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Output Selection with Fault Tolerance via Dynamic Controller Design ⋆

Z. Li * **A. C. Zolotas** * **I. M. Jaimoukha** ** **K. M. Grigoriadis** ***

* *Department of Electronic and Electrical Engineering, Loughborough University, LE11 3TU, UK (e-mail: z.li02@lboro.ac.uk; a.c.zolotas@lboro.ac.uk).*
** *Department of Electrical and Electronic Engineering, Imperial College London, SW7 2BT, UK (e-mail: i.jaimouka@imperial.ac.uk)*
*** *Department of Mechanical Engineering, University of Houston, USA (e-mail: karolos@uh.edu)*

Abstract: Input-Output selection/placement for control systems has been an attractive research topic in particular under fault-free conditions. In this paper we present a methodology of output selection in a closed-loop framework with a view of fault tolerance capability. The principles with regards to the selection of sensors are reduced hardware redundancy, reduced costs and easier implementation, and acceptable degraded performance when faults occur. The selection of sensors is based upon both closed-loop control and fault tolerance objectives by solving an $\mathcal{H}_\infty$ optimization problem for each group of sensors sets via Linear Matrix Inequalities (LMIs). The proposed scheme is applied to a practical example of ride quality improvement of a high speed rail vehicle.

1. INTRODUCTION

Modern control and monitoring systems that involve a large number of actuators and sensors prone to failure are becoming more complex and demanding in terms of maintenance. In a variety of practical engineering systems (aerospace, electro-mechanical systems, railway vehicle systems) for a given requirement the range of possible locations for sensors is usually known, the practical engineering issue is either to minimize the number of sensors to achieve a particular level of fault tolerance, or to optimise the location of a given number of sensors. The primary focus is that of optimised sensor selection for efficient robustness properties of the system,*assuming a consistent controller design*, with relation to faults prior to system reconfiguration. A survey on fault-free input/output selection methods can be found in van de Wal and de Jager [2001], where a review and some assessment is presented according to the desired properties meet in each selection method.

Control with fault tolerance has become a main concern in control system design procedures. It is desirable that fault tolerance needs to be included in sensor configuration/optimization. For this purpose, we investigate a framework which incorporates sensor faults and determine an appropriate set of criteria for the optimal selection relating to fault tolerance. An iterative approach is employed to lock on the sensor combination which provides the best $\mathcal{H}_\infty$ performance for closed-loop control and fault tolerance via a dynamic feedback controller. Each step in the iteration is solved by Linear Matrix Inequalities (LMIs).

The paper is organised as follows: Section 2 gives the problem formulation of the output selection problem with consideration of fault tolerance. Section 3 transforms the formulation into a state space framework under certain assumptions via LMI solutions and compares the performance index under different sensor combinations. A practical example is given in Section 4 while concluding remarks are made and future research directions are mapped out in Section 5.

2. PROBLEM FORMULATION

The proposed output selection scheme is based upon a $\mathcal{H}_\infty$ control performance index of closed-loop system transfer functions, integrated with a dynamic controller as depicted in Fig. 1.

Note that $d(s)$ characterises any exogenous inputs entering the system, $u(s)$ is the fixed set of control inputs, $y(s)$ the measurements (their number varies depending on the scenario considered) and $z(s)$ is the vector of regulated outputs (these can be related to ∞-norm or 2-norm or both types of the aforementioned norms). For the purposes of this work we consider only ∞-norm regulation, i.e. $z_\infty(s)$, for the control objectives.

Actuator faults and sensor faults can have channels to affect state dynamics and measured outputs directly, particularly in a feedback control system Jaimoukha et al. [2006], Patton and Chen [1997]. Here, consider a linear time invariant (LTI) dynamic system subject to disturbances, actuator and sensor faults modeled as

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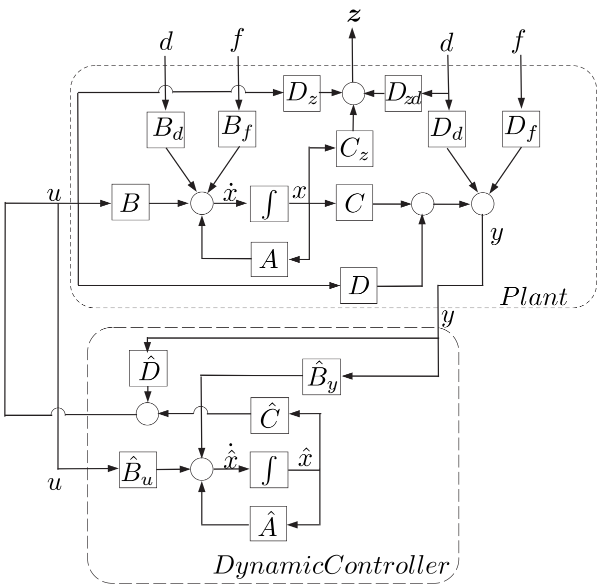

Fig. 1. Generic dynamic controller with fault tolerance

$$\dot{x}(t) = Ax(t) + B_d d(t) + Bu(t) + B_f f(t), \quad (1)$$

$$z(t) = C_z x(t) + D_{zd} d(t) + D_z u(t), \quad (2)$$

$$y(t) = Cx(t) + D_d d(t) + Du(t) + D_f f(t), \quad (3)$$

where $x(t) \in \mathcal{R}^{n_p}$, $u(t) \in \mathcal{R}^{n_{pu}}$ and $y(t) \in \mathcal{R}^{n_{py}}$ are the state, input and output vectors, respectively and $d(t) \in \mathcal{R}^{n_w}$ is the disturbance vector. The energy of the output signal $z(t) \in \mathcal{R}^{n_{p1}}$ is bounded for finite energy input signals by regulating the $\mathcal{H}_\infty$ norm of the system input-output gain (robustness metric). Here, $B_d \in \mathcal{R}^{n_p \times n_w}$, $D_{zd} \in \mathcal{R}^{n_{p1} \times n_w}$ and $D_d \in \mathcal{R}^{n_{py} \times n_w}$ are the corresponding disturbance distribution matrices, and $B \in \mathcal{R}^{n_p \times n_{pu}}$, $D_z \in \mathcal{R}^{n_{p1} \times n_{pu}}$ and $D \in \mathcal{R}^{n_{py} \times n_{pu}}$ are the corresponding control distribution matrices, respectively. Similarly, B_f and D_f are known and well-defined fault channel distribution matrices with appropriate dimensions. Without loss of generality, we can assume $D = 0$ (which is a generic assumption in $\mathcal{H}_\infty$ control problem).

The proposed generic dynamic controller (GDC) has the following model

$$\begin{aligned} \dot{\hat{x}}(t) &= \hat{A}\hat{x}(t) + \hat{B}_u u(t) + \hat{B}_y y(t), \\ u(t) &= \hat{C}\hat{x}(t) + \hat{D}y(t), \end{aligned} \quad (4)$$

where $\hat{A}, \hat{B}_u, \hat{B}_y, \hat{C}$ and $\hat{D}$ are constant controller gain matrices to be determined with appropriate dimensions.

Remark 1. It is worth noting that many standard controllers can be represented in this form. A static controller is not employed here since it is not sufficient to regulate both control and fault tolerance metrics.

Remark 2. In this paper a full-order controller dynamic is considered, namely, $\hat{x}$ has the same dimension as the plant state x. A reduced order controller is also possible subject to controllability of the original system.

By defining an augmented state $x_a(t) = \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix}$, it can be easily shown that the dynamics of the closed-loop system are given by

$$\dot{x_a} = A_{cl} x_a + B_{cl} d + F_{cl} f,$$

$$z = C_{zcl} x_a + D_{zcl} d + F_{zcl} f, \quad (5)$$

where

$$A_{cl} = \begin{bmatrix} A + B\hat{D}C & B\hat{C} \\ \hat{B}_u \hat{D}C + \hat{B}_y C & \hat{A} + \hat{B}_u \hat{C} \end{bmatrix},$$

$$B_{cl} = \begin{bmatrix} B_d + B\hat{D}D_d \\ \hat{B}_u \hat{D}D_d + \hat{B}_y D_d \end{bmatrix}, \; F_{cl} = \begin{bmatrix} B_f + B\hat{D}D_f \\ \hat{B}_u \hat{D}D_f + \hat{B}_y D_f \end{bmatrix},$$

$$C_{zcl} = \begin{bmatrix} C_z + D_z \hat{D}C & D_z \hat{C} \end{bmatrix}, \; D_{zcl} = D_{zd} + D_z \hat{D}D_d,$$

$$F_{zcl} = D_z \hat{D} D_f.$$

Taking Laplace transforms gives

$$n_{p1} \overset{\quad n_d \qquad n_f}{\begin{bmatrix} T_{zd}(s) & T_{zf}(s) \end{bmatrix}} \overset{s}{=} \left[\begin{array}{c|cc} A_{cl} & B_{cl} & F_{cl} \\ \hline C_{zcl} & D_{zcl} & F_{zcl} \end{array}\right].$$

Here, T_{zd} is the transfer function from d to z and T_{zf} is the transfer function from f to z, respectively.

2.1 Performance Index

For each output combination, the selection criteria is defined as follows: find the controller such that

$$\rho = \gamma_o := \inf_{\|T_{zd}\|_\infty < \gamma_d} \|T_{zf}\|_\infty \quad (6)$$

is obtained, where γ_d is a customer assigned bound for disturbance attenuation.

Remark 3. The consideration of fault tolerance from bounding T_{zf} is known as a simple passive method Patton [1997]. More sophisticated active approaches with reconfiguration are available today, however, a passive method is more straightforward to demonstrate the methodology behind for output selection.

2.2 Output Selection

Thus, the output selection problem under consideration can be formulated as follows:

Problem 4. Let all variables be defined as above. Find an optimal sensor set $k \in \mathcal{S}$ with $\mathcal{S}$ representing the entire set of sensor combinations, such that $\rho_k \le \rho_i, \; i = 1, 2, \ldots, N$, where ρ_i is the norm performance index selected as in (6) with the corresponding sensor set i deployed for a defined set of faults.

Remark 5. Note that the objective is in particular to find an appropriate sensor combination with the preferred control and fault tolerance properties, rather than directly to search for an optimal fault tolerant controller for a given system. Our purposes is that the selected sensor configuration can be used as an effective basis prior to reconfiguration schemes ultimately leading to a complete fault tolerant system configuration. Undoubtedly the minimum possible set of sensors will be attractive in terms of reducing complexity in a practical system relative to maintenance as well as sensor equipment and installation costs.

Earlier work on similar concepts of input/output selection was mainly focused on evaluating a single performance index such as nominal performance, robust performance and robust stability applied to fault-free environment Wal and Jager [1996], Wal et al. [1998]. In addition, work on the use of $\|.\|_\infty$ and $\|.\|_2$ for placing sensor/(actuator) pairs is addressed in Gawronski [1999] but in an open loop sense applied to flexible structures. Li et al. [2007] discussed output selection for control and fault detectability separately, with no consideration of fault tolerance and no integrated controller design. It is hence of interest to incorporate fault tolerance into initial system design and output selection. However, it should be noted that the term "fault tolerance" we are referring to in this paper is the robust performance of the effect of faults in the regulated output in terms of norms.

3. GENERIC DYNAMIC CONTROLLER (GDC) SYNTHESIS VIA LMIS

An analytical solution of Problem 4 is not straightforward due to the difficulty of incorporating all available sensor sets into one controller design setup. Here, we follow a tractable suboptimal solution using an iterative procedure to evaluate the performance index(es) for each chosen sensor combination.

3.1 Matrix Inequalities for Generic Dynamic Controller (GDC)

The problem in (6) is a constrained $\mathcal{H}_\infty$ optimization problem, which is possibly to be solved via constructing two equivalent linear matrix inequalities. One LMI is to minimize $\|T_{zf}\|_\infty$ and the other is to bound $\|T_{zd}\|_\infty$ with given γ_d.

By virtue of the Bounded Real Lemma Boyd et al. [1994], A_{cl} is stable and $\|T_{zf}\|_\infty < \gamma$ if and only if there exists a symmetric P with $P > 0$ and

$$T_{mi} := \begin{bmatrix} PA_{cl} + A_{cl}^T P & \star & \star \\ F_{cl}^T P & -\gamma I & \star \\ C_{zcl} & F_{zcl} & -\gamma I \end{bmatrix} < 0 \qquad (7)$$

where $\star$ denotes terms readily inferred from symmetry.

However, the matrix inequality in (7) cannot be solved directly by a convex optimization algorithm since nonlinear terms in the matrix inequalities will be encountered Scherer et al. [1997].

The following result using the technique change of variables, gives a bilinear formulation of the optimization problem of (6), which is solvable analytically via LMI toolbox. Boldface letters are used to indicate variables.

Lemma 3.1. Let all variables be defined as above, then a stabilizing dynamic controller exists such that $\|T_{zf}\|_\infty < \gamma$ is achieved if there exists X, Y, $\tilde{A}$, $\tilde{B}_y$, $\tilde{C}$ and $\hat{D}$ such that (8) is true.

Then, the stabilizing controller is given by

$$\begin{aligned} &\hat{C} = (\tilde{C} - \hat{D}CX)M^{-T},\ \bar{B}_y = N^{-1}(\tilde{B}_y - YB\hat{D}), \\ &\bar{A} = N^{-1}(\tilde{A} - YAX - YB\hat{D}CX \\ &\qquad - N\bar{B}_yCX - YB\hat{C}M^T)M^{-T}, \\ &\hat{A} = \bar{A} - \hat{B}_u\hat{C},\ \hat{B}_y = \bar{B}_y - \hat{B}_u\hat{D}, \end{aligned} \qquad (9)$$

where $\hat{B}_u$ is an arbitrary matrix with appropriate dimension, and square and nonsingular M and N should be chosen such that

$$MN^T = I - XY.$$

Proof We decompose the Lyapunov Matrix P in (7) as the following

$$P = \begin{bmatrix} Y & N \\ N^T & \hat{Y} \end{bmatrix},\ P^{-1} = \begin{bmatrix} X & M \\ M^T & \hat{X} \end{bmatrix},$$

where $X, Y, \hat{X}, \hat{Y} \in \mathcal{R}^{n\times n}$ are symmetric and nonsingular. Let

$$Q = \begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix},\ \tilde{Q} = \begin{bmatrix} I & Y \\ 0 & N^T \end{bmatrix},$$

from $P * P^{-1} = I$, we immediately have $PQ = \tilde{Q}$ and the following results:

$$\begin{aligned} &Q^T PA_{cl}Q = \tilde{Q}^T A_{cl}Q = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \\ &Q^T PF_{cl} = \tilde{Q}^T F_{cl} = \begin{bmatrix} B_{12} \\ B_{22} \end{bmatrix}, \\ &C_{cl}Q = \left[\, C_z\boldsymbol{X} + D_z\hat{\boldsymbol{D}}C\boldsymbol{X} + D_z\hat{\boldsymbol{C}}M^T \ \ C_z + D_z\hat{\boldsymbol{D}}C \,\right], \end{aligned}$$

where

$$\begin{aligned} A_{11} &= A\boldsymbol{X} + B\hat{\boldsymbol{D}}C\boldsymbol{X} + B\hat{\boldsymbol{C}}M^T,\ A_{12} = A + B\hat{\boldsymbol{D}}C, \\ A_{21} &= \boldsymbol{Y}A\boldsymbol{X} + \boldsymbol{Y}B\hat{\boldsymbol{D}}C\boldsymbol{X} + N(\hat{\boldsymbol{B}}_{\boldsymbol{y}} + \hat{\boldsymbol{B}}_{\boldsymbol{u}}\hat{\boldsymbol{D}})C\boldsymbol{X} \\ &\quad + \boldsymbol{Y}B\hat{\boldsymbol{C}}M^T + N(\hat{\boldsymbol{A}} + \hat{\boldsymbol{B}}_{\boldsymbol{u}}\hat{\boldsymbol{C}})M^T, \\ A_{22} &= \boldsymbol{Y}A + \boldsymbol{Y}B\hat{\boldsymbol{D}}C + N(\hat{\boldsymbol{B}}_{\boldsymbol{y}} + \hat{\boldsymbol{B}}_{\boldsymbol{u}}\hat{\boldsymbol{D}})C, \\ B_{12} &= B_f + B\hat{\boldsymbol{D}}D_f,\ B_{22} = \boldsymbol{Y}B_f + \boldsymbol{Y}B\hat{\boldsymbol{D}}D_f + N(\hat{\boldsymbol{B}}_{\boldsymbol{y}} + \hat{\boldsymbol{B}}_{\boldsymbol{u}}\hat{\boldsymbol{D}})D_f. \end{aligned}$$

Then we can define

$$\begin{aligned} &\bar{\boldsymbol{A}} = \hat{\boldsymbol{A}} + \hat{\boldsymbol{B}}_{\boldsymbol{u}}\hat{\boldsymbol{C}},\ \bar{\boldsymbol{B}}_{\boldsymbol{y}} = \hat{\boldsymbol{B}}_{\boldsymbol{u}}\hat{\boldsymbol{D}} + \hat{\boldsymbol{B}}_{\boldsymbol{y}}, \\ &\tilde{\boldsymbol{A}} = \boldsymbol{Y}A\boldsymbol{X} + \boldsymbol{Y}B\hat{\boldsymbol{D}}C\boldsymbol{X} + N\bar{\boldsymbol{B}}_{\boldsymbol{y}}C\boldsymbol{X} + \boldsymbol{Y}B\hat{\boldsymbol{C}}M^T + N\bar{\boldsymbol{A}}M^T, \\ &\tilde{\boldsymbol{B}}_{\boldsymbol{y}} = \boldsymbol{Y}B\hat{\boldsymbol{D}} + N\bar{\boldsymbol{B}}_{\boldsymbol{y}},\ \tilde{\boldsymbol{C}} = \hat{\boldsymbol{D}}C\boldsymbol{X} + \hat{\boldsymbol{C}}M^T \end{aligned}$$

If M and N are invertible, the variable $\bar{\boldsymbol{A}}, \bar{\boldsymbol{B}}_{\boldsymbol{y}}$ can be replaced by the new variables $\tilde{\boldsymbol{A}}, \tilde{\boldsymbol{B}}_{\boldsymbol{y}}$ without loss of generality. The constraint $P > 0$, can be expressed as an LMI as follows:

$$P > 0 \Leftrightarrow Q^T PQ > 0 \Leftrightarrow \begin{bmatrix} \boldsymbol{X} & I \\ I & \boldsymbol{Y} \end{bmatrix} > 0.$$

Also define $T = diag(Q, I)$, then (7) is true if and only if $T^T T_{mi} T < 0$, which results in (8) readily. □

Remark 6. From the above construction, we can observe that the controller gain $\hat{B}_u$ provides one extra degree of freedom in the design procedure, where the choice of $\hat{B}_u$ will not affect optimality. It can also be seen from (8) that $\hat{B}_u$ does not appear in the LMI iteration.

Similarly, we can also formulate the constraint $\|T_{zd}\|_\infty < \gamma_d$ into LMIs and hence has the following result for finding a controller to optimise (6):

$$\begin{bmatrix} A\boldsymbol{X}+B\tilde{\boldsymbol{C}}+(\star) & \star & \star & \star \\ \tilde{\boldsymbol{A}}+(A+B\hat{\boldsymbol{D}}C)^T & \boldsymbol{Y}A+\tilde{\boldsymbol{B}}_{\boldsymbol{y}}C+(\star) & \star & \star \\ (B_f+B\hat{\boldsymbol{D}}D_f)^T & (\boldsymbol{Y}B_f+\tilde{\boldsymbol{B}}_{\boldsymbol{y}}D_f)^T & -\gamma I & \star \\ C_z\boldsymbol{X}+D_z\tilde{\boldsymbol{C}} & C_z+D_z\hat{\boldsymbol{D}}C & D_z\hat{\boldsymbol{D}}D_f & -\gamma I \end{bmatrix} < 0, \quad \begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0. \tag{8}$$

Theorem 3.1. Let all variables be defined as above, then a stabilizing GDC exists such that (6) is achieved with a given γ_d if there exists X, Y, $\tilde{A}$, $\tilde{B}_y$, $\tilde{C}$ and $\hat{D}$ such that LMIs (8) and the following (10) are ture.

Then, the stabilizing GDC is similarly given by (9).

Remark 7. The constrained optimisation problem in (6) is solved here via dual LMIs (8) and (10) with LMI constraint for stabilizing the solution. Since γ_d is given, the variable to be minimised via LMIs is γ only, which can be easily handled with MATLAB Robust Toolbox.

3.2 Algorithm for Output Selection

As pointed out before, the controller design in Section 3.1 is solely for an individual candidate of sensor sets. Nevertheless, our objective is to find an output combination which achieves the best performance defined as in (6) among all available and reasonable candidates. An iterative procedure is employed to complete the search.

The following steps are important for sensor selection decision making:

Algorithm 1.

(1) Define fault conditions for the problem setup.
(2) Define sensor set.
(3) Solve for stabilizing dynamic controller, i.e. find γ, as in Theorem 3.1.
(4) Update and goto step 2 if fault conditions unchanged.
(5) Update and goto step 1 if fault conditions change.

4. SENSOR OPTIMISATION FOR A RAILWAY VEHICLE WITH FAULT TOLERANCE

The mathematical model of the application is based on the side-view of a railway vehicle as shown in Figure 2, considering both the bounce and pitch motions of the vehicle body and only the bounce motion of the bogie masses. The suspensions, which include the primary suspensions and secondary suspensions, are represented by dampers and springs in parallel. In fact, the primary suspension is mainly for providing guidance of the vehicle and the secondary suspension is aimed to improve the ride quality of the vehicle. Active control is provided by actuators placed across the front and rear secondary suspensions. The control objective is to achieve good ride quality while maintaining adequate suspension clearance, i.e. minimizing the acceleration of the vehicle body experienced by passengers without causing large suspension deflections. The dynamics of the model is given by Zheng et al. [2006]

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B_d}\mathbf{d} + \mathbf{B}\mathbf{u} \tag{11}$$

Only the rigid motion of the railway vehicle body is considered, where the states are chosen as translational velocities of the three masses, the rotational velocity of the vehicle body, and deflections across the various springs, represented as

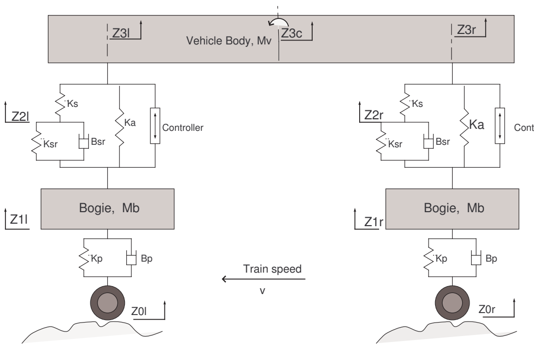

Fig. 2. A suspension system of a railway vehicle

$$x = [\dot{Z}_{3c} \;\; \dot{\theta} \;\; \dot{Z}_{1l} \;\; \dot{Z}_{1r} \;\; Z_{3l}-Z_{1l} \;\; Z_{3r}-Z_{1r} \;\; Z_{2l}-Z_{1l} \;\; Z_{2r}-Z_{1r} \;\; Z_{1l}-Z_{0l} \;\; Z_{1r}-Z_{0r}] \tag{12}$$

The suspension control inputs and track disturbance inputs are given as

$$\mathbf{u} = [U_l \;\; U_r], \; \mathbf{d} = [\dot{Z}_{0l} \;\; \dot{Z}_{0r}]. \tag{13}$$

The corresponding system matrices are given as follows. Since our control objective is to improve ride quality via minimizing the acceleration of the vehicle body, the regulated output is chosen to be bounce accelerations of the vehicle body, i.e., $\ddot{Z}_{3c}, \ddot{Z}_{3l}, \ddot{Z}_{3r}$. For this purpose of control, we have the following sensors available:

(1) bounce acceleration sensor at left to measure $\ddot{Z}_{3l}$
(2) bounce acceleration sensor at right to measure $\ddot{Z}_{3r}$
(3) deflection sensor to measure $Z_l = Z_{3l} - Z_{1l}$
(4) deflection sensor to measure $Z_r = Z_{3r} - Z_{1r}$

Therefore, the output equation with full measurements is given by

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D_d}\mathbf{d} + \mathbf{D}\mathbf{u} \tag{14}$$

where

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & -66.4312 & 12.9575 & 66.4312 & -12.9575 & 0 & 0 \\ 0 & 0 & 0 & 0 & 12.9575 & -66.4312 & -12.9575 & 66.4312 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$D = (1.0e-004) * \begin{bmatrix} -0.6539 & 0.1275 \\ 0.1275 & -0.6539 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \; D_d = 0.$$

Note that $D \neq 0$, although it is very small. The trick used here is to modify the output to include Du via loopshifting Safonov and Limebeer [1988], then we can apply the proposed controller design directly. We choose all distribution matrices for regulated signals from the output equation. In the remaining of this paper, case studies are carried out based on the selection of above measurements and potential source of sensor faults. First,

$$\begin{bmatrix} A\boldsymbol{X}+B\tilde{\boldsymbol{C}}+(\star) & \star & \star & \star \\ \tilde{\boldsymbol{A}}+(A+B\hat{\boldsymbol{D}}C)^T & \boldsymbol{Y}A+\tilde{\boldsymbol{B}}_{\boldsymbol{y}}C+(\star) & \star & \star \\ (B_d+B\hat{\boldsymbol{D}}D_d)^T & (\boldsymbol{Y}B_d+\tilde{\boldsymbol{B}}_{\boldsymbol{y}}D_d)^T & -\gamma_d I & \star \\ C_z\boldsymbol{X}+D_z\tilde{\boldsymbol{C}} & C_z+D_z\hat{\boldsymbol{D}}C & D_{zd}+D_z\hat{\boldsymbol{D}}D_d & -\gamma_d I \end{bmatrix} < 0, \; \begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0. \tag{10}$$

$$A=\begin{bmatrix} 0 & 0 & 0 & 0 & -26.7368 & -26.7368 & 26.7368 & 26.7368 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4.1784 & -4.1784 & -4.1784 & 4.1784 & 0 & 0 \\ 0 & 0 & -40.592 & 0 & 406.4 & 0 & -406.4 & 0 & -3948 & 0 \\ 0 & 0 & 0 & -40.592 & 0 & 406.4 & 0 & -406.4 & 0 & -3948 \\ 1 & -9.5 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 9.5 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 15.8478 & 0 & -23.7716 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15.8478 & 0 & -23.7716 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B=0.001*\begin{bmatrix} -0.0263 & -0.0263 \\ 0.0041 & -0.0041 \\ 0.4000 & 0 \\ 0 & 0.4000 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, B_d=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 40.5920 & 0 \\ 0 & 40.5920 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1.0000 & 0 \\ 0 & -1.0000 \end{bmatrix}.$$

extensive exploitations of a number of special cases are given. It is worth noting that we emphasise the importance of sensor selection, rather than strictly presenting the best ride quality improvement strategies (for the latter please refer to Zheng et al. [2006] and references within).

Full Measurements In this normal case, we assume that there are two faults occurring in the left and right deflection sensors with distribution matrices as

$$B_f=0, \; D_f=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T.$$

The control design objective is chosen as to ensure a worst case control performance via a generic $\mathcal{H}_\infty$ dynamic controller, with the presence of faults. The design parameter γ_d, is calibrated from classical $\mathcal{H}_\infty$ control design in a fault-free environment. In this case, γ_d is chosen to be 1 to gain satisfactory disturbance attenuation. For the full sensor set (namely, 4 measurable outputs), if we choose $\hat{B}_u = B$, Theorem 3.1 gives a generic $\mathcal{H}_\infty$ dynamic controller and an optimal $\gamma_0 = 0.0174$.

A time-domain simulation result is also given to verify that the controller design does not significantly lose acceptable control performance when faults occur while maintaining appropriate disturbance attenuation. Figure 3 shows the time response with the full set subject to left deflection sensor fault f_3 and right deflection sensor fault f_4, where f_3 is simulated by an abrupt jump from the $2nd$ second and f_4 is a negative unit step from the $6th$ second. Moreover, both track disturbances (i.e. the original and delayed versions) are Gaussian noises with mean zero and variance is $2\pi^2 Ar \times v$ (one-sided) for a speed of $v = 55(m/s)$ and a typical good quality track with track roughness $Ar = 2.5e-7(m)$.

Three Measurements We remove one measurement from the controller input and then observe the change of control performance with regard of fault tolerance. Note also that the controller is designed relative to the sensor set used, i.e. potential sensor fault could be excluded if the sensor is not used in the feedback. Here, left acceleration sensor is removed and for controller design we first assume that there is only one potential fault in the left deflection sensor, with

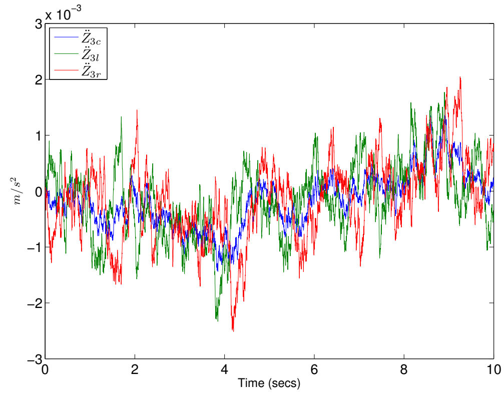

Fig. 3. Time response under fault tolerant design with four outputs y_1, y_2, y_3, y_4 and faults f_3, f_4.

$$B_f=0, \; D_f=\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T.$$

Similarly, Theorem 3.1 gives a generic $\mathcal{H}_\infty$ dynamic controller and an optimal $\gamma_0 = 0.0319$. Now, we increase a fault to the left deflection sensor and then identical calculation gives a corresponding optimal $\gamma_0 = 52.0663$. The result, combined with fundamental analysis Zheng et al. [2006], indicates that two deflection sensors contributes more significantly in the feedback control.

Simulated by the same disturbances as in above case, the time responses of regulated outputs subject to one sensor fault are shown in Figure 4.

Analysis and Comments Then, we investigate the remaining sensor sets following Algorithm 1. Since we have four available measurable outputs, there are $2^4 - 1 = 15$ combinations of sensor sets, of which only sets with more than one sensor are discussed. Thus, we continue in a similar way of controller design relative to other sensor selections and summarize the results in Table 1.

Note from Table 1 that all sensor combinations relate to both faults f_3, f_4. Thus, in a fault-free environment and for the defined control problem formulation it is appropriate to choose a three-sensor set as it is still possible to have proper robustness properties to the disturbances and faults affecting the system. Note that the aim is to choose the minimum set of sensors satisfying the objectives. This set

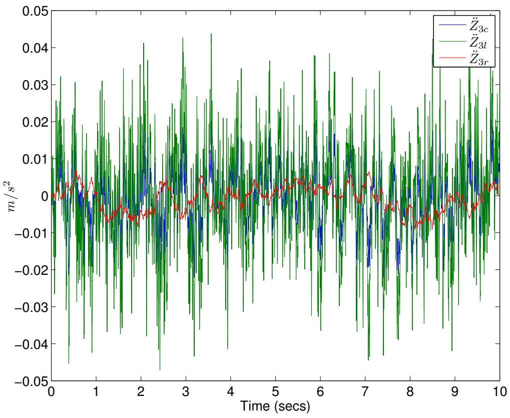

Fig. 4. Time response with y_2, y_3, y_4 and f_3.

Table 1. Output selection performance index considering faults f_3, f_4

Outputs ρ	f_3	f_4	f_3, f_4
y_1, y_2, y_3, y_4	0.0174	0.0174	0.0174
y_2, y_3, y_4	0.0319	0.0205	52.0663
y_1, y_3, y_4	0.0205	0.0319	52.0663
y_1, y_2, y_4	n/a	**0.0174**	n/a
y_1, y_2, y_3	**0.0174**	n/a	n/a
y_1, y_2	n/a	n/a	n/a
y_1, y_3	271.9498	n/a	n/a
y_1, y_4	n/a	52.2155	n/a
y_2, y_3	52.2155	n/a	n/a
y_2, y_4	n/a	271.9515	n/a
y_3, y_4	54.8313	54.8316	n/a

can be used as a basis for further designs in a complete fault tolerant framework.

A standard way of quantitative validation of time responses is to observe the root mean square (RMS) of regulated and measured outputs, shown in Table 2 (note that NF defines No Fault conditions).

Table 2. RMS of outputs

	$\ddot{Z}_{3c}$ ($\%g$)	$\ddot{Z}_{3l}$ ($\%g$)	$\ddot{Z}_{3r}$ ($\%g$)	Z_l (mm)	Z_r (mm)
Passive	1.52	2.78	3.50	7.96	11.4
H-inf (NF)	0.225	0.305	0.307	12.2	12.6
H-inf (f_3, f_4)	21.0	59.2	59.3	88546	84268
GDC (NF)	0.0050	0.0069	0.0073	34.0	35.4
GDC (f_3, f_4)	0.0050	0.0069	0.0073	34.0	35.4

The above table clearly indicates that classical $\mathcal{H}_\infty$ control design has insufficient fault tolerance under the presence of sensor faults. GDC design can maximally restrain the effects of sensor faults although there exists performance degradation of deflection control both in fault-free and faulty environment. (Due to space limitation, RMS results for other output sets are not presented here, although similar concepts apply).

Moreover, it is possible to follow a combinatorial decision making procedure if necessary. Thus, select different optimal sets of sensors corresponding to different (appropriate) fault considerations and utilise these as bases in the design of a re-configurable (e.g. switching between the different controllers) scheme for fault tolerant systems.

5. CONCLUSION

We discussed on a new setup to the output selection problem with consideration of passive fault tolerance. The performance index for decision making investigated combines both a generic $\mathcal{H}_\infty$ controller design and robustness design to faulty components. An iterative approach is then followed to testify different sensor sets under this performance index, with each of them solved by LMIs. The output selection algorithm has been applied to a practical railway vehicle rigid quality system with pre-set sensor fault scenarios.

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