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NONLINEAR CONTROL FOR UAV FORMATION FLYING

Z. Cheng, D.S. Necsulescu*, B. Kim and J.Z. Sasiadek*****

**Department of Mechanical Engineering, University of Ottawa, Ottawa, Ontario, Canada*
***DRDC, Department of National Defence, Ottawa, Ontario, Canada*
****Department of Mechanical and Aerospace Engineering, Carleton University, Ottawa, Ontario, Canada*

Abstract: Unmanned Aerial Vehicles (UAVs) became a technology that have attracted considerable interest in the commercial markets for the military and civilian uses, such as surveillance and reconnaissance, aerial surveys for natural sources, traffic monitoring, early forest fire detection etc. This paper deals with Nonlinear and Model Predictive Control (MPC) of Unmanned Aerial Vehicles (UAV) flying in formation. Although, UAVs present numerous advantages over the manned aircrafts, they face challenges in various aspects of control in autonomous mode, and even more, in formation flying. An advanced control system is demonstrated as a possible solution to improve and increase the level of the autonomous mode and flying capabilities of UAVs. This paper deals with advanced control system of Unmanned Aerial Vehicles (UAV) flying in formation. *Copyright © 2002 IFAC*

Keywords: Unmanned Aerial Vehicles, Model Predictive Control, Nonlinear Control, Formation Flying.

1. INTRODUCTION

Unmanned Aerial Vehicles (UAVs) have attracted considerable interest in the commercial markets for the military, mainly for surveillance and reconnaissance purposes, and for the civilian uses such as aerial surveys for agriculture, traffic monitoring, pollution control, meteorological data collection, pipeline survey and early forest fire detection, etc. Presently, UAVs prove to be cost-effective platforms for the military and civilian applications because they gather information without endangering the lives of the pilots, increase manoeuvrability without limitation due to human capabilities, result in much lower cost than the traditional aircraft and in elimination of on-board human-pilot interfaces. Although, UAVs present some advantages over the manned aircrafts, they require autonomous control and have more stringent size and capability constraints than conventional aircrafts. To enhance the safety and security level for performing some tasks, remotely controlled, semi-autonomous and autonomous vehicles are used in practice (Jiang *et al*, 2006). With recent developments in various aspects such as automatic flight control systems, guidance techniques, and navigation instruments, intensive research have been carried out to explore approaches to move the operations from ground stations and to the vehicle and realize complex operational objectives on-board with high reliability in aerial missions, at lower risk and lower cost. Therefore, complex requirements, especially related to vehicle dynamics, are imposed for UAV controller design. In most configurations, UAVs are designed to operate in two distinct modes: remote control from Ground Control Stations (GCS) and autonomous control (Jiang *et al*, 2006), (Stevens and Lewis, 1992), (McLean, 1990), (Nelson, 1998).

In GCS remote control mode, the UAV receives commands from the GCS, and at the same time, the motion and the states of the UAV are transmitted to the GCS. The commands are generated by an experienced pilot or by an expert system. Normally, the ground remote control mode is used to cope with highly uncertain or unpredicted conditions, such as takeoff and landing under complex environments, unpredicted failure of actuators during mission and unanticipated natural disturbances. In the ground remote control mode, UAVs are controlled by experienced pilots. Therefore, the complexity of the controller design and configuration are reduced and the failure of UAVs can be prevented under certain conditions. In the autonomous control mode, UAVs can perform pre-programmed operational missions without participation of pilots, and the states are transmitted to the GCS for monitoring and thus the decision of staying in autonomous control mode or switching to GCS control mode is made. One of the most important and challenging problems in guidance, navigation , and control of UAV vehicles, flying in formation is the trajectory tracking by the formation as well as preservation of formation itself [6]. The vehicles have to maintain constant distance, and often constant relative attitude between themselves, as well as, they have to follow certain changing pattern of velocities. This paper presents the concept of formation flying using a nonlinear control system and model predictive control (MPC). Model Predictive Control system was described by different authors in (Berlin and Frank, 1994) and (Cheng *et al*, 2007).

II. MATHEMATICAL MODEL OF UAV

The UAV nonlinear model can be expressed by the force equations, moment equations, kinematics equations and navigation equations in various frames of references. To simplify the expression of UAV model, different frame system is used for the above equations.

Force equation in the wind frame

$$\begin{bmatrix} \dot{V}_T \\ \dot{\beta} \\ \dot{\alpha} \end{bmatrix} = \frac{1}{m}\left\{ \begin{bmatrix} F_T\cos\alpha\cos\beta \\ \dfrac{-F_T\cos\alpha\sin\beta}{V_T} \\ \dfrac{-F_T\sin\alpha}{V_T\cos\beta} \end{bmatrix} + \begin{bmatrix} -d \\ \dfrac{y}{V_T} \\ \dfrac{-l}{V_T\cos\beta} \end{bmatrix} \right\} + \begin{bmatrix} 0 \\ r_w \\ \dfrac{q_w}{\cos\beta} \end{bmatrix} + \begin{bmatrix} g_1 \\ \dfrac{g_2}{V_T} \\ \dfrac{g_3}{V_T\cos\beta} \end{bmatrix} \tag{1}$$

where

$$\begin{bmatrix} p_w \\ q_w \\ r_w \end{bmatrix} = \begin{bmatrix} p\cos\alpha\cos\beta + q\sin\beta + r\sin\alpha\cos\beta \\ -p\cos\alpha\sin\beta + q\cos\beta - r\sin\alpha\sin\beta \\ -p\sin\alpha + r\cos\alpha \end{bmatrix} \tag{2}$$

and

$$\begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} g(-\cos\alpha\cos\beta\sin\theta + \sin\beta\sin\phi\cos\theta + \sin\alpha\cos\beta\cos\phi\cos\theta) \\ g(\cos\alpha\sin\beta\sin\theta + \cos\beta\sin\phi\cos\theta - \sin\alpha\sin\beta\cos\phi\cos\theta) \\ g(\sin\alpha\sin\theta + \cos\alpha\cos\phi\cos\theta) \end{bmatrix} \tag{3}$$

where: d, y l are aerodynamic forces along the reference frame axis and they are expressed by the dimensionless aerodynamic coefficients.

Details of the mathematical model could be found in (Cheng *et al*, 2007).

III. CONCEPT OF CONTROL SYSTEM

The single UAV is controlled in two stages. In the first stage, the Dynamic Inversion is applied to the 6-DOF nonlinear and then the Model Predictive Control for a linearized system is used in the second stage. This method applies to the dynamic systems that could be linearized using feedback linearization method. The control concept is given in Fig. 1. The purpose of dynamic inversion is to linearize the nonlinear system through active compensation using negative feedback control. In the second stage, the model predictive control is applied based on a prediction of the system controlled response and the current control input is computed as a linear combination of past input and past output to minimize the cost function of difference between the predicted system output and the desired output in the future.

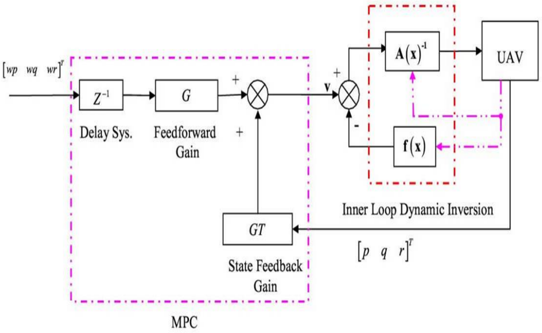

Fig.1 Model Predictive Control with Dynamic Inversion

To enable the UAV follow a pre-defined path, a guidance control algorithm has to be integrated with model predictive and dynamic inversion controller. In the UAV guidance control, the position of UAV is expressed by $(x_c \quad y_c \quad z_c)$ in the earth fixed coordinate and the waypoint is defined as $(x_w \quad y_w \quad z_w)$. The relative distance between the UAV and waypoint is named as Line Of Sight (LOS) which can be calculated as follow

$$\overrightarrow{\mathbf{LOS}} = \begin{bmatrix} x_c - x_w \\ y_c - y_w \\ z_c - z_w \end{bmatrix} \tag{4}$$

The cross product of LOS and airspeed can be expressed as

$$\mathbf{LOS} \times \mathbf{V_T} = \left(|\mathbf{V_T}||\mathbf{LOS}|\sin\varepsilon\right)\mathbf{n} \tag{5}$$

where

$$\varepsilon = \arcsin\left(\frac{\mathbf{LOS}\times\mathbf{V_T}}{|\mathbf{V_T}||\mathbf{LOS}|}\right) \tag{6}$$

$$\mathbf{n} = \frac{\mathbf{LOS}\times\mathbf{V_T}}{|\mathbf{V_T}||\mathbf{LOS}|} \tag{7}$$

The angular rate commands are generated by

$$\mathbf{y_c} = \begin{bmatrix} p_c \\ q_c \\ r_c \end{bmatrix} = K_N\varepsilon\mathbf{n} = K_N\varepsilon\begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \tag{8}$$

In order to hold the swarm formation, the distance between all the neighbours is not larger than the maximum formation distance and the member away from the groups is controlled as if attracted by the nearest member. On the other hand, the member will be controlled as if to repulse the member which is too close, to get minimum safe clearance from collision avoidance and the velocity of each member is aligned ([Okubo, 1985]). At this stage, the thrust force driving is controlled by the safety clearance among the formation members. The variation of command is defined by the following equation.

$$\delta_{th} = K_{th} \ln\left(\frac{d_{ij}}{d_0}\right) \qquad d_0 > d_{ij} > 0 \qquad (9)$$

where

δ_{th} : The variation of the throttle or one of angular rates.

K_{th} : The gain of controller

d_{ij} : The minimum clearance

d_0 : The threshold clearance

The configuration of two UAVs collision avoidance with fixed obstacle is shown in Fig 2.

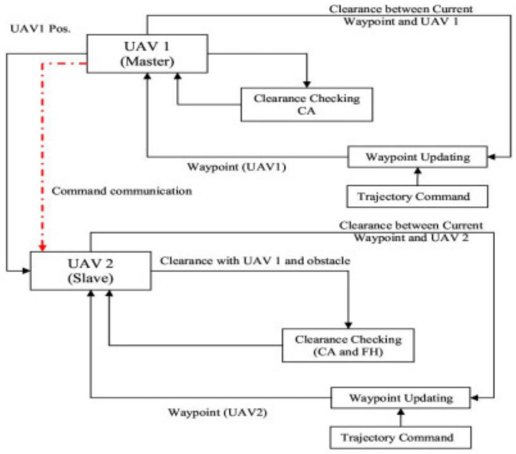

Fig 2 Collision Avoidance Control with Fixed Obstacle and Formation Hold

VI. EXPERIMENTAL RESULTS

The simulation for MPC with dynamic inversion runs using the controller shown in Fig 1.

The command inputs are defined as different wave forms to check the response of the systems for a variety of operational conditions. The desired angular rates are defined in the following:

Roll (p): square wave form

Pitch (q): square wave form

Yaw (r): sawtooth wave form (Fig. 3)

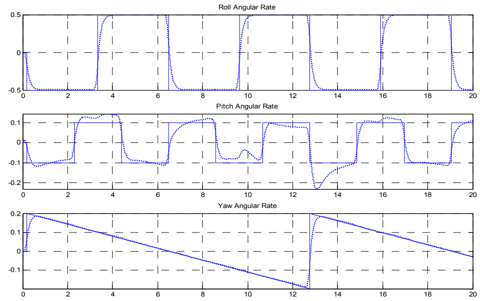

Fig. 3 Model Predictive Control with Dynamic Inversion (angle rates vs. time)

These results confirm that the simplified dynamic inversion is still effective in permitting MPC to achieve the desired roll, pitch and yaw angular rates. Also these results confirm the advantage of MPC due to predictive nature, shown by changes of angle rates that occur prior to actual changes in the desired angle rates.

In the following experiment, a single UAV is controlled to follow a predefined path which is described by a set of waypoints. The initial position and waypoint is set as follow:

	UAV
Initial position	[100 0 0]
Waypoints	[500,50,-50] [750,60,-80]

The simulation results are shown in Fig. 4a and 4b.

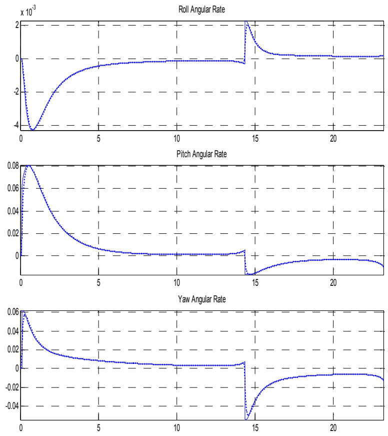

Fig 4a Angular Rates Response with MPC ($T_s = 0.005$ and $n_h = 20$)

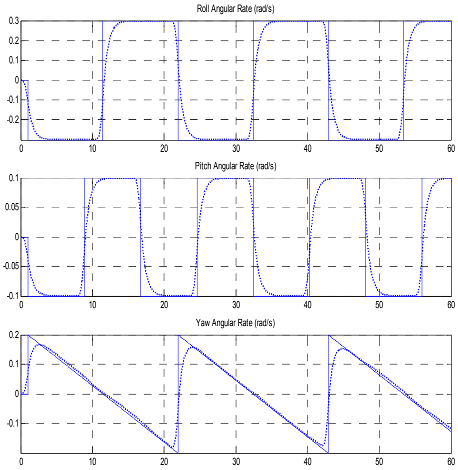

Fig 4b Angular Rates Response with MPC ($T_s = 0.005$ and $n_h = 200$)

where solid line is the future command and dot line is response of UAV.

It is observed that the long horizon (n_h) has a smoothing effect on the control surface deflection command and this can be used to preshape the command to suit the UAV's dynamics and avoid exciting various modes of vibration. Moreover, these results confirm that dynamic inversion is still effective in permitting model predictive to achieve the desired roll, pitch and yaw angular rates. Also these results confirm the advantage of model predictive control due to the predictive nature, shown by changes of angle rates that occur prior to actual changes in the desired angle rates. This advantage is important in achieving collision avoidance in UAVs formation flight starting the trajectory correction as soon as collision damage is sensed.

The UAV trajectory in the earth fixed coordinate is shown in Fig 5.

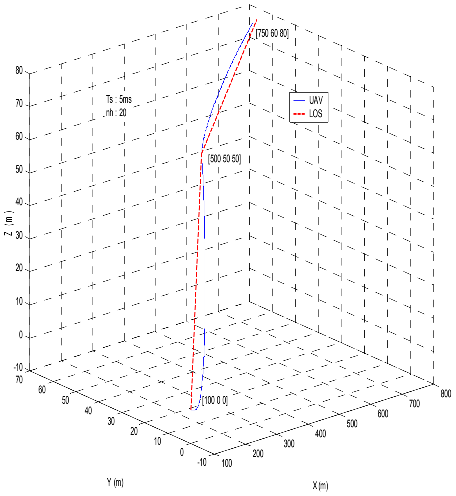

Fig. 5 UAV Trajectory in the Earth Fixed Coordinate ($T_s = 0.005$ and $n_h = 20$)

where, the dot line denotes to the "Line of Sight" (LOS) and the solid line to UAV. It is observed that the single UAV follows the pre-defined path under the control concept of LOS integrated with MPC.

In the next section, two UAVs fly in a formation group and the obstacle is set in the predefined path. The UAVs' initial point, waypoints and obstacle position are given in the following table.

	UAV1 (Master)	UAV2 (slave)
Initial position	[50 50 -50]	[50 100 -50]
Obstacle	[2050 2050 -50]	
Waypoints	[1050, 1050,-50] [3050, 3050,- 50] [4050, 4050,-50]	[1050, 1100,-50] [3050, 3100,- 50] [4050, 4100,-50]

Under this condition, the yaw angular rate of master UAV is modified to achieve collision avoidance when the clearance between master UAV and fixed obstacle is less than 250m. The clearance between mater UAV and obstacle is shown in Fig. 6.

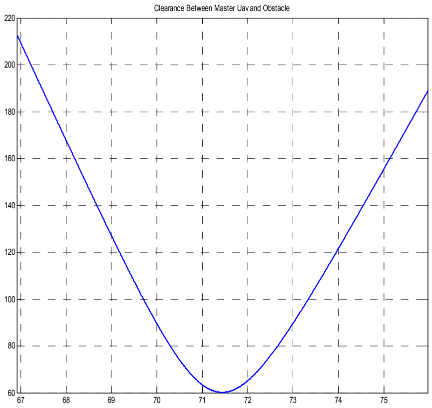

Fig. 6 Clearance between master UAV and obstacle ($T_s = 0.005$ $n_h = 10$)

The UAVs response is shown in the Fig. 7 and Fig. 8, where dot red line is the response of mater UAV and solid blue is of slave UAV.

The clearance distance between mater and slave UAVs is shown in Fig. 9 (distance versus time).

It is observed in Fig. 9 that two UAVs follow the pre-defined path and keep the flying formation. This confirms that model predictive control provides a valid concept for the UAV guidance and formation control. Figure 10 shows 3D trajectory in inertial coordinate frame for dynamic inversion controller.

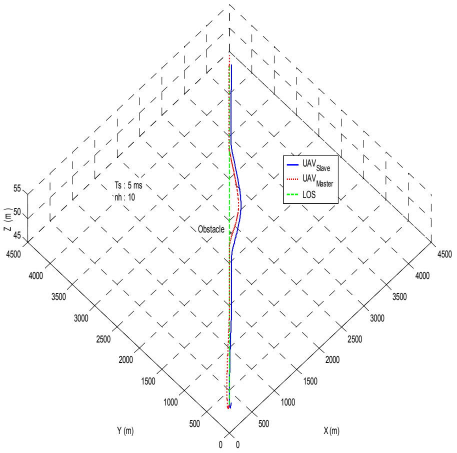

Fig. 7 Two UAVs Collision Avoidance and Formation Hold With MPC (T$_s$ = 0.005 n$_h$ = 10)

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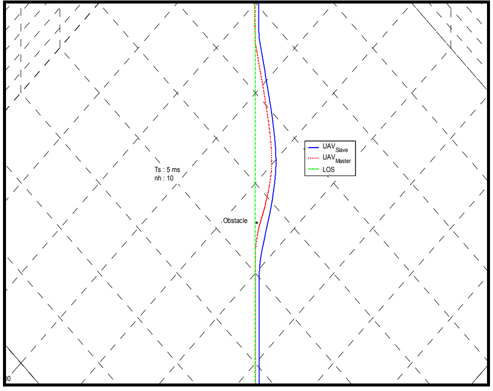

Fig. 8 Two UAVs Collision Avoidance and Formation Hold (Magnification of the central part of Fig. 7) (T$_s$ = 0.005 n$_h$ = 10)

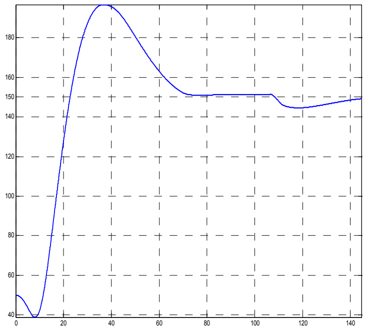

Fig. 9 Clearance between UAVs (T$_s$ = 0.005 n$_h$ = 10)

Furthermore, compared with classical controller, the inner and outer dynamic inversion controller can be used to control multiple variables simultaneously. The desired values are set as $\phi = 0.2 \quad \theta = 0.1 \quad \beta = 0$ rad. The results are shown in Fig. 10, 11and 12. The concept of UAV control system is shown in Fig. 13 and 14.

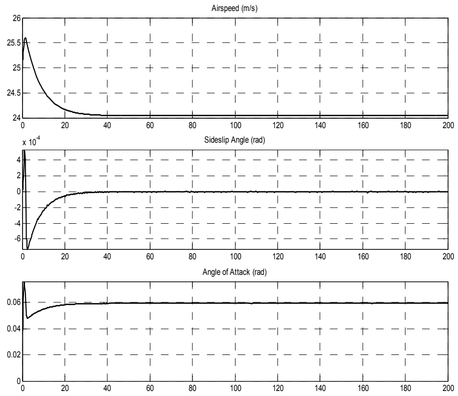

Fig.10 Airspeed and Aerodynamic Angles (Inner and Outer Loop Dynamic Inversion Controller $\phi = 0.2 \quad \theta = 0.1 \quad \beta = 0$)

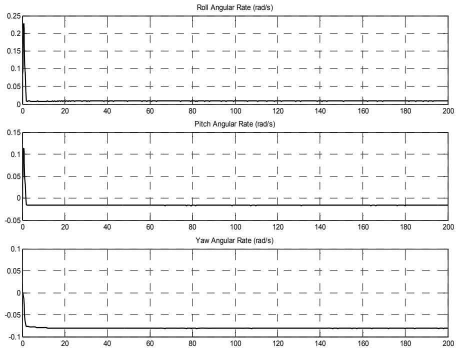

Fig. 11 Angular rates response (Inner and Outer Loop Dynamic Inversion Controller $\phi = 0.2 \quad \theta = 0.1 \quad \beta = 0$)

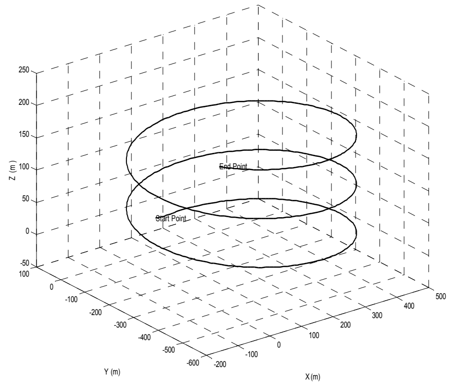

Figure 12 UAV 3D Trajectory in Earth Fixed Frame (Inner and Outer Loop Dynamic Inversion Controller $\phi = 0.2 \quad \theta = 0.1 \quad \beta = 0$)

In this paper an Unmanned Aerial Vehicle is modelled by force equations, moment equations, kinematics equations and navigation equations. The nonlinear controller is proposed based on the inner and outer loop dynamic inversion. A proportional controller is applied for inner loop to follow angular rates command and a proportional – integral controller is used for the outer loop to generate the command for the inner loop. Dynamic Inversion permits the use of a linear model predictive control for the inner loop.

The simulation results were compared with classical controller and verify that the inner / outer loop dynamic inversion controller is superior to the classical controller in the whole flight envelope. The combination of model predictive control with dynamic inversion proved to provide a solution for the autonomous control of an UAV navigating through obstacles.

In the dynamic inversion controller approach, it was assumed that the necessary states are observable at all time for the controller synthesis. In many cases, some states may not be observable and an appropriate observer has to be designed.

For the nonlinear UAV model, the aerodynamic coefficients are expressed by Taylor series approximation. In the multiple UAV formation, these coefficients are significantly different from single UAV flight case. The robustness of controller has to be considered to deal with the uncertainty of each coefficient. To complete the accurate path following, the separate navigation loop has to be added to generate the command reference as the inputs for the outer loop.

In order to improve performance a high performance predictive controller with operational constraints has to be investigated, as well as, the high level path planning system has to be included.

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VII. CONCLUSIONS

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Appendix 1

Nomenclature

x_b, y_b, z_b	body axes
x_e, y_e, z_e	earth axes
ϕ, ψ, θ	bank angle, yaw angle, pitch angle
β	angle of sideslip
α	angle of attack
I_x, I_y, I_z	moments of inertia about body axes
p, q, r	angular velocity components along body axes
u, v, w	linear velocity components along b
$\delta_{a,}\delta_{r,}\delta_e$	aileron, rudder, elevator deflections
W	weight of the aircraft
V	velocity of the aircraft
g	gravitational acceleration

Appendix 2

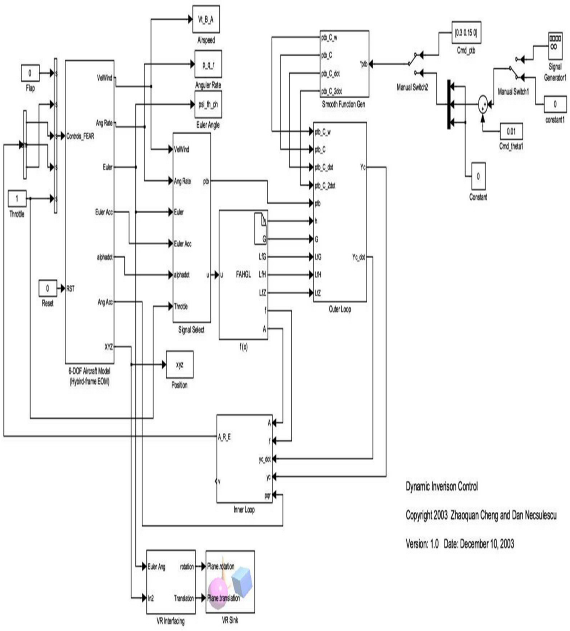

Fig. 13 Matlab and Simulink implementation of UAV dynamic inversion control system.