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Path tracking control of a flapping Micro Aerial Vehicle (MAV) ⋆

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Abstract: This paper deals with the path tracking control for a flapping wing Micro Aerial Vehicle (MAV) taking into consideration the input saturations. Based on the aerodynamic theory, a simplified model of a high frequency flapping MAV is presented and its average is calculated. Equivalence between the time varying and mean models is shown through averaging theory. Hence, a bounded non linear control, designed using the average model, is applied to the time varying system to drive the position and orientation to desired values. Finally, the robustness with respect to external disturbances is tested.

1. INTRODUCTION

Micro Aerial Vehicles (MAVs) have shown a large amount of interest in the last years, taking benefit from the progress in microelectronic technologies and materials. The most recent class of MAVs are the flapping wing airfoils, getting a growing interest within biology, aeronautic, robotic and control communities because of the fascination with the nature's flight mechanisms. Their development is also motivated by the advantages they present relative to the fixed and rotary airfoils (Kellogg et al. [2003]): flapping wing MAVs have a smaller size, produce less noise, develop more lift, show high maneuverability and theoretically consume less energy. The major drawback is still the complexity of reproducing the movements the insects develop during complex maneuvers (Dudley [2002]). Flapping MAVs can be used in many indoor/outdoor, civil/military applications like surveillance, monitoring, intervening in narrow and dangerous environments for searching and rescuing, investigating, spying, etc.
The present work is part of the OVMI ("Objet Volant Mimant l'Insecte") project aiming to design and develop a silicon based flapping MAV mimicking the insect in size and behavior.
The goal of this paper is to control the position of a flapping MAV taking off from a start point, following a nondescript definite trajectory in three dimensions then stabilize it at an end point in hovering mode. Few of previous works have dealt with the control problem in 3D. State feedback controllers were proposed; the first one acts directly on the position (Schenato et al. [2001]) and the second acts on the vertical force and torques, it is a bounded control calculated by poles placement using the linearized dynamics of the system (Schenato et al. [2002]). In Deng et al. [2006], a Linear Quadratic Gaussian (LQG) optimal state feedback controller is proposed.
The present paper proposes a nonlinear state feedback control law dedicated to stabilize the MAV at a desired position after tracking a predefined trajectory. Contrary to the linear control laws developed in previous works, this nonlinear control law is robust with respect to external disturbances. Moreover, it is bounded in order to take into consideration the input saturation, i.e. the saturation of the actuators driving the flapping wings. The attitude is represented by the quaternion to prevent the singularities induced by Euler angles.
The paper is organized as follows. In section 2, a simplified model is developed for control purpose and its average over a wingbeat period is computed taking into account the input saturations. Section 3 deals with the control problem, aiming to drive the MAV to desired position and orientation. Some simulations are shown in section 4 with robustness tests with respect to external disturbances. Finally, some conclusions and future works are presented in section 5.

2. FLAPPING FLIGHT MODELING

The movement of the MAV is determined through the dynamic equations of the body, the wing kinematics and the aerodynamic theory. A simplified model is then developed and its average is computed. This model is dedicated to the validation of the control law.

2.1 Wings movement parametrization

Flapping flight is governed by three major degrees of freedom (DOF) (Michelson et al. [1997]): *flapping*, *feathering*, *lagging*. Flapping is an up and down movement of the

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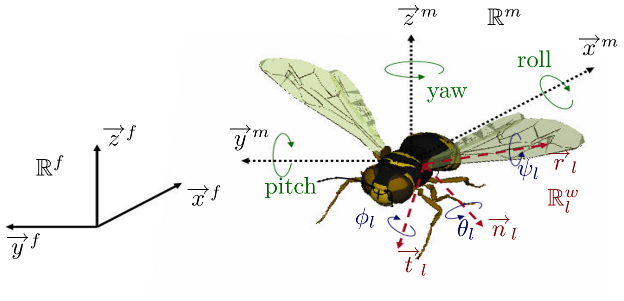


Fig. 1. Frames and wings angles

wing. It is a rotation of angle ϕ (flapping angle) about axis $\overrightarrow{t}$ parallel to the wing chord, oriented from trailing to leading edge (see Fig. 1). Feathering is a twist motion of the wing around its base axis, in order to change its angle of attack. It is a rotation of the wing of angle ψ (rotation angle) about axis $\overrightarrow{r}$ oriented from the wing base to its tip. Lagging is a forward and backward movement of the wing parallel to the MAV's body. It is a rotation of angle θ (deviation angle) about axis $\overrightarrow{n}$ perpendicular to the wing plane, oriented so that the three-sided frame $(\overrightarrow{r}, \overrightarrow{t}, \overrightarrow{n})$ attached to each wing is direct. ϕ, ψ, θ are known as the wings Euler angles. The wing is supposed to be oriented forward during downstroke, and backward during upstroke. Therefore, angle ϕ varies according to a sawtooth function and angle ψ to a pulse function, so that the wing changes its orientation at the end of each half stroke (see Fig. 2). Angle θ is taken to 0 in this work, making the wing beat in the mean stroke plane in order to use actuators for 2 DOF only. The temporal variation of the wings Euler angles is given by

$$
\begin{aligned}
\phi(t) &= \begin{cases} \phi_0(1 - \dfrac{2t}{\kappa T}) & 0 \leq t \leq \kappa T \\ \phi_0(2\dfrac{t - \kappa T}{(1-\kappa)T} - 1) & \kappa T < t \leq T \end{cases} \\
\psi(t) &= \psi_0 \operatorname{sign}(\kappa T - t) \qquad\qquad 0 \leq t \leq T \\
\theta(t) &= 0 \qquad\qquad\qquad\qquad\quad 0 \leq t \leq T
\end{aligned} \tag{1}
$$

where sign designates the classical sign function, T is the wingbeat period, κ is the ratio of downstroke duration to the wingbeat period, ϕ_0 and ψ_0 are respectively the amplitudes of flapping and rotation angles. ϕ_0 and ψ_0 considered for both left and right wings, will be taken as control variables later on.

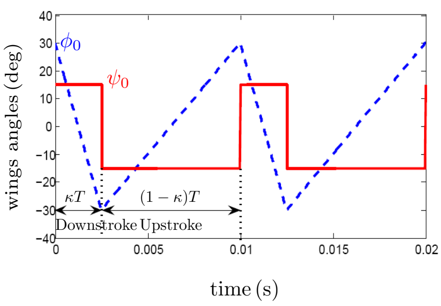


Fig. 2. Wings angles configuration over two wingbeat periods: flapping angle ϕ (dashed line) and rotation angle ψ (continuous line)

2.2 Body's dynamics

The flapping MAV is considered as a rigid body subject to external forces and torques. The equations of motion are given by

$$\dot{P}^f = V^f \tag{2}$$

$$\dot{V}^f = \frac{1}{m} R^T(q) f^m - cV^f - g \tag{3}$$

$$\begin{pmatrix} \dot{q}_0 \\ \dot{\overrightarrow{q}} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -\overrightarrow{q}^T \\ I_3 q_0 + \hat{q}^T \end{pmatrix} \omega^m \tag{4}$$

$$\dot{\omega}^m = J_m^{-1}(\tau^m - \omega^m \wedge J_m \omega^m) \tag{5}$$

$P^f \in \mathbb{R}^3$ and $V^f \in \mathbb{R}^3$ are respectively the linear position and velocity of the body's center of gravity relative to the fixed frame $\mathbb{R}^f$. ω^m is the angular velocity with respect to the mobile frame $\mathbb{R}^m$ attached to the insect's body on its center of gravity. c is the viscous damping coefficient and g the gravity vector. $f^m \in \mathbb{R}^3$ and $\tau^m \in \mathbb{R}^3$ are respectively the aerodynamic force and torque. $J_m \in \mathbb{R}^{3\times3}$ is the inertia matrix of the body relative to $\mathbb{R}^m$ and I_3 is the identity matrix. q is the quaternion defining the attitude of the body relative to $\mathbb{R}^f$ (Shuster [1993]), $q = [\cos\frac{\nu}{2}\ (\overrightarrow{e} \sin\frac{\nu}{2})]^T = [q_0\ \overrightarrow{q}^T]^T$ consisting of a rotation of angle ν about the Euler axis $\overrightarrow{e}$. $q_0 \in \mathbb{R}$ is the scalar part and $\overrightarrow{q} = [q_1\ q_2\ q_3]^T \in \mathbb{R}^3$ the vector part of the quaternion. $q \in \mathbb{H}$ where $\mathbb{H} = \{q \mid q_0^2 + \overrightarrow{q}^T\overrightarrow{q} = 1\}$ is the Hamilton space. $R(q) \in SO(3) = \{R(q) \in \mathbb{R}^{3\times3} : R^T(q)R(q) = I,\ \det R(q) = 1\}$ is the rotation matrix from the fixed frame $\mathbb{R}^f$ to the mobile frame $\mathbb{R}^m$, $R(q) = (q_0^2 - \overrightarrow{q}^T\overrightarrow{q})I_3 + 2(\overrightarrow{q}\,\overrightarrow{q}^T - q_0\hat{q}^T)$. $\hat{q}$ is the skew symmetric tensor associated to $\overrightarrow{q}$.
The wings are considered as rigid bodies subject to aerodynamic forces developed during the two wingbeats phases: downstroke and upstroke. In this work, only the delayed stall will be considered. It can be modeled by a quasi-steady equation of the wing kinematic position and velocity. In flapping flight, the circulation of the air on the wing edge created during the wing alteration phase produces additional aerodynamic forces: rotational lift and wake capture. These last two forces have a minor contribution to the global aerodynamic force (Schenato et al. [2003]), then can be neglected. The unique aerodynamic force considered in this work is applied on the wing's aerodynamic center, which coincides with its center of mass in practical cases. This center is located at the quarter distance of the wing's chord from the leading edge, and at 0.6 to 0.7 of the wing's length measured from the base (Schenato et al. [2003]). The force is perpendicular to the wing and has the opposite direction of the wing's velocity. The module of the force is considered proportional to the square of the wing's velocity relative to $\mathbb{R}^m$. This assumption has to be checked on the prototype under construction. The wings' inertial forces have a small effect because the mass of the insect's wings is less than 5% of the body's mass (Schenato et al. [2003]). The module f^w of the force is given by

$$f^w = -\frac{1}{2}\rho C_w S_w v^w |v^w| \tag{6}$$

ρ is the air density, S_w is the wing's surface, v^w is the wing's velocity, C_w is a coefficient of the aerodynamic force applied on a wing. $C_w = C(1 + C_f)$ during downstroke and $C_w = C(1 - C_f)$ during upstroke, where $C \approx 3.5$

is the delayed stall force coefficient derived empirically in Dickinson et al. [1999], Schenato et al. [2003] and C_f is a coefficient chosen so that the aerodynamic force is 20% greater during downstroke than during upstroke. This dissymmetry between the two half-strokes can be justified based on Dudley [2002]. During downstroke, the dorsal side of the wing is opposite to the air flow. The wing reversal, at the end of the downstroke, opposes the ventral side of the wing to the flow. Consequently, the orientation of air circulation about the wing reverses too, leading to a wing camber alteration, and the effective area of the wing is then reduced. Therefore, downstroke lift is likely to be higher than that of upstroke, such that the averaged force over a single wingbeat period should at least balance the body's weight. The wing's aerodynamic center's position is given by p^m which is the projection in $\mathbb{R}^m$ of the position $p^w = [L\ 0\ 0]$ relative to $\mathbb{R}^w$. The wing's velocity is the center of mass position's derivative $v^m = \dot{p}^m$ and v^w is the projection of v^m in $\mathbb{R}^w$ (Rifai et al. [2007a]).
The aerodynamic force relative to $\mathbb{R}^m$ is the sum of the forces developed by the left and right wings

$$f^m = f_l^m + f_r^m \tag{7}$$

The aerodynamic force has two components, a horizontal thrust that ensures a forward movement of the MAV, and a vertical lift that ensures a vertical movement.
The aerodynamic torque relative to $\mathbb{R}^m$ is defined as the vectorial product of the force f^m and the wing's aerodynamic center's position p^m. Angular viscous torques are negligible with respect to aerodynamic torques.

$$\tau^m(t) = p_l^m(t) \wedge f_l^m(t) + p_r^m(t) \wedge f_r^m(t) \tag{8}$$

2.3 Control constraints

The control of the flapping MAV's position and orientation is ensured by controlling the force and torque applied to the body. The flapping MAV model considered in this work has a high wingbeat frequency. Therefore, the aerodynamic force and torque affect the body by their averaged values as demonstrated in the averaging theory (Khalil [1996], Bullo [2002], Vela [2003]). Moreover, a stabilizing control law calculated using the averaged model will stabilize the high frequency oscillating model too (Bullo [2002], Schenato [2003]). Thereby, the averaged dynamics of the time varying model (1-8) are calculated (Rifai et al. [2007a]).

$$(\overline{f}_x, \overline{f}_z, \overline{\tau}_1, \overline{\tau}_3) = \Lambda(\phi_0^l, \phi_0^r, \psi_0^l, \psi_0^r) \tag{9}$$

The attitude stabilization of the MAV is ensured by the roll, pitch and yaw control torques in $\mathbb{R}^m$ ($\overline{\tau}_1$, $\overline{\tau}_2$, $\overline{\tau}_3$). The MAV is supposed to move forward due to the thrust control force $\overline{f}_x$, vertically due to the lift control force $\overline{f}_z$ ($\overline{f}_x$ and $\overline{f}_z$ are expressed in $\mathbb{R}^m$), laterally due to a coupling between the roll and the vertical movements. The thrust and lift forces ($\overline{f}_x, \overline{f}_z$) as well as the roll and yaw torques ($\overline{\tau}_1$, $\overline{\tau}_3$) are generated by the flapping wings (9). In particular, an increase in the amplitudes of the flapping angles of the two wings would result in a larger lift, and an increase in the amplitudes of the rotation angles in a larger thrust. A difference in amplitudes of the flapping angles of the two wings would result in a roll torque, and a difference in amplitudes of the rotation angles in a yaw torque. The pitch torque $\overline{\tau}_2$ will be generated by a small mass moving inside the MAV's body and changing its center of gravity. Input saturation supposes that

$$\begin{array}{rcl} 0 \leq & \phi_0 & \leq \overline{\phi_0} \\ -\overline{\psi_0} \leq & \psi_0 & \leq \overline{\psi_0} \end{array} \tag{10}$$

for left and right wings; system (9) defines a convex set Ω in the mean control variables ($\overline{f}_x$, $\overline{f}_z$, $\overline{\tau}_1$, $\overline{\tau}_3$) (see Fig. 3a and 3b, Ω_{τ_1,τ_3} and Ω_{f_x,f_z} are the projection of Ω on the planes (τ_1, τ_3) and (f_x, f_z) respectively). Therefore, anywhere in the set Ω, there exists a wing configuration (ϕ_0^l, ϕ_0^r, ψ_0^l, ψ_0^r) producing the mean desired forces and torques ($\overline{f}_x$, $\overline{f}_z$, $\overline{\tau}_1$, $\overline{\tau}_3$). Considering the mean behavior over a wingbeat period of system (2-5), the MAV is approximated by a rigid body subject to external forces and torques. Therefore, the averaged state of the time varying model $\overline{x}$ is equivalent to a rigid body state x^{rb}.
In this work, state feedback control laws, stabilizing the rigid body's position and orientation, will be proposed.

$$(\overline{f}_x, \overline{f}_z, \overline{\tau}_1, \overline{\tau}_3) = \mathcal{U}(x^{rb}) = \mathcal{U}(\overline{x}) \tag{11}$$

By inverting (9), (11) can be written

$$(\phi_0^l, \phi_0^r, \psi_0^l, \psi_0^r) = \Lambda^{-1}(\mathcal{U}(\overline{x})) \tag{12}$$

3. FLAPPING FLIGHT CONTROL

The control of the flapping MAV amounts from controlling its position and orientation. System (2-5) can be decomposed into two subsystems, one defining the translation and the other the rotation. The control of the global system will be ensured based on the theory of cascade (Sontag [1989]) specially that the translational subsystem depends on the rotational one, and the rotational subsystem is independent from the translational one. The system has then the following form

$$\begin{cases} \dot{x} = f(x, y) \\ \dot{y} = g(y, u) \end{cases} \tag{13}$$

3.1 Attitude control

The control law applied in this paragraph is supposed to drive the body to a desired orientation q_d, while the angular velocity should vanishes: $q \to q_d$, $\omega^m \to 0$ as $t \to \infty$. The error between the current and desired orientations of the body is quantified by the quaternion error: $q_e = q \otimes q_d^{-1}$, where q^{-1} is the quaternion conjugate given by $q^{-1} = [q_0 - \overrightarrow{q}^T]^T$, $\otimes$ is the quaternion product defined by $q \otimes Q = [(q_0 Q_0 - \overrightarrow{q}\,\overrightarrow{Q})\ \ (q_0 \overrightarrow{Q} + Q_0 \overrightarrow{q} + \overrightarrow{q} \wedge \overrightarrow{Q})^T]^T$, and $\wedge$ denotes the vectorial product.
The proposed attitude stabilizing control torque is a bounded state feedback based in its formulation on the model of a rigid body (Guerrero-Castellanos et al. [2007]) (equivalent to the averaged model of the flapping body) and applied to the time variant model (flapping MAV). This control is very simple, therefore suitable for an embedded implementation. Moreover, it is robust with respect to the aerodynamic coefficient (Rifai et al. [2007b]) and does not require the knowledge of the body's inertia. Let $\overline{\tau} = [\overline{\tau}_1\ \overline{\tau}_2\ \overline{\tau}_3]^T$ be the roll, pitch and yaw control torques.

$$\overline{\tau}_i = -\alpha_i \sigma_{M_{2,i}}(\lambda_i[\gamma_i \overline{\omega}_i + \mathrm{sign}(q_{e_0})\sigma_{M_{1,i}}(\overline{q}_{e_i})]) \tag{14}$$

where $i \in \{1, 2, 3\}$, $\mathrm{sign}(q_0)$ takes into account the possibility of 2 rotations to drive the body to its equilibrium orientation; the one of smaller angle is chosen. $\overline{\omega}_i$ and $\overline{q}_i$ are the averaged angular velocities and quaternion over a single wingbeat period (averaged state of the rotational subsystem $\overline{x}_r = \{\overline{\omega}_i, \overline{q}_i\}$, $i = 1, 2, 3$) representing the time varying angular velocities and quaternion of a rigid body. α_i, λ_i, γ_i are positive parameters. Differently from Guerrero-Castellanos et al. [2007], γ_i has been added in order to slow down the convergence of the torque relative to the angular velocity, to make it achievable in a wing-beat period. Moreover, the general case represented by the quaternion error (instead of the current quaternion) is considered. $\sigma_{M_{1,i}}$ and $\sigma_{M_{2,i}}$ are saturation functions with $M_{1,i}$ and $M_{2,i}$ the saturation bounds: $M_{1,i} \geq 1$, $M_{2,i} \geq \lambda_i(2M_{1,i} + \epsilon_i)$ and $\epsilon_i > 1$. The $M_{2,i}$'s are chosen in order to respect the input saturations: wings Euler angles and body's length. The saturation bounds $M_{2,1}$ and $M_{2,3}$ are adjusted in (14) so that $\overline{\tau}_1$ and $\overline{\tau}_3$ remain in the limits of Ω_{τ_1,τ_3} (see Fig. 3a), which guarantees not to exceed the maximum angles. $M_{2,2}$ should respect the saturation induced by the length of the body, since the pitch torque is generated by a small mass moving inside it.

The asymptotic stability of the closed loop system has been shown in Guerrero-Castellanos et al. [2007] for rigid bodies using the following Lyapunov function (the added parameter γ_i and the use of the quaternion error do not change the proof)

$$Vr = \frac{1}{2}\omega^{rb^T} J_m \omega^{rb} + \kappa((1 - q_{e0}^{rb})^2 + \overrightarrow{q}_e^{rb^T} \overrightarrow{q}_e^{rb}) \qquad (15)$$

Therefore, $\overline{\omega} \to 0$ and $\overline{q} \to q_d$ (based on the rigid body case). By means of the averaging theory, the stability of the high frequency flapping insect is guaranteed.

3.2 Position control

Subsystem (2-3) can be considered as a chain of integrators by neglecting the drag force represented by cV^f, therefore considering the system at low speeds. cV^f is considered as a disturbance term in simulations. Supposing that, after a sufficiently long time, subsystem (4-5) is stabilized over the pitch and yaw axis ($\eta_2 = 0$, $\eta_3 = 0$), then normalizing the translational subsystem (2-3), it can be written ($P^f = [P_x\ P_y\ P_z]^T$ is the current position)

$$\begin{cases} \dot{p}_1 = p_2 \\ \dot{p}_2 = v_x \end{cases} \qquad (16)$$

$$\begin{cases} \dot{p}_3 = p_4 \\ \dot{p}_4 = -v_h \sin(\eta_1) \\ \dot{p}_5 = p_6 \\ \dot{p}_6 = v_h \cos(\eta_1) - 1 \end{cases} \qquad (17)$$

$p = (p_1, p_2, p_3, p_4, p_5, p_6) = (\overline{P}_x, \overline{V}_x, \overline{P}_y, \overline{V}_y, \overline{P}_z, \overline{V}_z)$ is the averaged state of the translational subsystem, $\overline{x}_t = p$, $v_x = \frac{\overline{f}_x}{mg}$, $v_h = \frac{\overline{f}_z}{mg}$, η_1 the roll angle and 1 is the normalized gravity.
A bounded state feedback control law, calculated using the averaged model over a wingbeat period (equivalent to a rigid body model), is applied to the time variant model in order to drive the MAV's center of mass to a desired position $P_d = (x_d, y_d, z_d)$; the error in position is $(e_x, e_y, e_z) = (p_1 - x_d, p_3 - y_d, p_5 - z_d)$. The proposed controller is extremely low cost for an embedded implementation. Moreover, it is robust to measurement delays and to system model uncertainty (Marchand and Hably [2005]).

Stabilization of the forward movement System (16) defines a double integrator, and can be stabilized using the control developed in (Marchand and Hably [2005]). v_x can then be chosen as

$$v_x = \frac{\overline{v}_x}{\varepsilon_x + \varepsilon_x^2}(-\varepsilon_x \sigma(\dot{e}_x) - \varepsilon_x^2 \sigma(\varepsilon_x e_x + \dot{e}_x)) \qquad (18)$$

where ε_x is a positive parameter lower than 1 and $\sigma(.)$ is a twice differentiable function bounded between ± 1 parameterized by $0 < \mu < 1$ (Hably et al. [2006])

$$\sigma(s) = \begin{cases} -1 & s < -1 - \mu \\ e_1 s^2 + e_2 s + e_3 & s \in [-1-\mu, -1+\mu[\\ s & s \in [-1+\mu, 1+\mu] \\ -e_1 s^2 + e_2 s - e_3 & s \in]1-\mu, 1+\mu] \\ 1 & s > 1 + \mu \end{cases} \qquad (19)$$

with $e_1 = \frac{1}{4\mu}$, $e_2 = \frac{1}{2} + \frac{1}{2\mu}$, $e_3 = \frac{\mu^2 - 2\mu + 1}{4\mu}$ and μ sufficiently small.

$\overline{v}_x$ should respect the saturation bound represented by the set Ω_{f_x,f_z} (see Fig. 3b) in order to guarantee admissible flapping and rotation angles. The asymptotic stability of (p_1, p_2) is then ensured using Marchand and Hably [2005] on the error dynamics.

Stabilization of the lateral and vertical movements System (17) associates the lateral movement of the MAV to a roll movement, inspired from the works on PVTOLs (Planar Taking Off and Landing) aircrafts (Hably et al. [2006]). η_1 is considered as an intermediate input for system (17) and should converge to a desired angle η_{1_d} given by

$$\eta_{1_d} = \arctan(\frac{-v_y}{v_z + 1}) \qquad (20)$$

v_y and v_z will be determined later on.
The vertical normalized lift v_h is given by

$$v_h = \sqrt{v_y^2 + (v_z + 1)^2} \qquad (21)$$

An appropriate control torque as defined in (14) will drive the roll angle η_1 to the desired value η_{1_d} defined by the corresponding quaternion q_d, hence system (17) will be transformed into the form of two independent second order integrators (Hably et al. [2006]).

$$\begin{cases} \dot{p}_3 = p_4 \\ \dot{p}_4 = v_y \end{cases} \qquad \begin{cases} \dot{p}_5 = p_6 \\ \dot{p}_6 = v_z \end{cases} \qquad (22)$$

Therefore, the stability of the lateral and vertical movements can be ensured using the following control law:

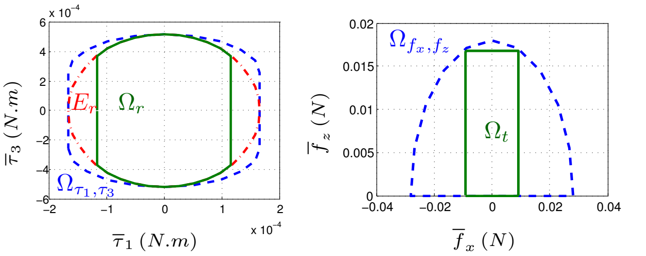
$$v_y = \frac{\overline{v}_y}{\varepsilon_y + \varepsilon_y^2}(-\varepsilon_y \sigma(\dot{e}_y) - \varepsilon_y^2 \sigma(\varepsilon_y e_y + \dot{e}_y)) \qquad (23)$$

$$v_z = \frac{\overline{v}_z}{\varepsilon_z + \varepsilon_z^2}(-\varepsilon_z \sigma(\dot{e}_z) - \varepsilon_z^2 \sigma(\varepsilon_z e_z + \dot{e}_z)) \qquad (24)$$

$0 < \varepsilon_y, \varepsilon_z < 1$, and $\sigma(.)$ is defined as in (19). $\overline{v}_y$ and $\overline{v}_z$ are chosen such that:

$$\overline{v}_h = \sqrt{\overline{v}_y^2 + (\overline{v}_z + 1)^2} \qquad (25)$$

and $\overline{v}_h$ should respect the saturation bounds represented by the set Ω_{f_x,f_z} (see Fig. 3b). The asymptotic stability of (p_3, p_4, p_5, p_6) is then ensured using Marchand and Hably [2005], Hably et al. [2006] on the error dynamics.



(a) Rotational saturation set (b) Translational saturation set

Fig. 3. Yaw torque versus roll torque (3a), defining the saturation set Ω_{τ_1,τ_3} approximated to an ellipse E_r then to a set Ω_r. Lift versus thrust (3b), defining the saturation set Ω_{f_x,f_z} approximated to the set Ω_t.

Stability of the translational movement of the time varying system Applying the proposed control law, $(\overline{P}^f - P_d) \rightarrow 0$ and $(\overline{V}^f - V_d) \rightarrow 0$. By means of the averaging theory, the stability of the high frequency flapping insect is guaranteed.

4. SIMULATIONS

The Hymenoptera insect model (Knospe [1998]) is adopted for simulation in the present work. The wingbeat frequency is $100\,Hz$ and the body mass $500\,mg$. The wing surface ($S_w \approx 1.5\,cm^2$) is computed so that a vertical ascendant movement can be achieved with a flapping angle remaining lower than $\overline{\phi}_0 = 50\,°$ (maximum flapping amplitude for Hymenoptera). The rotation angle amplitude is taken to its maximum value $\overline{\psi}_0 = 90\,°$. Based on these numerical values, the saturation sets Ω_{τ_1,τ_3} and Ω_{f_x,f_z} can be determined explicitly (10). Ω_{τ_1,τ_3} has been approximated to the largest ellipse E_r that fits inside Ω_{τ_1,τ_3} (see Fig. 3a) for calculus simplification reasons. Therefore, the control torques $\overline{\tau}_1$ and $\overline{\tau}_3$ should respect an ellipsoidal saturation defined by

$$y^T P y = 1 \tag{26}$$

where $y = (\overline{\tau}_1, \overline{\tau}_3)^T$ and P is a diagonal definite positive matrix representing the ellipse's semi-axes. Practically, to avoid a null yaw/roll control torque in case of saturating the roll/yaw torque, 70% of $\alpha_1 M_{2,1}$ will be attributed to $\overline{\tau}_1$, $\overline{\tau}_3$ will be computed using (26) defining a set Ω_r. This choice is justified by the necessity to bring the MAV to its flat position (horizontal plane) first.
The admissible set of thrust and lift forces Ω_{f_x,f_z} is drawn on Fig. 3b. Since $\overline{f}_z$ will be decomposed in mgv_y and mgv_z, rectangular saturation is chosen inside Ω_{f_x,f_z} in order to pass up of the computation of the bounds at each iteration. Saturation bounds are calculated so that more power is given to the lateral movement since it is associated to the roll movement: the MAV can be brought to the horizontal plane rapidly. Set Ω_t is then obtained.
The predefined trajectory presented in this work is a helix. The MAV should follow this path to reach a desired position and stay there in hovering mode. The proposed control is tested with respect to external disturbances (forces of $(2.10^{-3}, 10^{-5}, 8.10^{-3})$N and torques of $(10^{-5}, 10^{-6}, 10^{-5})$N.m) applied at $t = 20\,s$ during 5 wigbeats periods. Figure 4 shows the convergence of desired and current trajectories in 3D. Figure 5 shows the linear movements, velocities and control forces. The roll, pitch and yaw angles, angular velocities and control torques are plotted on figure 6.

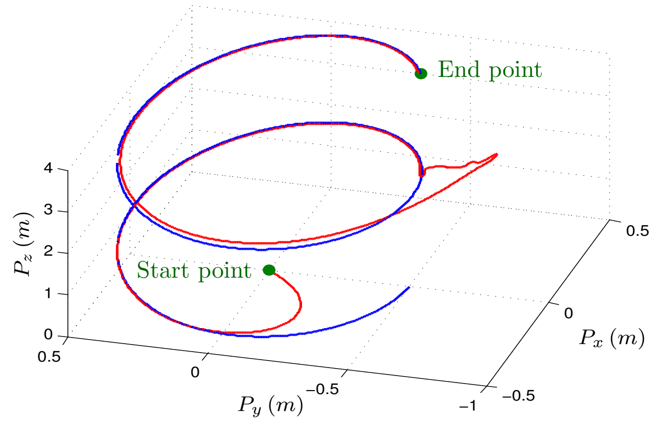


Fig. 4. The linear movement of the MAV in 3D (red), going from a start point to an end point, tracking a desired trajectory (blue), in presence of external disturbances applied at $t = 20\,s$ during 5 wingbeats.

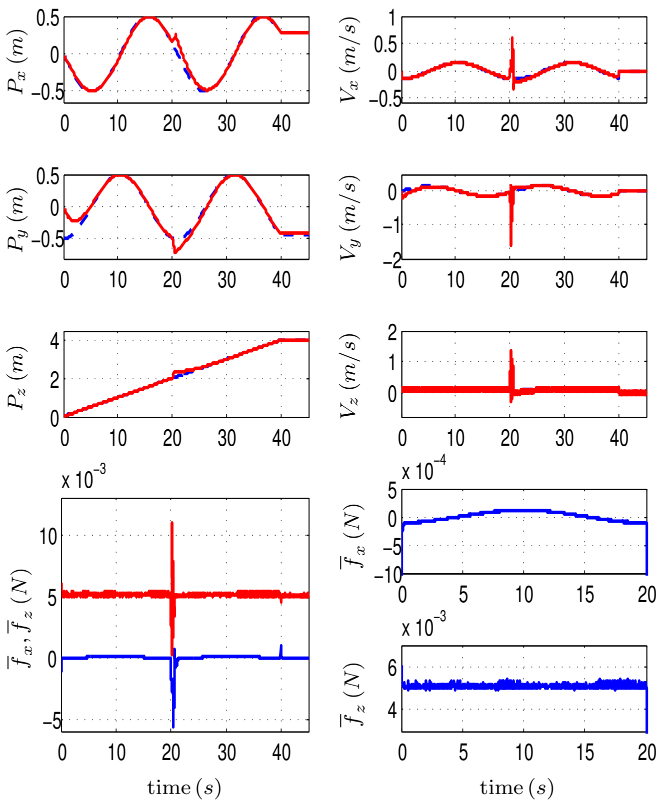


Fig. 5. The linear displacements, velocities (current in red and desired in blue) and corresponding control forces in presence of external disturbances applied at $t = 20\,s$ during 5 wingbeats. The control forces zoomed to $20\,s$ to show the shape of the curves (bottom right).

5. CONCLUSIONS AND FUTURE WORKS

A bounded nonlinear control law has been presented in this paper in order to control the trajectory in 3D of a flapping MAV and to stabilize it in hovering mode at the end of the path. The control is based on the theory of cascade, aiming to globally stabilize the attitude of the MAV while driving the body to the desired position. The proposed control law takes into account the saturation of the actuators driving the flapping wings. It is robust with respect to external disturbances, body's inertia, etc. Moreover, it has a low computational cost, therefore it is suitable for an embedded implementation. The proposed control will be tested with respect to some modeling errors, parameters uncertainties, etc.

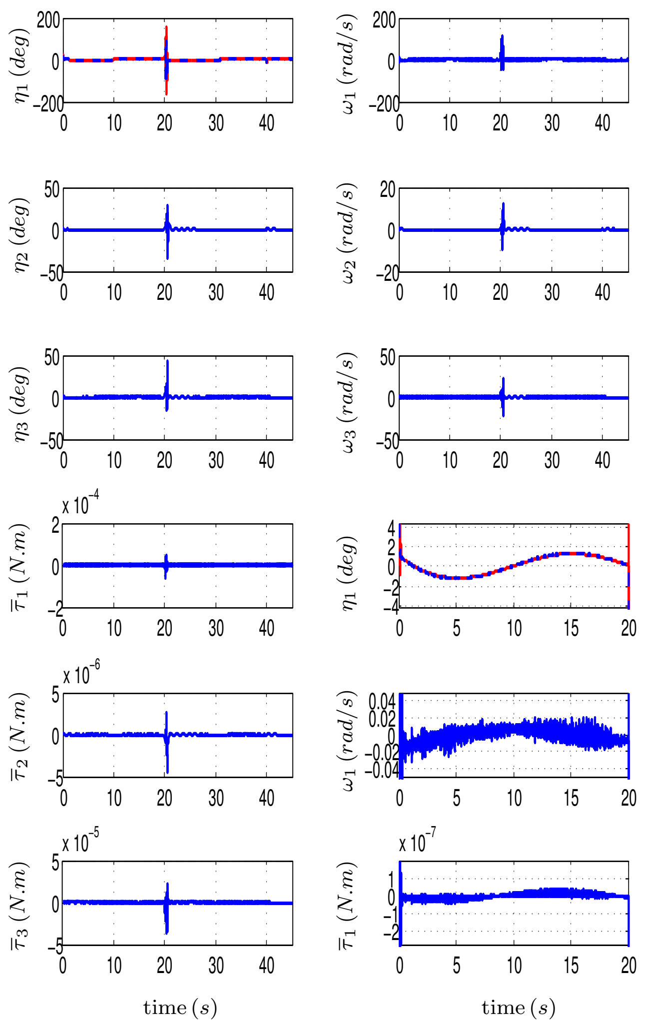


Fig. 6. The roll, pitch and yaw angles, angular velocities and control torques in presence of external disturbances applied at $t = 20\,s$ during 5 wingbeats. The roll angle (current in red and desired in blue), angular velocity and control torque zoomed to 20 s to show the sinusoidal shape of the curves (bottom right).

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