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# Cooperative Surveillance of a Moving Target using a Formation Framework

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**Abstract:** In this paper we address the formation control problem of generating a formation for a group of nonholonomic mobile agents. The formation control scheme which is proposed in this paper is based on a fusion of leader-follower and the virtual referenced approaches. This scheme gives a formation error representation that is independent of the number of agents in the formation and the resulting control algorithm is scalable. The proposed controller is based on feedback linearization, and formation errors are guaranteed to be globally asymptotically stable. As a possible application, the proposed algorithm is implemented on the cooperative ground moving target surveillance problem. The controller design algorithm is verified through computer simulations.

## 1. INTRODUCTION

There are several threads of study in cooperative control problems; 1.task allocation also known as the assignment problem (Shehory et al. (1998); Kinston et al. (2005)), 2.formation control problem, and 3.path planning problem (Betts (1998); Milam et al. (2000); Faiz et al. (2001)). Moreover, the formation control problems can be further categorized as formation generating, formation keeping and maneuvering problems. Some applications of cooperative control include the following; exploration and mapping(Fox et al. (2000)), search and rescue(Kitano et al. (1999)), automated highways(Bender (1991)), surveillance(Collins et al. (2001)), formation flying(Giulietti et al. (2000)), box pushing(Lewis et al. (1997); Mataric et al. (1995)), scouting(Balch et al. (1998); Cook et al. (1996)), and radar deception(Maithripala et al. (2007)).

In this paper we are interested in the formation generating problem. The formation generating problem is similar to formation keeping except, in that the formation error is stabilized globally instead of locally. The key issue in formation generating is the design of a control law for each agent so that all agents fall within a preassigned formation. Designing such a control law for each agent requires the reference position of each of the agents, and the reference position is typically specified roughly by three methods in the literature: leader-follower, virtual reference, and neighbor reference. The leader-follower approach is used for the formation generating problem in Das et al. (2002); Desai et al. (2001); Wang (1991). A single agent can be designated as a leader (Wang (1991)), or there can exist multiple leaders (Das et al. (2002); Desai et al. (2001)). The desired position of the follower in the preassigned formation is simply decided by a geometric relation between the leader and its follower. A drawback of the leader-follower approach is that the group behavior depends highly on the physical constraints of the leader. Also, the leader-follower approach to formation control is not suitable when the group behavior has to undergo rotational motion.

The weakness of the leader following approach at times can also be an advantage. Namely, the group behavior can be directed by just the behavior of a leader. The virtual reference approach involves what we term as the virtual structure(VS) concept. The virtual reference point is computed by averaging the positions of all agents in Balch et al. (1998), or the virtual reference frame can be fixed at the virtual center of the formation, or at the center of the virtual structure(VS) as in Ren et al. (2004). The main strength of the virtual reference approach is that the guidance of group behavior is easier than the other approaches since all agents in the formation are treated as a single object. However, the physical constraints of each of the agents do not appear explicitly in the group behavior. Therefore, it is a challenging problem to meet control bounds of each agent with the virtual reference approach. In the neighbor reference approach, each agent makes an effort to decrease the formation error (Yamaguchi et al. (2001)). Since there is no explicit reference point or frame in the latter, it is challenging to stabilize the formation. However, this approach is suitable for decentralized, autonomous control. This approach does not require a global reference point, and each agent can make a preassigned formation without central communication.

The leader-follower reference approach itself does not introduce a restriction on the group behavior except rotational motion. However, if each agent has nonholonomic constraints, the group behavior is restricted in order to maintain a formation. However, the virtual reference approach introduces non-scalability in computation. The computational time increases exponentially with the number of agents, and adding just one agent to the formation results in a recalculation of the formation error and the control law. In this paper we approach the formation forming problem using a mixture of leader-follower and virtual

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reference methods in order to overcome the drawbacks of these approaches. First, we discuss scalable formation constraints and controller design based on feedback linearization in section $II$. As a preliminary step, collision avoidance among agents is not considered. Next, cooperative surveillance of a moving target problem is considered as an application for the proposed approach in section $III$.

## 2. APPROACH TO GENERATING FORMATION

A formation of multi-agents is defined as follows.

*Definition 2.1.* (Formation). A formation of multi-agents comprises $N$ number of mobile agents and $M$ number of virtual leaders where $N \geq 2$ and $M \geq 1$, and where each of the $N$ agents maintains a specific geometric distance from the other agents in the formation.

All the $N$ agents in the formation are assumed restricted to the two dimensional plane and may be considered a single rigid body. The virtual leaders are assumed to be in the same plane. Also, $N$ mobile agents are assumed given to build a certain formation. This formation accomplished through a control strategy can be viewed as a Virtual Structure(VS). A local frame $B$ is assumed fixed to the VS with its origin $O_B$ located at the first virtual leader. $\mathbf{b}_i$ is the position vector of the $i^{th}$ agent with respect to $O_B$. $\mathbf{r}_i$ is the position vectors of $i^{th}$ agent with respect to an inertial frame $I$. Let $[\#]_B$ denote the coordinates of a vector in the local body frame $B$ and $[\#]_I$ denote the coordinates of a vector in an inertial frame $I$.

Suppose that each agent motion is governed by the following state equations;

$$\begin{aligned} \dot{\mathbf{z}}_i &= f_i(\mathbf{z}_i) + g_i(\mathbf{z}_i)\mathbf{u}_i \\ \mathbf{r}_i &= h(\mathbf{z}_i) \end{aligned} \tag{1}$$

where $\mathbf{z}_i = [x_i, y_i, \theta_i, v_i]^T$ is the state of the system, $\mathbf{u}_i = [\omega_i, u_i]^T$ is the control, $\mathbf{r}_i = [x_i, y_i]^T$ are geometric variables used for defining the formation in $\mathcal{R}^2$,and $f_i = [v_i cos\theta_i, v_i sin\theta_i, 0, 0]^T$, $g_i = \begin{bmatrix} 0,0,1,0 \\ 0,0,0,1 \end{bmatrix}^T$. Only the rotational velocity and translational acceleration are controlled in this system.

### 2.1 Formation constraints and formation errors

*Definition 2.2.* (Function $\angle$).

$$\angle\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{cases} \arctan(y/x), & \text{for } x > 0 \\ \arctan(y/x) + \pi, & \text{for } y \geq 0, x < 0 \\ \arctan(y/x) - \pi, & \text{for } y < 0, x < 0 \\ \dfrac{\pi}{2}, & \text{for } y > 0, x = 0 \\ -\dfrac{\pi}{2}, & \text{for } y < 0, x = 0 \\ undefined, & \text{for } y = 0, x = 0 \end{cases}$$

A formation comprising $N$ agents without a virtual leader can be uniquely specified by at least $2N-3$ constraints in $\mathcal{R}^2$ when $N \geq 2$ and by at least $3N-6$ constraints in $\mathcal{R}^3$ when $N \geq 3$.

One can suggest the following formation constraints in $\mathcal{R}^2$;

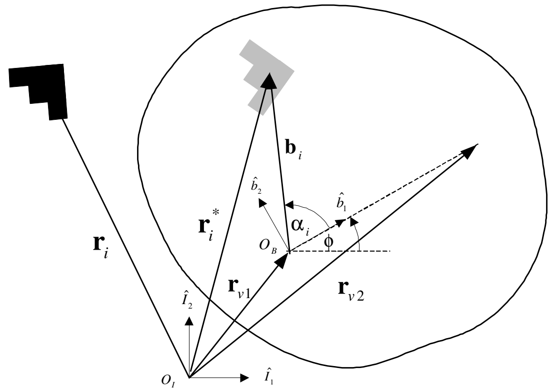


Fig. 1. Configuration of Formation

$$Q_i = \begin{cases} \|\mathbf{r}_i^* - \mathbf{r}_{i+1}^*\|^2 - d_{i,i+1}^2 = 0, & \text{for } i = 1,\ldots,N-1 \\ \|\mathbf{r}_{i-N+1}^* - \mathbf{r}_{i-N+3}^*\|^2 - d_{i-N+1,i-N+3}^2 = 0, & \\ & \text{for } i = N,\ldots,2N-3 \end{cases}$$

where $d_{a,b}$ is the desired distance between the $a^{th}$ and $b^{th}$ agents, and the desired position of $i^{th}$ agent is denoted by $\mathbf{r}_i^*$. Constraints $Q_i$ may be replaced by the following;

$$Q_i = \begin{cases} \|\mathbf{r}_i^* - \mathbf{r}_{i+1}^*\|^2 - d_{i,i+1}^2 = 0, & \text{for } i = 1,\ldots,N-1 \\ \angle([\mathbf{r}_{i-N+1}^* - \mathbf{r}_{i-N+2}^*]_I) - \angle([\mathbf{r}_{i-N+3}^* - \mathbf{r}_{i-N+2}^*]_I) & \\ \quad -\alpha_{i-N+1,i-N+3} = 0, & \text{for } i = N,\ldots,2N-3 \end{cases}$$

where $\alpha_{a,a+2}$ is the desired angle between the two vectors, $\mathbf{r}_a^* - \mathbf{r}_{a+1}^*$ and $\mathbf{r}_{a+2}^* - \mathbf{r}_{a+1}^*$.

These $2N-3$ constraints are sufficient to specify the formation, however, each agent is connected to each other in sequence through intermediate agents. Namely, the constraint $Q_k$ is not independent of the constraint $Q_{k-1}$. Consequently, this formalism is not scalable.

Another idea for realization of formation constraints is given in Egerstedt et al. (2001). Here the formation constraints were defined through a single function. However, this representation depends on the number of agents. Therefore we need to find a new representation of formation constraints which satisfies the following conditions;

- The number of formation constraints should be a multiple of $N$ when the formation comprises $N$ real agents.
- Each formation constraint should contain the configuration of only one agent.

By introducing virtual leaders, formation constraints can be made independent of each other, thereby making the formalism for formation generation scalable in the number of agents. The configuration of a formation itself cannot be specified by one virtual leader or by an agent on its own. In other words, at least two virtual leaders or agents are required to specify a unique geometric formation in $\mathcal{R}^2$.

Suppose the positions of two virtual leaders $VL_1$ and $VL_2$ are denoted by the position vectors $\mathbf{r}_{v1}$ and $\mathbf{r}_{v2}$ with respect to $I$ in Fig.1. The first virtual leader is assumed to coincide with $O_B$ and the second virtual leader is assumed fixed at $\begin{bmatrix} b1 \\ 0 \end{bmatrix}_B$. Then the position of each of the agents can be specified by $d_i$ and $\alpha_i$, where $d_i = \|\mathbf{b}_i\|$ and

$\alpha_i = \angle([\mathbf{b}_i]_B)$ are as shown in Fig.1. One can choose new constraints for the formation, which contains two virtual leaders $VL_1$, $VL_2$, as follows;

$$\begin{aligned} Q_i &= \begin{bmatrix} \|\mathbf{r}_i^* - \mathbf{r}_{v1}\|^2 - d_i^2 \\ \angle([\mathbf{r}_i^* - \mathbf{r}_{v1}]_I) - \angle([\mathbf{r}_{v2} - \mathbf{r}_{v1}]_I) - \alpha_i \end{bmatrix} \\ &= \begin{bmatrix} q_{i,l} \\ q_{i,\theta} \end{bmatrix} = 0, \qquad i = 1, \ldots, N \end{aligned} \tag{2}$$

Hence this representation of formation constraints satisfies the two required conditions discussed above.

The error in formation is defined by substituting $\mathbf{r}_i$ for $\mathbf{r}_i^*$ in the formation constraints as follows;

$$\begin{aligned} E_i &= \begin{bmatrix} \|\mathbf{r}_i - \mathbf{r}_{v1}\|^2 - d_i^2 \\ \angle([\mathbf{r}_i - \mathbf{r}_{v1}]_I) - \angle([\mathbf{r}_{v2} - \mathbf{r}_{v1}]_I) - \alpha_i \end{bmatrix} \\ &= \begin{bmatrix} e_{i,l} \\ e_{i,\theta} \end{bmatrix}, \qquad i = 1, \ldots, N \end{aligned} \tag{3}$$

### *2.2 Control law by feedback linearization*

Sliding mode control can be used for stabilizing formation error as in Barth (2006) or Galzi et al. (2006). Sliding mode control has the feature that it is robust to small disturbances, although the control law suffers from chattering(Khalil (1996)). In Barth (2006) the formation error is stabilized asymptotically even when the target changes its moving direction suddenly. However, the algorithm there cannot be expanded to more than two agents. In this paper, we propose a scheme for this scenario which is applicable to $N(\geq 1)$ agents and which is also scalable. Potential field methods can lead to collision-free control laws for each agent (Olfati et al. (2002)). However, it is quite challenging to incorporate nonholonomic constraints in a potential field method. Since nonholonomic constraints capture the limitation of actual agents well, for example the no slip constraints of ground vehicles, we consider nonholonomic constraints in the formalism of the approach. However this introduces nonlinearities into the control design.

One can establish a controller which stabilizes these errors using a suitable Lyapunov function. Let us define $s_{i,l} = \dot{e}_{i,l} + \lambda_{i,l} e_{i,l}$ , $s_{i,\theta} = \dot{e}_{i,\theta} + \lambda_{i,\theta} e_{i,\theta}$ where $\lambda_{i,l} > 0$ and $\lambda_{i,\theta} > 0$.

Consider $\mathcal{V} = \frac{1}{2}\sum_1^n (s_{i,l}^2 + s_{i,\theta}^2)$ a Lyapunov function candidate. Then $\dot{\mathcal{V}} = \sum_1^n (s_{i,l}\dot{s}_{i,l} + s_{i,\theta}\dot{s}_{i,\theta})$. If $\mathbf{u}_i$ satisfies $\dot{s}_{i,l} = -\gamma_{i,l}s_{i,l}$ and $\dot{s}_{i,\theta} = -\gamma_{i,\theta}s_{i,\theta}$ where $\gamma_{i,l} > 0$ and $\gamma_{i,\theta} > 0$, $\dot{\mathcal{V}}$ will be $-\sum_1^n (\gamma_{i,l}s_{i,l}^2 + \gamma_{i,\theta}s_{i,\theta}^2)$. Since $\dot{\mathcal{V}} = -\sum_1^n (\gamma_{i,l}s_{i,l}^2 + \gamma_{i,\theta}s_{i,\theta}^2)$ is always negative and $\mathcal{V} = \frac{1}{2}\sum_1^n (s_{i,l}^2 + s_{i,\theta}^2) > 0$, the errors will be asymptotically stabilized in the sense of Lyapunov with these controllers.

The control inputs appear in the second derivatives of the errors, and each control law for the actual agents can be calculated by solving two linear equations as follows;

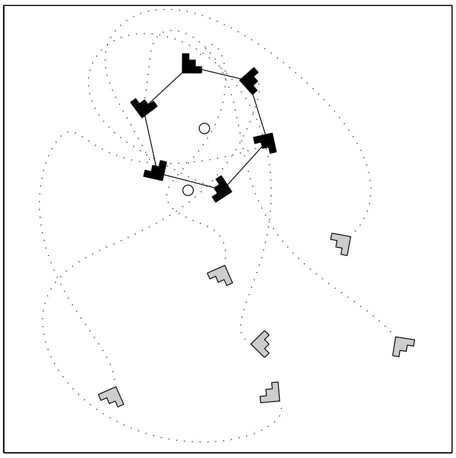

Fig. 2. Forming hexagonal formation. White circles stand for virtual leaders, gray shows initial configuration of agents and black shows accomplished formation

$$\begin{aligned} \ddot{e}_{i,l} &= A_i + B_i u_i + C_i \omega_i \\ \ddot{e}_{i,\theta} &= AA_i + BB_i u_i + CC_i \omega_i \end{aligned} \tag{4}$$

where

$$\begin{aligned} A_i &= 2\dot{\tilde{x}}_i^2 + 2\dot{\tilde{y}}_i^2 - 2\tilde{x}_i\ddot{x}_{v1} - 2\tilde{y}_i\ddot{y}_{v1}, \\ B_i &= 2\tilde{x}_i \cos\theta_i + 2\tilde{y}_i \sin\theta_i, \\ C_i &= -2\tilde{x}_i\dot{y}_i + 2\tilde{y}_i\dot{x}_i, \end{aligned}$$

$$\begin{aligned} AA_i =& -\frac{\tilde{x}_i\ddot{y}_{v1} - \ddot{x}_{v1}\tilde{y}_i}{\tilde{x}_i^2 + \tilde{y}_i^2} - \frac{\tilde{x}_{v2}\ddot{\tilde{y}}_{v2} - \ddot{\tilde{x}}_{v2}\tilde{y}_{v2}}{\tilde{x}_{v2}^2 + \tilde{y}_{v2}^2} \\ &- \frac{(\tilde{x}_i\dot{\tilde{y}}_i - \dot{\tilde{x}}_i\tilde{y}_i)(2\tilde{x}_i\dot{\tilde{x}}_i + 2\tilde{y}_i\dot{\tilde{y}}_i)}{(\tilde{x}_i^2 + \tilde{y}_i^2)^2} \\ &+ \frac{(\tilde{x}_{v2}\dot{\tilde{y}}_{v2} - \dot{\tilde{x}}_{v2}\tilde{y}_{v2})(2\tilde{x}_{v2}\dot{\tilde{x}}_{v2} + 2\tilde{y}_{v2}\dot{\tilde{y}}_{v2})}{(\tilde{x}_{v2}^2 + \tilde{y}_{v2}^2)^2}, \\ BB_i =& \frac{\tilde{x}_i\cos\theta_i - \tilde{y}_i\sin\theta_i}{\tilde{x}_i^2 + \tilde{y}_i^2}, \qquad CC_i = \frac{\tilde{x}_i\dot{x}_i + \tilde{y}_i\dot{y}_i}{\tilde{x}_i^2 + \tilde{y}_i^2}, \\ \tilde{x}_i =& x_i - x_{v1}, \qquad \tilde{y}_i = y_i - y_{v1}, \\ \tilde{x}_{v2} =& x_{v2} - x_{v1}, \quad \tilde{y}_{v2} = y_{v2} - y_{v1}. \end{aligned}$$

Therefore, the control law $\mathbf{u}_i$ is determined as follows;

$$\begin{bmatrix} C_i & B_i \\ CC_i & BB_i \end{bmatrix} \mathbf{u}_i = \begin{bmatrix} F_i \\ G_i \end{bmatrix} \tag{5}$$

where $F_i = -\lambda_{i,l}\dot{e}_{i,l} - \gamma_{i,l}(\dot{e}_{i,l} + \lambda_{i,l}e_{i,l}) - A_i$, and $G_i = -\lambda_{i,\theta}\dot{e}_{i,\theta} - \gamma_{i,\theta}(\dot{e}_{i,\theta} + \lambda_{i,\theta}e_{i,\theta}) - AA_i$. The paths of the two virtual leaders $VL_1$, $VL_2$ coordinates the motion of the VS, and the control law $\mathbf{u}_i$ forces the agents to remain in formation. Figure 2 shows a simulation generating a formation through the controls $\mathbf{u}_i$ which satisfies (5), with the formation error stabilized in the sense of Lyapunov. Note that the matrix $P = \begin{bmatrix} C_i & B_i \\ CC_i & BB_i \end{bmatrix}$ can be singular when the $i^{th}$ agent coincides with the virtual leaders or when the $i^{th}$ agent is stationary. The former case can be avoided by choosing $\alpha_i \neq 0$, $d_i > 0$ while in the latter case, no smooth time-invariant control law can stabilize the error. This well known result is found in Brockett (1983).

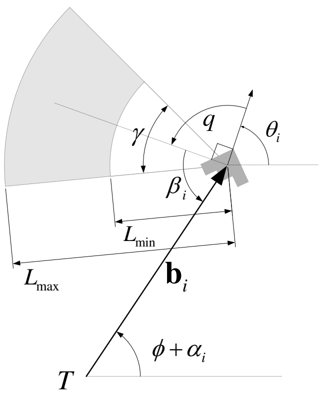


Fig. 3. Characterization of $\beta_i$: Gray sector stands for the footprint of the sensor.

## 3. APPLICATION: COOPERATIVE GROUND MOVING TARGET SURVEILLANCE SCENARIO

Now we consider the following scenario as an application of the formation control scheme proposed in section $II$. Consider a task where multiple agents are assigned to track a moving target. Each agent has a specific sensor footprint provided by a sensor fixed on it. By assumption, since the sensor is fixed to the body of the agent, the footprint of the sensor can only be moved by moving the agent. It is also assumed that each sensor has limitations in range and angle. The scenario considered is one where a single agent cannot track the moving target for all time on its own.

Let $\mathbf{r}_t$ be the position vector of target $T$. If $\mathbf{r}_t$ coincides with $\mathbf{r}_{v1}$, as shown in Fig.1, and the value of $d_i$ is chosen to lie between $L_{min}$ and $L_{max}$, as in Fig.3, then $N$ agents will lie on the circle whose radius is $d_i$ and whose center coincides with the target. Next we consider the distribution of the $N$ agents on this circle so that the target falls within at least one of the sensor footprints of the cooperating agents. The goal is to propose a scheme that determines the minimum number of agents required to this task.

Let us assume that $e_{i,l} = 0$, $e_{i,\theta} = 0$, and let $[\mathbf{R}]$ be the coordinate transformation matrix from frame $B$ to the inertial frame $I$. Let $[\mathbf{r}_t]_I = [x_t, y_t]^T$ and $[\mathbf{r}_i]_I = [x_i, y_i]^T$. Then, $[\mathbf{r}_i]_I = [\mathbf{r}_t]_I + [\mathbf{R}][\mathbf{b}_i]_B$ which yields;

$$\begin{aligned} \dot{x}_i &= \dot{x}_t - \dot{\phi} d_i \sin(\phi + \alpha_i) \\ \dot{y}_i &= \dot{y}_t + \dot{\phi} d_i \cos(\phi + \alpha_i) \end{aligned} \tag{6}$$

Suppose the target as well as the $i^{th}$ agent has nonholonomic kinematics of a uni-cycle given by;

$$\begin{cases} \dot{x}_t = V_t \cos\theta_t \\ \dot{y}_t = V_t \sin\theta_t \\ \dot{\theta}_t = \omega_t \end{cases} \qquad \begin{cases} \dot{x}_i = V_i \cos\theta_i \\ \dot{y}_i = V_i \sin\theta_i \\ \dot{\theta}_i = \omega_i \end{cases} \tag{7}$$

where $V_t > 0$ and $V_i > 0$.

Let us define the separation angle $\beta_i$ as

$$\beta_i = q - \theta_i + (\phi + \alpha_i) \tag{8}$$

The separation angle $\beta_i$ in Fig.3 shows how much the center of the sensor footprint deviates off of the target.

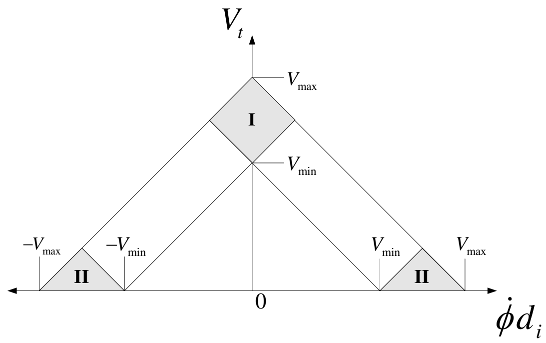


Fig. 4. Feasible region of $V_t$ and $\dot{\phi}$

If $\beta_i \leq \frac{\gamma}{2}$, the moving target falls in the sensor footprint of the $i^{th}$ agent. In this paper, $q$ is considered a constant value of $\frac{\pi}{2}$ for simplicity.

Suppose that $V_{max}$ and $V_{min}$ are the maximum and minimum values of $V_i$ respectively. Equation (6) and (7) yield the following;

$$V_i^2 = V_t^2 + \dot{\phi}^2 d_i^2 + 2V_i \dot{\phi} d_i \sin(\theta_t + \phi + \alpha_i)) \tag{9}$$

One can find the feasible region for $V_t$ and $\dot{\phi}$ from (9) leading to the following;

$$\begin{aligned} (V_t - |\dot{\phi}| d_i)^2 &\geq V_{min}^2 \\ (V_t + |\dot{\phi}| d_i)^2 &\leq V_{max}^2 \end{aligned} \tag{10}$$

This region is shown graphically in Fig.4.

Since regions $I$ and $II$ in Fig.4 are disconnected, the controller will be discontinuous if the values of $V_t$ varies from region $I$ to region $II$ or vice versa. So, the value of $V_t$ is assumed to remain in region $I$ or $II$ exclusively. Let us call case-$I$ situation where the pair $(V_t, \dot{\phi} d_i)$ remains exclusively in region $I$ and case-$II$ when it remains exclusively in region $II$.

The separation angle $\beta_i$ can be calculated by the following;

$$\beta_i = \angle \left( \begin{bmatrix} \dot{\phi} d_i - V_t \sin\kappa_i \\ V_t \cos\kappa_i \end{bmatrix} \right) \tag{11}$$

where $\kappa_i = \phi + \alpha_i - \theta_t$.

First, let us consider case-$I$. Here $V_t$ is between $V_{max}$ and $V_{min}$, and $\beta_i$ is not limited and can be any value in $(-\pi, \pi]$.

Let us consider the situation where $N$ agents are distributed uniformly on the circle whose radius is $d_i$ and whose center coincides with the target. This can be expressed by $N = \lceil \frac{2\pi}{\gamma} \rceil$ and $\alpha_{i+1} = \alpha_i + \frac{2\pi}{N} (i = 1, 2, ..., N-1)$.

*Definition 3.1.* (Ceiling Function). $\lceil x \rceil$ is the ceiling function that returns the smallest integer not less than $x$. $\lceil x \rceil = min\{n \in \mathcal{Z} | x \leq n, x \in \mathcal{R}\}$.

$N = \lceil \frac{2\pi}{\gamma} \rceil$ is not enough when $\dot{\phi} \neq 0$ because the partial derivative of $\beta_i$ with respect to $\kappa_i$ is not always 1 except when $\dot{\phi} = 0$. Figure 5(a) shows that all of the agents fail to detect the moving target when $\gamma = \frac{\pi}{2}$, $N = 4$ and $\alpha_{i+1} = \alpha_i + \frac{\pi}{2} (i = 1, 2, 3)$. The moving target is shown in Fig.5 as $*$, and the white sectors stand for the footprints of the sensors.

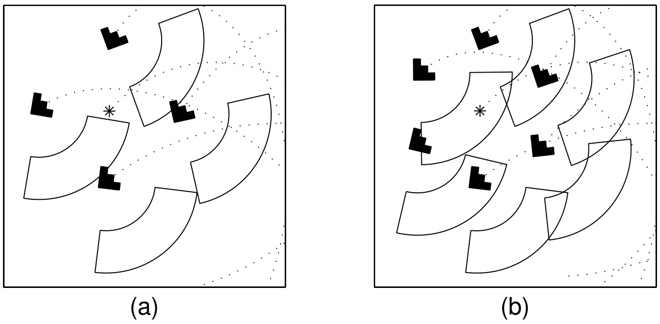


Fig. 5. (a) shows that $N(=\lceil\frac{2\pi}{\gamma}\rceil)$ agents fail to detect the moving target. (b) shows that $N_I^*$ agents succeed in detecting the moving target.

So we need to increase number of agents, considering the maximum value of $\left|\frac{\partial\beta_i}{\partial\kappa_i}\right|$.

$$\frac{\partial\beta_i}{\partial\kappa_i}=\frac{V_t^2-V_t\dot{\phi}d_i sin\kappa_i}{\dot{\phi}^2d_i^2+V_t^2-2V_t\dot{\phi}d_i\sin\kappa_i} \tag{12}$$

$\frac{\partial\beta_i}{\partial\kappa_i}$ always has a positive value in case-$I$, since $V_t\geq|\dot{\phi}|d_i$.

$$\begin{aligned}\max_{case-I}\left(\left|\frac{\partial\beta_i}{\partial\kappa_i}\right|\right)&=\max_{case-I}\left(\frac{V_t{}^2-V_t\dot{\phi}d_i\sin\kappa_i}{\dot{\phi}^2d_i^2+V_t^2-2V_t\dot{\phi}d_i\sin\kappa_i}\right)\\&=\max_{case-I}\left(\frac{V_t}{V_t-|\dot{\phi}|d_i}\right)\\&=\frac{V_{max}+V_{min}}{2V_{min}}>1\end{aligned} \tag{13}$$

$\left|\frac{\partial\beta_i}{\partial\kappa_i}\right|$ has the maximum value in case-$I$ where $V_t=\frac{V_{max}+V_{min}}{2}$ and $|\dot{\phi}|d_i=\frac{V_{max}-V_{min}}{2}$.

Now let us revise the number $N$ by using the maximum value of $\left|\frac{\partial\beta_i}{\partial\kappa_i}\right|$ in (13).

$$\begin{aligned}N_I^*&=\lceil\frac{2\pi\max_{case-I}\left(\left|\frac{\partial\beta_i}{\partial\kappa_i}\right|\right)}{\gamma}\rceil\\&=\lceil\frac{\pi(V_{max}+V_{min})}{\gamma V_{min}}\rceil\\\alpha_{i+1}&=\alpha_i+\frac{2\pi}{N_I^*}(i=1,2,...,N_I^*-1)\end{aligned} \tag{14}$$

Figure 5 (b) shows that $N_I^*$ number of agents can detect the moving target successfully.

Next consider case-$II$. From (12) $|\beta_i|$ has the maximum value when $\sin\kappa_i=\frac{V_t}{\dot{\phi}d_i}$. And the maximum value of $|\beta_i|$ in case-$II$ is the following;

$$\begin{aligned}\max_{case-II}(|\beta_i|)&=\arctan\left(\max_{case-II}\left(\frac{V_t}{\sqrt{(\dot{\phi}d_i)^2-(V_t)^2}}\right)\right)\\&=\arctan\left(\frac{V_{max}-V_{min}}{2\sqrt{V_{max}V_{min}}}\right)\end{aligned} \tag{15}$$

$\beta_i$ can vary only between $-\arctan\left(\frac{V_{max}-V_{min}}{2\sqrt{V_{max}V_{min}}}\right)$ and $\arctan\left(\frac{V_{max}-V_{min}}{2\sqrt{V_{max}V_{min}}}\right)$ in case-$II$. If half of the sensor coverage angle $\frac{\gamma}{2}$ is greater than $\max(|\beta_i|)$, then only one agent is required to track the moving target. However, if $\kappa_i$ lies in $[-\frac{\pi}{2}-\frac{\Delta_1}{2},-\frac{\pi}{2}+\frac{\Delta_1}{2}]$ or $[\frac{\pi}{2}-\frac{\Delta_2}{2},\frac{\pi}{2}+\frac{\Delta_2}{2}]$, then the $i^{th}$

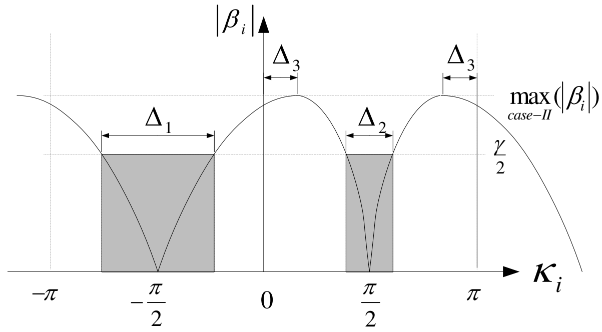


Fig. 6. $|\beta_i|$ in case-$II$

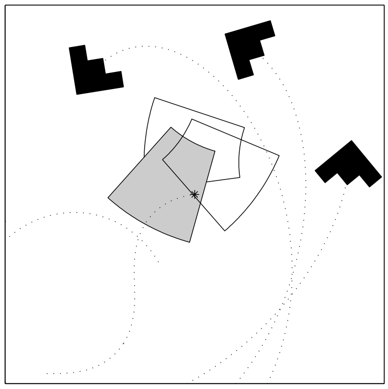

Fig. 7. $N_{II}^*$ agents succeed in detecting the moving target.

agent can detect the moving target when $\frac{\gamma}{2}<\max_{case-II}(|\beta_i|)$. Since $\Delta_3=\sin^{-1}(\frac{V_t}{\dot{\phi}d_i})$ is always greater than zero in case-$II$, $\Delta_1$ is always greater than $\Delta_2$. Also an $|\beta_i|$ is symmetric with respect to $\kappa_i=\frac{\pi}{2}+n\pi$ where $n$ is integer. If $N_{II}^*$ agents are distributed equally spaced on the half circle, and the angular displacement is smaller than $\Delta_2$, and at least one agent can detect the moving target, then we can propose the following scheme for case-$II$

$$\begin{aligned}N_{II}^*&=\begin{cases}\lceil\frac{\pi}{\Delta_2}\rceil & \text{if } \frac{\gamma}{2}<\max_{case-II}(|\beta_i|)\\1 & \text{if } \frac{\gamma}{2}\geq\max_{case-II}(|\beta_i|)\end{cases}\\\alpha_{i+1}&=\alpha_i+\frac{\pi}{N_{II}^*}(i=1,2,...,N_{II}^*-1)\end{aligned} \tag{16}$$

where $\Delta_2=2\arcsin(\frac{\dot{\phi}d_i}{V_t}\sin\frac{\gamma}{2})-\gamma$. Figure 7 shows $N_{II}^*$ agents successfully detecting the moving target when $\frac{\gamma}{2}<\max_{case-II}(|\beta_i|)$.

Additionally, $\dot{\phi}$ must be positive in case-$II$. The reason being that $q$, as shown in Fig.3, is a constant equal to $\frac{\pi}{2}$. If $\dot{\phi}$ has a negative value, $\beta_i\notin(-\frac{\pi}{2},\frac{\pi}{2})$ in case-$II$ which can easily be determined using (11), and none of the agents can detect the moving target.

## 4. CONCLUSION

Most leader following schemes do not scale up well since each of the functions describing formation errors, are expressed by relationships involving more than just one agent. The formalism which minimizes these formation errors is not scalable, since a change in the number of agents in the formation will cause all the mathematical expressions of formation errors to change. A new formation error representation was proposed in this paper. This representation is independent of the number of agents in the formation and the resulting control algorithm is scalable.

The proposed approach is based on a fusion of leader following and virtual structure approaches. The group behavior is directed by specifying the behavior of two virtual leaders in the method proposed. Also the configuration of all agents in the formation can be expressed easily since all the agents are assumed to form a virtual structure which behaves like a rigid body.

The proposed control scheme enables the determination of the minimum number of agents required for surveillance of the moving target. However, the number of agents returned by this scheme is not optimal and hence is a conservative solution. However, this is somewhat justified by the computational savings the scheme offers. The reason being that the computations require only the controller bounds for each of the agents and is independent of the motion of the moving target.

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