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HAMMERSTEIN SYSTEMS IDENTIFICATION IN PRESENCE OF HYSTERESIS-BACKLASH NONLINEARITY

F. GIRI [*] **, Y. ROCHDI, E. ELAYAN, A. BROURI and F. Z. CHAOUI**

[*] *GREYC, UMR 6072 CNRS, Universite de Caen,*
UFR des sciences, Campus 2,
BP 5186, 14032 Caen Cedex, FRANCE.
(e-mail: fouadgiri@yahoo.fr)

Abstract: The identification of Hammerstein systems is discussed for the systems that include memory nonlinearities. The focus is made on nonlinearities of the hysteresis-backlash type. The linear subsystem and the nonlinear element are identified separately. The identification of the former is dealt with combining an appropriate system parametrization and a specific input signal. The latter is designed so that it provides persistent excitation and makes the internal signal measurable in the considered parametrization. When the model of linear subsystem becomes available, the determination of the nonlinear element turns out to be easier. This is coped with using two appropriate parameterizations and specific input signals. The whole identification method is shown to be consistent.

1. INTRODUCTION

The nonlinear feature of physical systems may be captured using block-oriented models including linear dynamic blocks and nonlinear memory or memoryless elements [1]. The most known examples are Hammerstein and Wiener models. In this paper, we are considering the problem of nonlinear system identification based on Hammerstein model as shown in (Fig. 1). In most previous works devoted to Hammerstein system identification, the nonlinear element is supposed to be memoryless. Then, such an element is characterized by an algebraic expression $u = F(v, \theta)$ where the function $F(v, \theta)$ is supposed to be continuous in v and linear in the unknown parameter vector θ. Moreover, $F(v, \theta)$ is generally assumed to be a (truncated) polynomial or Fourier series in the variable v (e.g. [2]-[7]). Hammerstein system identification in presence of memory nonlinearities is a more challenging problem. It has been dealt with in [8] using a separable nonlinear least squares method. However, the proposed solution only applies to symmetric hysteresis backlash nonlinearities flanked by straight lines and involving unknown parameters.
In this paper, a new solution is developed for Hammerstein systems that contain not-necessarily symmetric backlash nonlinearities. Specifically, the considered hysteresis-backlash is flanked by two different polynomials P_1 and P_2 that involve several unknown parameters (Fig. 2). The purpose is to get consistent estimates of the linear subsystem parameters as well as estimates of the nonlinear parameters, i.e. the coefficients of the polynomials $P_1(v)$ and $P_2(v)$. The linear subsystem is first identified using a least squares estimator, based on an appropriate system parameterizations. A specific input signal is designed in such a way that persistent excitation is guaranteed and the involved internal signal becomes measurable. The consistent estimation of the linear subsystem makes it possible to obtain consistent estimation of the coefficient of the polynomials P_1 and P_2. These are separately estimated based on appropriate system parameterizations. The key step in the developing of these parametrizations is the design of an appropriate exciting signal that enforce the system to remain all the time on the polynomial borders of the nonlinearity and makes it possible to know, at each instant, on which branch of the nonlinearity the system is. The whole identification method is shown to be consistent.
The paper is organized as follows: Section (2) summarizes useful notations and technical lemma, the identification problem is formally presented in Section (3), Section (4) is devoted to identifying the linear subsystem parameters, identification of the nonlinear element is dealt with in Section (5),a conclusion and a reference list end the paper.

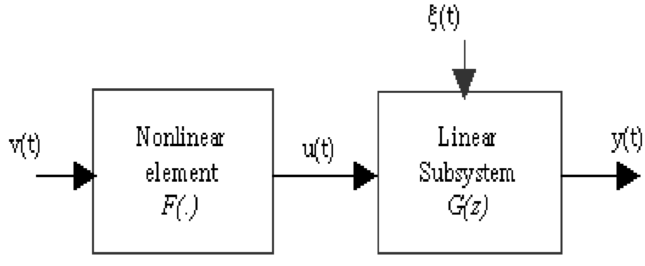

Fig. 1. Hammerstien model

2. NOTATIONS AND TECHNICAL LEMMA

2.1 Notations and acronyms

$F(.) = \mathbf{HB}(P_1, \quad P_2)$ is a Hysteresis-Backlash flanked by two polynomials $\{P_1(v), \quad P_2(v)\}$ of the form:

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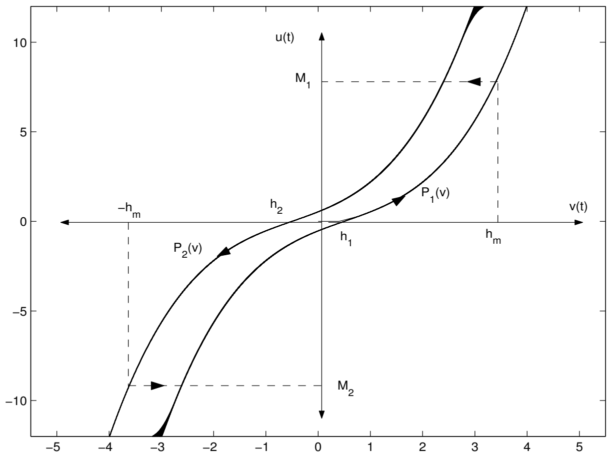

Fig. 2. An exemple of nonlinear (Backlash Hysteresis) element

$$P_1(v) = \sum_{i=0}^{m-1} c_{1i} v^i; \quad P_2(v) = \sum_{i=0}^{m-1} c_{2i} v^i \tag{1}$$

C_1: Vector of coefficients of $P_1(v)$.
C_2: Vector of coefficients of $P_2(v)$.
m: The dimension of the vectors C_1 and C_2, supposed to be known
t : discrete-time $(t = 0, 1, 2, ...)$
q^{-1} : delay operator (i.e. $q^{-1}x(t) = x(t-1)$)
PE : Persistent Excitation, Persistently Exciting
E(x(t)) : Ensemble mean of a stochastic process x(t)
$\bar{x}(N)$: Time average of a sequence x(t) i.e.:

$$\bar{x}(N) = \frac{1}{N}\sum_{i=1}^{N} x(i)$$

w.p.1 : with probability one

It is worth recalling that ; if x(t), y(t) are ergodic stationary stochastic processes, then one has w.p.1:

$$\lim_{N\to\infty} \bar{x}(N) = E(x(t)),$$

$$\lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} x(t)y(t+\tau) = E(x(t)y(t+\tau))$$

2.2 A class of persistently exciting sequences for linear systems

Consider a controllable linear system described by:

$$A(q^{-1})y(t) = B(q^{-1})u(t) + \xi(t) \tag{2}$$

where u(t), y(t) are the input and output, $\xi(t)$ accounts for external disturbances and $A(q^{-1})$, $B(q^{-1})$ are polynomial operators of the form:

$$A(q^{-1}) = 1 + a_1 q^{-1} + ... + a_n q^{-n}$$
$$B(q^{-1}) = b_1 q^{-1} + ... + b_n q^{-n}$$

Since $A(q^{-1})$ and $B(q^{-1})$ are coprime, due to system controllability, there exists a unique pair of polynomials $P(q^{-1})$, $Q(q^{-1})$ such that:

$$A(q^{-1})P(q^{-1}) + B(q^{-1})Q(q^{-1}) = 1 \tag{3}$$

Let us introduce the internal state z(t):

$$z(t) = P(q^{-1})u(t) + Q(q^{-1})x(t) \tag{4}$$

where x(t) denotes the undisturbed output, i.e.:

$$A(q^{-1})x(t) = B(q^{-1})u(t) \tag{5}$$

Then, operating $A(q^{-1})$ on (3) yields, using (4) and (2):

$$A(q^{-1})z(t) = u(t) \tag{6}$$

Let us introduce the state vector:

$$Z(t) = [z(t) \quad ... \quad z(t-2n+1)]^T \in \Re^{2n} \tag{7}$$

Then, one has the following technical lemma.

Lemma 1. Let the system (1) be submitted to an input signal of the form:

$$u(t) = \begin{cases} any \;\; value \quad if \quad t = t_k + 2n; \quad k = 0, 1, 2, ... \\ 0 \qquad\qquad\qquad otherwise \end{cases} \tag{8}$$

where t_k is any integer sequence such that $t_k \geq t_k - 1 + 4n$. Then, there exists a real constant such that, for all integers k:

$$\sum_{i=0}^{4n-1} Z(t_k + i)Z(t_k + i)^T \geq \lambda [u(t_k + 2n)]^2 I_{2n} \tag{9}$$

***Remarks** 1:*

(1) The above lemma is in fact a part of a more general result established in [9].
(2) The input sequence u(t) consists of a train of impulses. The time-interval separating two successive impulses cannot be smaller than 4n-1. The k^{th} impulse is applied at the center of the time-interval $[t_k, t_k + 4n - 1]$ and has an undefined amplitude. In view of (8), it produces its exciting effect in the same interval. This is referred to interval-excitation property, [9].
(3) In the case where $t_k = t_{k-1} + 4n$, and $|u(t_k + 2n)| \geq \gamma > 0$ (for all k and some $\gamma > 0$) then, inequality (8) provides the vector sequence *Z(t)* with the well known persistent excitation property (e.g. [10]). An interesting situation is when $|u(t_k + 2n)| = \gamma(-1)^k$: then the input sequence u(t) turns out to be periodical i.e. (easily realizable) and zero-mean. This idea is exploited later in this paper.

3. IDENTIFICATION PROBLEM STATEMENT

3.1 Class of identified systems

We are considering systems that can be described by the Hammerstein model (Fig. 1):

$$A(q^{-1})y(t) = B(q^{-1})u(t) + \xi(t) \quad and \quad u(t) = F(v(t)) \tag{10}$$

with,

$$A(q^{-1}) = 1 + a_1 q^{-1} + ... + a_n q^{-n}$$
$$B(q^{-1}) = b_1 q^{-1} + ... + b_n q^{-n}$$

where $\xi(t)$ is a bounded, stationary and ergodic sequence of zero-mean and stochastically independent variables. The function $F(.)$ is a hysteresis-backlash characterized by two polynomials $\{P_1(v), P_2(v)\}$ (Fig. 2). The system description is made more precise by the following assumptions:
A1. The order n of the linear subsystem is known.
A2. $A(z^{-1})$ and $B(z^{-1})$ are coprime.
A3. All zeroes of $q^n A(q^{-1})$ are strictly inside the unit circle.
A4. The polynomials $P_1(v)$, $P_2(v)$ are of known degree *(m-1)*. Furthermore, there is a known real h_m such that $h_m >$

$max(|h_1|, |h_2|)$.
where h_1 and h_2 are such that $P_1(h_1) = P_2(h_2) = 0$
A5. $B(1) \neq 0$ i.e. the static gain of the linear subsystem is nonzero.
A6. $\{\xi(t)\}$ is a zero-mean ergodic stationary independent stochastic process.
Remarks 2:

(1) The structure assumption A1 is a usual one, even in linear system identification.
(2) Assumption A2 ensures the controllability of the transfer function $B(z^{-1})/A(z^{-1})$ of the linear subsystem.
(3) Assumption A4 is not restrictive because h_m may be chosen arbitrarily large. This assumption is also required in [8], where symmetric hysteresis-backlash and hysteresis-relay have been considered, i.e. $h_1 = h_2 = a$ and $M_1 = M_2 = 1$. The proposed identification method involves a minimization problem with respect to a. A graphical minimum search has been resorted to get an estimate of the unknown value of a. In fact, this graphical search should be initialized in an interval including the unknown parameter a in order to converge to the latter (otherwise, it will converge to a local minimum in the search interval).
(4) An advantage of the present work with respect to [8], is to allow h_1 to be different of $-h_2$ and M_1 to be different of $-M_2$.
(5) Assumption 4 will prove to be useful when designing an exciting input signal that makes it possible to know, at each instant, on which side of the hysteresis the system operates i.e. it will be possible to know wether the couple $(v(t), u^*(t))$ is on the superior polynomial or on the inferior polynomial.
(6) Systems for which *B(1)=0* include the derivative operator $(1-q^{-1})$. This feature is relatively rare in practical applications and is easily recognizable. In such situations, assumption A3 can be complied with taking as input signal the quantity $(1-q^{-1})u(t)$.
Except for assumptions A1-A5, the system is arbitrary. Thus, the dynamic parameters (a_i, b_i) are unknown and the leading coefficients $(b_1, b_2, ...)$ may be null i.e. the true plant delay is also unknown (but is not greater than n).

3.2 Identification objective

Our purpose is to design an identification scheme that provides consistent estimates of both the linear subsystem $B(z^{-1})/A(z^{-1})$ and the nonlinear element $F(.) = \mathbb{HB}(P_1, P_2)$. Tree major difficulties have to be overcome: (i) the memory nature of the involved nonlinearity, (ii) the fact that the internal sequence u(t) is not measurable i.e. the external sequences *v(t)* and *y(t)* are the only information to be based-on in the identification process (Fig. 1), (iii) The system output is perturbed by $\xi(t)$.

4. IDENTIFICATION OF THE LINEAR DYNAMIC SUBSYSTEM

The problem of identifying the linear subsystem is dealt with in this section. The proposed identification solution is designed in three steps. First, an adequate system rescaling is introduced. The obtained system representation is further transformed to cope with the unavailability of the internal signal u(t). The transformed representation involves linearly the linear subsystem parameters and, therefore, is based upon to estimate these parameters. Finally, a persistently exciting input is resorted to ensure the consistency of the estimates.

4.1 Model reforming

The initial model of the system is characterized by the set $(A(q^{-1}),\ B(q^{-1}),\ F(.))$. As the interval signal $u(t)$ is not measurable, the system can be also represented by $(A(q^{-1}),\ B^*(q^{-1}),\ F^*(.))$, with;

$$B^*(q^{-1}) = \mu B(q^{-1}), \quad F^*(.) = F(.)/\mu \tag{11}$$

and $\mu = M_2 - M_1$. Then

$$\begin{aligned} B^*(q^{-1}) &= (M_2 - M_1)b_1 q^{-1} + ... + (M_2 - M_1)b_n q^{-n} \\ &= b_1^* q^{-1} + b_2^* q^{-2} + ... + b_n^* q^{-n} \end{aligned} \tag{12}$$

Note also that $F^*(.)$ is in turn a Hysteresis-Backlash that is flunked by polynomials:

$$P_1^*(v) = P_1(v)/\mu \quad , \quad P_2^*(v) = P_2(v)/\mu$$

Furthermore, if $F(v) = M_1$ (resp M_2), then, $F^*(v) = M_1/\mu \triangleq M_1^*$ (resp M_2^*) and $M_2^* - M_1^* = 1$.
Using equations (11) and (12), the system (10) can be described as follows:

$$\begin{aligned} A(q^{-1})y(t) &= B^*(q^{-1})u^*(t) + \xi(t) \\ u^*(t) &= F^*(v(t)) \end{aligned} \tag{13}$$

The internal signal $u^*(t)$ is still unavailable. Therefore, the system representation (13) will further be transformed in the next Subsection.

4.2 Design of a regression form for the linear subsystem

Let $\{y_1(t)\}$ denotes the solution of (13) corresponding to the following input sequence:

$$v_1(t) = \begin{cases} 0 & if \quad t = 0 \\ h_m & for \quad t > 0 \end{cases} \tag{14}$$

The signal $\{y_1(t)\}$ undergoes (for $t > 0$) the equation; :

$$\begin{aligned} A(q^{-1})y_1(t) &= B^*(q^{-1})u_1^*(t) + \xi_1(t) \\ u_1^*(t) &= F^*(h_m) \end{aligned} \tag{15}$$

where $\xi_1(t)$ denotes the realization of $\xi(t)$ during the present experiment. It readily follows from Fig. 2 and (11)-(13) that $u_1^*(t) = M_1/\mu$; $(\forall t \geq 1)$. Then, time-averaging of both sides of (15), over the interval $1 \leq t \leq L$, yields:

$$A(q^{-1})\bar{y}_1(L) = B^*(q^{-1})M_1^* + \bar{\xi}_1(L) \tag{16}$$

The ergodicity of $\{\xi_1(t)\}$ implies that $\bar{\xi}_1(L) \to 0$, as $L \to \infty$ (w.p. 1). Also, let $\bar{y}_1$ denotes the limit of $\bar{y}_1(L)$ when $L \to \infty$. It follows from (16) and (14) that such a limit exists and satisfies the equation:

$$A(1)\bar{y}_1 = B^*(1)M_1^* \tag{17}$$

Practically, $\bar{y}_1$ is computed from a sufficiently large sample $\{y(t); t = 1, \quad \cdots, \quad L\}$. Now, subtracting (17) from (13) gives:

$$A(q^{-1})[y(t) - \bar{y}_1] = B^*(q^{-1})[u^*(t) - M_1^*] + \xi(t) \qquad (18)$$

For convenience, let us introduce the following notations:

$$\tilde{y}(t) = y(t) - \bar{y}_1 \qquad (19)$$

$$\tilde{u}(t) = u^*(t) - M_1^* \qquad (20)$$

Using (18, 19, 20), it follows that the identified system (13) can be given the compact form:

$$A(q^{-1})\tilde{y}(t) = B^*(q^{-1})\tilde{u}(t) + \xi(t) \qquad (21)$$

On the other hand, using (11), (20) and fig. 2, it follows that if $v(t) \in \{h_m, -h_m\}$, then:

$$v(t) = h_m \quad \Rightarrow \quad \tilde{u}(t) = 0$$
$$v(t) = -h_m \quad \Rightarrow \quad \tilde{u}(t) = 1 \qquad (22)$$

That is the internal sequence in (21) turns out to be perfectly measurable, as long as the input sequence v(t) takes its values in the set $\{h_m, -h_m\}$. Therefore, identification of $A(q^{-1})$ and $B^*(q^{-1})$ can be performed based upon the equation error (21). To this end, the latter is given the following regressive form:

$$\tilde{y}(t) = \tilde{\phi}(t)^T \theta^* + \xi(t) \qquad (23)$$

with:

$$\tilde{\phi}(t)^T = [-\tilde{y}(t-1) \ldots - \tilde{y}(t-n) \quad \tilde{u}(t-1) \ldots \tilde{u}(t-n)] \qquad (24)$$

$$\theta^* = [a_1 \quad \ldots \quad a_n \quad b_1^* \quad \ldots. \quad b_n^*]^T \qquad (25)$$

where b_i^* (i=1, ..., n) denote the coefficients of $B^*(q^{-1})$

4.3 Estimation of linear subsystem parameters

The regression form (23) is now based upon to estimate the unknown parameter vector θ^*. It is well understood that the input sequence $v(t)$ takes its values only in the set $\{-h_m, h_m\}$ so that the internal sequence $\tilde{u}(t)$ becomes measurable. Given a sufficiently large set of data $\{v(t), \quad y(t); \quad 1 \le t \le N\}$, parameter estimation can be performed using the well known least-squares algorithm:

$$\hat{\theta}(N) = \left[\frac{1}{N}\sum_{i=1}^{N} \tilde{\phi}(i)\tilde{\phi}(i)^T\right]^{-1} \left[\frac{1}{N}\sum_{i=1}^{N} \tilde{\phi}(i))\tilde{y}(i)^T\right] \qquad (26)$$

4.4 A persistently exciting input sequence

The choice of the input *v(t)* will be appropriate if the following three requirements are fulfilled:
(i) *v(t)* should be easily realizable.
(ii) It must take its values in the set $\{-h_m, h_m\}$ so that the sequence $\tilde{u}(t)$ can be measurable.
(iii) The resulting regression vector $\tilde{\phi}(t)$ should satisfy the persistent excitation (PE) property.
Bearing these in mind, the following periodic sequence (with period $T = 4n$) is proposed, where k is any integer, $t_k = kT$, $t_k \le t < t_{k+1}$:

$$v_2(t) = \begin{cases} -h_m & for \quad t = t_k + 2n \\ h_m & otherwise \end{cases} \qquad (27)$$

In view of (21) the resulting internal signal $\tilde{u}(t)$, denoted $\tilde{u}_2(t)$ takes the following values:

$$\tilde{u_2}(t) = \begin{cases} 1 & for \quad t = t_k + 2n \\ 0 & otherwise \end{cases} \qquad (28)$$

4.5 Convergence analysis of the parameter estimation

Proposition 1: Consider the system (10), submitted to Assumptions A1-A6, with a nonlinear element $F(.) = \mathbb{HB}(P_1, P_2)$. If the system is excited by the input sequence (27), then:

(1) The system can be described by the equation error (21) and the regression form (23).
(2) he sequence $\{\tilde{\phi}(t)\}$ is PE in the mean i.e. there exists a positive real β such that:

$$\lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \tilde{\phi}(i)\tilde{\phi}(i)^T > \beta \quad (w.p.1) \qquad (29)$$

(3) The estimation algorithm (26) when applied to the regression (23), yields a consistent estimate $\hat{\theta}(N)$ i.e $\hat{\theta}(N) \to \infty$ as $N \to \infty$ (w.p.1) □.
Proof: see [11] (chapter 4, Lemma 1).

5. NONLINEAR PARAMETERS IDENTIFICATION

The identification of the nonlinearity is now dealt with using the system representation (13). Accordingly, one has to identify the function $F^*(.)$ which, due to (1) and (11), is characterized by the polynomials:

$$P_1^*(v) = P_1(v)/\mu \triangleq \sum_{i=0}^{m-1} c_{1i}^* v^i \text{ and}$$

$$P_2^*(v) = P_2(v)/\mu \triangleq \sum_{i=0}^{m-1} c_{2i}^* v^i \, .$$

It is obvious that the identification of the coefficients of the polynomials $\{P_1^*(v), P_2^*(v)\}$ amounts to determining the corresponding coefficient vectors C_1^* and C_2^* defined as follows:

$$C_1^* \triangleq C_1/\mu = [c_{10}^* \quad c_{11}^* \quad \cdots \quad c_{1(m-1)}^*] \qquad (30)$$

$$C_2^* \triangleq C_2/\mu = [c_{20}^* \quad c_{21}^* \quad \cdots \quad c_{2(m-1)}^*] \qquad (31)$$

5.1 Generation of an exciting input

Let us consider the periodic input sequence, with period $T = 2m(n+1)$, denoted $v_3(t)$, defined as follows:

for $k = 0, \ 1, \ 2, \ldots \quad ; \quad j = 0, 1, 2, \quad \cdots, \quad 2m-1;$
$i = 0, 1, 2, \quad \cdots, n \quad and \quad t = 2m(n+1)k + j(n+1) + i:$

$$v_3(t) = \begin{cases} (h_m + jh_m/m) & for \quad j = 0, \ldots, m-1 \\ (-h_m - (j-m)h_m/m) & for \quad j = m, \ldots, 2m-1 \end{cases} \qquad (32)$$

Equation (32) defines a periodical triangular-like signal illustrated by figure (3) for *m*=6, *n*=2 and $h_m = 4$. To understand the behavior of $v_3(t)$, let us examine it over one period of time. i.e. $[2m(n+1)k \quad , \quad 2m(n+1)k + 2m(n+1)]$. From $t = 2m(n+1)k$ to $t = 2m(n+1)k + (m-1)(n+1) + n$:
the signal $v_3(t)$ ascends from h_m to $2h_m$, Furthermore, $v_3(t)$ is constant in any subinterval of the form $[2m(n+1) + j(n+1) \quad , \quad 2m(n+1) + j(n+1) + n]$. During this ascending phase the couple $(v_3(t), u^*(t))$ describes part (1) of the nonlinearity showed in figure (4).

On the other hand, from $t = 2m(n+1)k + m(n+1)$ to $t = 2m(n+1)k + (2m-1)(n+1) + n$,
the signal $v_3(t)$ descends from $-h_m$ to $-2h_m$. During this descending phase, the couple $(v_3(t), u^*(t))$ describes the part (2) of the nonlinearity presented in figure (4).
From the above observations, it becomes clear that $u^*(t)$ can be expressed as follows:

$$u^*(t) = \begin{cases} \sum_{i=0}^{m-1} c^*_{1i}(h_m + jh_m/m)^i \quad for \quad j = 0, ..., m-1 \\ \sum_{i=0}^{m-1} c^*_{2i}(-h_m - (j-m)h_m/m)^i \quad for \quad j = m, ..., 2m-1 \end{cases} \tag{33}$$

where $t = 2m(n+1)k + j(n+1) + i$, $i = 0, 1, \quad \cdots \quad n$ and k=1, 2, ...
Fig 3 also shows that the signal $v_3(t)$ is discontinuous as it jumps instantaneously from $-2h_m$ to $+h_m$ (at the instant $t = 2m(n+1)k$) and from $2h_m$ to $-h_m$ (at $t = 2m(n+1)k + m(n+1)$).
Such a discontinus feature is necessary to enforce the couple $(v_3(t), u^*(t))$ rejump from one branch of the hysteresis to the other. Doing so, we ensure that the couple $(v_3(t), u^*(t))$ never evolve on the horizontal part of the nonlinearity (Fig 4). It means all the time on the polynomial branches (1) and (2), jumping periodically.

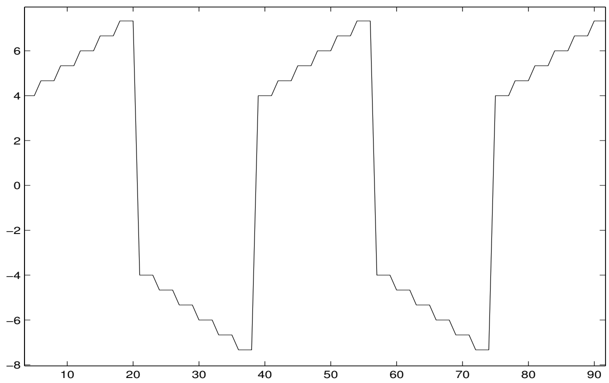

Fig. 3. The period sequence $v_3(t)$ defined in (32), for m=6, n=2 and $h_m = 4$

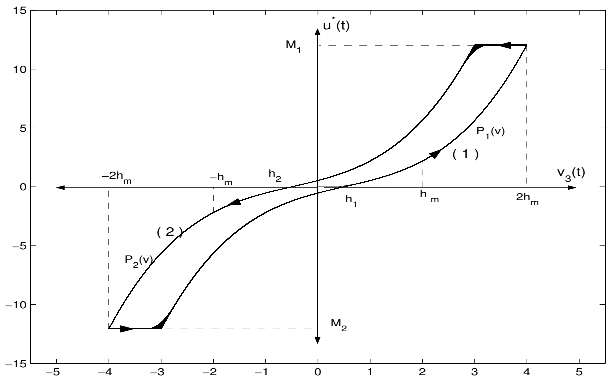

Fig. 4. The output $u^*(t)$ of the sequence $v_3(t)$

5.2 Identification of $\mathbf{C}_1^*$

We will now establish a system representation that involves linearly the unknown parameters i.e. the component of the vector C_1^*. To alleviate the equations, let us introduce the integer sequence $t_{kj} = 2m(n+1)k + j(n+1)$ with $k = 0, 1, 2, ...,$ $j = 0, 1, 2, \cdots, m-1$. With this notation, it follows from (13) that:

$$A(q^{-1})y(t_{kj}) = B^*(q^{-1})u^*(t_{kj}) + \xi(t_{kj}) \tag{34}$$

On the other hand, it follows from (32) that

$$u^*(t_{kj} - 1) = u^*(t_{kj} - 2) = \quad ....$$
$$= u^*(t_{kj} - (n+1)) = \sum_{i=0}^{m-1} c^*_{1i}(h_m + jh_m/m)^i \tag{35}$$

Using (40), it follows from (39) that:

$$A(q^{-1})y(t_{kj}) = B^*(1)\left[\sum_{i=0}^{m-1} c^*_{1i}(h_m + jh_m/m)^i\right] + \xi(t_{kj}) \tag{36}$$

Letting $Z_{ji}(k) = y(t_{kj} - i)$ and $\gamma_j(k) = \xi(t_{kj})$ then, for any k and j=1, ..., m, (41) becomes:

$$Z_{j0}(k) + a_1 Z_{j1}(k) + ... + a_n Z_{jn}(k) =$$
$$B^*(1)\left[\sum_{i=0}^{m-1} c^*_{1i}(h_m + jh_m/m)^i\right] + \gamma_j(k) \tag{37}$$

Introduce the average quantities:
$\bar{Z}_{ji}(T) = \frac{1}{N}\sum_{K=0}^{N} Z_{ji}(k)$ and $\bar{\gamma}_j(T) = \frac{1}{N}\sum_{k=0}^{N} \gamma_j(k)$

It follows, taking the average of both sides of (37):

$$\bar{Z}_{j0}(T) + a_1\bar{Z}_{j1}(T) + ... + a_n\bar{Z}_{jn}(T)$$
$$= B^*(1)\left[\sum_{i=0}^{m-1} c^*_{1i}(h_m + jh_m/m)^i\right] + \bar{\gamma}_j(T) \tag{38}$$

or, equivalently (using assumption A5):

$$\left[\sum_{i=0}^{m-1} c^*_{1i}(h_m + jh_m/m)^i\right] =$$
$$\frac{\bar{Z}_{j0}(T) + a_1\bar{Z}_{j1}(T) + ... + a_n\bar{Z}_{jn}(T)}{B^*(1)} - \frac{\bar{\gamma}_j(T)}{B^*(1)} \tag{39}$$

Writing equation (39) for $j = 1, \quad ..., \quad m$, yields the following vector equation:

$$M_1 C_1^* = \bar{Z}(T) - \bar{\gamma}(T) \tag{40}$$

with;

$$M_1 = \begin{bmatrix} 1 & (h_m + jh_m/m) & \cdots & (h_m + jh_m/m)^{m-1} \\ 1 & (h_m + 2h_m/m) & \cdots & (h_m + 2h_m/m)^{m-1} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & (2h_m) & \cdots & (2h_m)^{m-1} \end{bmatrix} \quad (41)$$

$$C_1^* = [c_{10}^* \quad c_{11}^* \quad \cdots \quad c_{1(m-1)}^*]^T \quad (42)$$

$$\bar{\gamma}(T) = \left[\frac{\bar{\gamma}_1(T)}{B^*(1)} \quad \frac{\bar{\gamma}_2(T)}{B^*(1)} \cdots \frac{\bar{\gamma}_m(T)}{B^*(1)}\right]^T \quad (43)$$

,

$$\bar{Z}(T) = \begin{bmatrix} \dfrac{\bar{Z}_{10}(T) + a_1\bar{Z}_{11}(T) + \ldots + a_n\bar{Z}_{1n}(T)}{B^*(1)} \\ \dfrac{\bar{Z}_{20}(T) + a_1\bar{Z}_{21}(T) + \ldots + a_n\bar{Z}_{2n}(T)}{B^*(1)} \\ \vdots \\ \dfrac{\bar{Z}_{m0}(T) + a_1\bar{Z}_{m1}(T) + \ldots + a_n\bar{Z}_{mn}(T)}{B^*(1)} \end{bmatrix} \quad (44)$$

Since $\bar{\gamma}(T)$ is not measurable, equation (40) suggests the following estimate of C_1 :

$$\hat{C}_1(T) = M_1^{-1}\bar{Z}(T) \quad (45)$$

Proposition 2: Consider the system (10), submitted to Assumptions A1-A6, with a nonlinear element $F(.) = \mathbb{HB}(P_1, P_2)$. If the system is excited by the input sequence (33), then:
1- The system can be represented by the equation-error (36) that implies the vector equation (45) .
2- The estimator (45) is consistent i.e. $\hat{C}_1(T) \to C_1^*$ as $T \to \infty$ (w.p.1). □

Proof: Part (1) has already been constructively been proved by equations (34) to (44).
Part (2) follows from comparing (40) and (45). These yields $C_1^* - \hat{C}_1 = M_1^{-1}\bar{\gamma}(T)$, which establish the proposition using the fact that M_1 est invertible and $\bar{\gamma}_j(T) \to 0$ as $T \to \infty$ (w.p.1) (assumption A6). □□□

*5.3 **Identification of** C_2^**

The vector C_2^* can be identified using the estimation procedure that used for C_1^* with the following substitutions:
$C_1^* \quad \to \quad C_2^*$
$j \to \quad j = m, \ldots, 2m-1$
$u^* \to \sum_{i=0}^{m-1} c_{2i}^*(-h_m - (j-m)h_m/m)^i$
Then, C_2^* will be obtained from the following formula:

$$\hat{C}_2 = M_2^{-1}\bar{Z}(T) \quad (46)$$

6. CONCLUSIONS

We have considered the problem of identifying Hammerstein systems in presence of memory and not-necessarily symmetric, nonlinearities. Specifically, the focus has been made on the hysteresis-backlash nonlinearity (fig(2)). The identification of the linear subsystem has first been dealt with in Section (IV). The first step consisted in designing the system parameterizations (21) that presents three key properties: (i) the unknown linear parameters come in linearly; (ii) the involved internal signal ($\tilde{u}(t)$) is known as long as $v(t) \in \{-h_m, +h_m\}$; (iii) the particular input $v_2(t)$ provides the persistent excitation (Proposition 1). Then, the least square algorithm (26) turned out to be consistent. The identification of the nonlinearity is based upon the system parametrization (13), that involves the backlash-hysteresis $F^*(.)$. The latter is flanked by two polynomials $P_1^*(v)$ and $P_2^*(v)$ that are characterized by two vectors C_1^* and C_2^* containing the coefficients of there polynomials. The estimation of these coefficients is dealt with in section (V). The main feature of the proposed method is the input signal $v_3(t)$ defined by (32). The latter has three key properties: (i) $v_3(t)$ is such that the couple $(v_3(t), u_3^*(t))$ remains along the polynomial borders of the nonlinearity $F^*(.)$. (ii) This makes it possible to build up the system parametrization (41) that involves linearly the parameters of the nonlinearity. (iii) The exciting effect of $v_3(t)$ guarantees a consistent estimate of the unknown parameters, using the algorithms (45)-(46).

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