Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Dynamic Feedback Tracking Control of Nonholonomic Mobile Robots with Unknown Camera Parameters ⋆

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Abstract: This paper investigated the tracking control of nonholonomic mobile robots with uncertainties. Nonholonomic kinematic systems with visual feedback are uncertain and more involved in comparison with common kinematic systems. Barbalat theorem and Lyapunov techniques were exploited to craft a dynamic feedback robust controller that enables the mobile robot configuration tracking despite the lack of depth information and the lack of precise visual parameters. The most interesting feature of this paper is that the problem was discussed in the image frame and the inertial frame, which made the problem easy and useful. The convergence of the error system by using the proposed method was rigorously proved. The simulation was given to show the effectiveness of the presented controllers.

1. INTRODUCTION

Considerable attention has been paid toward the motion tracking control of nonholonomic mechanical systems in the last decade and different control algorithms can be found in the literature Bloch et al. [1992],Chang et al. [1996],Colbaugh et al. [1996], Sampei et al. [1995]. The interest in such nonlinear control systems originates from the fact that such problems are not amenable to usual methods of linear control theory.

Since the chained-form system was first introduced by Murray et al. [1993], it has been extensively studied by many researchers as a benchmark example in the area of control of nonholonomic systems. Simple approaches using PID controllers Normey-Rico et al. [2001],Figueiredo et al. [2005] and nonholonomic constraints[10a] were proposed. Earlier research includes the work of Jiang Jiang et al. [1997], who developed a methodology using time-varying feedback and backstepping applied to a differential driven mobile robot. This approach was extended to chained form systems Jiang et al. [1999] and to systems with uncertainties Jiang [2000]. Trajectory tracking of underactuated ships by Lyapunov's direct method is presented in Jiang Jiang [2002]. The guidance of marine vehicles using models in polar coordinates has also received attentionAicardi et al. [2001],Aicardi et al. [2001]. Sliding mode trajectory tracking strategies for differential driven mobile robots are treated in Yang et al. [1999],Yang et al. [1999], and Chwa Chwa et al. [2002]. With reference to the popular unicycle kinematics, Oriolo and co-workers Oriolo et al. [2002] shown that dynamic feedback linearization is an efficient design tool leading to a solution simultaneously valid for both trajectory tracking and set-point regulation problems. Motivated by a famous theorem of Brockett Brockett [1983], which implies that there is no smooth or even continuous time-invariant state feedback law to asymptotically stabilize such a nonholonomic system, most efforts have been devoted to the stabilization problem (see e.g., Astolfi [1996],Kolmanovsky et al. [1995],Samson [1995],Tian et al. [2002]), which still remains to be a very interesting topic today. However, recent years have also seen increasing interests in the tracking control problem of the chained-form system Walsh et al. [1994]. Global K-exponential controllers are constructed for the tracking control problem of nonholonomic systems in chained-form whose reference targets are allowed to converge to a point exponentially in Tian et al. [2007]. A switching scheme was proposed to solve Lyapunov stability and exponential convergence for uncertain chained form systems using sate feedback in Xi et al. [2003]. In Xi et al. [2004], robust global exponential regulation and Lyapunov stability with out feedback were discussed for a class of disturbed nonlinear chained systems.

A visual servo tracking controller was developed in Chen et al. [2006] for a monocular camera system mounted on an underactuated wheeled mobile robot (WMR) subject to nonholonomic motion constraints (i.e., the camera-in-hand problem). A Lyapunov-based analysis is used to develop an adaptive update law to actively compensate for the lack of depth information required for the translation error system. However, the proposed arguments in that paper were somewhat of problem, which was described in the remark in detail. In this paper, a new controller was proposed and strict proof was given.

The paper is organized as follows. Section 2 introduced the problem statement. In Section 3, the controllers were synthesized. In Section 4, the simulation results were carried out to validate the theoretical framework. Finally, in Section 5 the major contribution of the paper is summarized.

⋆ This paper was supported by The National Nature Science Foundation(60474009), Shu Guang Program(05SG48) and The Science Technology Program of Shanghai Education Committee(F78573)).

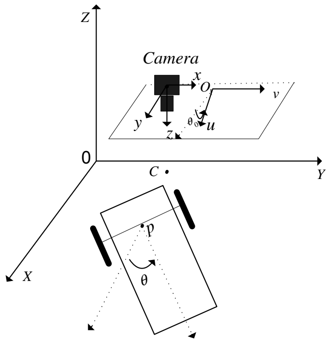


Fig. 1. Wheeled Mobile Robots with Monocular Camera

2. PROBLEM STATEMENT

2.1 System Configuration

In the Figure 1 of the mobile robot shown below. Assume that a pinhole camera is fixed to the ceiling and the camera plane and the mobile robot plane are parallel. There are three coordinate frames, namely the inertial frame X-Y-Z, the camera frame x-y-z and the image frame u-O_1-v. Assume that the x-y plane of the camera frame is the identical one with the plane of the image coordinate plane. C is the crossing point between the optical axis of the camera and X-Y plane. Its coordinate relative to X-Y plane is (p_x, p_y), the coordinate of the original point of the camera frame with respect to the image frame is defined by (O_{c1}, O_{c2}), (x, y) is the coordinate of the mass center of the robot with respective to X-Y plane. Suppose that (x_m, y_m) is the coordinate of (x, y) relative to the image frame. Pinhole camera model yields

$$\begin{bmatrix} x_m \\ y_m \end{bmatrix} = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} R \left[\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} p_x \\ p_y \end{bmatrix} \right] + \begin{bmatrix} O_{c1} \\ O_{c2} \end{bmatrix} \qquad (1)$$

where α_1, α_2 are constant which are dependent on the depth formation, focus length, scalar factors along x axis and y axis respectively.

$$R = \begin{bmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{bmatrix} \qquad (2)$$

where θ_0 denotes the angle between u axis and X axis with a positive anticlockwise orientation.

2.2 Problem Description

Assume that the geometric center point and the mass center point of the robot are the same. The nonholonomic constraint is defined by

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0 \qquad (3)$$

By this formula, nonholonomic kinematic equation is written by

$$\begin{cases} \dot{x} = vcos\theta \\ \dot{y} = vsin\theta \\ \dot{\theta} = \omega \end{cases} \qquad (4)$$

where v and ω denote the velocity of the heading direction of the robot and the angle velocity of the rotation of the robot, respectively.

In the image frame, the kinematic model can be deduced by (1)

$$\begin{aligned} \begin{bmatrix} \dot{x}_m \\ \dot{y}_m \end{bmatrix} &= \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} R \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \\ &= \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} R\nu \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \\ &= \nu \begin{bmatrix} \alpha_1 \cos(\theta - \theta_0) \\ \alpha_2 \sin(\theta - \theta_0) \end{bmatrix} \end{aligned} \qquad (5)$$

Generally, (x, y) can be obtained from the encoders of motors and other sensors such as ultrasonic sensors, infrared sensors, etc.. However, for complex environment, it is difficult to do it. But vision information can be easily exploited to deal with this problem.

In this paper, the camera is used to measure (x, y) and determine the desired target. A kind of effective method is that the error between the mass center point of the robot and its desired point in the image frame can be used in the closed-loop feedback control of the robot. As for the angle, θ, it can be obtained easily from the angle sensor. Therefore, θ is still included in the error model.

In contrary to the general model of nonholonomic mobile robots, three new parameters, α_1, α_2 and θ_0, are added. Suppose that the three parameters are available. Then the system can be reduced by the following transformation as

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{\alpha_1}\cos(\theta - \theta_0) & \frac{1}{\alpha_2}\sin(\theta - \theta_0) \\ -\frac{1}{\alpha_1}\sin(\theta - \theta_0) & \frac{1}{\alpha_2}\cos(\theta - \theta_0) \end{bmatrix} \begin{bmatrix} x_m \\ y_m \end{bmatrix} \qquad (6)$$

Let

$$\begin{cases} u_1 = \omega \\ u_2 = -\omega x_2 + \nu \end{cases} \qquad (7)$$

(6) can be deduced

$$\begin{aligned} \dot{x}_1 &= -\omega x_2 + \nu = u_2 \\ \dot{x}_2 &= \omega x_1 = u_1 x_1 \\ \dot{\theta} &= u_1 \end{aligned} \qquad (8)$$

This is a common nonholonomic chained form system. Lots of methodsMurray et al. [1993]Sordalen et al. [1995]Jiang et al. [1997]Aicardi et al. [1994] can be used to investigate it.

But when α_1, α_2 and θ_0 are unknown, the transformation (6) cannot be used for state feedback.

Considered next are controller designs for unknown α_1 and α_2.

Assumption: θ_0 known and $\alpha_1 = \alpha_2 = d$ unknown. In general, for a CCD camera, the scalar factor on the x axis is almost identical to that on the y axis.

Under this case, system (5) can be rewritten as

$$\begin{bmatrix} \dot{x}_m \\ \dot{y}_m \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \nu d \cos(\theta - \theta_0) \\ \nu d \sin(\theta - \theta_0) \\ \omega \end{bmatrix} \qquad (9)$$

Substituting θ by $(\theta - \theta_0)$, it follows that

$$\begin{bmatrix} \dot{x}_m \\ \dot{y}_m \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \nu d \cos\theta \\ \nu d \sin\theta \\ \omega \end{bmatrix} \quad (10)$$

Set

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_m \\ y_m \end{bmatrix} \quad (11)$$

Consider the tracking problem of a nonholonomic mobile robot with a fixed camera. The desired trajectory is obtained from a prerecorded set of images of a stationary target viewed by the fixed camera as the WMR moves. For example, the desired WMR motion could be obtained as an operator drives the robot via a teach pendant, with the fixed camera capturing and storing the sequence of images of the stationary target.

Let $x_3 = \theta$. The kinematics of the robot can be completely described in the image frame and inertia frame as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -d\nu + x_2\omega \\ -x_1\omega \\ \omega \end{bmatrix} \quad (12)$$

where x_3 means θ, which denotes the right-handed rotation angle about the rotation of the actual robot and the reference robot. x_1, x_2 and x_3 can be obtained by using calibration. d is a unknown time-varying parameter, corresponding to certain variable relevant to distance in physics. ν and ω denote the respective linear and angular velocity of the actual WMR. In fact, equation (12) is equivalent to commonly nonholonomic chained form systems with two inputs if $-d\nu + x_2\omega$ is substituted by another new input. However, in practical engineering, d is usually unknown. Hence, the methods available in the literature cannot be directly used to investigate these kinds of systems.

In addition, given that the desired trajectory is generated from a prerecorded set of images taken by the fixed camera as the WMR was moving, a similar expression (12) can be developed as follows:

$$\begin{bmatrix} \dot{x}_{d1} \\ \dot{x}_{d2} \\ \dot{x}_{d3} \end{bmatrix} = \begin{bmatrix} -d\nu_d + x_2\omega_d \\ -x_1\omega_d \\ \omega_d \end{bmatrix} \quad (13)$$

where ν_d and ω_d denote the respective linear and angular velocity of the desired WMR. In practical application, x_{1d}, x_{2d}, x_{3d}, ω_d, ν_d are available.

Let

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_{1d} \\ x_2 - x_{2d} \\ x_3 - x_{3d} \end{bmatrix}$$

By using (12) and (13), the derivatives of the equation above can be written by

$$\begin{bmatrix} \dot{e_1} \\ \dot{e_2} \\ \dot{e_3} \end{bmatrix} = \begin{bmatrix} -d(\nu - \nu_d) + e_2\omega + x_{d2}(\omega - \omega_d) \\ -e_1\omega - x_{d1}(\omega - \omega_d) \\ \omega - \omega_d \end{bmatrix} \quad (14)$$

Then the tracking problem of the nonholonomic WMR with an on -board camera is how to design controller ν and ω such that e_1, e_2, and e_3 converge to zero as t goes to infinity if d and ν_d are unknown.

3. CONTROLLER DESIGN

Assumption 1 ν_d and ω_d are bounded. x_{1d}, x_{2d}, x_{3d} and their derivatives and the second derivative of x_{2d} are bounded. There exists a known positive number V_d such that $|\nu_d(t)| \leq V_d$.

Assumption 2 $\dot{x}_{1d} \nrightarrow 0$ as $t \to \infty$.

Theorem Given arbitrarily positive k_1 and k_2, the controller is chosen as follows:

$$\begin{bmatrix} \dot{p} \\ \omega \\ \nu \end{bmatrix} = \begin{bmatrix} -k_2p - e_3 + e_2x_{d1} - e_1x_{d2} \\ \omega_d + p \\ k_1e_1 + V_d \mathrm{sgn}\ e_1 \end{bmatrix} \quad (15)$$

Then, under assumption1, the states of the closed loop system consisted of (14) and (15) converge to zero as t goes to infinity for unknown d and ν_d. ■

Proof. By using (15), consider the extended closed loop system written by

$$\begin{bmatrix} \dot{e_1} \\ \dot{e_2} \\ \dot{e_3} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} -d(k_1e_1 + V_d\ \mathrm{sgn}\ e_1 - \nu_d) + e_2\omega + x_{d2}p \\ -e_1\omega - x_{d1}p \\ p \\ -k_2p - e_3 + e_2x_{d1} - e_1x_{d2} \end{bmatrix} \quad (16)$$

Choose the candidate Lyapunov function of the system above as follows:

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + p^2)$$

Differentiating it along (16), we have

$$\begin{aligned} \dot{V} =& -k_1de_1^2 - dV_d|e_1| + d\nu_de_1 + e_2e_1\omega + x_{d2}pe_1 \\ & -e_1e_2\omega - x_{d1}pe_2 + pe_3 - k_2p^2 - e_3p + e_2x_{d1}p \\ & -e_1x_{d2}p \\ =& -k_1de_1^2 - dV_d|e_1| + d\nu_de_1 - k_2p^2 \\ \leq& -k_1de_1^2 - dV_d|e_1| + d|\nu_d||e_1| - k_2p^2 \\ \leq& -k_1de_1^2 - k_2p^2 \end{aligned} \quad (17)$$

The last inequality above is obtained by using $|\nu_d(t)| \leq V_d$ in assumption 1.

By (17), V is monotonously decreasing. Therefore, $e_1 \in L_\infty$, $e_2 \in L_\infty$, $e_3 \in L_\infty$ and $p \in L_\infty$, and $e_1 \in L_2$, $p \in L_2$. According to (16) and assumption 1, we know that $\dot{e_1}$, $\dot{e_2}$, $\dot{e_3}$, and $\dot{p}$ are bounded and therefore e_1, e_2, e_3, and p are uniformly continuous. By Barbalat theorem, $e_1 \to 0$ and $p \to 0$ as $t \to \infty$. It is easy to see that

$$\frac{\mathrm{d}\,(-e_3 + e_2x_{d1})}{\mathrm{d}t} = -p + (-e_1\omega - x_{d1}p)\,x_{d1} + e_2\dot{x}_{d1}. \quad (18)$$

The right hand of equation (18) is bounded by assumption 1 and hence, $-e_3 + e_2x_{d1}$ is uniformly continuous. In addition, it is noted that $-k_2p - e_1x_{d2} \to 0$ as $t \to \infty$. By using the last equation of (16) and the Extended Barbalat lemma in Appendix, we have that $e_3 - e_2 x_{d1} \to 0$ as $t \to \infty$. According to (16), it is easy to deduce that

$$\frac{\mathrm{d}\left(e_2 \dot{x}_{d1}\right)}{\mathrm{d}t} = \left(-e_1 \omega - x_{d1} p\right) \dot{x}_{d1} + e_2 \ddot{x}_{d1} \qquad (19)$$

The right hand of equation (19) is bounded by assumption 2 and hence, $e_2 \dot{x}_{d1}$ is uniformly continuous. It is noted that $(-e_1\omega - x_{d1}p)\, x_{d1} \to 0$ as $t \to \infty$ because e_1 and $p \to 0$ as $t \to \infty$. Extended Barbalat lemma is again applied to equation (18), we know that $e_2 \dot{x}_{d1} \to 0$ as $t \to \infty$. By using $e_3 - e_2 x_{d1} \to 0$, we have $e_3 \dot{x}_{d1} \to 0$ as $t \to \infty$. It is easily seen that $p\dot{x}_{d1} \to 0$ and $e_1 \dot{x}_{d1} \to 0$ as $t \to \infty$. Therefore, $V\dot{x}_{d1} \to 0$ as $t \to \infty$. Because V is monotonous and has a limit as $t \to \infty$, it deduces that $V \to 0$ as $t \to \infty$ by using assumption 2. Hence, e_1, e_2, e_3 and $p \to 0$ as $t \to \infty$. ■

Remark. The controller proposed in Chen et al. [2006] is different from one given here. The adaptive law was not used in this paper. The method is much simpler. In addition, under the condition of assumption 2, in Chen et al. [2006], it is difficult to prove that

$$\lim_{t\to\infty} e_2(t) = 0$$

provided that the only condition

$$\lim_{t\to\infty} \dot{x}_{d1} e_2(t) = 0.$$

is satisfied. However, the proof proposed here is much strict.

4. SIMULATION

The simulation will be done based on the system (14).

Take the initial value $(-0.3, 1.4, 0.4)$ and the controller is chosen like (15). Choose $d = 0.2$, $w_d = 4$, $v_d = 0.3$, $k_1 = 54$, $k_2 = 11$, $/nu_d = 1$. The responses of states of the system with respect to time are shown in Fig. 2-9.

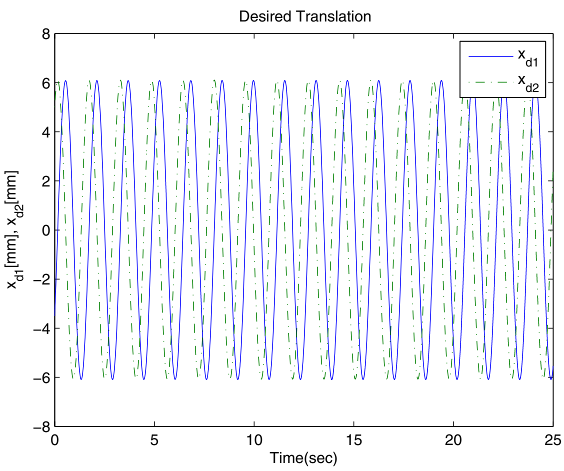


Fig. 2 Desired translation with respect to time

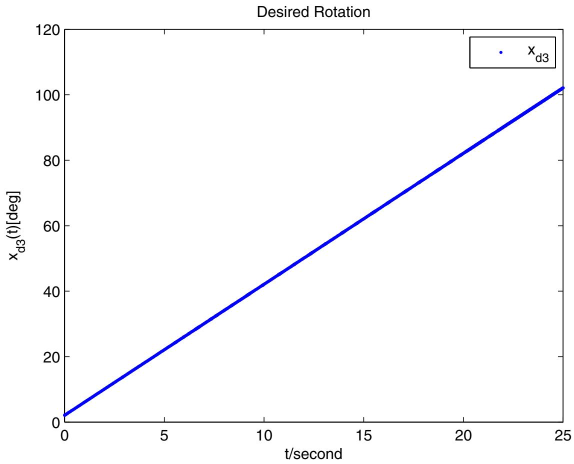
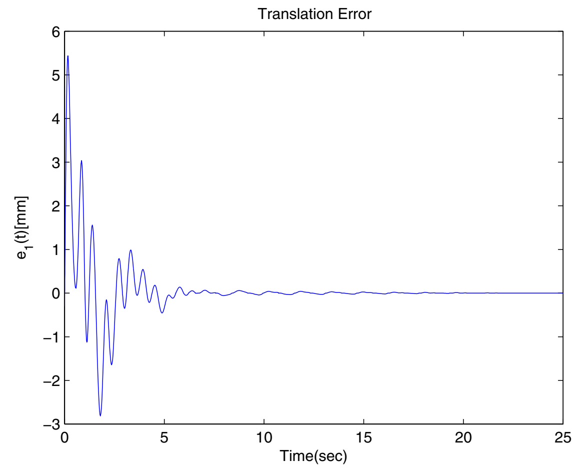


Fig. 3 Desired rotation with respect to time



translation Fig. 4 Translation error with respect to time

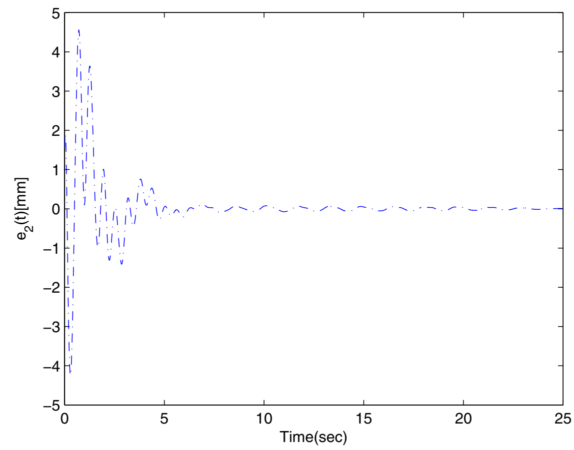


Fig. 5 Translation error with respect to time

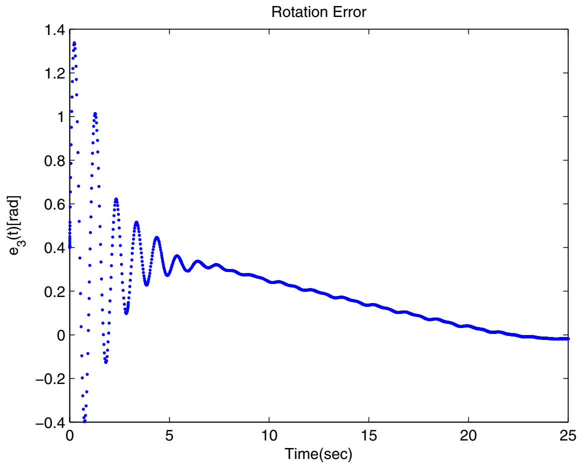


Fig. 6 The rotation error with respect to time

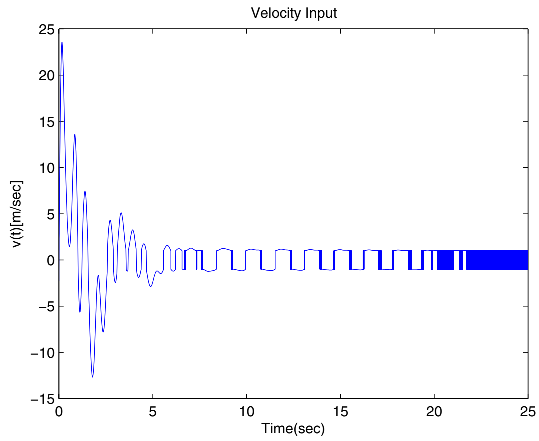


Fig. 7 The response of the input $v(t)$ with respect to time

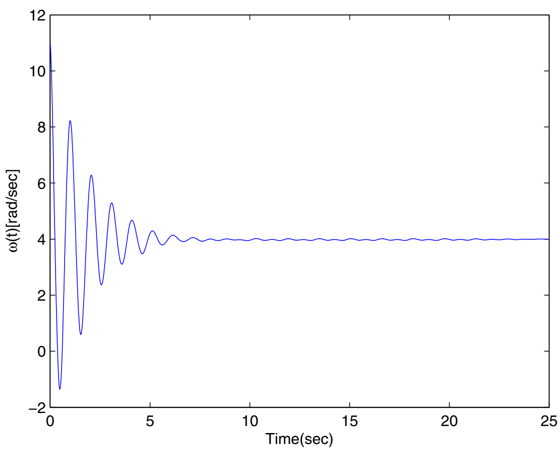


Fig. 8 The response of the input $\omega(t)$ with respect to time

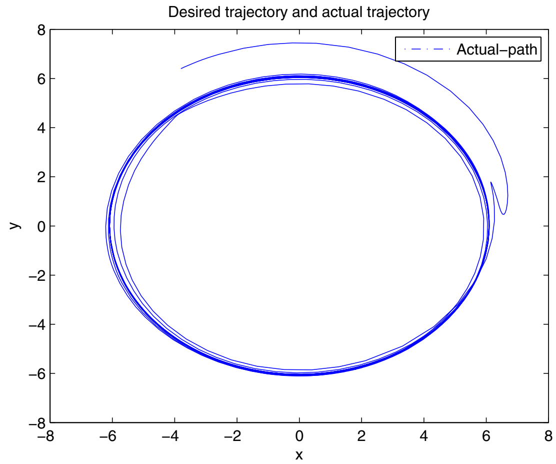


Fig. 9 The trajectory of the actual mobile robot tracking the desired mobile robot with respect to time

Generally, large $k1, k2$ will be helpful to increase the speed of convergence. Too small $p(0)$ will probably increase the time of simulation. Too small $p(0)$ will do it also too. Good $p(0)$ will be near 7.

5. CONCLUSION

This paper investigated the visual servoing tracking of nonholonomic mobile robots. Barbalat theorem and Lyapunov techniques were exploited to craft the dynamic robust controllers that enable the mobile robot configuration tracking despite the lack of depth information and the lack of precise visual parameters. The most interesting feature of this paper is that the problem was discussed in the image frame and the inertial frame, which made the problem easy and useful. The convergence of the error system by using the proposed method was rigorously proved. In this way, tracking problems can be developed for chained form systems and dynamic control systems.

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Lemma (Extended Barbalat's Lemma): If a differential function $f(t) \in$R has a finite limit as $t \to \infty$ and its time derivative can be written as follows:

$$\dot{f}(t) = g_1(t) + g_2(t)$$

where $g_1(t)$ is a uniformly continuous function and

$$\lim_{t\to\infty} g_2(t) = 0$$

then

$$\lim_{t\to\infty} \dot{f}(t) = 0 \text{ and } \lim_{t\to\infty} g_1(t) = 0$$

Its proof was seen in Slotine et al. [1991].