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Frequency Identification of Biased Harmonic Disturbance

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Abstract: The paper is dedicated to problem of unknown frequency identification of an unmeasured biased harmonic disturbance $\bar{y}(t)=\bar{\sigma}_0+\bar{\sigma}\sin(\bar{\omega}t+\bar{\phi})$ affecting a nonlinear system. Unlike known analogues, this approach allows to regulate time of estimation of unknown frequency $\bar{\omega}$.

1. INTRODUCTION

This paper deals with problem of frequency identification of an unmeasured harmonic disturbance $\bar{y}(t)=\bar{\sigma}_0+\bar{\sigma}\sin(\bar{\omega}t+\bar{\phi})$ affecting a nonlinear system; $\bar{\sigma}_0$, $\bar{\sigma}$, $\bar{\phi}$ are unknown constant values. Problem of frequency identification of a sinusoidal signal is a very important basic problem, which has different applications in theoretical and engineering disciplines, see (Clarke, 2001). Today there are many different approaches to identification of unknown frequency of a sinusoidal function, see (Bodson and Douglas, 1997; Hsu, *et al.*, 1999; Mojiri and Bakhshai, 2004; Marino and Tomei, 2002; Xia, 2002; Obregón-Pulido, *et al.*, 2002; Bobtsov, *et al.*, 2002; Hou, 2005). Let us note that today approaches to frequency identification are not limited with studying the case of a single sinusoid, see (Bodson and Douglas, 1997; Hsu, *et al.*, 1999; Mojiri and Bakhshai, 2004). In particular, papers (Hou M., 2005; Aranovskiy, *et al.*, 2007; Bobtsov, 2007) consider problem of frequency identification of a biased sinusoidal signal, and papers (Marino and Tomei, 2002; Xia, 2002; Obregón-Pulido, *et al.*, 2002; Bobtsov, *et al.*, 2002) show common case of a harmonic signal, which is a sum of *n* sinusoidal functions with different frequencies.

Algorithm proposed in Aranovskiy, *et al.*, 2007 has dynamic order equal to three, and in its turn, that is better than the most known results, published in (Marino and Tomei, 2002; Xia, 2002; Obregón-Pulido, *et al.*, 2002; Bobtsov, *et al.*, 2002; Bobtsov, 2007; Hou, 2005). In (Xia, 2002; Obregón-Pulido, *et al.*, 2002; Bobtsov, *et al.*, 2002; Bobtsov, 2007; Hou, 2005) minimal dimension of dynamic order of the algorithm is four, and in (Marino and Tomei, 2002) dimension of the algorithm amounts to nine. Besides, algorithm of identification, proposed in Aranovskiy, *et al.*, 2007 allows to regulate rate of convergence of tuned parameter (estimation of frequency of signal $\bar{y}(t)=\bar{\sigma}_0+\bar{\sigma}\sin(\bar{\omega}t+\bar{\phi})$).
This paper develops result of Aranovskiy, *et al*, 2007 for the case of frequency identification of an unmeasured harmonic disturbance $\bar{y}(t)=\bar{\sigma}_0+\bar{\sigma}\sin(\bar{\omega}t+\bar{\phi})$ presenting in a nonlinear system.

2. PROBLEM STATEMENT

Consider nonlinear system of the form

$$g(t)=\frac{b(p)}{a(p)}u(t)+\frac{d(p)}{a(p)}f(g(t))+\frac{c(p)}{a(p)}\bar{y}(t) \qquad (1)$$

where p is differentiation operator; $g(t)$ is output; $u(t)$ is control; $f(g(t))$ is nonlinearity; $\bar{y}(t)$ is unknown biased harmonic signal; $a(p)$ is unstable polynomial, $\deg a(p)=n$; $b(p)$, $d(p)$, $c(p)$ are stable polynomials; degrees of polynomials $b(p)$, $d(p)$, $c(p)$ are less then n; coefficients of polynomials $a(p)$, $b(p)$, $d(p)$ and $c(p)$ are known; $g(t)$, $u(t)$ and $f(g)$ are known; $\bar{y}(t)=\bar{\sigma}_0+\bar{\sigma}\sin(\bar{\omega}t+\bar{\phi})$ is harmonic disturbance with unknown bias $\bar{\sigma}_0$, amplitude $\bar{\sigma}$, frequency $\bar{\omega}$ and phase $\bar{\phi}$.

Let us formulate purpose of control as design of identification algorithm, which should ensure realization of condition

$$\lim_{t\to\infty}\left|\bar{\omega}-\hat{\bar{\omega}}(t)\right|=0, \qquad (2)$$

where $\hat{\bar{\omega}}(t)$ is a current estimation of parameter $\bar{\omega}$ for any $\bar{\sigma}_0$, $\bar{\sigma}$, $\bar{\phi}$ and $\bar{\omega}>0$.

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Passing to Laplace images in (1) we obtain

$$G(s)=\frac{B(s)}{A(s)}U(s)+\frac{D(s)}{A(s)}F(s)+ +\frac{C(s)}{A(s)}Y(s)+\frac{H(s)}{A(s)} \quad (3)$$

where s is complex variable, $G(s)=L\{g(t)\}$, $U(s)=L\{u(t)\}$, $F(s)=L\{f(g(t))\}$, $Y(s)=L\{\bar{y}(t)\}$, are Laplace images of functions $g(t)$, $u(t)$, $f(g(t))$, $\bar{y}(t)$ respectively, polynomial $H(s)$ denotes sum of all terms containing nonzero initial conditions.
Let us transform model (3) the following way

$$(s+\lambda)^n G(s)=a_1(s)G(s)+B(s)U(s)+ +D(s)F(s)+C(s)Y(s)+H(s),$$

whence

$$G(s)=\frac{a_1(s)}{(s+\lambda)^n}G(s)+\frac{B(s)}{(s+\lambda)^n}U(s)+ +\frac{D(s)}{(s+\lambda)^n}F(s)+\frac{C(s)}{(s+\lambda)^n}Y(s)+\frac{H(s)}{(s+\lambda)^n}, \quad (4)$$

where parameter $\lambda>0$ и $A(s)=(s+\lambda)^n-a_1(s)$.
After inverse Laplace transform equation (4) takes the form

$$g(t)=\frac{a_1(p)}{(p+\lambda)^n}g(t)+\frac{b(p)}{(p+\lambda)^n}u(t)+ +\frac{d(p)}{(p+\lambda)^n}f(g(t))+\frac{c(p)}{(p+\lambda)^n}\bar{y}(t)+\varepsilon_y(t), \quad (5)$$

where $\varepsilon_y(t)=L^{-1}\left\{\frac{H(s)}{(s+\lambda)^n}\right\}$ is exponentially decaying function of time caused by nonzero initial conditions, and it is possible to accelerate its convergence to zero by increasing parameter λ.
Neglecting exponentially decaying item $\varepsilon_y(t)=L^{-1}\left\{\frac{H(s)}{(s+\lambda)^n}\right\}$, let us parameterize model (5).
Consider auxiliary filters of the following form

$$\upsilon_1(t)=\frac{1}{(p+\lambda)^n}g(t), \quad (6)$$

$$\upsilon_2(t)=\frac{1}{(p+\lambda)^n}u(t), \quad (7)$$

$$\upsilon_3(t)=\frac{1}{(p+\lambda)^n}f(g(t)), \quad (8)$$

Substituting (6)-(8) into (5), we obtain

$$g(t)=a_1(p)\upsilon_1(t)+b(p)\upsilon_2(t)+d(p)\upsilon_3(t)+y(t), \quad (9)$$

where $y(t)=\frac{c(p)}{(p+\lambda)^n}\bar{y}(t)$.
Assumption. Polynomial $c(p)$ does not have pure imaginary roots $\pm j\bar{\omega}$.
For (9) we have

$$g(t)=z(t)+y(t), \quad (10)$$

where $z(t)=a_1(p)\upsilon_1(t)+b(p)\upsilon_2(t)+d(p)\upsilon_3(t)$.
From (10) we have

$$y(t)=g(t)-z(t), \quad (11)$$

Consider signal $y(t)=\frac{c(p)}{(p+\lambda)^n}\bar{y}(t)$. As polynomial $(p+\lambda)^n$ is Hurwitz and $\bar{y}(t)=\bar{\sigma}_0+\bar{\sigma}\sin(\bar{\omega}t+\bar{\phi})$, signal $y(t)$ can be rewritten the following way: $y(t)=\sigma_0+\sigma\sin(\omega t+\phi)$ and $\omega=\bar{\omega}$. So, problem of frequency identification of the signal $\bar{y}(t)=\bar{\sigma}_0+\bar{\sigma}\sin(\bar{\omega}t+\bar{\phi})$ can be turned into problem of frequency identification of measured biased harmonic signal

$$y(t)=g(t)-z(t)=\sigma_0+\sigma\sin(\omega t+\phi), \quad (12)$$

and new purpose of control is

$$\lim_{t\to\infty}\left|\omega-\hat{\omega}(t)\right|=0, \quad (13)$$

where $\hat{\omega}(t)$ is a current estimation of parameter ω for any σ_0, σ, ϕ and $\omega>0$.

3. MAIN RESULT

It is known that for generating of signal (12) it is possible to use differential equation of the view (3), see Aranovskiy, *et al.*, 2007

$$\dddot{y}(t)=-\omega^2\dot{y}(t)=\theta\,\dot{y}(t), \quad (14)$$

where $\theta=-\omega^2$ is a constant parameter.

Lemma. Consider an auxiliary second-order filter

$$\begin{cases}\dot{\varsigma}_1(t)=\varsigma_2(t),\\ \dot{\varsigma}_2(t)=-2\alpha\varsigma_2(t)-\alpha^2\varsigma_1(t)+y(t),\\ \varsigma(t)=\varsigma_1(t)\end{cases} \quad (15)$$

or

$$\varsigma(t) = \frac{1}{(p+\alpha)^2} y(t) , \quad (16)$$

where p is differentiation operator and number $\alpha > 0$.
Then differential equation (14) can be rewritten in the form

$$\dot{y}(t) = 2\alpha\ddot{\varsigma}(t) + \alpha^2\dot{\varsigma}(t) + \theta\dot{\varsigma}(t) + \varepsilon_y(t) , \quad (17)$$

where $\varepsilon_y(t)$ is exponentially decaying function of time caused by nonzero initial conditions.
Proof. After Laplace transform of (14) we obtain

$$sY(s) = \frac{s}{(s+\alpha)^2}\theta Y(s) + \frac{2\alpha s^2 + \alpha^2 s}{(s+\alpha)^2} Y(s) + + \frac{D(s)}{(s+\alpha)^2} , \quad (18)$$

where s is complex variable, $Y(s) = L\{y(t)\}$ is Laplace image of signal $y(t)$, and polynomial $D(s)$ denotes sum of all terms, containing nonzero initial conditions.
From (18) we find

$$\dot{y}(t) = \frac{p}{(p+\alpha)^2}\theta\, y(t) + \frac{2\alpha p^2 + \alpha^2 p}{(p+\alpha)^2} y(t) + +\varepsilon_y(t) , \quad (19)$$

where exponentially decaying function of time $\varepsilon_y(t) = L^{-1}\{D(s)/(s+\alpha)^2\}$ is determined by nonzero initial conditions.
Substituting (16) into (19) we obtain

$$\dot{y}(t) = 2\alpha\ddot{\varsigma}(t) + \alpha^2\dot{\varsigma}(t) + \theta\dot{\varsigma}(t) + \varepsilon_y(t) ,$$

which was to be proved.

Remark 1. As exponentially decaying function $\varepsilon_y(t) = L^{-1}\{D(s)/(s+\alpha)^2\}$ depends on parameter α, it is possible to accelerate convergence of $\varepsilon_y(t)$ to zero by increasing α.

Now, on base of lemma results one can formulate scheme of unknown parameter θ identification. First let us suppose that function $\dot{y}(t)$ is measured. Then, neglecting exponentially decaying item $\varepsilon_y(t)$, ideal identification algorithm can be written the following way

$$\dot{\hat{\theta}}(t) = k\dot{\varsigma}^2(t)(\theta - \hat{\theta}(t)) = k\dot{\varsigma}(t)\,\xi(t) - k\dot{\varsigma}^2(t)\hat{\theta}(t) ,$$
$$\hat{\omega}(t) = \sqrt{\left|\hat{\theta}(t)\right|} , \quad (20)$$

where function $\xi(t) = \dot{y}(t) - 2\alpha\ddot{\varsigma}(t) - \alpha^2\dot{\varsigma}(t)$ and number $k > 0$.
The following statement proves efficiency of ideal identification algorithm for achieving purpose (2).

Proposition. Let algorithm of identification of unknown parameter θ have the view

$$\dot{\hat{\theta}}(t) = k\dot{\varsigma}^2(t)(\theta - \hat{\theta}(t)) ,$$

where number $k > 0$, and function $\varsigma(t)$ is solution of differential equation (15).
Then purpose of the view (13) is achieved.

Proof of the proposition. Consider estimation error of parameter θ of the following form

$$\tilde{\theta}(t) = \theta - \hat{\theta}(t) . \quad (21)$$

After differentiation of equation (21) we have

$$\dot{\tilde{\theta}} = \dot{\theta} - \dot{\hat{\theta}}(t) = 0 - k\dot{\varsigma}^2(t)\tilde{\theta}(t) = -k\dot{\varsigma}^2(t)\tilde{\theta}(t) . \quad (22)$$

Solving differential equation (22) we obtain

$$\tilde{\theta}(t) = \tilde{\theta}(t_0)e^{-k\gamma(t,t_0)} , \quad (23)$$

where function

$$\gamma(t,t_0) = \int_{t_0}^{t} \dot{\varsigma}^2(\tau)d\tau . \quad (24)$$

It is obvious that as polynomial $(p+\alpha)^2$ is Hurwitz, function $\varsigma(t)$ takes the view

$$\varsigma(t) = \tilde{\sigma}_0 + \tilde{\sigma}\sin(\omega t + \tilde{\phi}) + \Delta , \quad (25)$$

where $\tilde{\sigma}_0$, $\tilde{\sigma}$ and $\tilde{\phi}$ are constant coefficients depending on parameters of signal $y(t) = \sigma_0 + \sigma\sin(\omega t + \phi)$ and number α; Δ is an exponentially decaying item, caused by transients.
Let us neglect Δ, then after differentiating (25) we obtain

$$\dot{\varsigma}(t) = \tilde{\sigma}\omega\cos(\omega t + \tilde{\phi}) .$$

Substituting $\dot{\varsigma}(t) = \tilde{\sigma}\omega\cos(\omega t + \tilde{\phi})$ into (24) we have

$$\gamma(t,t_0) = \int_{t_0}^{t} \dot{\varsigma}^2(\tau)d\tau = \tilde{\sigma}^2\omega^2 \int_{t_0}^{t} (\cos(\omega\tau + \tilde{\phi}))^2 d\tau =$$

$$= \frac{\tilde{\sigma}^2\omega^2 t}{2} - \frac{\tilde{\sigma}^2\omega^2 t_0}{2} + \frac{\tilde{\sigma}^2\omega^2 \sin(2\omega t + 2\tilde{\phi})}{4\omega} - \\ - \frac{\tilde{\sigma}^2\omega^2 \sin(2\omega t_0 + 2\tilde{\phi})}{4\omega} = \gamma_0 t + \gamma_1(t,t_0) , \quad (26)$$

where function

$$\gamma_1(t,t_0) = -\frac{\tilde{\sigma}^2\omega^2 t_0}{2} + \frac{\tilde{\sigma}^2\omega^2 \sin(2\omega t + 2\tilde{\phi})}{4\omega} - \\ - \frac{\tilde{\sigma}^2\omega^2 \sin(2\omega t_0 + 2\tilde{\phi})}{4\omega}$$

is bounded for any *t*, and number $\gamma_0 = \frac{\tilde{\sigma}^2\omega^2}{2}$.
Let us substitute (26) into (23)

$$\tilde{\theta}(t) = \tilde{\theta}(t_0) e^{-k\gamma_0 t} e^{-k\gamma_1(t,t_0)} . \quad (27)$$

It follows from (27) that $\lim_{t\to\infty} \tilde{\theta} = 0$, and hence $\hat{\omega}(t) = \sqrt{|\hat{\theta}(t)|} \to \omega(t)$ for $t \to \infty$. Proposition is proven.

Remark 2. It follows from equation (27) that function $\hat{\theta}(t)$ converges to parameter θ faster as coefficient *k* is increased. It means that change of coefficient *k* leads to reducing or increasing of convergence rate of the tuned parameter to its real value in identification algorithm (20).

However, in our case only signal *y(t)* is measured but not its derivatives. To derive realizable scheme of identification algorithm let us consider the following variable

$$\chi(t) = \hat{\theta}(t) - k\dot{\varsigma}(t) y(t) . \quad (28)$$

Differentiating (28) we obtain

$$\dot{\chi}(t) = \dot{\hat{\theta}}(t) - k\ddot{\varsigma}(t) y(t) - k\dot{\varsigma}(t)\dot{y}(t) = \\ = k\dot{\varsigma}(t)(\dot{y}(t) - 2\alpha\ddot{\varsigma}(t) - \alpha^2\dot{\varsigma}(t)) - k\dot{\varsigma}^2(t)\hat{\theta}(t) - \\ -k\ddot{\varsigma}(t) y(t) - k\dot{\varsigma}(t)\dot{y}(t) = \\ = k\dot{\varsigma}(t)(-2\alpha\ddot{\varsigma}(t) - \alpha^2\dot{\varsigma}(t)) - k\dot{\varsigma}^2(t)\hat{\theta}(t) - \\ -k\ddot{\varsigma}(t) y(t) . \quad (29)$$

From equations (28), (29) we receive realizable identification algorithm of the following view

$$\dot{\chi}(t) = k\dot{\varsigma}(t)(-2\alpha\ddot{\varsigma}(t) - \alpha^2\dot{\varsigma}(t)) - k\dot{\varsigma}^2(t)\hat{\theta}(t) - \\ -k\ddot{\varsigma}(t) y(t) , \quad (30)$$

$$\hat{\theta}(t) = \chi(t) + k\dot{\varsigma}(t) y(t) , \quad (31)$$

$$\begin{cases} \dot{\varsigma}_1(t) = \varsigma_2(t), \\ \dot{\varsigma}_2(t) = -2\alpha\varsigma_2(t) - \alpha^2\varsigma_1(t) + y(t), \\ \varsigma(t) = \varsigma_1(t). \end{cases} \quad (32)$$

4. EXAMPLE

Consider nonlinear system described by Duffing equation (see for instance Fradkov *et al.,* 1997)

$$\ddot{g}(t) + a_1\dot{g}(t) + a_0 g(t) - d_0 f(g) - c_0\bar{y}(t) = b_0 u(t) , \quad (33)$$

where a_1, a_0, d_0, c_0 and b_0 are known numbers, nonlinear function $f(g(t)) = g^3(t)$ and $\bar{y}(t) = \bar{\sigma}_0 + \bar{\sigma}\sin(\bar{\omega}t + \bar{\phi})$ is unmeasured biased harmonic signal.
Let us use filters of the view (6)-(8) to generate signal $z(t) = a_1(p)\upsilon_1(t) + b(p)\upsilon_2(t) + d(p)\upsilon_3(t)$

$$\upsilon_1(t) = \frac{1}{(p+5)^n} g(t) , \quad (34)$$

$$\upsilon_2(t) = \frac{1}{(p+5)^n} u(t) , \quad (35)$$

$$\upsilon_3(t) = \frac{d(p)}{(p+5)^n} f(g(t)) . \quad (36)$$

Frequency estimation of signal $y(t) = g(t) - z(t)$ (11) is fulfilled by algorithm (20), (30)-(32).
Results of computer simulation for different values of parameters of model (33) are shown in Fig. 1-Fig. 6.
Fig. 1-Fig. 3 show simulation results for $a_1 = 0$, $a_0 = -0.75$, $d_0 = -0.75$, $c_0 = 1$, $b_0 = 1$, $u(t) = 2$, $\bar{y}(t) = 1 + 7\sin(5t)$, $g(0) = 1$, $\dot{g}(0) = 0$, $\varsigma(0) = \dot{\varsigma}(0) = 0$.

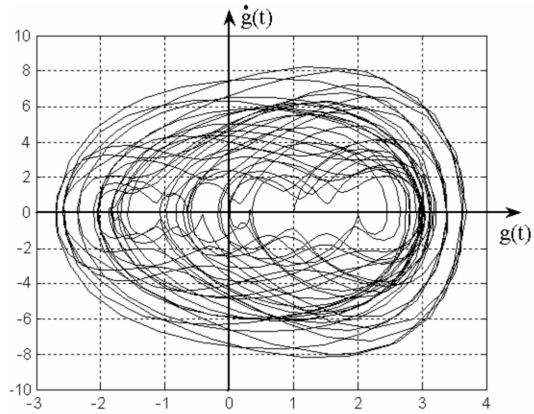

Fig. 1. Phase-plane portrait of system (33)

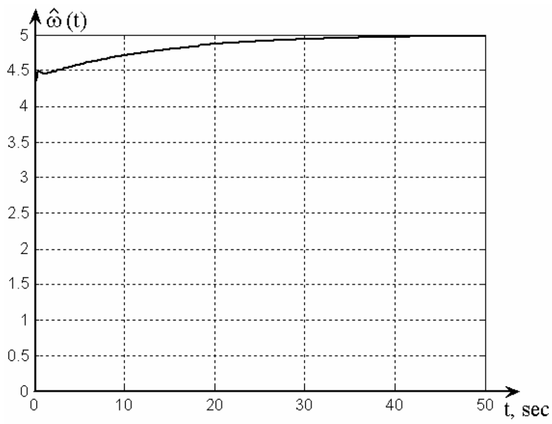

Fig. 2. Frequency estimation ($\hat{\omega}(t)$) for parameter $k = 750$

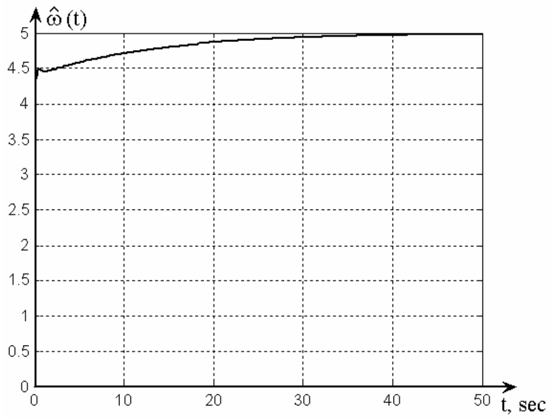

Fig. 3. Frequency estimation ($\hat{\omega}(t)$) for parameter $k = 1000$

Fig. 4-Fig. 6 show simulation results for $a_1 = 0.4$, $a_0 = -1.1$, $d_0 = -1$, $c_0 = 1$, $b_0 = -1$, $u(t) = 1$, $\overline{y}(t) = 1 + 1.8\sin(1.8t)$, $g(0) = 1$, $\dot{g}(0) = 0$, $\varsigma(0) = \dot{\varsigma}(0) = 0$.

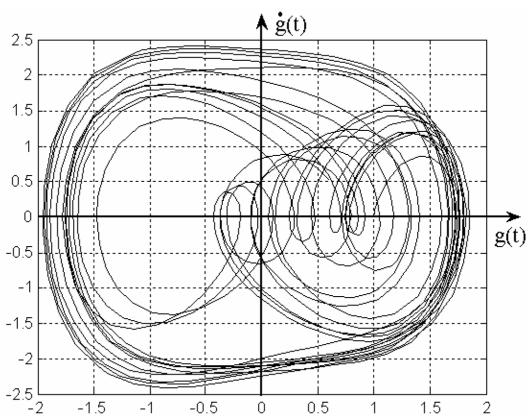

Fig. 4. Phase-plane portrait of system (33)

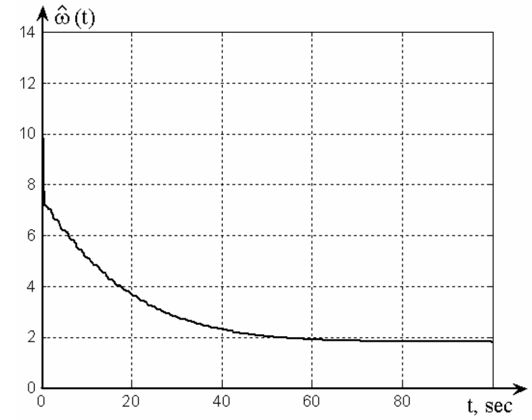

Fig. 5. Frequency estimation ($\hat{\omega}(t)$) for parameter $k = 10000$

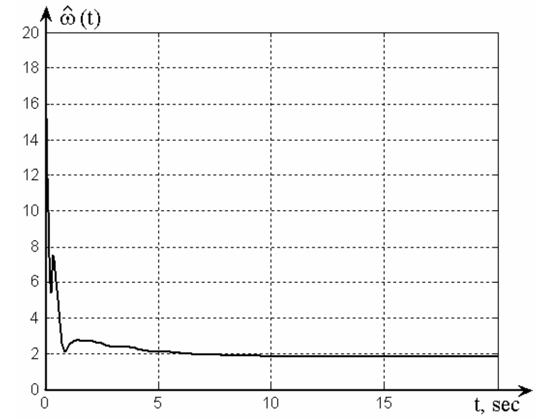

Fig. 6. Frequency estimation ($\hat{\omega}(t)$) for parameter $k = 50000$

Simulation results show that problem of frequency identification of unknown harmonic signal $\overline{y}(t) = \overline{\sigma}_0 + \overline{\sigma}\sin(\overline{\omega}t + \overline{\phi})$ has been solved. Time required for identification of unknown harmonic signal can be reduced by increasing parameter k of algorithm (30)-(32).

5. CONCLUSION

Problem of frequency identification of an unmeasured harmonic disturbance $\overline{y}(t) = \overline{\sigma}_0 + \overline{\sigma}\sin(\overline{\omega}t + \overline{\phi})$ has been considered for any unknown constant values $\overline{\sigma}_0$, $\overline{\sigma}$, $\overline{\phi}$, $\overline{\omega} > 0$. Designed algorithm of identification (6)-(8), (30)-(32):

– has been shown to allow accelerating rate of convergence of estimate $\hat{\theta}(t)$ to θ thanks to increasing coefficient k (see remarks 1 and 2 and example);
– does not require measurements of the disturbance $\overline{y}(t)$.

REFERENCES

Aranovskiy S., Bobtsov A., Kremlev A., Nikolaev N., Slita O. (2007). Identification of frequency of biased harmonic signal. *9th IFAC Workshope "Adaptation and Learning in Control and Signal processing"*. Saint-Petersburg, Russia.

Bobtsov A., Lyamin A., Romasheva D. (2002). Algorithm of parameter's identification of polyharmonic function. *15th IFAC World Congress on Automatic Control*. Barcelona, Spain.

Bobtsov A. (2007). New approach to the problem of globally convergent frequency. *International Journal of Adaptive Control and Signal Processing*, DOI: 10.1002/acs.971.

Bodson M., Douglas S.C. (1997). Adaptive algorithms for the rejection of periodic disturbances with unknown frequencies. *Automatica*, **33**, 2213-2221.

Clarke D. W. (2001). On the design of adaptive notch filters. *Int. J. Adapt. Control*, **15**, 715–744.

Fradkov A.L., Druzhinina M.V. (1997). Reduced order shunt nonlinear adaptive controller. *Proc. 4th European Control Conference*. Brussels.

Hou M. (2005). Amplitude and frequency estimator of a sinusoid. *IEEE Transactions on Automatic Control*, **50**, 855–858.

Hsu L., Ortega R., Damm G. (1999). A globally convergent frequency estimator. *IEEE Transactions on Automatic Control*, **46**, 967-972.

Mojiri M. and Bakhshai A. R. (2004). An Adaptive Notch Filter for Frequency Estimation of a Periodic Signal. *IEEE Transactions on Automatic Control*, **49**, 314-318.

Marino R. and Tomei R. (2002). Global Estimation of Unknown Frequencies. *IEEE Transactions on Automatic Control*, **47**, 1324–1328.

Obregón-Pulido G., Castillo-Toledo B. and Loukianov A. A. (2002). Globally Convergent Estimator for n–Frequencies. *IEEE Transactions on Automatic Control*, **47**, 857–863.

Xia X. (2002). Global Frequency Estimation Using Adaptive Identifiers. *IEEE Transactions on Automatic Control*, **47**, 1188–1193.