraints

$$\sigma = \begin{cases} c_i(x) \le 0, & i = 1, \cdots, n_1, \\ ceq_i(x) = 0, & i = 1, \cdots, n_2, \end{cases} \tag{8}$$

where $O(x) = [O_1, O_2, \cdots, O_n]^T$ and $x = [x_1, x_2, \cdots, x_k]^T$.

2.4 Optimization Criteria

This work is based on optimization of two important controller performance indices, namely, jitter margin and ITAE which are briefly described in this subsection.

Jitter Margin Let $G(s)$ be an FOPDT plant system as shown in (1) and $C(s)$ be the FO-PI controller given by (5). Let $\Delta(t)$ be the time-varying delay of the system as shown in Fig. 2. closed-loop system can have while still maintaining its stability and performance. Furthermore, the condition of stability for continuous-time varying delay systems can be verified by (9). This paper takes into account the form of equation given in [Eriksson and Johansson, 2007a,b] for finding the stability condition for SISO continuous systems, though information provided in [Marti, 2002] is also quite useful in regard to jitter margin.

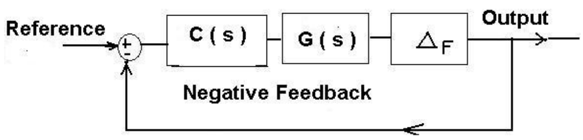

Fig. 2. Block diagram of closed-loop system with delay

$$\left|\frac{G(j\omega)C(j\omega)}{1+G(j\omega)C(j\omega)}\right| < \frac{1}{\delta_{max}\omega}. \tag{9}$$

stability, consider the transformed system shown in Fig. 3 which is equivalent to system represented by Fig. 2 [Kao and Lincoln, 2004], where signals $m(t)$ and $n(t)$ are marked between two dashed blocks Δ_F and G_{nm}. Following [Kao and Lincoln, 2004], let us denote the operator Δ as

$$\Delta m(t) = m(t - \delta(t)) \text{ s.t. } 0 \le \delta(t) \le \delta_{max} \tag{10}$$

and obviously, via the operator Δ_F of the left dashed box in Fig. 2,

$$n(t) = \Delta_F m(t) = (\Delta - 1)\frac{1}{s}. \tag{11}$$

Then, $y(t)$, the output signal of the plant $G(s)$, can be expressed as

$$y(t) = \int_0^t m(\nu)\mathrm{d}\nu. \tag{12}$$

Therefore, Δ_F can be expressed as

$$n(t) = \Delta_F m(t) = y(t - \delta(t)) - y(t) = \int_{t-\delta(t)}^t m(\nu)\mathrm{d}\nu. \tag{13}$$

Thus,

$$\Delta_F m(t)^2 \le \delta(t)\int_0^t m(\nu)^2 \mathrm{d}\nu \le \delta_{max}\int_0^t m(\nu)^2\mathrm{d}\nu. \tag{14}$$

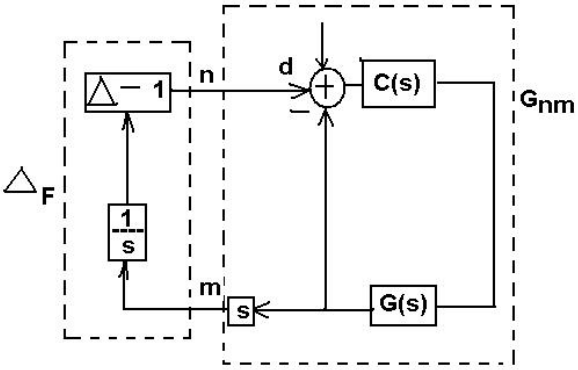

Fig. 3. Block diagram of the equivalently transformed system

Now, the L_2 norm of $n(t)$ is bounded as

$$\parallel \Delta_F \parallel_{L_2}^2 \leq \int_0^{\infty} \delta_{max}\Big(\int_{t-\delta(t)}^{t} m(\nu)\mathrm{d}\nu\Big)\mathrm{d}t \tag{15}$$

$$= \delta_{max}\int_0^{\infty}\int_{-\delta_{max}}^{0} m(t+s)^2\mathrm{d}s\mathrm{d}t$$

$$= \delta_{max}\int_{-\delta_{max}}^{0}\int_0^{\infty} m(t)^2\mathrm{d}t\mathrm{d}s$$

$$= \delta_{max} \parallel m(t) \parallel_{L_2}^2 \int_{-\delta_{max}}^{0}\mathrm{d}s = \delta_{max}^2 \parallel m(t) \parallel_{L_2}^2 .$$

Thus, the L_2 gain of operator Δ_F is bounded by δ_{max}. The stability criterion in (9) is from the small gain theorem applied to transformed system block diagram. Thus, one can conclude that the transformed system is stable if L_2 induced norm of linear part of system from n to m is bounded by $1/\delta_{max}$, i.e,

$$\parallel G_{nm} \parallel_{L_2} = \sup_{\omega\epsilon[0,\infty]} |\frac{G(j\omega)C(j\omega)}{1+G(j\omega)C(j\omega)}| < \frac{1}{\delta_{max}\omega}. \tag{16}$$

This paper deploys above stability condition for computing jitter margin of KLT systems.

ITAE Criterion ITAE stands for integral of time weighted absolute error, that is,

$$ITAE = \int_0^{\infty} t|e(t)|\mathrm{d}t. \tag{17}$$

Optimum ITAE is used as a deciding factor in design and tuning of controllers by many researchers, e.g., [Caceres et al., 2000, Shrivastava, 1992]. implies better performance of the system.

3. TEST BENCH SIMULATION AND OPTIMIZATION

The objective of this study is to design an optimum FOPI controller such that the jitter margin and system performance are maximized and yet the closed-loop feedback system is stable. For our numerical simulation and optimization studies, a set of 100 FOPDT systems are used with 10 delay values $L = [1, 2, \cdots, 10]^T$ and 10 time-constant values $T = [1, 2, \cdots, 10]^T$ and $K = 1$. These values are substituted in equation of first order plus time delay systems as in (1) to get 100 different systems. Further two objective functions are targeted as:

$$O_1(x) = ITAE = \int_0^{\infty} t|e(t)|\mathrm{d}t$$

and

$$O_2(x) = \frac{1}{\delta_{max}}.$$

Note that, here δ_{max} can be computed from (9) as

$$\delta_{max} = \min_{\omega} \epsilon[0,\infty]|\frac{1+G(j\omega)C(j\omega)}{j\omega G(j\omega)C(j\omega)}|. \tag{18}$$

Hence the multi-objective optimization problem takes the following form:

$$\min_{x=[K_p,K_i,\alpha]} O(x) \tag{19}$$

where $O(x) = [O_1, O_2]$ subject to the following equality and inequality constraints

$$\sigma: \begin{cases} |C_0 + G(j\omega)C(j\omega)|^2 \geq R_0^2, & i = 1, \cdots, n_1 \\ \dfrac{\partial|C_0 + G(j\omega)C(j\omega)|^2}{\partial\omega} = 0, & i = 1, \cdots, n_2 \end{cases} \tag{20}$$

where the objective function $O_1(x)$ is the ITAE criterion and $O_2(x)$ is the inverse of jitter margin. These values should be minimized while still ensuring robust stability of the system. The set of constraint equations defined by σ ensures the robustness and stability. The inequality constraint $|C_0+G(j\omega)C(j\omega)|^2$ is the sensitivity constraint which is also a function of K_p, K_i, α and ω and must be greater than R_0^2. Here, C_0 and R_0 are the center and radius of the circle which encloses both the M_s and M_p circles described by

$$C_0 = \frac{M_s - M_sM_p - 2M_sM_p^2 + M_p^2 - 1}{2M_s(M_p^2 - 1)}, \tag{21}$$

$$R_0 = \frac{M_s + M_p - 1}{2M_s(M_p^2 - 1)}, \tag{22}$$

where M_s and M_p are the maximum absolute values of sensitivity and complementary sensitivity functions, respectively, Furthermore, as remarked in [Bhaskaran et al., 2007a, Eriksson and Johansson, 2007a,b], $|1 + G(j\omega)C(j\omega)|^2 = 0$ is the stability region of the sensitivity constraint and satisfies the boundary condition at critical point or the point at which $C_0 = 1$ and $R_0 = 0$. In our implementation, a MATLAB command `fgoalattain` is used to get optimized values of x by multi-objective goal attainment. `fgoalattain` command finds the minimum of a multi-objective optimization problem by minimizing γ such that $O(x) - W\gamma \leq O_{goal}$, where x are the optimized gain parameter values, W is the weight (generally equal to absolute value of goal function) and O_{goal} is the target values of the objective functions. In our case, goal and weight values are given by

$$\mathbf{O_{goal}} = \begin{bmatrix} J_{FMIGO} & \dfrac{1}{T+L} \end{bmatrix} \tag{23}$$

$$\mathbf{W} = \begin{bmatrix} J_{FMIGO} & \dfrac{2T}{T+L} \end{bmatrix}$$

where J_{FMIGO} is the ITAE cost criterion value if FMIGO tuning rules are used and since $(T + L)$ gives relatively large jitter margin for both delay dominated and lag dominated systems, it is chosen as goal for jitter margin. For systems with $T > L$, setting goal equal to weight results in poor performance because of trade-off between the objectives. To avoid this situation, weight is different from goal. Optimization is run for many iterations until minimum values of Ox are obtained. The final value of x which is the result of `fgoalattain` command is the optimized gain parameters and α value. At these values, the system has maximum jitter margin and good performance ensuring robustness and stability.

Furthermore, extensive simulations were performed to investigate the behavior of fractional order proportional integral controller after optimization. A plot of jitter margin as a function of τ, before and after optimization, of FOPI controller is shown in Fig. 4. The jitter margin of KLT systems after optimization is comparatively larger (up to two fold for $\tau \leq 0.8$) than the jitter margin of systems prior to optimization. Similar increase in jitter margin for delay dominated systems ($\tau > 0.6$) is accompanied with very slight decrease in performance. This is in contrast to lag-dominated systems that show an increase in the jitter margin without adversely affecting the system performance, as shown in Fig. 5. Nonetheless, the jitter margin is improved in all the cases studied above. Thus, it could be inferred that the optimal F-MIGO tuning is a better option over simple FMIGO tuning in increasing the jitter margin of closed-loop systems controlled by fractional order controllers.

4. OPTIMAL FOPI TUNING RULES & VERIFICATION

This section describes the methods used for derivation of optimal FO-PI tuning rules. The optimized gain parameters and α obtained as result of `fgoalattain` command in previous section were plotted in MATLAB and analyzed carefully to find any hidden pattern or there dependence on the delays L and the time-constants T of the systems. For example, it was found that the optimal proportional gain parameter K_p^o increases with increasing T and decreases with increasing L.

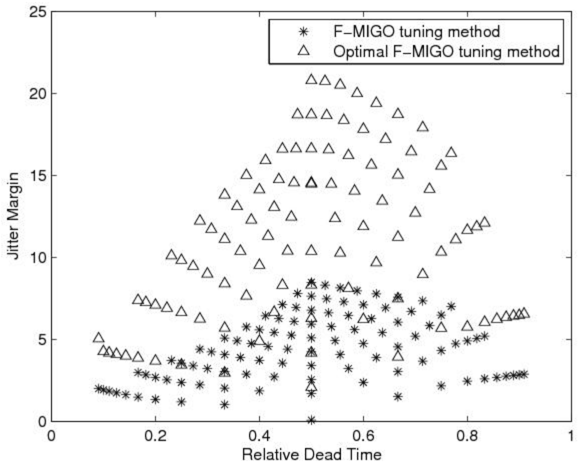

Fig. 4. Improvement of jitter margin of KLT systems after optimization of FOPI controller

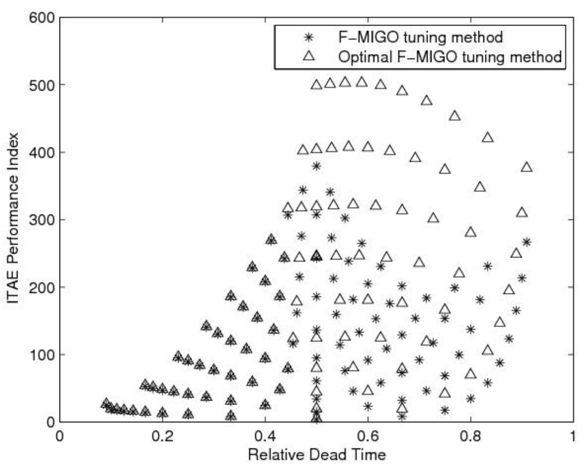

Fig. 5. Performance of KLT systems before and after optimization of FOPI controller

Whereas optimal integral gain parameter K_i^o shows a decrease with increasing values of T and L, though this decrease is more profound for small values of L and becomes almost constant for large values of L. Furthermore, it should be noted here that the integral gain parameters so obtained from multi-objective optimization method ensure the stability of the systems, but do not result in true jitter margin when tested in Simulink. Thus, to tighten the constraint, the optimized K_i values were increased by some integer factor which was determined by simulation results. The optimal fractional order α^o of the integrator was a function of the relative dead time τ and delay L of the system. These optimal tuning rules are expressed mathematically as:

$$K_p^o = \frac{0.2T}{L} + 0.16, \tag{24}$$

$$K_i^o = \frac{0.25}{TL} + \frac{0.19833}{L} + 0.09, \tag{25}$$

$$\alpha^o = \tau - 0.04L + 1.2399. \tag{26}$$

To verify the tuning rules obtained above, three different types of systems are considered. These are a lag dominated system with $\tau = 0.2$ $(K = 1, L = 2, T = 8)$, an intermediate delayed system with $\tau = 0.5$ $(K = 1, L = 8, T = 8)$ and a delay dominated system with $\tau = 0.8$ $(K = 1, L = 8, T = 2)$. The optimal gain parameters K_p^o, K_i^o and α^o are computed using (24), (25) and (26), respectively. These are then used to compute the jitter margins using stability criteria in (9). The step response of the systems are plotted at various input delays as shown in Figs. 6, 7 and 8.

It can be observed that for all the three cases considered, systems are stable at jitter margin (shown by `JM` in the figures) and become unstable if the jitter margin is increased by just 20 per cent. Several other systems were simulated to see the validity of the tuning rules and it was found that tuning rules are quite accurate.

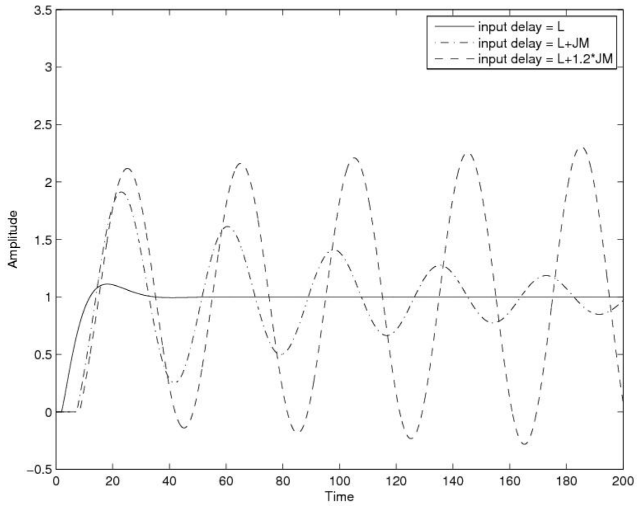

Fig. 6. Step response of lag dominated system $(K = 1, L = 2, T = 8)$ at different delays

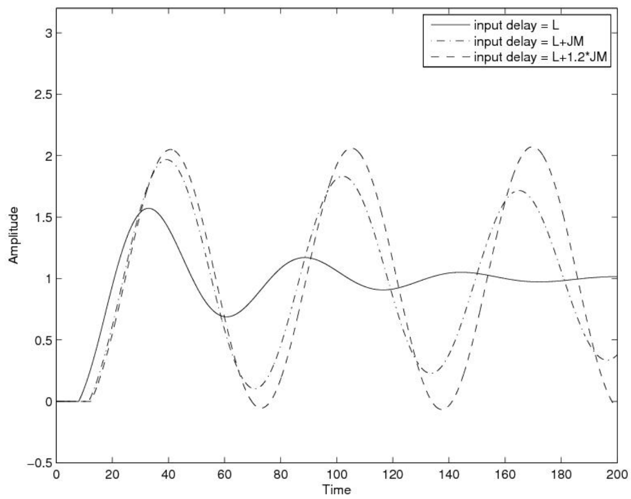

Fig. 7. Step response of an intermediate system $(K = 1, L = 8, T = 8)$ at different delays

5. COMPARISON BETWEEN OFOPI & OPID CONTROLLERS

In addition to designing of an optimal FOPI (OFOPI) controller and developing optimal FOPI tuning rules, we also compare the OFOPI controller and the optimal PID controller (OPID) studied in [Eriksson and Johansson, 2007a,b]. Briefly summarizing, the OPID controller is represented in time-domain as

$$u(t) = k(py_r(t) - y_f(t)) + k_i \int_0^t (y_r(\tau) - y_f(\tau))\mathrm{d}\tau$$
$$+k_d(q\frac{\mathrm{d}y_r(t)}{\mathrm{d}t} - \frac{\mathrm{d}y_f(t)}{\mathrm{d}t}), \tag{27}$$

where k, k_i and k_d are the gain parameters of the controller given by AMIGO tuning rules, p and q are the set-point weights and y_f is the filtered process variable. The output is considered to pass through a low pass filter having a transfer function $G_f(s)$ given as

$$G_f(s) = \frac{1}{(T_f s + 1)^2}. \tag{28}$$

The other controller parameters are defined mathematically as

$$p = \begin{cases} 0, & \text{if } \tau \le 0.5, \\ 1, & \text{if } \tau > 0.5; \end{cases} \tag{29}$$

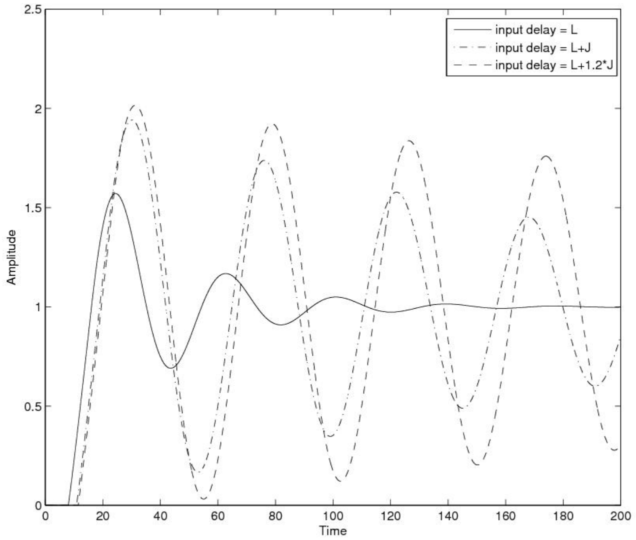

Fig. 8. Step response of delay dominated system ($K = 1, L = 8, T = 2$) at different delays

$$q = 0;$$

$$T_f = \begin{cases} \dfrac{0.05}{\omega_{gc}}, & \text{if } \tau \leq 0.2, \\ 0.1L, & \text{if } \tau > 0.2; \end{cases}$$

where ω_{gc} is the cut off frequency of the filter. Further, the tuning rules proposed in [Eriksson and Johansson, 2007a,b] on PID controller were

$$k = \frac{0.4T - 0.04}{K_p L} + \frac{0.16}{K_p} \tag{30}$$

$$k_i = 0.01\Big(\frac{-0.11T^3 + 1.5T^2 - 1.5}{K_p L^2} + \frac{0.35T^2 + 4T + 50}{K_p L}\Big) \tag{31}$$

$$k_d = 0.01\Big(\frac{0.4T^2 + 11T}{K_p}\Big). \tag{32}$$

It should be noted that the OPID controller proposed in [Eriksson and Johansson, 2007a,b] uses a low pass filter which enhances the performance of the controlled system whereas the OFOPI controller designed in this paper uses no filter for process output. Jitter margins and ITAE indices were calculated for the test batch of hundred KLT systems by using these optimal AMIGO tuning rules (OPID controller) and optimal F-MIGO tuning rules (OFOPI controller). These are shown in Fig. 9 and Fig. 10, respectively.

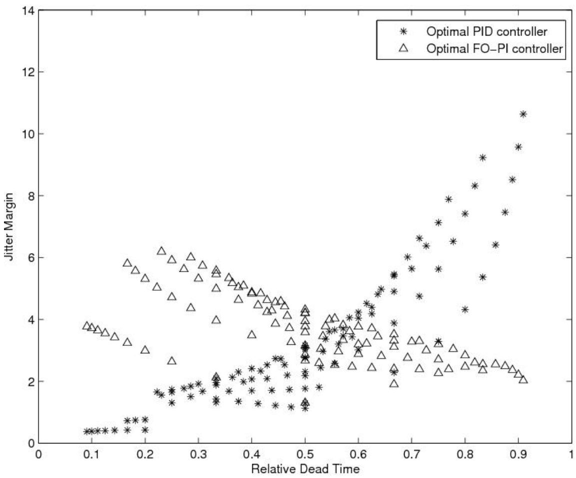

Fig. 9. Jitter margin of KLT systems controlled by OPID and OFOPI controllers

Thorough investigation of these figures reveals that OFOPI is a better controller than OPID for systems with $\tau < 0.5$. These systems have larger jitter margin and lower ITAE values than that obtained by OPID controller. This is in contrast to OPID controller which have a better performance for systems with $\tau > 0.5$.

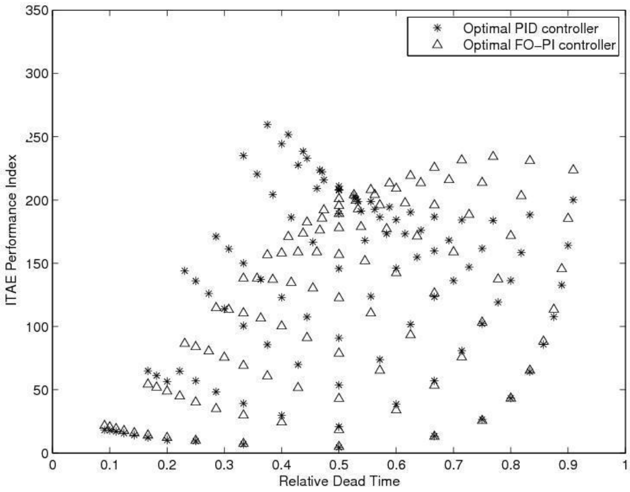

Fig. 10. Performance of KLT systems controlled by OPID and OFOPI controllers

6. CONCLUSION & FUTURE WORKS

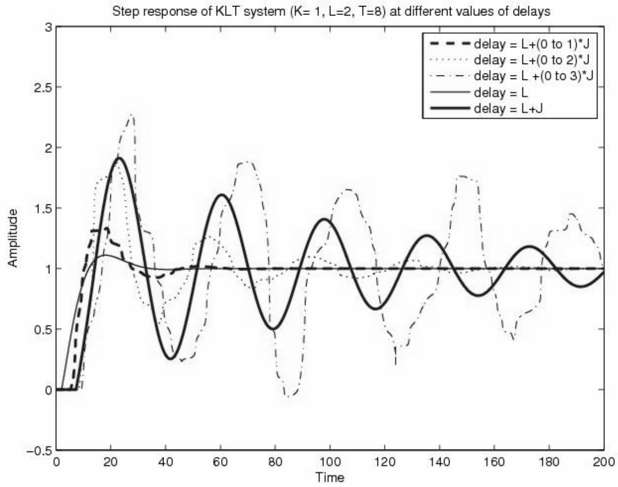

Fig. 11. Step response of lag dominated system ($K = 1, L = 2, T = 8$) at different delays

This paper provided a detailed explanation of design of a robust-jitter controller called optimum fractional proportional integral controller. The efficiency of controller in providing higher jitter margin when compared to simple FOPI controller and PID controller (for $\tau < 0.5$) was proved by simulating 100 KLT systems and making a comparison. Finally, tuning rules were given to determine the gain parameters and α of OFOPI controller. This kind of controller could prove to be a better option than OPID controller for systems with small value of τ and when large jitter margin and better controller performance are desirable.

Present work considers a special case when $\delta(t) = \delta_{max}$, for all values of t. In other words, $\parallel \Delta_F \parallel_{L_2} = \delta_{max}$. For such a case, the tuning rules give the gain parameters of the OFOPI controller at which the jitter margin is maximized for the system. In other cases when $\delta(t) \neq \delta_{max}$, these tuning rules no longer hold. This is shown in Fig. 11, Fig. 12 and Fig. 13 by simulating three different systems when $\delta(t) \neq \delta_{max}$ and $\delta(t)$ is uniformly distributed in a a given range.

Future research work will include design of OFOPI tuning rules for systems when $\delta(t) \neq \delta_{max}$. We also plan to investigate several other sets of 100 KLT test batches for validation purpose and engineering

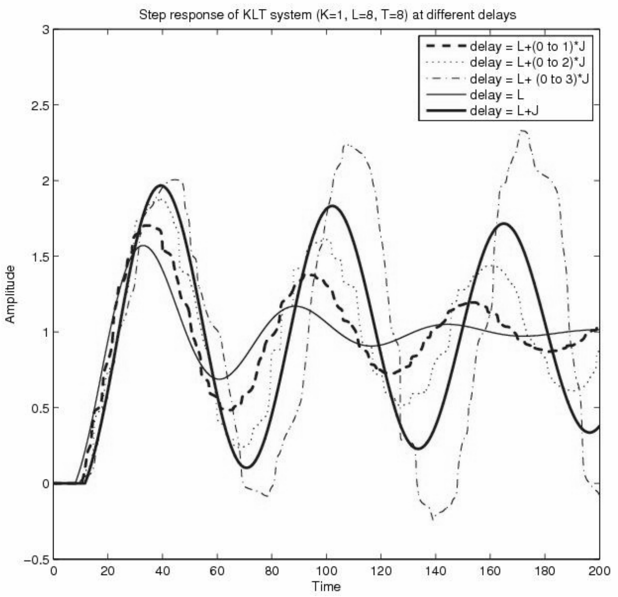

Fig. 12. Step response of an intermediate system ($K = 1, L = 8, T = 8$) at different delays

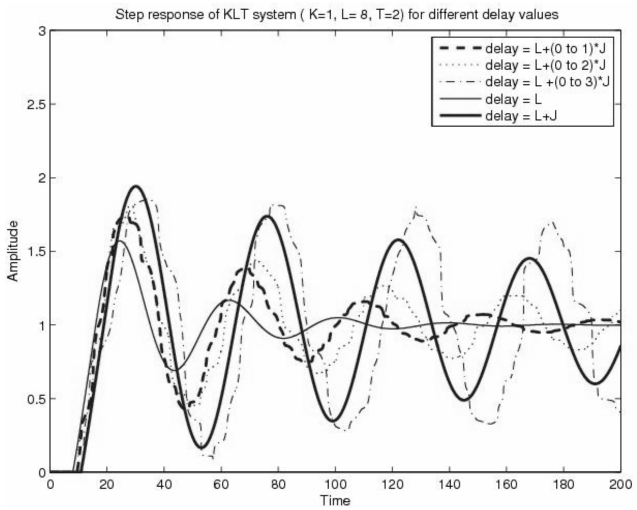

Fig. 13. Step response of delay dominated system ($K = 1, L = 8, T = 2$) at different delays

an embedded and telepresence control of a three-axis T2 Stand-alone Smart wheel control at CSOIS using the OFOPI controller/tuning rules.

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