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# Robustness Issues Related to the Application of Distributed Model Predictive Control Strategies

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**Abstract:** In this paper, a methodology is proposed to address robustness aspects related to the application of distributed model predictive control. Two problems are studied: the *decomposition problem* and the *coordination problem* in the presence of model errors. Three different MPC strategies are considered: centralized, fully decentralized, and Nash equilibrium based MPC. The methodology requires the computation of closed-loop system's variability via the solution of generalized eigenvalue problem which is formulated as a finite set of linear matrix inequalities. To select the best model decomposition or control strategy based on robust performance, the worst variability for each candidate is minimized by manipulating the input weights of the controller. Two case studies are presented to illustrate the application of the methodology.

## 1. INTRODUCTION

Model predictive control MPC is a widely accepted technology for the control of multivariable processes in the process industry (Qin and Badgwell, 2003). A key drawback of MPC is related to the high computational effort required when dealing with processes with relatively a large number of inputs and outputs. These intensive computations are due to the need to solve a large optimization problem on-line. An additional drawback with the use of fully centralized controllers for high dimensional processes is their low resilience with respect to partial equipment failure or partial plant shutdowns (Li *et al.*, 2005). To deal with these drawbacks, engineers have generally resorted to partitioning the original process inputs and outputs into smaller subsystems and have applied MPC controllers to each one of these subsystems. The operations of several MPC controllers in such fashion have been referred to in the literature as Distributed MPC strategies. However, although distributed MPC applications result in less computation, when the individual MPC controllers for the different subsystems are operated in a completely decentralized fashion, closed loop performance may be significantly reduced since interactions between variables are ignored. In order to account for these interactions researchers have proposed the use of some form of coordination between the MPC controllers for the different subsystems. Coordination strategies based on Nash equilibrium (Li *et al.,* 2005) or cooperative schemes based on weighted cost functions have been reported (Venkat, 2006). Common to these coordination strategies is that they required exact knowledge of the process models to provide the designed optimal or near optimal closed loop performance. However, in reality, linear models are never accurate due to nonlinearity or inaccurate identification. The robustness of distributed control strategies to model error has been identified as one of the major factors for the successful application of distributed MPC strategies (Rawlings and Stewart, 2007).

This paper proposes the application of robust control tools to address two different aspects of the application of distributed MPC control strategies in the presence of model error or uncertainty: (1) *The decomposition problem*: This part of the study consists of searching for the best partitioning of the original process model into sub-systems that will result in the best closed loop robust performance, i.e. performance in the presence of model error. In previous studies this decomposition has been done in an ad-hoc fashion. An example of a multi-unit process is used as a case study. (2) *The coordination problem*: This part of the study addresses the sensitivity of strategies with different degrees of coordination to model error. A high purity distillation column is used as a case study. Three strategies are used and compared to one another: i- centralized control, ii- fully decentralized control and iii- a coordinated distributed control based on *Nash equilibrium*. For the current preliminary study, constraints have not been considered. This topic is left for future work. The different strategies are compared on the basis of the computation of a performance index obtained from a generalized eigenvalue problem (GEVP) in the presence of model error. Each control strategy is optimized based on this index with respect to the manipulated variables weights of the MPC controllers to permit a comparison based on the best possible controllers.

## 2. DEFINITIONS AND METHODOLOGY

### *2.1 Process Model*

In this work, it is assumed that the nominal model of the process used by the MPC is a discrete linear time-invariant (LTI) state-space model of the following form:

$$x(k+1) = Ax(k) + Bu(k) \quad (1)$$

$$y(k) = Cx(k) \quad (2)$$

where $x(k) \in \Re^{nx}$ is an *nx*-dimensional state vector; $u(k) \in \Re^{nu}$ is an *nu*-dimensional input vector; $y(k) \in \Re^{ny}$ is an *ny*-dimensional output vector; $A \in \Re^{nx \times nx}$ is the state matrix, $B \in \Re^{nx \times nu}$ is the input matrix, and $C \in \Re^{ny \times nx}$ is the measurement matrix; *k* is the time interval. In addition, due to model uncertainty, it is assumed that the actual process to be controlled is as follows:

$$x_p(k+1) = A_p x_p(k) + B_p u(k) \quad (3)$$

$$y_p(k) = C_p x_p(k) \quad (4)$$

The number of states in (3)-(4), $nx_p$, can be different from the number of states *nx* in (1)-(2) but the number of inputs and outputs is the same. The models are assumed to be available and obtaining such models is out of the scope of this paper.

### *2.2 MPC Strategies*

Three MPC strategies are considered herein; *viz.*, centralized control and distributed strategies: i-Nash-based distributed MPC, and ii- fully decentralized MPC. A brief description of their structure follows in order.

#### *2.2.1 Centralized MPC*

In the present study, the centralized MPC is based on the formulation proposed by Maciejowski (2002). The cost function is defined as follows:

$$\min_{\Delta U} J(k) = \left\| Y(k) - T(k) \right\|_Q^2 + \left\| \Delta U(k) \right\|_\lambda^2 \quad (5)$$

where $Y(k) = [y(k+1|k),\ldots,y(k+Hp|k)]^T$ is the vector of predicted outputs; *Hp* is the prediction horizon; $T(k) = [R(k+1),\ldots,R(k+Hp)]^T$ is the vector of set-points; *Q* is the output weights matrix; $\Delta U(k) = [\Delta u(k|k),\ldots, \Delta u(k+Hu-1|k)]^T$; *Hu* is the control horizon; $\Delta u(k|k) = u(k|k)-u(k-1|k-1)$; $\lambda$ is the input weights matrix. The set-point signal $R(k)$ is obtained by filtering the original set-point signal $r(k) = [r_1(k),\ldots,r_{ny}(k)]^T$ according to the following exponential filter:

$$R(k+1) = \alpha R(k) + (1-\alpha) r(k) \quad (6)$$

where $\alpha$ is the filter parameter that is specified by the user based on the desired set-point bandwidth and *r* is assumed to be white noise.

From the nominal model (1)-(2), the predicted output vector is calculated as follows:

$$Y(k) = \Psi x(k) + \Gamma u(k-1) + \Theta \Delta U(k) \quad (7)$$

The matrices $\Psi, \Gamma,$ and $\Theta$ are defined in (Maciejowski, 2002) and are not given here for brevity.

The tracking error vector of the free-response $E(k)$ is defined as:

$$E(k) = T(k) - \Psi x(k) - \Gamma u(k-1) - \Xi(k) \quad (8)$$

where the term $\Xi(k)$ accounts for unmeasured disturbances and/or model errors due to the difference between the nominal model in (1)-(2) and the plant model in (3)-(4) . This difference is assumed to remain constant along the horizon of *Hp* time intervals. Thus, $\Xi(k)$ is defined as follows:

$$\Xi(k) = Lc\left[ y_p(k) - y(k|k) \right] \quad (9)$$

In the above expression, $Lc = \left[ I_{ny}, \cdots, I_{ny} \right]^T_{ny \times Hp}$, where *I* is the identity matrix, and $y(k|k)$ is equal to the output of the model in (2) .

The optimal moves at the current step (k), $\Delta u^*(k|k)$, are calculated from the solution of (5) as follows:

$$\Delta u^*(k|k) = K_{MPC} E(k) \quad (10)$$

where $K_{MPC} = [I_{nu}, 0_{nu}, \ldots, 0_{nu}]_{nu \times Hu} (\Theta^T Q \Theta + \lambda)^{-1} \Theta^T Q$ .

At this point, the closed-loop system of the plant model given in (3)-(4) and the centralized MPC can be obtained. To simplify the notations, it is assumed without loss of generality that $u(k) = u(k|k)$ and $u(k-1) = u(k-1|k-1)$. The resulting closed-loop system is:

$$\begin{bmatrix} x_p(k+1) \\ x(k+1) \\ u(k) \\ \underline{R(k+1)} \\ e(k) \end{bmatrix} = \begin{bmatrix} A_{CL} & B_{CL} \\ C_{CL} & D_{CL} \end{bmatrix} \begin{bmatrix} x_p(k) \\ x(k) \\ u(k-1) \\ \underline{R(k)} \\ r(k) \end{bmatrix} \quad (12)$$

where the closed-loop matrices are defined as follows:

$$A_{CL} = \begin{bmatrix} A_p - B_p K_{MPC} LcC_p & B_p K_{MPC}(LcC - \Psi) \\ -BK_{MPC} LcC_p & A + BK_{MPC}(LcC - \Psi) \\ -K_{MPC} LcC_p & K_{MPC}(LcC - \Psi) \\ 0 & 0 \end{bmatrix}$$

$$\begin{matrix} . & B_p - B_p K_{MPC}\Gamma & B_p K_{MPC} Lc \\ . & B - BK_{MPC}\Gamma & BK_{MPC} Lc \\ . & I_{nu} - K_{MPC}\Gamma & K_{MPC} Lc \\ . & 0 & \alpha I_{ny} \end{matrix} \Bigg];$$

$$B_{CL} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ (1-\alpha) I_{ny} \end{bmatrix};\ C_{CL} = \begin{bmatrix} -Cp \\ 0 \\ 0 \\ I_{ny} \end{bmatrix}^T;\ D_{CL} = [0]$$

### 2.2.2 *Distributed MPC: Nash-based coordinated and Fully decentralized MPC*

For distributed MPC, the nominal model given in (1)-(2) is decomposed into $N$ subsystems. The model of subsystem $i \in \{1,.....,N\}$ can be written as:

$$x_i(k+1) = A_i x_i(k) + B_{ii} u_i(k) + \sum_{j \neq i} B_{ij} u_j \qquad (13)$$

$$y_i(k) = C_i x_i(k) \qquad (14)$$

where $x_i(k) \in \Re^{nx_i}$ is an $nx_i$-dimensional state vector of the subsystem $i$ including the effect of states from other subsystems; $u_i \in \Re^{nu_i}$ is an $nu_i$-dimensional input vector estimated by the $i^{th}$ MPC assigned to subsystem $i$; $u_j \in \Re^{nu_j}$ is an $nu_j$-dimensional input vector estimated by the $j^{th}$ MPC assigned to subsystem $j$ and affects subsystem $i$; $y_i \in \Re^{ny_i}$ is an $ny_i$-dimensional output vector; $A_i$, $B_{ii}$, $B_{ij}$, and $C_i$ are matrices of appropriate dimensions. For analysis purposes, all equations for the individual subsystems are grouped together to formulate an overall model as follows:

$$x(k+1) = A_o x(k) + B_o u(k) \qquad (15)$$

$$y(k) = C_o x(k) \qquad (16)$$

where $A_o$ = *block-diag*($A_1$,…,$A_N$); $C_o$ = *block-diag*($C_1$,…,$C_N$);

$$B_o = \begin{bmatrix} B_{11} & \cdots & B_{1N} \\ \vdots & \ddots & \vdots \\ B_{N1} & \cdots & B_{NN} \end{bmatrix}.$$

The state vector x and the input vector u are obtained by appending all the state vectors and the input vectors of the $i$ subsystems respectively. For the closed-loop system in (12) of distributed MPC, the matrices $A$, $B$, and $C$ are replaced by $A_o$, $B_o$, and $C_o$; respectively.

The formulation of the Nash-based distributed MPC strategy is generally based on the work reported in (Li *et al.,* 2005). However, since Li's formulation was based on input/output models, in the current work a formulation of the Nash-equilibrium based MPC based on state-space models had to be developed. In the Nash-based MPC, the $\Delta U_i$ manipulated variable action is calculated by minimizing the local cost function of the $i^{th}$ subsystem as follows:

$$\min_{\Delta U_i} J_i(k) = \left\| Y_i(k) - T_i(k) \right\|_Q^2 + \left\| \Delta U_i(k) \right\|_\lambda^2 \qquad (17)$$

The predicted output vector of subsystem $i$ is obtained by solving (15)-(16) recursively and given as:

$$Y_i(k) = \Psi_i x_i(k) + \Gamma_{ii} u_i(k-1) + \Theta_i \Delta U_i(k) + \sum_{j \neq i} \Gamma_{ij} u_j(k-1) + \sum_{j \neq i} \Theta_{ij} \Delta U_j(k) \qquad (18)$$

The matrices $\Psi_i$, $\Gamma_i$, $\Gamma_{ij}$, $\Theta_i$, and $\Theta_{ij}$ can be obtained using the same definitions as in (Maciejowski, 2002) from the model given in (13)-(14).

The tracking-error vector $E_i(k)$ and the optimal solution of (17) $\Delta U_i^*(k)$ for subsystem $i$ are given in the following equations:

$$E_i(k) = T_i(k) - \Psi_i x_i(k) - \Gamma_{ii} u_i(k-1) - \sum_{j \neq i} \Gamma_{ij} u_j(k-1) - \sum_{j \neq i} \Theta_{ij} \Delta U_j(k) - \Xi_i(k) \qquad (19)$$

$$\Delta U_i^*(k) = K_{ii} \left[ \Omega_i(k) - \sum_{j \neq i} \Theta_{ij} \Delta U_j(k) \right] \qquad (20)$$

with $K_{ii} = \left( \Theta_i^T Q_i \Theta_i + \lambda_i \right)^{-1} \Theta_i^T Q_i$ and $\Omega_i(k)$ contains all the right hand side terms in (19) except the term $\sum_{j \neq i} \Theta_{ij} \Delta U_j(k)$.

For numerical convenience, equation (20) is generally solved for large systems by iterations. In the current work a closed form solution derived from (20) is used as follows:

$$\Delta U(k) = (I - K_0)^{-1} K_1 \Omega(k) \tag{21}$$

where $K_0 = \begin{bmatrix} 0 & -K_{11}\Theta_{12} & \cdots & -K_{11}\Theta_{1N} \\ -K_{22}\Theta_{21} & 0 & \cdots & -K_{22}\Theta_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -K_{NN}\Theta_{N1} & -K_{NN}\Theta_{N2} & \cdots & 0 \end{bmatrix}$,

$K_1 = \begin{bmatrix} K_{11} & & 0 \\ & \ddots & \\ 0 & & K_{NN} \end{bmatrix}$.

In equation (21), the term $\Omega(k)$ is similar to $E(k)$ in (8) with:

$T(k) = [T_1(k),...,T_N(k)]^T$ ; $\Psi = block-diag[\Psi_1,\cdots,\Psi_N]$;

$\Gamma = \begin{bmatrix} \Gamma_{11} & \cdots & \Gamma_{1N} \\ \vdots & \ddots & \vdots \\ \Gamma_{N1} & \cdots & \Gamma_{NN} \end{bmatrix}$; $L_C = \begin{bmatrix} 1_{(Hp*ny_1)\times 1} & 0_{(Hp*ny_1)\times(N-1)} \\ & \ddots \quad \ddots \\ 0_{(Hp*ny_N)\times(N-1)} & 1_{(Hp*ny_N)\times 1} \end{bmatrix}$

The current control moves are given by:

$$\Delta u(k) = K_{MPC}\Omega(k) \tag{22}$$

with $K_{MPC} = L(I - K_0)^{-1} K_1$

where $L = block\text{-}diag(L_1,...,L_N)$; $L_i = [1,0,\cdots,0]_{1\times(nu_i Hu)}$.

The aforementioned formulation for Nash-based distributed MPC strategy can be also used to analyze the specific case of fully decentralized MPC where all the interactions are ignored, i.e. all the terms corresponding to the interaction between the subsystems are eliminated in (14). Accordingly, $K_0$ in (22) is omitted. Thus, a closed-loop system representation can be obtained for either a Nash-based MPC or a fully decentralized MPC based on the formulation presented in (12). Once the closed-loop system is formulated for each MPC strategy, a performance index can be calculated as shown in the next section.

### *2.3 Methodology*

The closed-loop system given in (12) can be re-written in the following compact form:

$$\begin{bmatrix} \eta(k+1) \\ e(k) \end{bmatrix} = \begin{bmatrix} A_{CL} & B_{CL} \\ C_{CL} & D_{CL} \end{bmatrix} \begin{bmatrix} \eta(k) \\ r(k) \end{bmatrix} \tag{23}$$

where $\eta(k) = [x_p(k),x(k),u(k-1),R(k)]^T$ is the states vector of the closed-loop system.

Gao and Budman (2005) used a robust performance index to design gain-scheduled PI controllers for nonlinear processes. This index corresponds to the variability of the closed-loop system. In the current study the variability of the system is given as an upper bound of the effect of set-point input signals $r(k)$, defined in (6), on the output error signals $e(k) = R(k) - y(k)$, according to the following inequality:

$$\frac{\|e\|_{\ell_2}}{\|r\|_{\ell_2}} < \gamma \tag{24}$$

A bound on γ can be obtained from the solution of a Generalized Eigenvalue Problem, GEVP (Boyd *et al.*, 1994):

$$\gamma^2_{\min} = \min_P \gamma^2 \tag{25}$$

subject to $\begin{bmatrix} A_{CL}^T P A_{CL} - P & A_{CL}^T P B_{CL} & C_{CL}^T \\ B_{CL}^T P A_{CL} & B_{CL}^T P B_{CL} & D_{CL}^T \\ C_{CL} & D_{CL} & -I \end{bmatrix} < \gamma^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}$

where $P$ is positive definite matrix.

In the presence of model uncertainty, the plant model given in (3)-(4) can be represented by a set of linear plants where the state-space matrices are defined as follows:

$$[A_p\ B_p\ C_p] = \sum_{i=1}^{l} \beta_i [A_{pi}\ B_{pi}\ C_{pi}] \quad ; \sum_{i=1}^{l} \beta_i = 1 \tag{26}$$

Then, the problem (25) has to be solved with $l$ inequalities each corresponding to a model $i$ included in the set of plants defined in (26). To find the best controller, the input weights (λ) are optimized to produce the MPC controller with the best performance as follows:

$$\gamma^2_{opt} = \min_{\lambda} \gamma^2_{\min} \tag{27}$$

All other tuning parameters, such as the prediction and control horizons and the output weights are fixed a priori for simplicity. A smaller value of $\gamma_{opt}$ implies a better closed loop performance following definition (24). The problem in (26) is formulated in MATLAB® and solved using MATLAB® linear matrix inequalities (LMI) toolbox and the problem in (27) is solved using the MATLAB® optimization toolbox. In the present work, for the MPC strategies discussed in the previous section, two different problems can be addressed.

Firstly, the *decomposition problem* in which the objective is to find the best partitioning into subsystems that result in the best closed-loop robust performance. Secondly, the *coordination problem* in the presence of model error is studied by comparing the robust performance in terms of minimum variability for the three MPC strategies discussed in the previous section.

## 3. CASE STUDIES

### *3.1 Decomposition Problem*

Samyudia *et al.*, (1994) studied a multi-unit process composed of two CSTRs connected in series with a perfect separator. The unreacted substance is recycled and fed-back to the first reactor. The following is a continuous-time linearized model:

$$\begin{bmatrix} dC_1/dt \\ dC_2/dt \\ dT_1/dt \end{bmatrix} = \begin{bmatrix} -1.1002 & 0.4463 & 0 \\ 0.6695 & -1.1369 & 0 \\ 11.7337 & 0 & -0.0214 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ T_1 \end{bmatrix} + \begin{bmatrix} -0.0368 & 0 \\ 0.0552 & 0 \\ 0 & -0.0026 \end{bmatrix} \begin{bmatrix} F_R \\ P_s \end{bmatrix} \quad (28)$$

$$\begin{bmatrix} C_2 \\ T_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ T_1 \end{bmatrix}$$

where $C_1$ is the concentration in the first reactor, $C_2$ is the concentration in the second reactor, $T_1$ is the temperature in the first reactor, $F_R$ is the recycle flow rate, $P_s$ is the steam pressure. The objective is to control $C_2$ and $T_1$ by manipulating $F_R$ and $P_S$. To give equal importance to errors in $C_2$ and $T_1$, the errors in $C_2$ were multiplied by 100. The above model was discretized using a sample time of 0.1. By examining the model in (28), $C_2$ should be paired with $F_R$ and $T_1$ should be paired with $P_s$. Samyudia *et al.* (1994) proposed two plant decompositions for decentralized control; namely, a *physical* decomposition based on material and energy balances around each reactor, and a *mathematical* decomposition in which each decomposed subsystem is composed from either the material balances or energy balances of the two units. The $\gamma_{opt}$ was calculated for each decomposition to compare their closed-loop performance. At this stage, only the fully decentralized MPC is considered. The results obtained from the proposed methodology as well as the results obtained from simulations are summarized in table 1 for a unit step-change in $C_2$. The following parameters were used $Hp$=300, $Hu$=100, $\alpha$ = 0.99 and $Q_i=I_{2x2}$. The uncertainty consisted in 20% perturbations in the parameter -0.0368 and 5% in the parameter -0.0214 in (26). Calculations were done with ($\gamma_{opt,\ uncer}$) and without uncertainty ($\gamma_{opt,\ nom}$). Since the analysis produces the worst case scenario, a worst signal for set-point tracking was sought for the purpose of simulation and comparison to analysis. A pulse of 20 intervals was found to give large values of $\gamma$. For comparison purposes, simulations were also conducted for step changes. The simulation results both for step and pulse are given in table 1.

**Table 1. Results of analysis and simulation**

| | Mathematical decomposition | Physical Decomposition |
|---|---|---|
| $\gamma_{opt,\ uncer}$ | 1.035 | 1.297 |
| $\gamma_{opt,\ nom}$ | 1.006 | 1.276 |
| $\gamma_{pulse}$ | 0.220 | 0.331 |
| $\gamma_{step}$ | 0.143 | 0.147 |

Although the analysis is conservative, the simulation results are consistent with the analysis and they show that the mathematical decomposition is better. Samyudia *et al.* (1994) obtained similar conclusion based on open-loop considerations. By comparing ($\gamma_{opt,\ nom}$) and ($\gamma_{opt,\ uncer}$) for the two decompositions it is clear that uncertainty has a larger effect on the mathematical decomposition due to the fact that the uncertain parameter -0.0368 appears only in this decomposition.

### *3.2 Coordination Problem*

To illustrate the coordination problem the example used is a high-purity column studied by Skogestad and Morari (1988). The example is challenging due to the high condition number of the process and its sensitivity to model error. The simplified model of the system is given by the following transfer function (Skogestad and Morari, 1988):

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \frac{1}{75s+1} \begin{bmatrix} 0.878 & 0.864 \\ 1.082 & 1.096 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (29)$$

where $y_1$ and $y_2$ are the top and bottom product compositions and $u_1$ and $u_2$ are the reflux flowrate and the boilup.

A state-space realization with only two states can be easily obtained from the model above and is not shown for brevity.

The $\gamma_{opt}$ was calculated for each one of the 3 MPC strategies: centralized, fully decentralized and Nash-based. The following parameters were assumed for the 3 controllers as follows: $Hp$ = 20, $Hu$ = 5, $\alpha$ = 0.99 and $Q_i=I_{2x2}$. As done for the previous example, simulations were carried out for a pulse of 40 time intervals and step set-points. To assess the effect of uncertainty, two cases were considered: i. a change of 0.2% and 0.5% on the steady-state gains in elements (1,1) and (1,2) in (29); respectively, ii. a change of 2.5% and 2% on the steady-state gains in elements (1,1) and (1,2) in (29); respectively. The analytical and simulation results are summarized in table 2 and the dynamic response of the system to unit set-point change in $y_1$ for case ii is shown in figure 1.

**Table 2. Results of analysis and simulation for different MPC strategies**

| | Centralized | Decentralized | Nash-Based |
|---|---|---|---|
| $\gamma_{opt,\ nom}$ | 0.013 | 0.33 | 0.015 |
| $\gamma_{opt,\ uncer,\ i}$ | 0.153 | 0.372 | 0.123 |
| $\gamma_{pulse,\ i}$ | 0.077 | 0.200 | 0.032 |
| $\gamma_{step,\ i}$ | 0.014 | 0.076 | 0.006 |
| $\gamma_{opt,\ uncer,\ ii}$ | 0.546 | 0.541 | 0.535 |
| $\gamma_{pulse,\ ii}$ | 0.082 | 0.240 | 0.065 |
| $\gamma_{step,\ ii}$ | 0.015 | 0.108 | 0.012 |

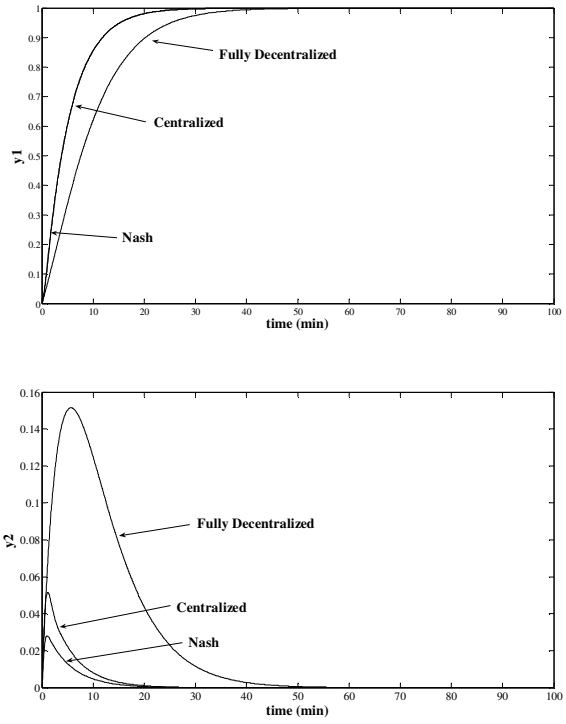


*Fig.1. Dynamic response of the system to unit set-point change in $y_1$ for case ii*

Generally, the results of the analysis are consistent with the results of the simulation. The results are highly sensitive to model uncertainty ($\gamma_{opt,\ uncer}$) as seen from the large increase between the nominal case ($\gamma_{opt,\ nom}$), i.e. the case without uncertainty, and the two cases with uncertainty specified above. For case ii corresponding to larger uncertainty, ($\gamma_{opt,\ uncer,ii}$) becomes similar for the three controllers indicating that the fully decentralized MPC strategy may give similar performance to the other two strategies. Although in the simulation for the case with larger model error the fully decentralized control has the worst performance, the error is of similar magnitude as the others. It should be also remembered that the pulse or step set-points changes may not be the worst scenarios and there is no systematic way to find the worst case. In addition, the Nash-based controller is the less sensitive to model errors in both cases. This is due to the fact that the Nash-based controller does not require a full inverse of the ill-conditioned model of the process as it is the case in the centralized controller. For case ii as given in figure 1, Nash-based MPC gives similar response to centralized MPC for $y_1$ whereas the fully decentralized MPC results in a sluggish response and for $y_2$ Nash-based MPC gives slightly better performance by maintaining the lowest overshoot compared to centralized MPC whereas fully decentralized MPC resulted in the largest overshoot.

## 4. CONCLUSION

In this work, a methodology is developed to address robustness issues related to distributed MPC strategies. The methodology allows for selecting the best model decomposition and for comparing the performance of three MPC strategies: centralized, fully decentralized, and Nash-Based, in the presence of model errors. The approach is based on the minimization of robust performance index of the closed-loop system for different MPC strategies with different uncertainty levels. Two case studies were selected to illustrate the application of the methodology. The analytical results obtained in this study are consistent with simulation. However, since the methodology calculates bounds on the closed-loop system's variability the results are somewhat conservative. This conservatism increases when the uncertainty level is high.

The applicability of the proposed methodology to systems with higher dimension and consideration of process constraints is currently being investigated.

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